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ON TRADING ON THE STOCK MARKET WITH THE SHORTAGE OF THE LIQUIDITY

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Abstract: In the article the model of the market with the transaction costs is considered with the market participant who intends to sell the shares of the stock with the presence of the liquidity shortage. The shortage in the liquidity can manfest itself in the occurrence of the market impact which can siginficantly decrease the profit from the stock trade. If the trading velocity is above some level, the market impact can occure and increase the cost of the trade. However the transaction cost can be present even in case of a small transaction on the stock market. The problem of maximization of the expected amount of money obtained from the sale of the stock shares is solved for the case of strategies with the constant trade speed and the particular range of the stock price drift. The example of numerical computations with the use the formulas from the paper, is included.

Keywords: stock price drift, transaction cost, liquidity shortage, market impact, trading speed

JEL classification: C6, G11

INTRODUCTION

The level of liquidity is an important characteristic of the stock market. The liquidity shortage can manifest itself by the occurrence of the market impact which is change in the stock price, induced by trading. The market impact (also called the price impact), if occurs, is unfavorable to the initiator of the trade – the price grows when the market participant is an initiator of the stock purchase and drops if the trade initiator is selling the stock shares. Thus, the price impact can be seen as the source of transaction costs. The problem of the precise assessment of the

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transaction cost of the planned trade execution is important theoretically and practically.

Studying the market impact has become popular in quantitative finance [Tóth et al. 2011]. Price impact can be an important factor influencing the result of the investment on the stock market. The empirical study of the market impact is described, for example, in [Zarinelli et al. 2014]. It seems that the price impact assessment software can be an important tool for the financial investors [Gatheral 2010]. Another important factor affecting the profitability of the transaction is the bid-ask spread. The bid-ask spread can be defined as the difference between the highest bid and the lowest ask stock prices on the market. Occurring the positive difference between the highest bid price of the stock and the lowest ask stock price causes the transaction cost even for very slow trade execution. In the model considered in the paper the level of transaction costs depends on the speed of the execution of the trade. The dependence of transaction costs on the velocity of the trade of the trade speed was considered, for example, in [Almgren et al. 2005] and [Kociński 2018]. In the article, if the trade speed does not exceed some level then, the transaction cost ratio is constant but starting from this speed level, if the trade speed increases, then the ratio of the trade cost also increases. Thus reducing the velocity of the trade can reduce the transaction cost induced by the price impact. However, an important factor influencing the profitability of the investment on the stock market is the stock price drift. The drift may be the effect of the aggregate activity of the stock market participants which is a reaction on the information about the perspectives of profits resulting from the stock trading. For example, the positive value of the drift may be caused be the information that stock dividend paid by the company that emitted the stock shares will be higher than expected by the participants of the stock market. The negative value of the drift may be, for example, generated by the announcement that the stock dividend will be lower than it was forecasted by the stock investors. The optimization of the trade execution was considered, for example, in [Almgren, Chriss 2000].

The aim of the article is to determine the sale strategy which maximizes the expected amount of money obtained from selling the stock shares in the described in the article market model with the constraint of constant trade velocity.

THE MODEL OF THE STOCK PRICE AND SELLING

Let S_t denote the price of the stock at time t. Assume that the expected stock price $E(S_t)$ at time t is given as follows:

$$E(S_t) = S_0(1 + \mu t) \text{ for } t \in [0, T], \tag{1}$$

where μ denotes the stock price drift. It is assumed that $\mu < 0$.

Consider the market participant who has Y shares of the stock and wants to maximize the amount of money received from the sale of the stock in the interval $(0,T)$. It is assumed that the speed of the given trade is constant. It seems that on the market where the attained trading velocity can significantly differ from the trade speed planned by the stock seller, the assumption of the constant speed of the trade can be moderate with respect to the influence on the stock sale. The strategy with constant speed is characterized by the moment of the start of the sale t_0, the moment of the end of the sale t_1 and the number of the stock shares sold X. The values of t_0 and t_1 are in the interval $(0,T)$. Consider the trading strategy χ characterized by the triple (t_0,t_1,X) where $t_1 < t_2$. Notice that the trade speed of the strategy χ equals $\frac{X}{t_2 - t_1}$. It is assumed that

$$1-\gamma+\mu t-\beta\frac{X}{t_2-t_1}>0 \tag{2}$$

and the expected trade price at time $t \in (t_1,t_2)$ is given as follows:

$$E(S_t)=\begin{cases} S_0(1-\alpha+\mu t)\,\text{for}\,\frac{X}{t_2-t_1}\le v \\ S_0\left(1-\gamma+\mu t-\beta\frac{X}{t_2-t_1}\right)\text{for}\,\frac{X}{t_2-t_1}>v \end{cases}, \tag{3}$$

where α,β,γ are the nonnegative constants.

According to (3), if the trade speed doesn't exceed the level v, the trade cost is proportional to the stock price S_0 with the constant of proportionality equal to α. Moreover, by (3) it follows that if the velocity of trade is greater than v, then the cost of trading is the sum of the part proportional to the stock price S_0 with the proportionality constant equal to γ, and the part proportional to the speed of the trade with the constant of proportionality equal to β.

The meaning of the inequality (2) is clear in view of (3): if (2) holds then the negative and zero values of the expected trade price are excluded. The existing of the parameter α in the model implies that transaction cost can be paid even if the market impact effect is not affecting the stock price.

The constraint (2) is imposed in order the expected trade price of execution of the strategy χ to be positive for each trading moment from the interval (t_1,t_2).

It assumed that:

$$1-\alpha+\mu T>0 \tag{4}$$

and

$$1-\alpha = 1-\gamma-\beta v\,. \tag{5}$$

The inequality (4) implies that there exists a trading velocity such that it is possible to have a positive expected trade price for each trading moment from the interval $[0,T]$. The inequality (5) is for the expected trade price to be a continuous function of the trade speed for a fixed trading moment.

TIHE STOCK SALE OPTIMIZATION

Let $EA(\chi)$ symbolize the expected amount of money obtained by executing the strategy χ. The following formula holds:

$$EA(\chi) = \frac{X}{t_1 - t_0}\int_{t_0}^{t_1} E(S_t)dt\,. \tag{6}$$

Denote the selling strategy that maximizes $EA(\chi)$ by χ^*. By (3), (6) and by the fact that he drift is negative it follows that for the selling strategy χ, which maximizes $EA(\chi)$, the sale starts from the moment 0, which means that $t_0 = 0$. Consequently, by (3) and (6) the strategy χ^* is obtained by values of t_1 and X such that $EA(\chi)$ is maximized.

Let the function φ of two variables t_1 and X be defined as follows:

$$\varphi(t_1, X) = \begin{cases} S_0\left((1-\alpha)X + \dfrac{\mu t}{2}X\right) \text{for} \dfrac{X}{t_1} \le v \\ S_0\left((1-\gamma)X + \dfrac{\mu t}{2}X - \dfrac{\beta}{t_1}X^2\right) \text{for} \dfrac{X}{t_1} > v \end{cases}. \tag{7}$$

The strategy χ^* is obtained by determining the values of t_1 and X maximizing the function φ with the constraints:

$$0 \le t_1 \le T\,, \tag{8}$$

$$0 \le X \le Y\,, \tag{9}$$

$$1-\gamma+\mu t_1 - \beta\frac{X}{t_1} > 0\,. \tag{10}$$

However, for the pair (t_1, X) maximizing the function φ with the constraints (8) and (9) the constraint (10) is also satisfied. Therefore, the problem of determining the selling strategy χ^* is solved by finding the values t_1 and X maximizing the function φ with the constraints (8) and (9). Denote by $t^*{}_1$ and X^* the moment of the end of the sale for the strategy χ^* and the number of the stock shares sold for the strategy χ^*, respectively.

Let τ denote the function of X such that $\tau(X)$ is the value of t_1 which maximizes the function φ with the constraint (8) and X as the parameter. Moreover, let η denote the function of X defined as follows:

$$\eta(X) = \varphi(\tau(X), X). \tag{11}$$

The problem of finding X^* is solved by finding the maximum of the function $\eta(X)$ with the constraint (9) and the value of $t^*{}_1$ equals $\tau(X^*)$.

In order to determine the strategy χ^*, three cases will be considered.

1. $\beta \geq \frac{1}{2v}\left(1-\gamma+\frac{\mu T}{2}\right)$

Then,

$$X^* = \min(vT, Y), \tag{12}$$

$$t^* = \frac{X^*}{v}. \tag{13}$$

2. $\beta \leq \frac{1}{2v}\left(1-\gamma+\frac{\mu T}{2}\right)$ and $-\mu T \leq \frac{2}{3}(1-\gamma)$.

Then,

$$X^* = \min\left(\frac{T}{2\beta}\left(1-\gamma+\frac{\mu T}{2}\right), Y\right), \tag{14}$$

$$t^* = \begin{cases} T \text{ for } X^* > Tv \\ \frac{X^*}{v} \text{ for } X^* \leq Tv \end{cases}. \tag{15}$$

3. $\beta \leq -\frac{\mu T}{2v}$ and $-\mu T \geq \frac{2}{3}(1-\gamma)$.

Then,

$$X^* = \min\left(-\frac{2}{\mu\beta}\left(\frac{1-\gamma}{3}\right)^2, Y\right), \tag{16}$$

$$t^* = \begin{cases} \sqrt{-\dfrac{2\beta X^*}{\mu}} \text{ for } X^* > -\dfrac{2\beta v^2}{\mu} \\ \dfrac{X^*}{v} \text{ for } X^* \le -\dfrac{2\beta v^2}{\mu} \end{cases}. \tag{17}$$

NUMERICAL EXAMPLE

In this section the following values of the parameters T, S_0, Y γ and v are used in computations: for $T = 0.125$, $S_0 = 1$, $Y = 0.5$, $\gamma = 0.05$, and $v = 0.1$. The number of the stock shares Y is expressed as the fraction of the average traded volume of the stock in the interval $(0,1)$. Thus, the market participant has 0,5 of the average traded number of the stock shares in the interval $(0,1)$. The considered values of the model parameters in this example are within of the reasonable choices to the exemplary calculations.
In Table 1 there are computed the values of X^* for 180 pairs of (β,μ).

Table 1. The number of the stock shares to sell for the strategy maximizing the expected amount of money from selling the stock of the market participant

β \ μ	-5.00	-4.50	-4.00	-3.50	-3.00	-2.50	-2.00	-1.50	-1.00
0.01	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.02	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.03	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.04	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.05	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.06	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.07	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.08	0.498	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.09	0.443	0.464	0.486	0.500	0.500	0.500	0.500	0.500	0.500
0.10	0.398	0.418	0.438	0.457	0.477	0.496	0.500	0.500	0.500
0.11	0.362	0.380	0.398	0.415	0.433	0.451	0.469	0.487	0.500
0.12	0.332	0.348	0.365	0.381	0.397	0.413	0.430	0.446	0.462
0.13	0.306	0.322	0.337	0.352	0.367	0.382	0.397	0.412	0.427
0.14	0.285	0.299	0.313	0.326	0.340	0.354	0.368	0.382	0.396

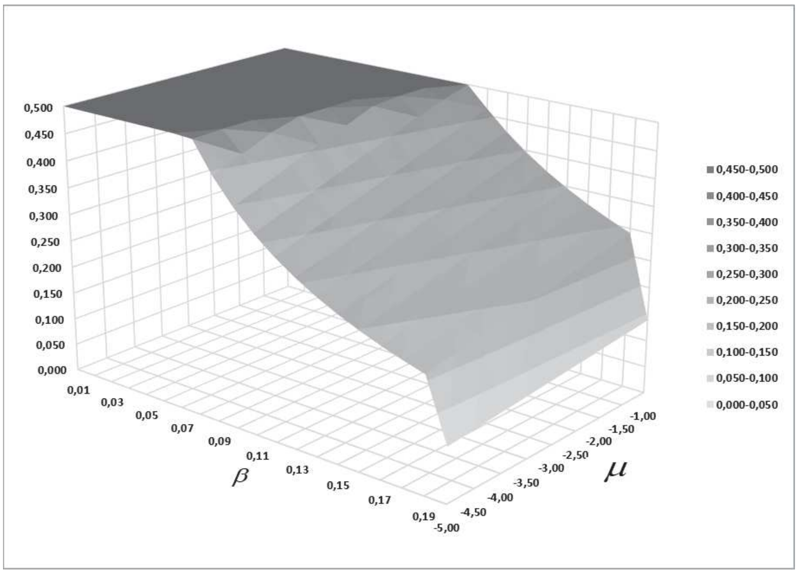
Table 1. (continued)

β \ μ	-5.00	-4.50	-4.00	-3.50	-3.00	-2.50	-2.00	-1.50	-1.00
0.15	0.266	0.279	0.292	0.305	0.318	0.331	0.344	0.357	0.370
0.16	0.249	0.261	0.273	0.286	0.298	0.310	0.322	0.334	0.347
0.17	0.234	0.246	0.257	0.269	0.280	0.292	0.303	0.315	0.326
0.18	0.221	0.232	0.243	0.254	0.265	0.276	0.286	0.297	0.308
0.19	0.210	0.220	0.230	0.241	0.251	0.261	0.271	0.282	0.292
0.20	0.100	0.104	0.109	0.114	0.119	0.124	0.129	0.134	0.139

Source: own computation

In Figure 1 it is shown how the value of X^* depends on the parameters β and μ.

Figure 1. The values of X^* depending on β and μ



Source: Table 1 and own preparation

Table 2 contains the results of computing the values of the expected amount of money obtained by executing the strategy χ^* for 180 pairs of (β, μ).

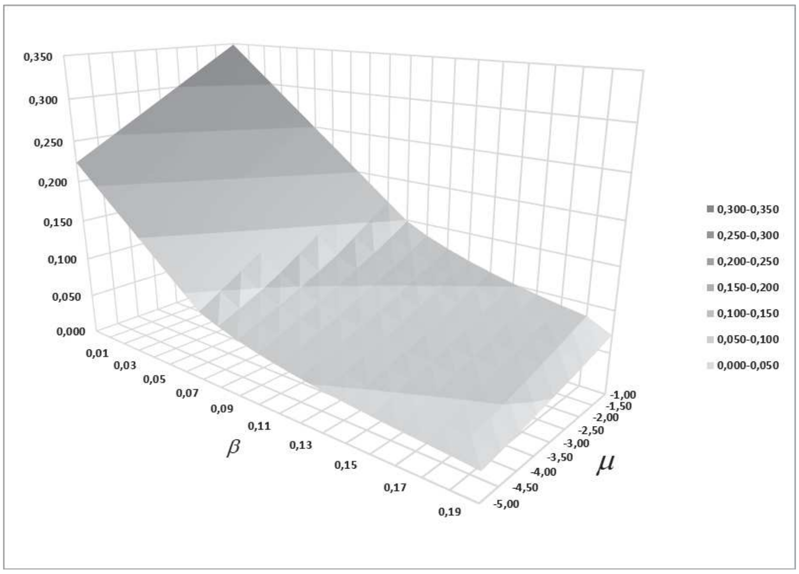
Table 2. The expected amount of money from executing the strategy χ^* by the market participant

β \ μ	-5.00	-4.50	-4.00	-3.50	-3.00	-2.50	-2.00	-1.50	-1.00
0.01	0.224	0.239	0.255	0.271	0.286	0.302	0.318	0.333	0.349
0.02	0.204	0.219	0.235	0.251	0.266	0.282	0.298	0.313	0.329
0.03	0.184	0.199	0.215	0.231	0.246	0.262	0.278	0.293	0.309
0.04	0.164	0.179	0.195	0.211	0.226	0.242	0.258	0.273	0.289
0.05	0.144	0.159	0.175	0.191	0.206	0.222	0.238	0.253	0.269
0.06	0.124	0.139	0.155	0.171	0.186	0.202	0.218	0.233	0.249
0.07	0.104	0.119	0.135	0.151	0.166	0.182	0.198	0.213	0.229
0.08	0.084	0.099	0.115	0.131	0.146	0.162	0.178	0.193	0.209
0.09	0.075	0.086	0.097	0.111	0.126	0.142	0.158	0.173	0.189
0.10	0.067	0.077	0.088	0.099	0.110	0.122	0.138	0.153	0.169
0.11	0.061	0.070	0.080	0.090	0.100	0.111	0.123	0.135	0.149
0.12	0.056	0.064	0.073	0.082	0.092	0.102	0.113	0.124	0.136
0.13	0.052	0.059	0.067	0.076	0.085	0.094	0.104	0.114	0.125
0.14	0.048	0.055	0.063	0.070	0.079	0.087	0.097	0.106	0.116
0.15	0.045	0.051	0.058	0.066	0.073	0.082	0.090	0.099	0.109
0.16	0.042	0.048	0.055	0.062	0.069	0.077	0.085	0.093	0.102
0.17	0.040	0.045	0.051	0.058	0.065	0.072	0.080	0.088	0.096
0.18	0.037	0.043	0.049	0.055	0.061	0.068	0.075	0.083	0.091
0.19	0.035	0.041	0.046	0.052	0.058	0.064	0.071	0.078	0.086
0.20	0.033	0.037	0.041	0.046	0.050	0.055	0.060	0.066	0.072

Source: own computation

In Figure 2 it is shown how the value of $EA\left(\chi^*\right)$ depends on the parameters β and μ.

Figure 2. The values of the expected amount of money from executing the strategy χ^* depending on β and μ



Source: Table 2 and own preparation

From the considered numerical computations it can be seen that the market impact and the negative drift in the price of the stock may negatively influence the financial profit of selling the shares of the stock.

SUMMARY

In the article the model of the market with the market impact, transaction costs and the drift in the stock price is considered. In a framework of this model, the stock sale strategy which maximizes the expected amount of money obtained from selling the market participant's shares of the stock with the constraint of constant trading velocity is determined. From the numerical computations included in the article it can be concluded that the market impact and the drift in the price of the stock may significantly affect the financial profit of investing in the stock market.

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