THE FOSTER-HART FORMULA FOR CASES OF SUPPLY CHAIN PROJECTS WITH VARIOUS CONSTRAINTS

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Abstract: In the publication it was mentioned that the tool supporting the process of reducing of the risk of bankruptcy within the framework of supply chain projects is the measure of the Foster-Hart, whose most important features make it unique. It takes into account the possibility of bankruptcy. The main purpose of the publication is to conduct a spatial graphic interpretation of the Foster-Hart formula for such projects, however, with entering into consideration different constraints, that should be taken into account by the companies (principal contractors).

INTRODUCTION

All projects are both unique and complex, so managing them, can be considered a complicated and problematic process, as it always involves planning, organizing and coordinating resources so that project goals can be successfully completed [Vanhoucke 2012]. Practice clearly indicates that the value of implemented projects is getting higher, and therefore they are exposed to various types of risks. They also include financial risk, which may significantly expose enterprises to reduced financial liquidity and, consequently, also to bankruptcy. All this makes project risk management more and more often supported by many tools

and techniques, which include, among others, various risk models and active management of it [Boyce 2003]. This approach makes it easier to complete the project on time, taking into account its specificity and budget. This specificity is the result of the fact that in practice different types of projects are implemented, which includes also comprehensive supply chain projects, that can be considered interesting due to their features, because they are particularly exposed to various types of risks (including to the already mentioned financial risk), which should be managed to avoid bankruptcy.

In order to support the financial risk management process of these projects, an instrument seems to be necessary to measure risk, taking into account domestic and foreign currencies, which may distort its level, while taking into account the possibility of bankruptcy. Based on the literature research, it was found that such an instrument could be the Foster-Hart measure, taking the form of the formula presented in further parts of the work. Although its basic features have been studied in modern literature, no spatial graphic interpretation of this formula has been made for cases of complex supply chain projects, taking into account various constraints. Due to the possibility of bankruptcy for companies (principal contractors) implementing such projects, it may seem desirable, which is why it is the purpose of the work. For its needs, literature devoted to the Foster-Hart measure was used, as well as hypothetical data, whose analysis was carried out in the MATLAB software environment.

SUPPLY CHAIN PROJECTS – GENERAL CHARACTERISTICS

In the subject literature, projects are usually defined in a similar way. However, the most well-known is the definition of the Project Management Institute [PMI 2013], according to which „A project is a temporary endeavor undertaken to create a unique product, service, or result. The temporary nature of projects indicates that a project has a definite beginning and end. The end is reached when the project's objectives have been achieved or when the project is terminated because its objectives will not or cannot be met, or when the need for the project no longer exists. A project may also be terminated if the client (customer, sponsor, or champion) wishes to terminate the project". Practice shows that there are also projects in supply chain management, here the term "supply chain projects" is used to denote them. According to Chopra and Meindl (2010), a supply chain „consists of all parties involved, directly or indirectly, in fulfilling a customers request" and the Supply Chain Process Cycle consists of the order, replenishment, manufacturing and procurement cycle.

On the whole, there are no precisely definitions of supply chain projects. To understand a meaning of such projects, it is necessary to characterize a project production, for which this chain is temporarily build. However, such a production is connected with the production of product (service) made by temporary organization which is created for a project. After finishing this project the

organization is liquidate [Modig 2007]. The most important element of this organization is a principal contractor which manages the project [Parrod, Thierry, Fargier, Cavaille 2007] and deliver it. It means that the whole temporary organization delivers the project product from raw materials to customer. All in all, supply chain project involves principal contractor delivering project to which belong complicated and non-routine tasks [Modig 2007]. A delivery of it may expose to the risk of bankruptcy, because the cost control is problematic. That is why assuming payments generated by such project we are able to use a Foster-Hart measure or riskiness in order to avoid a risk of bankruptcy. Such approach is not widely described in the literature. All in all, each supply chain project is typically unique whereas the customers are changing, and the wealth of a company can be too less for implementing it.

FOSTER-HART MEASURE AND SUPPLY CHAIN PROJECTS

The Foster-Hart measure [Foster, Hart 2009], due to its properties, seems to be appropriate for supporting the financial risk management process of the supply chain project. This is mainly due to the fact that it allows us to indicate one of them, the implementation of which may lead to bankruptcy of the enterprise. It's enough to mention that this is a feature not found among other measures. In addition, it is universal and objective, while being monotonic, which allows us to analyze investments and projects from various currency areas. Its form is expressed by the following formula:

$$E\left[\log\left(1 + \frac{1}{R(g)}g\right)\right] = 0, \tag{1}$$

whereas $E[X]$ is the expected value of the random variable X, g is the income that is generated by the investment (or project) with a certain probability (p_i) at the end of the given period and $R(g)$ means the critical value of the investor's (or business) wealth and, at the same time, the measure of investment risk.

The presented measure is also based on the following assumptions:

$$\sum_{i=1}^{n} p_i = 1 \ , \sum_{i=1}^{n} p_i g_i > 0, \text{ and} \tag{2a}$$

$$g_i < 0 \text{ then } g_j > 0, \text{ where} \tag{2b}$$

$$i \neq j \text{ and also,} \tag{2c}$$

$$i, j = 2, 3, \dots \tag{2d}$$

This critical value is calculated in order to compare with the current level of wealth of the investor (or company). It allows the division of investments (or projects) into two types because, if $R(g)$ is higher than the investor's current level of wealth, then it is considered too risky and therefore leading the investor to bankruptcy. If, on the contrary, $R(g)$ equals or is lower than this level, then undertakings can be considered as acceptable and increase the assets of the

company. Based on the research carried out in Polish and foreign literature, it was found that there is a need to support the financial risk management process of supply chain projects, using the Foster-Hart measure, moreover, it is a little-considered subject. Literature is focused on this measure in the context of the use for investment in shares [Leiss, Nax 2018], and not for other projects that may or even are always associated with income's generation. The problem that is associated with the use of this measure concerns the adaptation of its form to the mentioned process. However, assuming that it can be constructed using the classical theory of the decision tree [Aczel 2008], then an equivalent form of formula (1) can be used, which for four final payoffs is as follows:

$$\left(1 + \frac{g_1}{R(g)}\right)^{p_1} \times \left(1 + \frac{g_2}{R(g)}\right)^{p_2} \times \left(1 + \frac{g_3}{R(g)}\right)^{p_3} \times \left(1 + \frac{g_4}{R(g)}\right)^{p_4} = 1. \qquad (3)$$

The presented dependence is possible assuming that during the implementation of the project, and thus after its start, at some point t_1 a specific scenario may occur, represented by two decision nodes, so that at the end of the project implementation period (t_2) four final can be assumed payoffs, two of which are the result of a given node. In this way one can get a payoff table, for which there is a dependence (3), where for example g_1 and g_2 are the result of occurrence at the moment t_1 of the positive scenario, and g_3 and g_4 - the negative scenario. Therefore, spatial graphic interpretation of the solution of Foster-Hart formula should help some main contractors evaluate supply chain project. Thanks to the spatial graphic a company can determine, whether to implement this project or not. For this purpose is needed hypothetical income of a project and hypothetical wealth of a proncipal contractor ($R(g)$). As it was shown, the too low value of wealth of a company. which does this implementation, can cause bankruptcy.

GRAPHIC INTERPRETATION OF THE FOSTER-HART FORMULA FOR SUPPLY CHAIN PROJECTS

The graphical interpretation of the Foster-Hart formula in the process of evaluating of supply chain projects riskiness, is based on the analysis of selected project cases. These cases relate to four projected payments with different probabilities and different scenarios. This analysis is based on a hypothetical supply chain project, which may take the form of two different variants (Table 1). Simulation tests were carried out in the MATLAB programming environment, while payoff was given without specifying the currency, because this is not important for the considerations. Three-dimensional drawings are prepared in such a way that they contain a hyper-plane that is the right side of the Foster-Hart formula and a plane with a value of "0", where their common part creates a trace that is the solution to this formula. On the other hand, in this two-dimensional drawings, this trace has been presented, taking certain values depending on the size of the withdrawals.The solution of the Foster-Hart formula is also presented on the three-dimensional charts. It is needed, because considering various constraints

there are more than one combination of payments, which can solve the Foster-Hart formula for one $R(g)$.

Table 1. Parameters of the sample supply chain project and its variants analyzed

| Project parameters in the base version	
Parameter	Value
$R(g)$	250,000
Value of p_i ($p_1 = p_2 = p_3 = p_4$)	0.25
Range of g_1	(2,400;3,000) – change with step 20
Range of g_2	(-3,000;-2,500) – change with step 20
Value of g_3	-600
Value of g_4	800
Variant Number 1*	
Value of p_1	0.30
Value of p_2	0.25
Value of p_3	0.25
Value of p_4	0.20
Variant Number 2**	
$R(g)$	500,000
Value of g_3	600
Value of g_4	-800
Value of p_1	0.30
Value of p_2	0.25
Value of p_3	0.25
Value of p_4	0.20

* For variant 1, the values of $R(g)$ and g are not given because they are the same as for the base version.

**For variant 2, the values g_1 and g_2 are not given because they are the same as for the base version. The most important features differentiating the presented investment cases are expressed using two-dimensional and three-dimensional charts.

Source: own study

The first numerical experiment of the project in the base version, of which $R(g)$ is immutable, but with replacement payout values, is shown in Figure 1.

Figure 1. Three-dimensional graph of dependencies between the parameters of the analyzed
            project in the base version
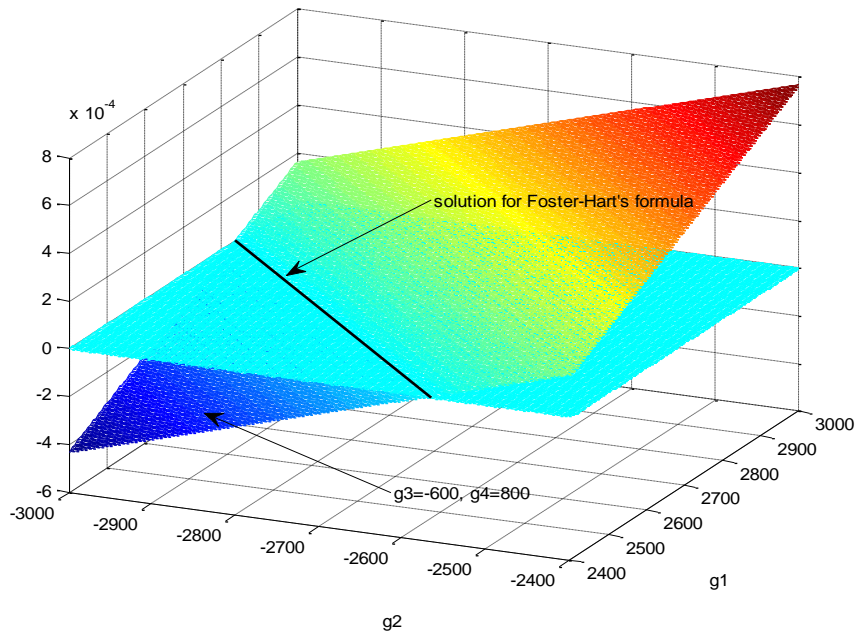
Figure 1 presents the fact that not every combination of payments allows for the solution of the Foster-Hart formula. Therefore, not every project can be analyzed by enterprises. The rightness of the Hart-Foster rule occurs only for certain parameter ranges. Figure 1 shows a continuous and intermittent solution, which cover different ranges of variability g_1 and g_2 for accepted input assumptions. What's more, the changed payout values for the negative scenario significantly change the solution of the formula being described, making this case a completely separate project. The details of solution from Figure 1 is more convenient to analyze in a two-dimensional graph, as shown in Figure 2 including the variant of the base version. Thus, by changing the input parameters of the project, different solutions are obtained using the Foster-Hart measure.
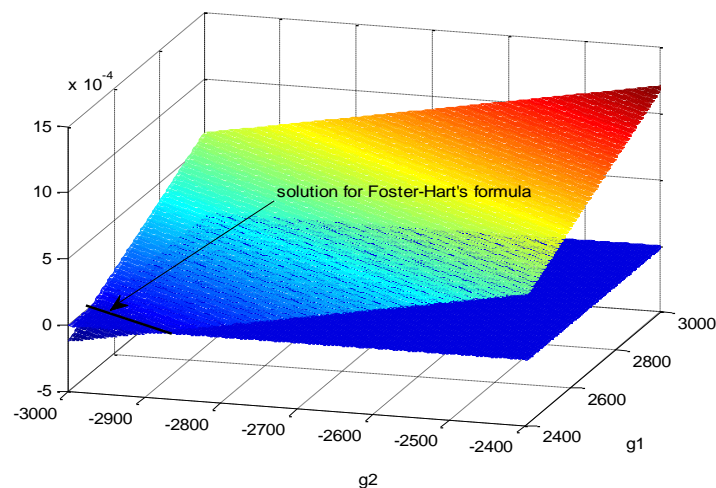
Figure 2. Two-dimensional graph presenting the solution of the Foster-Hart formula depending on the size g_1 and g_2 for the design in the base version
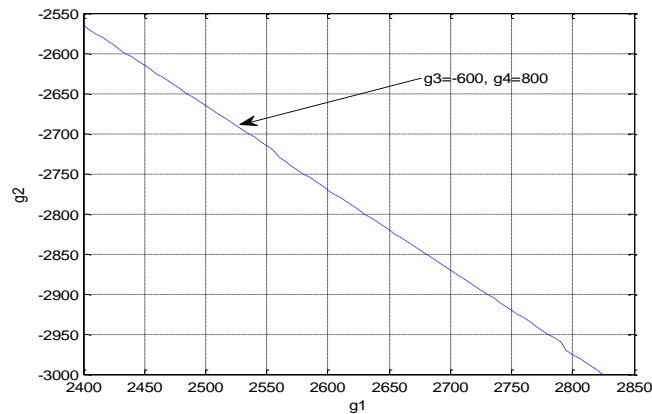
Source: own study based on the results of the MATLAB program

The first variant of the project is distinguished by the fact that the probabilities associated with the negative scenario are relatively low, and therefore the change in the size of payments generated by projects under such a scenario does not play a significant role for the enterprise. It is worth noting, however, that with the increase of p_1 and with the same p_2, a higher loss g_2 can be accepted for a given payment value g_1. This means that such a specific project should be treated as less risky and therefore more attractive for the company (Figure 3).
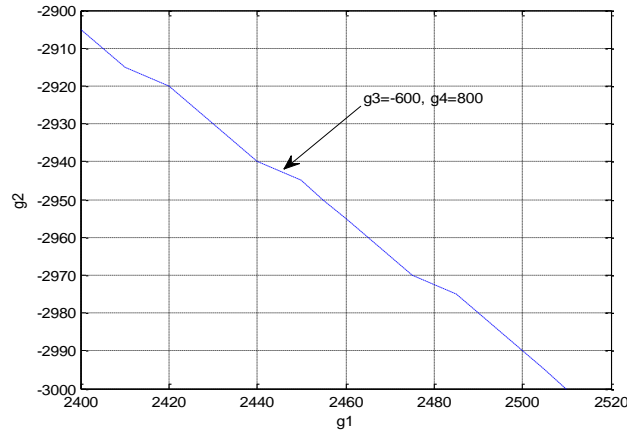
Figure 3. Three-dimensional graph of dependencies between the parameters of the analyzed project in the first variant

Source: own study based on the results of the MATLAB program

The reduction of the project risk is also illustrated in the two-dimensional graph (Figure 4).
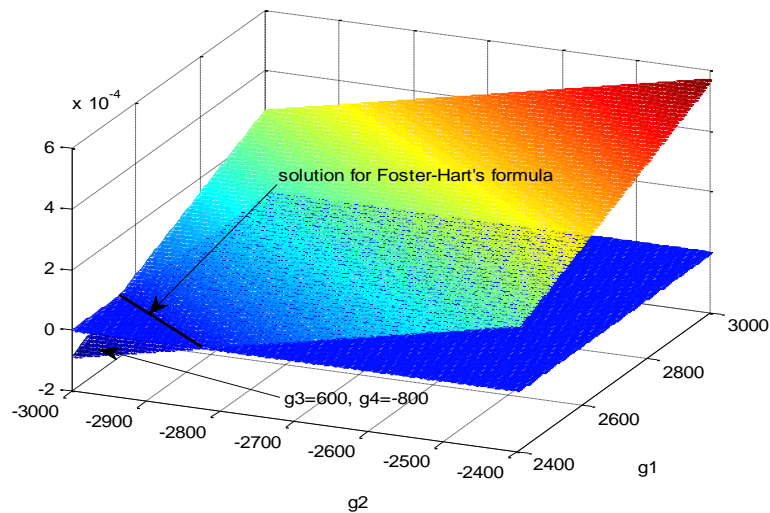
Figure 4. Two-dimensional graph presenting the solution of the Foster-Hart formula depending on the size g_1 and g_2 for the first variant

Source: own study based on the results of the MATLAB program

The second variant of the analyzed project assumes that $R(g)$ increases, but at the moment t_2 a company can get very low value of a project payment (Figure 5).

Figure 5. Three-dimensional graph of dependencies between the parameters of the analyzed project in the second variant with changed values of $R(g)$

Source: own study based on the results of the MATLAB program

The simulations carried out, reflect the fact that as the value of $R(g)$ increases, the project becomes less and less risky, and therefore the company may become less sensitive to changes in payments in hypothetical cases – even with low g_4, because the hyper-plane and the "0" plane create with it an increasingly smaller angle.

Figure 6. Two-dimensional graph presenting the solution of the Foster-Hart formula depending on the size g_1 and g_2 for the second variant of the project
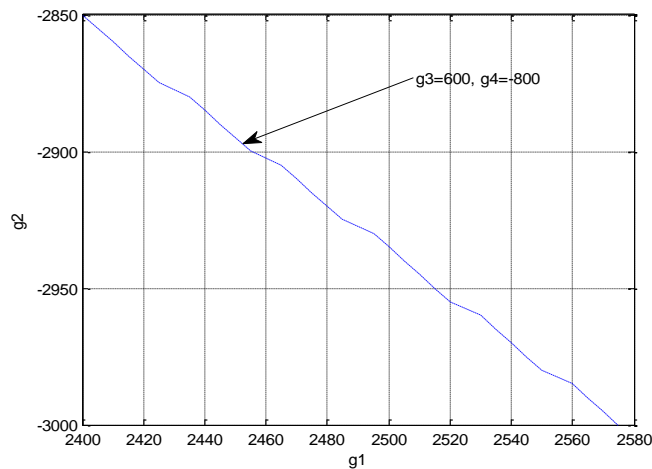
Source: own study based on the results of the MATLAB program

It should be added that the location of these hyper-planes also changes (Figure 6), which indicates that changes $R(g)$ may have to carry out other projects that the company originally set up. This is because, with the increase of $R(g)$, increasingly large ranges of values g_2 are acceptable to the company (in compare to the basic version).

CONCLUSIONS

The main problem with supply chain projects is that they are characterized by so many risks that the company realizing them, may lose liquidity. For this reason, the process of financial risk management of these projects should be supported by an instrument that measures risk, taking into account the possibility of bankruptcy. The work showed that it could be the Foster-Hart measure. Summarizing, in this paper it has been shown that with the appropriate assumption, this measure can be used in the process of evaaluating of the supply chain project from the perspective of bankruptcy avoiding, however it is not always possible to resolve the Foster-Hart formula. Its graphic analysis confirmed this fact. In addition, it allowed to state that changes in the value of payments, generated by the project, change the solution of the formula. In addition, even a small decrease in

the probability level of the payout values, in the case of a positive scenario means that the project should be treated as more risky. It is logical, however, that with the increase in the value of $R(g)$ of a company, the supply chain project should be treated as less risky. Let it be pointed out, therefore, that the graphical interpretation of the Foster-Hart formula can be treated as a tool supporting the analysis of projects, because it is to possible to compare a $R(g)$ of a project with a asset of company and to reject such project for which too high level of $R(g)$ is a need.

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