

**IS THERE STILL ROOM FOR INCREASING SPEED
IN ALGORITHMIC AND HIGH-FREQUENCY TRADING?
THE CASE OF EUROPEAN OPTIONS PRICED
IN THE HESTON MODEL**

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Abstract: The purpose of the article was to investigate the possibility of increasing speed in transactions made within algorithmic and high-frequency trading. The analysis carried out for this purpose concerned the European options priced in the Heston model. Among issues being discussed, a new scheme of calculating Fourier and inverse Fourier transforms was proposed. It guarantees an increase of computational speed in relation to existing methods of generating final results.

Keywords: European options, the Heston model, Fourier transform

JEL classification: C02, G13

INTRODUCTION

In the financial literature algorithmic trading is variously defined. According to P. Teleaver, M. Galas and V. Lalchand [Teleaven et al. 2013] this term should be understood as any form of exchange of capital assets using advanced algorithms (computer systems) in order to automate entire or part of the transaction process. A. Cartea and S. Jaimungal [Cartea, Jaimungal 2013] are of a similar opinion. According to them algorithmic trading refers to the use of computer algorithms that make trading decisions, submit orders and manage those orders. In a slightly different way algorithmic trading is defined by A. P. Chaboud, B. Chiquoine, E. Hjalmarsson and C. Vega [Chaboud et al. 2014]. In their opinion this term is inextricably linked to the monitoring of markets, management and exchange of financial assets that are traded highly frequently. Such approach is in line with

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views of M. A. Goldstein, P. Kumar and F. C. Graves [Goldstein et al. 2014], who directly associate algorithmic trading with submitting, executing and cancelling buy or sell orders, analyzing financial data to identify short-term price opportunities, and trading via computers. In distinguishing between algorithmic and high-frequency trading helps the U.S. Securities and Exchange Commission (SEC). In one of the researches carried out by the SEC it was pointed out that despite the lack of a precise definition of high-frequency trading, algorithmic trading is a superior category in relation to high-frequency trading¹. In other studies performed by the SEC it is stated that high-frequency trading is a significant but not the only part of algorithmic and computer-aided trading². In this approach algorithmic trading covers wide range of activities including execution of orders on behalf of institutional clients and market makers by algorithms. Due to the fact that placed orders are of a large value they are placed on the market after dividing them into smaller suborders (or child orders) using appropriately designed queuing systems. For the purpose of this article, however, it is assumed that algorithmic trading is a trading system based on precise, previously prepared computer instructions in the process of capital assets' exchange [Yadav 2015].

The aim of the article is to show that there is still room for increasing speed in algorithmic and high-frequency trading. The article is organized as follows. In the first section development of algorithmic and high-frequency trading is presented. In the second section, the Heston model is briefly described. In the third section, some new approaches to the valuation of options in the Heston framework are proposed and an analysis of the speed of pricing European options is performed. Finally, in fourth section, the article is summarized and major conclusions are drawn.

DEVELOPMENT OF ALGORITHMIC AND HIGH-FREQUENCY TRADING

Algorithmic and high-frequency trading changed the functioning of contemporary capital markets. First of all, the automation of trades on stock exchanges significantly transformed the investment process. The involvement of financial resources in the transactions on the stock market to a lesser extent began to be related to the search for underestimated or overvalued securities and the selection of the appropriate moment of their purchase or sale. Greater weight began to be attributed to the continuous opening and closing positions at very short time intervals with the intention of generating short-term arbitrage profits. Secondly, some of the responsibilities related to making investment decisions were passed to

¹ <http://www.sec.gov/rules/concept/2010/34-61358.pdf> [access: 10.02.2016].

² https://www.sec.gov/marketstructure/research/hft_lit_review_march_2014.pdf [access: 15.02.2016].

the algorithms, in particular in areas related to the analysis of data and the selection of information that is most important for future changes in the prices of financial instruments. What's more, algorithms responsible for the purchase and sale began to take into account the impact of orders placed by other market participants, as well as the consequences of the use of algorithms by the buyers and sellers for the valuation of individual assets. Thirdly, investing financial resources based on mathematical instructions reduced the role of the human factor in the investment process to the development of financial models and the programming of computers.

The automation of stock trading, the use of algorithms to open and close positions, and the reduction of importance of the human factor in the investment process transformed algorithmic and high-frequency trading from a niche investment strategy into a dominant form of trade on many capital markets. This seems to be confirmed by M. A. Goldstein, P. Kumar and F. C. Graves [Goldstein et al. 2014]. According to them in the period of 2000-2012 the share of high-frequency transactions on the U.S. stock market increased from less than 10% to over 50%. On other segments of U.S. financial market a similar phenomenon was observed. The volume of high-frequency trading on the U.S. option and currency markets at the end of 2012 fluctuated from 40% to 60% of the total volume. According to N. Popper [Popper 2012] a similar tendency was observed in the most developed countries of the European Union, Japan and the remaining part of Asia. In their case, in 2012, high-frequency trading was responsible for 45%, 40% and 12% of stock trading respectively.

Development of algorithmic and high-frequency trading increased the speed of trade and shortened the time of holding open positions in financial instruments. As noted by A. G. Haldane [Haldane 2016], within 15 years starting from 2000, the average length of time for stock ownership was reduced from a few seconds to a few milli- or even microseconds. Moreover, as was noted by M. Narang [Narang 2016], the competition between traders on the financial market decreased the profit margin on the U.S. equity market to the value of one-tenth of a cent per share.

According to N. Popper [Popper 2016], in recent years, the role of algorithmic and high-frequency trading on the U.S. capital market has been gradually decreasing. In the period of 2009-2012, the annual number of shares bought or sold in computer-assisted transactions decreased from around 6 billion to around 3 billion. This meant a decrease of the share of algorithmic and high-frequency trading in the total volume from 61% to 51%. In the analyzed period, this phenomenon was accompanied by the decrease in profits of companies using these forms of trading from 4.9 billion USD in 2009 to 1.25 billion USD in 2012. It is worth noting that one of the reasons for the drop in profitability of algorithmic and high-frequency trading was the growing costs of both the maintenance of the ICT infrastructure and the acquisition and processing of market data. On the other side, in the period of 2009-2012, in the U.S., relatively stable increase in stock indices was observed. It was also a period of low market interest rates. Such circumstances could not ensure adequate profitability of momentum and reversal

strategies. In addition, as noted by L. Cardella, J. Hao, I. Kalcheva and Y. Ma [Cardella et al. 2014], the development of algorithmic and high-frequency trading and their profitability should be considered in a broader sense, i.e. they should include other financial instruments, and market segments. It was partly confirmed by N. Popper [Popper 2016] who observed an increase of the high-frequency trading in the total volume on the currency market (from 12% in 2009 to 28% in 2012).

Higher frequency of stock exchange transactions, increase in the speed of trade and shortening the time of holding open positions as well as popularization of algorithmic and high-frequency trading influenced the level of both informational and operational efficiency of the financial market.

In the next part of the article construction of the Heston model is briefly discussed and then alternative Fourier transform schemes are presented.

THE HESTON MODEL

There are two processes underlying the Heston model, i.e.:

$$dS_t = \mu S_t dt + \sqrt{\sigma_t^2} S_t dW_{1,t}, \quad (1)$$

$$d\sigma_t^2 = \kappa(\theta - \sigma_t^2)dt + v\sqrt{\sigma_t^2}dW_{2,t}, \quad (2)$$

where: S_t is the price of the underlying asset in period t , μ is the (constant) drift, σ_t^2 is the variance of the rates of return on the underlying asset, κ is the mean-reverting speed, θ is the average long-term variance of the rate of return on underlying asset (long-run mean), v is the volatility of volatility, and $W_{1,t}$, $W_{2,t}$ are two correlated Wiener processes such that $E(W_{1,t}W_{2,t}) = \rho$.

From eq. 2 it can be easily concluded that the main difference between the Black-Scholes [Black, Scholes 1973] and the Heston models [Heston 1993] refers to the variance of the rate of return on the underlying asset. In the Black-Scholes model volatility is fixed over time, while in the Heston model it is stochastic and given by the CIR process.

Pricing European options in the Heston model is based on the martingale approach to determining theoretical values of the contracts, i.e.:

$$\begin{aligned} C(S, \sigma^2, t) &= e^{-r(T-t)} E^{\mathbb{Q}^H}((S_T - K)^+) = \\ &= e^{-r\tau} E^{\mathbb{Q}^H}(S_T 1_{S_T > K}) - e^{-r\tau} K E^{\mathbb{Q}^H}(1_{S_T > K}) = \\ &= e^{x_t} P_1(x, \sigma^2, \tau) - e^{-r\tau} K P_2(x, \sigma^2, \tau), \end{aligned} \quad (3)$$

where: $\tau = T - t$, r is the risk-free rate, K is the exercise price, \mathbb{Q}^H is a martingale measure in the Heston model, $P_1(x, \sigma^2, \tau)$, $P_2(x, \sigma^2, \tau)$ are probabilities of expiring options in-the-money and the remaining notation is consistent with previously introduced.

According to S. L. Heston [Heston 1993], analytical formulas for $P_1(x, \sigma^2, \tau)$ and $P_2(x, \sigma^2, \tau)$ are not known. However, it can be concluded that the knowledge of the characteristic functions $\phi_1(\xi, x, \sigma^2)$ and $\phi_2(\xi, x, \sigma^2)$ corresponding to P_j , for $j = 1, 2$, allows to calculate $P_1(x, \sigma^2, \tau)$ and $P_2(x, \sigma^2, \tau)$. For this purpose, it is enough to calculate the inverse Fourier transforms according to the following formula (the method is referred to as H-H):

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \Re \left(\frac{e^{-\mathbb{I}\xi \ln K} \phi_j(\xi, x, \sigma^2)}{\mathbb{I}\xi} \right) d\xi, \quad (4)$$

where: $\Re(\cdot)$ is the real part of the subintegral function, \mathbb{I} is the imaginary part of the complex number and the remaining notation is consistent with previously introduced.

In order to find the formula for the price of the European option it is necessary to introduce assumption concerning general form of the characteristic function corresponding to the probabilities P_j , for $j = 1, 2$, i.e.:

$$\phi_j(\xi, x, \sigma) = e^{C_j(\xi, \tau) + D_j(\xi, \tau)\sigma^2 + \mathbb{I}\xi x}, \quad (5)$$

where:

$$C_j(\xi, \tau) = r\mathbb{I}\xi\tau + \frac{a}{v^2} \left[(b_j - v\rho\mathbb{I}\xi + d_j)\tau - 2\ln \left(\frac{1 - g_j e^{d_j\tau}}{1 - g_j} \right) \right], \quad (6)$$

$$D_j(\xi, \tau) = \frac{b_j - v\rho\mathbb{I}\xi + d_j}{v^2} \left(\frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}} \right), \quad (7)$$

$$u_1 = \frac{1}{2}, \quad (8)$$

$$u_2 = -\frac{1}{2}, \quad (9)$$

$$a = \kappa\theta, \quad (10)$$

$$b_1 = \kappa + \lambda - v\rho, \quad (11)$$

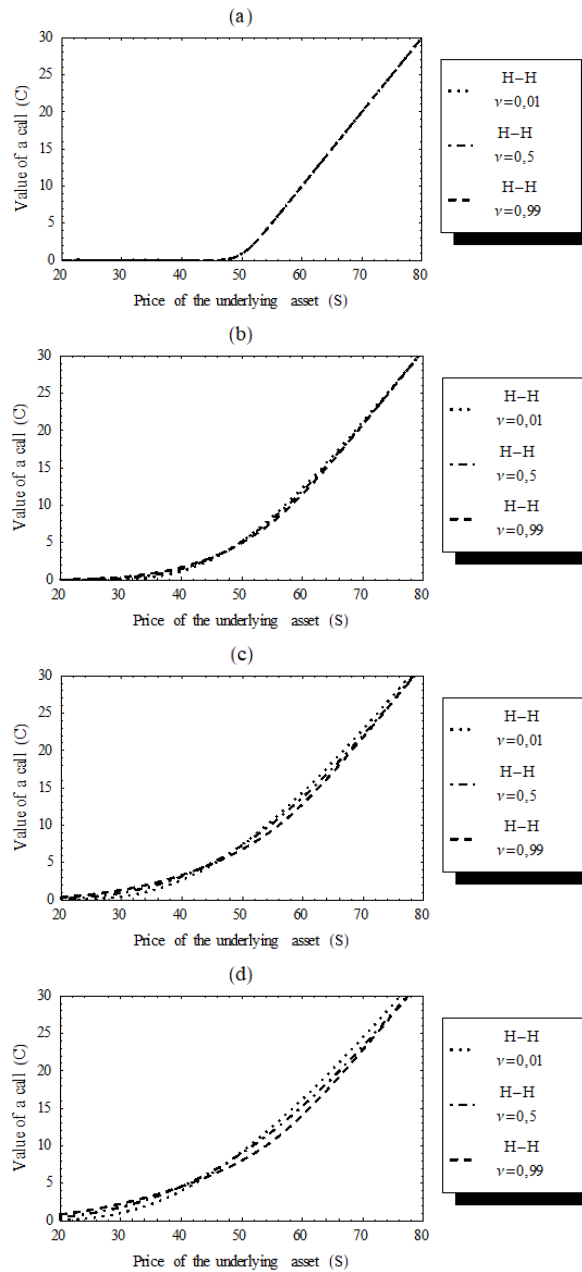
$$b_2 = \kappa + \lambda, \quad (12)$$

$$g_j = \frac{b_j - v\rho\mathbb{I}\xi + d_j}{b_j - v\rho\mathbb{I}\xi - d_j}, \quad (13)$$

$$d_j = \sqrt{(v\rho\mathbb{I}\xi - b_j)^2 - v^2(2u_j\mathbb{I}\xi - \xi^2)}. \quad (14)$$

The payoff functions of the analyzed derivatives are presented in Figure 1. The figure is prepared assuming that: $S \in [20, 80]$, $K = 50$, $\sigma^2 = 0.2$, $r = 5\%$, $\frac{T-t}{T} \in \{0.01, 0.3, 0.6, 0.9\}$, $\kappa = 0.05$, $\lambda = 0.08$, $\theta = 0.15$, and $\rho = 0.8$.

Figure 1. Payoff functions of the European call in the Heston model assuming that: $\kappa = 0.05$, $\lambda = 0.08$, $\theta = 0.15$, $\rho = 0.8$ and v takes various values for: (a) $\frac{T-t}{T} = 0.01$, (b) $\frac{T-t}{T} = 0.3$, (c) $\frac{T-t}{T} = 0.6$ i (d) $\frac{T-t}{T} = 0.9$



Source: own elaboration

SCHEMES OF THE FOURIER TRANSFORM

There are many methods of calculating the Fourier and inverse Fourier transforms. In consequence, there are many ways of determining value of the European options in the Heston model. In the article special attention in this matter will be directed to the formulas developed by P. Carr and D. Madan [Carr, Madan 1999], G. Bakshi and D. Madan [Bakshi, Madan 2000], M. Attari [Attari 2004], and A. Orzechowski [Orzechowski 2018], i.e.:

1. P. Carr and D. Madan [Carr, Madan 1999] for $\alpha = 1$ (the method is referred to as H-CM):

$$C(S_t, t) = \frac{e^{-\alpha k}}{\pi} \int_0^\infty \Re \left(e^{-\mathbb{I}\xi k} \frac{e^{-r\tau} \phi_2(\xi - (\alpha+1)\mathbb{I}, x, \sigma^2)}{\alpha^2 + \alpha - \xi^2 + \mathbb{I}(2\alpha+1)\xi} \right) d\xi. \quad (15)$$

2. P. Carr and D. Madan [Carr, Madan 1999] for $\alpha = 1$ (the method is referred to as H-CMTV):

$$C(S_t, t) = \frac{1}{\sinh(\alpha k)} \frac{1}{\pi} \int_0^\infty \Re \left(e^{-\mathbb{I}\xi k} \frac{\zeta(\xi - \mathbb{I}\alpha, x, \sigma^2) + \zeta(\xi + \mathbb{I}\alpha, x, \sigma^2)}{2} \right) d\xi, \quad (16)$$

$$\text{where: } \zeta(\xi, x, \sigma^2) = e^{-r\tau} \left(\frac{1}{1 + \mathbb{I}\xi} - \frac{\phi_2(-\mathbb{I}, x, \sigma^2)}{\mathbb{I}\xi} + \frac{\phi_2(\xi - \mathbb{I}, x, \sigma^2)}{\mathbb{I}\xi(1 + \mathbb{I}\xi)} \right).$$

3. G. Bakshi and D. Madan [Bakshi, Madan 2000] (the method is referred to as H-BM):

$$C(S_t, t) = \frac{1}{2}(S_t - Ke^{-r\tau}) + \frac{S_t}{\pi} \int_0^\infty \Re \left(\frac{e^{-\mathbb{I}\xi k} \phi_1(\xi, x, \sigma^2)}{\mathbb{I}\xi} \right) d\xi + \\ -Ke^{-r\tau} \frac{1}{\pi} \int_0^\infty \Re \left(\frac{e^{-\mathbb{I}\xi k} \phi_2(\xi, x, \sigma^2)}{\mathbb{I}\xi} \right) d\xi. \quad (17)$$

4. M. Attari [Attari 2004] (the method is referred to as H-A):

$$C(S_t, t) = S_t \left(1 + \frac{e^l}{\pi} \int_0^\infty \Re \left(\frac{e^{-\mathbb{I}\xi l}}{\mathbb{I}(\xi + \mathbb{I})} \phi_3(\xi, x, \sigma^2) \right) d\xi \right) + \\ -e^{-r\tau} K \left(\frac{1}{2} + \frac{1}{\pi} \int_0^\infty \Re \left(\frac{e^{-\mathbb{I}\xi l}}{\mathbb{I}\xi} \phi_3(\xi, x, \sigma^2) \right) d\xi \right), \quad (18)$$

$$\text{where: } \phi_3(\xi, x, v) = e^{C_2(\xi, \tau) + D_2(\xi, \tau)\sigma^2 + \mathbb{I}\xi r\tau}, \text{ and } l = \frac{K}{S_t e^{r\tau}}.$$

5. A. Orzechowski [Orzechowski 2018] (the method is referred to as H-Au1):

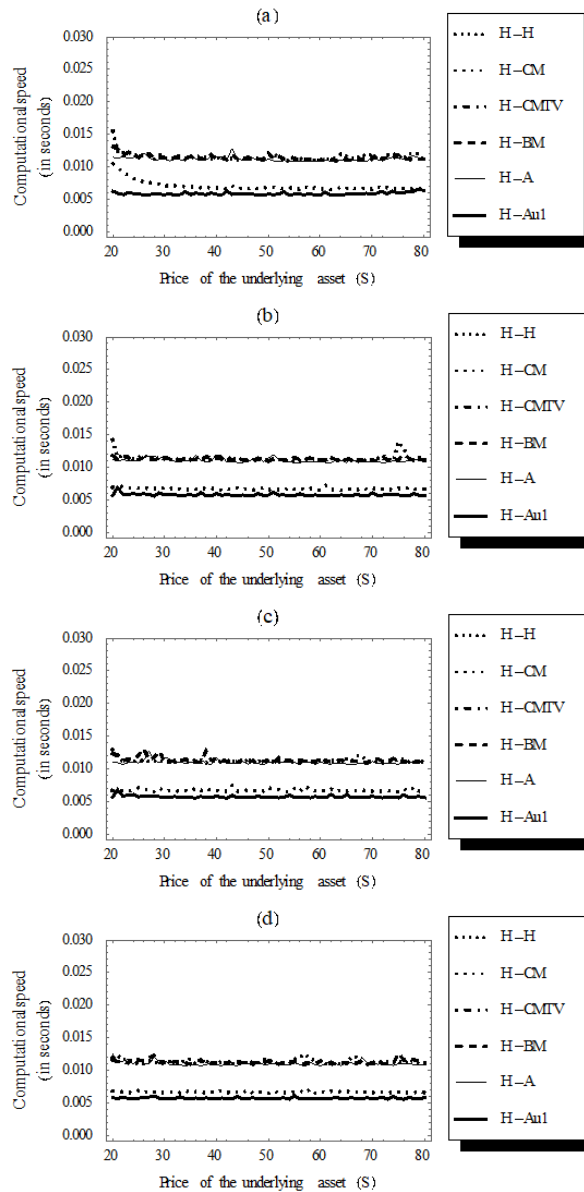
$$C(S_t, t) = \frac{1}{2}S_t - e^{-r\tau} \frac{1}{\pi} \int_0^\infty \Re \left(e^{-\mathbb{I}\xi k} \frac{\phi_2(\xi - \mathbb{I}, x, \sigma^2)}{\mathbb{I}\xi(\mathbb{I}\xi + 1)} \right) d\xi. \quad (19)$$

It is worth noting that above mentioned schemes can be applied to pricing options in diffusion, jump-diffusion as well as pure jump models.

In order to investigate which of the methods described above is the best in terms of computational speed, appropriate codes are developed in the Mathematica 10.2. The package being used is run on a computer with Intel i5-4210U CPU @ 1.70 GHz processor with RAM memory of 6 GB. It is important that each time, before the codes are started, cache memory is deleted. It is done in

order to force the written blocks of commands to be restarted by the computer. The results of the tests carried out are expressed in the graphical form - see Figure 2.

Figure 2. Computational speed in the Heston model assuming that: $\kappa = 0.05$, $\lambda = 0.08$, $\theta = 0.15$, $\rho = 0.8$ and $v = 0.2$ for: (a) $\frac{T-t}{T} = 0.01$, (b) $\frac{T-t}{T} = 0.3$, (c) $\frac{T-t}{T} = 0.6$ i (d) $\frac{T-t}{T} = 0.9$



Source: own elaboration

Based on obtained results it can be easily seen that the slowest methods of pricing European options in the Heston model are H-H, H-CMTV, H-BM and H-A. It should be noted that in the case of H-CMTV the number of bisections in the implemented numerical scheme is reduced in relation to other methods. The H-CM, and H-Au1 methods are among the fastest methods of determining theoretical values of the European options (the second one is slightly faster) regardless of the the time remaining to expiration and the moneyness of the contracts. It is impossible to disregard the fact that the fastest methods of pricing European options have two common features, i.e.: one characteristic function of the variable S_t in the formula for the price of the option and the value ξ in the denominator of the subintegral function squared.

SUMMARY

The main purpose of this article was to show that the pricing of the European options can be speeded up. Information of how to do it can be important for the possibility of developing high-frequency and algorithmic trading strategies. What is especially important is that the increase in the computational speed of the pricing of the European options is obtained not via technological advances in the computer hardware, information processing or telecommunications but by developing new method of calculating the Fourier and inverse Fourier transforms. It is also worth noting that the results can be used to develop numerical schemes based of the Fourier transform, i.e. the discrete and fast Fourier transforms.

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