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ON THE CHOICE OF SYNTHETIC MEASURES FOR ASSESSING ECONOMIC EFFECTS

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Abstract: Multidimensional analysis uses various measures for assessing economic effects. However, no single synthetic measure, regardless how popular, can give a satisfactory solution to the above problem. In general, various approaches of combining measures can lead to stable outcomes. Nevertheless, when combining "weak" classifiers one can obtain inevitably poorer classification. We propose here a new approach to construct doubly synthetic measures. The main goal of this work is to analyse the influence these new synthetic measures on the ranking of multidimensional objects.

Keywords: multidimensional analysis, synthetic measure, ranking methods

INTRODUCTION

Many scientific studies use multidimensional analysis to process their empirical data. It is also widely used in an enterprise environment. It is applied to compare objects defined as a set of *n*-indicator variables. Usually, the goal of such analysis is to reduce a large quantity of gathered data to a small number of simple categories (a few synthetic indicators) which is a subject to further analysis and allows the creation of uniform groups obtained and defined by the values of these categories. The bibliography in this area is extensive (i.a. Aczel 1989, Morrison

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1990, Hair et al. 1995). Among the group of methods discussed in the related literature, the basic group contains the methods that utilize the so called model objects. Breiman proved in his works [Breiman, 1994, 1996, 1998] that using a single synthetic measure to either rank objects or to classify those objects can be far from optimal. Furthermore, a superposition of many measures gives a stable and close to optimal result. It should be noted, however when combining "weaker" classifiers one can obtain weaker classifier either.

According to Jackson [Jackson 1969, 1969a, 1970] our problem is correctly stated when:

- as a result of the applied algorithm we obtain a single result,
- the resulting classification is stable. The latter means that the resulting classification or order does not change "drastically" when the inputs are slightly varied,
- the applied algorithm is invariant with respect to the permutation of variables and names of objects that are to be ordered and classified,
- the applied algorithm scale insensitive in all cases when the values of variables belong to a scale with an absolute zero. The latter means that the algorithm is indifferent to multiplication of the matrix of distances.

In general, ranking methods can be split into model and non-model ones.

Non-model methods rely on constructing a synthetic aggregate measure based only on normalized values of features. Model methods rely on constructing taxonomic measures of growth (artificial reference points) and measuring distances from these models and on are based on creating a synthetic measure.

Naturally, models can also play an important part in normalization of variables (see [Kukula 2000]). Noticeably, most techniques that are commonly classified as non-model can be ultimately reduced to a form relying on the chosen explicit model.

The choice of a model in an automated reporting system is especially important in assessing, ordering and classifying objects according to the value of a synthetic measure within a specified period. Consequently, the goal of this work is to illustrate the influence of a choice of a proposed model on multidimensional objects' ranking.

MODEL MEASURES

Let, $X = \Re^n$, $\Re = (-\infty, \infty)$, $n \in N$, denote n-dimensional vector space. Consider now a problem of classifying $m \in N$ objects $Q_1, Q_2, ..., Q_m$ of a studied phenomenon based on their variables (features). Without loss of generality, we assume that all features have the character of a stymulant.

Assume that vector $\mathbf{x_i} = (x_{i1}, x_{i2}, \ldots, x_{in}) \in X$, $i = 1,2,\ldots,m$, describe the i-th object. If $x_{ik} > x_{jk}\ (x_{ik} \geq x_{jk})$ for $k = 1,2,\ldots n$ then we will write:

$$\mathbf{x_i} > \mathbf{x_j}\ (\mathbf{x_i} \geq \mathbf{x_j}),\ \text{where } i,j \in [1,m].$$

It is easy to see that if $\mathbf{x}_i \geq \mathbf{x}_j$ and $\mathbf{x}_i \neq \mathbf{x}_j$, then in some cases it is natural to say that object $\mathbf{x_i}$ is better (more highly rated) than object $\mathbf{x_j}$. This means that none of components of vector $\mathbf{x_i}$ is less than a corresponding component of vector $\mathbf{x_j}$, and at least one of them is greater, which implies the existence a k belonging to [1, n] such that $x_{ik} > x_{jk}$.

Let us use the following denotations:

$$x_{0,k} = \min_{1 \leq i \leq m} x_{ik}, \quad x_{m+1,k} = \max_{1 \leq i \leq m} x_{ik}, \quad k = 1,2 \ldots n,$$

and

$$\mathbf{x_0} := \left(x_{0,1}, x_{0,2}, \ldots, x_{0,n}\right),$$
$$\mathbf{x_{m+1}} := \left(x_{m+1,1}, x_{m+1,2}, \ldots, x_{m+1,n}\right).$$

It is obvious that objects $\mathbf{Q}_0$ - described by vector $\mathbf{x}_0$, $\mathbf{Q}_{m+1}$ described by vector $\mathbf{x}_{m+1}$ (perhaps fictitiously) are not worse nor better than the rest of objects $\mathbf{Q}_1$, $\mathbf{Q_2}$,...,$\mathbf{Q}_m$,. That is:

$$\mathbf{x}_{m+1} \geq \mathbf{x}_i \text{ and } \mathbf{x}_i \geq \mathbf{x}_0 \text{ for each } i : m \geq i \geq 1$$

In conjunction with the above let us denote by

$$\left\langle \mathbf{x_0}, \mathbf{x_{m+1}} \right\rangle := \left\{ \mathbf{x} \in \Re_+^n : \mathbf{x_0} \leq \mathbf{x} \leq \mathbf{x_{m+1}} \right\}$$

as an interval (hypercube) in an n-dimensional Euclidean space.

In the case when objects $\mathbf{Q}_0$ and $\mathbf{Q}_{m+1}$ are different from considered objects $\mathbf{Q}_1$, $\mathbf{Q}_2$,...,$\mathbf{Q}_m$, they fulfill the roles the worst and the best, respectively objects. Objects $\mathbf{Q}_0$ and $\mathbf{Q}_{m+1}$ can be treated as models.

Suppose X is a empty set. We say that a function d which projects a Cartesian product into a set of non-negative numbers $\Re_+^1 = \langle 0, +\infty)$ defines a distance between any elements $x, y \in X$ belonging to X if it fulfills the following criteria:

1. d(x, y) =d(y, x) (symmetric)
2. d(x, x) = 0.

A distance d(x, y) is a metric if it fulfills the triangle inequality

3. d(x, y) ≤ d(x, z) + d(z, y) for all $x, y, z \in X$

Let $\mathbf{x, y} \in \Re_+^n, \mathbf{x} = \left(x_1, x_2, \ldots, x_n\right)$, $\mathrm{y} = \left(y_1, y_2, \ldots, y_n\right)$ The following function is related to construction of radar measures [Binderman, Borkowski, Szczesny 2012]

$$d_{rad}(\mathbf{x,y}) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} |x_i - y_i| |x_{i+1} - y_{i+1}|} \tag{1}$$

where $x_{n+1} := x_1$, $y_{n+1} := y_1$ is a distance but not a metric. It can be easily verified that this function fulfills 1 and 2, but not 3.

Indeed, let $\mathbf{x}=(n,1,0,0,\ldots,0)$, $\mathbf{y}=0:=(0,0,\ldots,0)$, $\mathbf{z}=(0,1,0,0,\ldots,0)$, then $d_{rad}(\mathbf{x},\mathbf{y})=1$, $d_{rad}(\mathbf{x},\mathbf{z})=0$, $d_{rad}(\mathbf{z},\mathbf{y})=0$. Hence $1=d_{rad}(\mathbf{x},\mathbf{y})>d_{rad}(\mathbf{x},\mathbf{z})+d_{rad}(\mathbf{z},\mathbf{y})=0$.
On the other hand, the function

$$d_p(\mathbf{x},\mathbf{y})=\left[\sum_{j=1}^{n}\left|x_j-y_j\right|^p\right]^{\frac{1}{p}},\ \ \infty>p\geq 1, \tag{2}$$

is an example of a metric, and is also known as Minkowski's metric [Kukuła K, 2000].

Note 1. If $X=X_1\times X_2\times\ldots\times X_k\left(\sum_{j=1}^{k}dim\ X_j=n\right)$, ρ_i are distances in spaces $X_i\ (i=1,2,\ldots,k)$ then a distance in space X can be defined by using distance $\rho_i\ (i=1,2,\ldots,k)$. For example, a standard distance in space X is defined by:

$$\rho(\mathbf{x},\mathbf{y})=\sqrt{\sum_{i=1}^{k}\rho_i^2(\mathbf{x},\mathbf{y})}\ ,$$

$\mathbf{x}=(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_k),\mathbf{y}=(\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_k)\in X;\mathbf{x}_i,\mathbf{y}_i\in X_i;\ i=1,2,\ldots,k$,
Especially, if $X=X_1\times X_2,\ X_1=X_2=\Re^2;\ \rho_1=d_{rad},\ \rho_2=d_p$ where functions d_{rad},d_p are defined by equations (1) and (2), respectively, then a standard distance in space X is defined by

$$\rho(\mathbf{x},\mathbf{y})=\sqrt{\sum_{i=1}^{2}(x_i-y_i)^2+\frac{1}{2}\sum_{i=3}^{4}|x_i-y_i||x_{i+1}-y_{i+1}|}\ ,$$

$\mathbf{x}=(\mathbf{x}_1,\mathbf{x}_2),\mathbf{y}=(\mathbf{y}_1,\mathbf{y}_2)\in X,\mathbf{x}_1=(x_1,x_2),\mathbf{y}_1=(y_1,y_2)\in X_1$,
$\mathbf{x}_2=(x_3,x_4),\mathbf{y}_2=(y_3,y_4)\in X_2;\ x_5:=x_3,\ y_5:=y_3$.

Let $\rho^*(\mathbf{x},\mathbf{y})$ denote distance between vectors $\mathbf{x},\mathbf{y}\in\Re_+^n$ and $\rho^*(\mathbf{x_0},\mathbf{x_{m+1}})\neq 0$. In literature, classification of objects is performed by utilizing the following equations defining synthetic measures of the given vector $\mathbf{x}\in\langle\mathbf{x_0},\mathbf{x_{m+1}}\rangle$.

$$\mu_1(\mathbf{x})=\frac{\rho^*(\mathbf{x_0},\mathbf{x})}{\rho^*(\mathbf{x_0},\mathbf{x_{m+1}})}, \tag{3}$$

$$\mu_2(\mathbf{x})=1-\frac{\rho^*(\mathbf{x_{m+1}},\mathbf{x})}{\rho^*(\mathbf{x_0},\mathbf{x_{m+1}})}, \tag{4}$$

$$\mu_3(\mathbf{x})=\frac{\mu_1(\mathbf{x})+\mu_2(\mathbf{x})}{2}=\frac{1}{2}+\frac{\rho^*(\mathbf{x_0},\mathbf{x})-\rho^*(\mathbf{x_{m+1}},\mathbf{x})}{2\rho^*(\mathbf{x_0},\mathbf{x_{m+1}})}, \tag{5}$$

$$\mu_4(\mathbf{x}) = \frac{\mu_1(\mathbf{x})}{1+\mu_1(\mathbf{x})+\mu_2(\mathbf{x})} = \frac{\rho^*(\mathbf{x_0},\mathbf{x})}{\rho^*(\mathbf{x_0},\mathbf{x})+\rho^*(\mathbf{x_{m+1}},\mathbf{x})}, \tag{6}$$

It can be easily shown that measures μ_1 and μ_2 use one model, while measures μ_3 and μ_4 resort to two models. These measures can be treated as tools for solving multi-criteria decision problems. Each of measures μ_1 and μ_2 uses only one criterion while measures μ_3 and μ_4 - two criteria.

In his work [Hellwig 1968] gave a measure that utilized only the best objects. The theory behind and applications of measure μ_3 were discussed in a series of works by Binderman [Binderman A. 2006] as well as in [Binderman Z. et al. 2012, 2013]. Measure μ_4 is linked with the TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution, see [Hwang, Yoon 1981]).

Note 2. If $X_1, \dots, X_k$ are variables with values from an interval scale and variables $Z_1, \dots, Z_k$ are derived from them by normalizing them with a zero unitarization method, then we receive

$$w_i = \frac{1}{n}\sum_{j=1}^{n} z_{ij} = \frac{\rho_1[(0,\dots,0),(z_{i1},\dots,z_{in})]}{\rho_1[(0,\dots,0),(1,\dots,1)]} = 1 - \frac{\rho_1[(1,\dots,1),(z_{i1},\dots,z_{in})]}{\rho_1[(0,\dots,0),(1,\dots,1)]}, \tag{7}$$

where $i = 1,\dots,m$ and a ρ_1 denotes a Minkowski metric as defined by (2).

With this a typical synthetic measure which construction is based on variables normalized with a zero unitarization method is also an indicator which is received by using a standard technique of comparison with a negative model. It can be shown that if the zero unitarization method is replaced with standardization as the tool to normalize variables, then the values of the indicator can be expressed by using distances from a negative model, namely:

$$w_i = \frac{1}{n}\sum_{j=1}^{n} z_{ij} = \frac{1}{n}\left(\rho_1[(z_{01},\dots,z_{0n}),(z_{i1},\dots,z_{in})] - \rho_1[(0,\dots,0),(z_{i1},\dots,z_{0n})]\right),$$

where $(z_{01},\dots,z_{0n})$ denotes a vector of values of the negative model. Which means that this indicator is also a synthetic indicator, which is constructed as a distance from the negative model.

In the next step we normalize the distance of vectors $\rho^*(\mathbf{x},\mathbf{y})$ to the established model vectors $\mathbf{x_0}$, $\mathbf{x_{m+1}}$, with (3):

$$\rho(\mathbf{x},\mathbf{y}) := \frac{\rho^*(\mathbf{x},\mathbf{y})}{\rho^*(\mathbf{x_0},\mathbf{x_{m+1}})},$$

Then $\rho^*(\mathbf{x_0},\mathbf{x_{m+1}}) = 1$ and equations (3)-(6) become:

$$\mu_1(\mathbf{x}) = \rho(\mathbf{x_0},\mathbf{x}), \tag{3'}$$

$$\mu_2(\mathbf{x}) = 1 - \rho(\mathbf{x_{m+1}},\mathbf{x}), \tag{4'}$$

$$\mu_3(\mathbf{x}) = \frac{1}{2}\left[1 + \rho(\mathbf{x_0}, \mathbf{x}) - \rho(\mathbf{x_{m+1}}, \mathbf{x})\right], \tag{5'}$$

$$\mu_4(\mathbf{x}) = \frac{\rho(\mathbf{x_0}, \mathbf{x})}{\rho(\mathbf{x_0}, \mathbf{x}) + \rho(\mathbf{x_{m+1}}, \mathbf{x})}, \tag{6'}$$

In the special case when vectors $\mathbf{x_0} = 0 = (0,0,\ldots,0)$, $\mathbf{x_{m+1}} = 1 := (1,1,...,1)$ then

$$\rho^*(0,1) = \begin{cases} 1 & dla \quad \rho^* = d_{rad} \\ n^{1/p} & dla \quad \rho^* = d_p \end{cases}, \tag{8}$$

Noticeably, the considered measures, as defined by (3')-(6') are normalized in terms of established models, that is:

$$\mu_i(\mathbf{x_0}) = 0, \ \ \mu_i(\mathbf{x_{m+1}}) = 1, \ dla \ \ i = 1,2,3,4, \tag{9}$$

The above measures are ones of the most commonly used measures to order objects. Nevertheless, one can give other measures based on averages, which utilize distances from models.

Let $\mathbf{x} \in \langle x_0, x_{m+1} \rangle$, numbers be defined by (3'), (4'). For a given vector **x** we can define the following measures:

$$\mu_5(\mathbf{x}) = \begin{cases} \dfrac{2\mu_1(\mathbf{x})\mu_2(\mathbf{x})}{\mu_1(\mathbf{x}) + \mu_2(\mathbf{x})} & \text{dla} \quad \mu_1(\mathbf{x}) + \mu_2(\mathbf{x}) \neq 0 \\ 0 & \text{dla} \quad \mu_1(\mathbf{x}) + \mu_2(\mathbf{x}) = 0 \end{cases} \text{- harmonic average,}$$

$$\mu_6(\mathrm{x}) = \sqrt{\mu_1(\mathrm{x})\mu_2(\mathrm{x})} \text{ - geometric average,}$$

$$\mu_7(\mathrm{x}) = \sqrt{\frac{\mu_1(\mathrm{x})\mu_2(\mathrm{x})}{2}} \text{ - root mean square.}$$

It can be shown [Mitrinovic 1993] that for a given vector the following inequalities hold:

$$\min(\mu_1, \mu_2) \leq \mu_5 \leq \mu_6 \leq \mu_3 \leq \mu_7 \leq \max(\mu_1, \mu_2).$$

NON-STATIONARY MODEL MEASURES

The described measures can be understood of as functions of any vector $\mathbf{x} \in \langle \mathbf{x}_0, \mathbf{x}_{m+1} \rangle$ (functions of n real variables) or as functions of vector $\mathbf{x} \in \langle \mathbf{x}_0, \mathbf{x}_{m+1} \rangle$ and vectors that define equivalent objects - (functions of $(n+1)\cdot m$ real variables) because model vectors are functions' values, depending on vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m$.

If μ denotes any set measure of vector $\mathbf{x} \in \langle \mathbf{x}_0, \mathbf{x}_{\mathrm{m+1}} \rangle$, defined by one of (3)-(6) then in the second case we should have: $\mu = \mu(\mathbf{x}, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)$.

As a result of the above, the considered measures can be used, as necessary, in one of two ways: static and dynamic. If objects Q_1, Q_2,...,Q_m of the studied phenomenon are considered in a time interval $\langle T_0, T_1 \rangle$ then their describing vectors should be treated as functional vectors $x_1, x_2, \ldots, x_m$ dependent on time.

Let $t_1, t_2, \ldots, t_q \in \langle T_0, T_1 \rangle, q \in N$. To order the objects Q_1, Q_2,...,Q_m with the considered measures at a given point in time $t_j \ (j \in \{1,2,\ldots,q\})$ or to order them based on their descriptions at points in time $t_1, t_2, \ldots, t_q$, we must compute the coordinates of model vectors $\mathbf{x}_0(t_j), \mathbf{x}_{m+1}(t_j)$:

$$x_{0,k}(t_j) = \min_{1 \le i \le m} x_{ik}(t_j), \quad x_{m+1,k}(t_j) = \max_{1 \le i \le m} x_{ik}(t_j), \qquad (10)$$
$$k = 1,2,\ldots,n; \ j = 1,2,\ldots,q$$

or coordinates of model vectors $\mathbf{x}_0, \mathbf{x}_{m+1}$:

$$x_{0,k}(t_j) = \min_{1 \le i \le q} x_{0,k}(t_j), \quad x_{m+1,k}(t_j) = \max_{1 \le i \le q} x_{m+1,k}(t_j),$$
$$k = 1,2,...,n$$

As a direct result of the definition, the following inclusions are sound:

$$\underset{j \in \{1,2,\ldots,q\}}{\forall} \mathbf{x}_0(t_j), \mathbf{x}_{m+1}(t_j) \in \langle x_0, x_{m+1} \rangle$$

In the case when we want to order objects in the entire time interval (T_0, T_1), we must choose models $\mathbf{x}_0$, $\mathbf{x}_{m+1}$, such that the following inequalities hold:

$$x_{0,k} \le \inf_{1 \le i \le m} x_{ik}(t), \quad x_{m+1,k} \ge \sup_{1 \le i \le m} x_{ik}(t), \quad k = 1,2,...,n \qquad (11)$$
$$\text{for each } t \in \langle T_0, T_1 \rangle$$

Especially, if the functional vectors $\mathbf{x}_i(t), i \in \{1,2,\cdots,m\}, t \in \langle T_0, T_1 \rangle$ are continuous then:

$$x_{0,k} = \min_{1 \le i \le m} x_{ik}(t), \quad x_{m+1,k} = \max_{1 \le i \le m} x_{ik}(t), \quad k = 1,2,...,n,$$

Note 3. In a dynamic approach to the problem of ordering and classifying objects in the entire time interval $\langle T_0, T_1 \rangle$, we should assume that the obtained result, which uses "partial" results - got from pairs of model vectors $\mathbf{x}_0(t_j), \mathbf{x}_{m+1}(t_j); j = 1,2,\ldots,q$ can be significantly different from the result received by means of "integral" models $\mathbf{x}_0, \mathbf{x}_{m+1}$.

Naturally, the choice of a model depends on the way of presenting/reporting the concrete phenomenon in a given period. However, the scale of differences can prove to be substantial, as the following analysis shows.

ILLUSTRATION OF CONSEQUENCES OF A CHOICE OF A MODEL

Ranking of objects is determined by, (in addition to the feature transformation method), the choice of model object. In this theoretical example we present different results of ordering objects in a dynamic approach depending on the method defining the model object.

In order to do that we created the following simulation which generates values for variables X_1-X_4, at given points in time T_1-T_5, with the distributions of their values:

$$X_1(T_1) \approx N(6{,}2),\ X_1(T_j) = X_1(T_{j-1}) + U_j,\ j = 2,\ldots,5,\ U_j \approx N(0{,}5;0{,}2),$$

$$X_2(T_1) \approx N(6{,}2),\ X_2(T_j) = X_2(T_{j-1}) + V_j,\ j = 2,\ldots,5,\ V_j \approx N(-0{,}5;0{,}2),$$

$$X_3(T_1) \approx N(6{,}2),\ X_3(T_j) = X_3(T_{j-1}) + \xi_j W_j,\ j = 2,\ldots,5,\ W_j \approx N(0{,}4;2),$$

$$X_4(T_1) \approx N(6{,}2),\ X_4(T_j) = X_4(T_{j-1}) + \eta_j Z_j,\ j = 2,\ldots,5,\ Z_j \approx N(0{,}4;2),$$

$$\xi_2 = \xi_3 = \eta_4 = \eta_5 = -1,\quad \eta_2 = \eta_3 = \xi_4 = \xi_5 = 1$$

$X_1(T_1),\ i = 1,\ldots,4,\ U_j, V_j, W_j, Z_j,\ j = 2,\ldots,5$, which are independent.

By using this model we generated data tables for 10 objects. One of these simulation is presented in Table 1. The last two rows of Table 1 contain the negative and positive model for each time point T_1-T_5, respectively.

Table 1. Sample data for simulations of 10 objects

	T_1				**T_2**				**T_3**				**T_4**				**T_5**			
	X_1	**X_2**	**X_3**	**X_4**	**X_1**	**X_2**	**X_3**	**X_4**	**X_1**	**X_2**	**X_3**	**X_4**	**X_1**	**X_2**	**X_3**	**X_4**	**X_1**	**X_2**	**X_3**	**X_4**
o01	3.17	5.76	6.64	3.79	3.79	5.38	6.2	5.41	3.96	4.99	10.33	2.16	4.56	4.27	12.99	-1.5	5.5	3.83	15.38	-0.11
o02	5.89	9.26	3.8	2.04	6.17	8.87	6.81	1.37	6.92	8.5	7.41	-2.93	7.32	7.94	6.82	-7.27	7.91	7.66	10.48	-9.63
o03	8.57	6.26	4.07	1.63	8.9	5.71	2.57	4.47	9.72	5.29	2.61	8.08	10.18	4.82	4.13	7.2	10.97	4.51	8.57	5.88
o04	7.04	2.2	9.56	3.19	7.68	2.45	6.69	6.48	8.01	1.89	6.06	5.09	8.29	1.17	9.06	6.01	8.61	0.65	5.01	7.81
o05	9.96	7.22	6.13	2.94	10.54	6.68	4.6	1.37	10.65	6.18	4.12	1.08	11.12	5.68	6.35	-0.19	11.5	4.87	7.99	-4.15
o06	8.37	4.83	5.91	1.97	8.82	4.51	8.42	3.29	9.27	3.99	6.77	5.23	9.87	3.37	8.5	6.55	10.55	2.85	8.78	8.62
o07	2.87	6.55	6.86	4.1	3.1	5.75	7.81	5.52	3.86	4.98	6.8	6.01	4.47	4.53	5	5.04	4.96	4.28	4.26	5.56
o08	3.24	7.22	6	4.59	3.89	6.84	9.26	7.11	4.65	6.25	8.72	7.06	5.03	5.91	11.49	7.6	5.75	5.64	12.09	11.95
o09	5.66	4.92	7.02	3.79	6.11	4.35	8.17	2.14	6.41	3.64	6.58	3.66	6.94	3.04	6.76	7.47	7.37	2.52	7.26	11.06
o10	3.08	6.1	9.75	4.78	3.75	5.44	12.88	7.73	3.99	4.83	10.2	10.08	4.61	4.53	11.49	10.55	5.25	3.86	12.6	12.7
min	**2.87**	**2.2**	**3.8**	**1.63**	**3.1**	**2.45**	**2.57**	**1.37**	**3.86**	**1.89**	**2.61**	**-2.93**	**4.47**	**1.17**	**4.13**	**-7.27**	**4.96**	**0.65**	**4.26**	**-9.63**
max	**9.96**	**9.26**	**9.75**	**4.78**	**10.54**	**8.87**	**12.88**	**7.73**	**10.65**	**8.5**	**10.33**	**10.08**	**11.12**	**7.94**	**12.99**	**10.55**	**11.5**	**7.66**	**15.38**	**12.7**

Source: own research

Naturally, the integral models are (2.87, 0.65, 2.57, -9.63) and (11.5, 9.26, 15.38, 12.7), respectively. We have used the most common synthetic indicator, defined by (7). For each of the objects, in each of the time periods, we have calculated the value of the indicator as well as the rank of the values in two approaches:

i. by normalizing the data by two integral models (W_1),
ii. by normalizing the data by five models from each period (W_2).

The obtained results are presented in Table 2. In the last 5 columns of Table 2 we have put the changes in positions in the order of objects according to the values of indicators W_1 and W_2 (a positive number means that object moves up the ranking by the number of spots, while a negative one means that object falls in the ranking). The differences are significant, but the simulation is specially chosen for this differentiating characteristic.

Table 3 presents the changes of order's positions in a ranking of objects for 8 simulations done, according to the values of indicators W_1 and W_2, same as in Table 2.

The changes to the order are not substantial in all cases, see, for example, simulation number 6. The choice of a model (an integral one for the entire period or a different one for each interval) has an impact regardless of the choice of a "distance from the model"-based on the indicator from the list $\mu_1 - \mu_7$.

For the given simulation from Table 1, we calculated the values of these 7 indicators by using an Euclidean metric (ρ_2). The latter is done after normalizing the data with a zero unitarization method. The changes in the order of the objects between these two applications of models are presented in Table 4. In each case we can see significant differences. These are similar to the differences observed in Table 2 when we have used distance ρ_1.

Table 2. Values of synthetic indicators defined by (7) using an integral model (W_1) and models for individual periods (W_2)

	W_1 (integral pattern)					W_2 (5 different patterns)					W_1 - ranks					W_2 - ranks					Change of position				
	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5
o01	0.387	0.403	0.441	0.448	0.525	0.427	0.374	0.469	0.449	0.491	10	10	7	9	6	7	9	7	7	7	3	1	0	2	-1
o02	0.492	0.540	0.515	0.450	0.504	0.389	0.433	0.518	0.433	0.503	2	4	6	8	8	9	6	6	9	6	-7	-2	0	-1	2
o03	0.483	0.479	0.532	0.552	0.637	0.356	0.432	0.556	0.552	0.638	3	6	2	5	2	10	7	4	5	1	-7	-1	-2	0	1
o04	0.446	0.452	0.418	0.474	0.409	0.513	0.455	0.419	0.469	0.352	6	7	8	6	9	4	4	9	6	9	2	3	-1	0	0
o05	0.606	0.560	0.536	0.565	0.540	0.630	0.449	0.538	0.579	0.546	1	2	1	3	5	2	5	5	4	5	-1	-3	-4	-1	0
o06	0.476	0.543	0.531	0.579	0.612	0.403	0.482	0.570	0.601	0.598	4	3	3	2	3	8	3	3	2	3	-4	0	0	0	0
o07	0.409	0.427	0.412	0.371	0.369	0.479	0.407	0.424	0.321	0.300	9	9	10	10	10	6	8	8	10	10	3	1	2	0	0
o08	0.428	0.527	0.521	0.582	0.656	0.518	0.570	0.584	0.612	0.626	8	5	5	1	1	3	2	2	1	2	5	3	3	0	-1
o09	0.442	0.442	0.416	0.461	0.508	0.501	0.334	0.415	0.443	0.458	7	8	9	7	7	5	10	10	8	8	2	-2	-1	-1	-1
o10	0.466	0.560	0.523	0.563	0.608	0.646	0.628	0.612	0.587	0.563	5	1	4	4	4	1	1	1	3	4	4	0	3	1	0

Source: own research

Table 3. Changes in the order of objects for a few sample data simulations

	Simulation 2					Simulation 3					Simulation 4					Simulation 5					Simulation 6					Simulation 7					Simulation 8				
	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5
o01	1	1	0	0	0	1	0	-1	0	1	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	0	2	0	-1	0	1	1	-1	0	-1	0
o02	4	0	1	0	0	0	0	0	0	-4	1	0	0	0	1	-2	0	0	0	0	1	0	0	-1	0	1	1	2	1	0	3	2	0	1	0
o03	-1	0	0	0	0	0	0	0	0	-1	-2	0	0	0	0	-1	0	0	0	0	0	0	0	0	-1	-1	-1	-2	2	1	0	0	1	0	0
o04	-5	-1	0	0	0	-1	0	0	0	0	3	0	0	0	0	2	-1	1	0	-1	1	-1	-1	0	0	2	0	-2	-4	-1	0	-1	0	-1	0
o05	-1	0	-2	0	0	-1	0	0	1	3	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	-5	-1	0	1	1	-1	-1	0	0	0
o06	0	0	-1	0	0	1	0	0	0	-1	0	0	-1	0	0	-3	-2	-1	0	0	0	0	0	0	0	5	2	2	0	-2	-1	0	0	0	0
o07	0	0	0	0	0	1	0	1	0	1	0	0	1	0	-1	0	2	0	0	0	-1	0	0	1	0	-2	0	0	0	-1	0	0	-1	0	0
o08	6	1	2	0	0	0	0	0	0	1	-1	0	0	0	0	1	2	0	-1	0	0	1	1	0	1	0	0	1	-1	0	0	-3	0	1	0
o09	-2	-1	0	0	0	0	0	0	-1	-1	-2	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
o10	-2	0	0	0	0	-1	0	0	0	1	0	0	-1	0	0	0	-2	-1	0	1	0	0	0	0	0	-2	-1	0	1	1	-2	2	0	0	0

Source: own research

Table 4. Changes in the order of objects from Table 1 for individual indicators μ_1- μ_7 (using zero unitarization method)

	Indicator μ_1					Indicator μ_2					Indicator μ_3					Indicator μ_4					Indicator μ_5					Indicator μ_6					Indicator μ_7				
	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5
o01	1	0	0	2	-1	3	2	-3	0	0	3	1	0	2	-1	3	1	0	2	-1	3	1	0	0	-1	3	1	0	1	-1	3	1	0	2	-1
o02	-4	-2	0	0	1	-2	-4	-4	0	0	-6	-2	0	-1	2	-6	-2	0	0	2	-5	-2	0	0	2	-7	-3	0	-1	2	-6	-2	0	0	2
o03	-4	-2	-1	-1	1	-7	3	1	0	0	-7	-1	-1	0	1	-7	-1	-1	0	1	-8	1	1	0	1	-7	0	-1	0	1	-7	-1	-2	0	1
o04	5	4	-1	-2	0	0	3	0	1	1	2	3	-1	0	0	2	2	-1	0	0	1	3	-1	0	0	1	3	-1	0	0	2	2	-1	0	0
o05	-1	-1	-3	-1	0	0	-7	1	0	1	0	-4	-4	-1	0	0	-3	-4	0	0	0	-6	-3	0	1	0	-4	-3	0	0	-1	-2	-4	-1	0
o06	-4	0	0	0	0	-6	-2	0	1	1	-5	0	-1	0	0	-5	0	-1	1	0	-5	0	0	1	0	-4	0	-2	1	0	-5	0	0	0	0
o07	2	-1	1	0	0	3	3	2	0	-1	3	1	1	0	0	3	1	1	0	0	3	1	2	0	0	3	1	2	0	0	3	1	1	0	0
o08	1	2	2	2	0	4	4	0	-1	-1	5	3	3	0	-1	4	3	3	-1	-1	4	3	1	-1	-1	5	3	3	-1	-1	4	3	3	0	-1
o09	2	0	0	0	0	2	-4	2	-1	0	2	-2	0	-1	-1	3	-2	0	-2	-1	4	-2	-1	0	-1	3	-2	-1	0	-1	3	-2	0	-2	-1
o10	2	0	2	0	-1	3	2	1	0	-1	3	1	3	1	0	3	1	3	0	0	3	1	1	0	-1	3	1	3	0	0	4	0	3	1	0

Source: own research

SUMMARY

Most often the comparison of effectiveness of corporate units uses data for a single reporting period. It is performed by an automated reporting system (as part of a centralized management information system) which usually utilizes objects defined by (10) as models. This research (based on theoretical data) indicates that a better approach is to choose a dynamic model based on longer time input data (as defined by (11)). Such solution more accurately captures the dynamics of changes to the values of individual variables which constitute a synthetic measure. Our simulations confirm that regardless of the choice of our measures, the differences in rankings of examined objects can still be substantial. Our research needs further verification on empirical samples.

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