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TESTING FOR TRADING-DAY EFFECTS IN PRODUCTION IN INDUSTRY: A BAYESIAN APPROACH

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Abstract: The aim of this paper is to construct a parametric method in a Bayesian framework to identify trading-day frequency for monthly data. The well-known visual spectral test (implemented, for example, in X-12-ARIMA) is a popular tool in the literature. In the article's proposed method, the assumption concerning the almost periodicity of the mean function plays a central role. We use a set of frequencies that corresponds to the trading-day effect for monthly data. As an illustration, we examine this effect in production in industry in European economies for data adjusted by working days and for gross data.

Keywords: trading-day effect, production in industry, almost periodic function, AR model

INTRODUCTION

The trading-day effect (or calendar effect) in both monthly and quarterly macroeconomic time series is well known [Cleveland et al. 1980, Cleveland et al. 1982, Bell et al. 1983, Dagum et al. 1993, Bell et al. 2004, Soukup et al. 1999, Ladiray 2012]. The work of [Ladiray 2012] describes the present state of advances in this field.

The calendar effect is caused by different numbers of working days during months or quarters. For example, each February (in non-leap-years) has four weeks, which means that in the month we have four Mondays, Tuesdays, Wednesdays, etc. For other months, the number of days is not a multiple of 7, which means that the number of working days (from Monday to Friday) varies from month to month. This periodic phenomenon in numbers of working days in

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months and quarters can be a source of additional variability for macroeconomic time series called the "trading-day effect" or the "calendar effect".

Let f(t) denote the deterministic function defining the number of working days (or Mondays, Tuesdays, etc.) in month t. As the most popular Georgian calendar is periodic with periods equal to 400 years, the function f can be represented by a Fourier series with a known theoretical set of frequencies. However, the number of such frequencies is quite large. In [Ladiray 2012], the periodograms of the number of weekdays (from Monday to Friday) were evaluated using a theoretical path that covers 400 years. This theoretical set of frequencies contains two frequencies of 2.18733 and 2.71093 with dominating amplitude and many others with much lower amplitude. In practice, the real frequencies corresponding to the calendar effect must be estimated because the length of the sample is much shorter than that used in theoretical evaluation. Therefore, to address the estimation problem in this paper, we use the idea of almost periodic (*ap*) functions. An almost periodic function is a generalization of a well-known periodic case.

Note that for such macroeconomic time series as production in industry or GDP, the calendar adjustment is a preliminary step (beside seasonal adjustment) in real data analysis. The estimation of trading-day effects is possible using regression variables (see implementation in X-13-ARIMA-SEATS and other procedures).

The fundamental problem connected with calendar adjustment is a diagnostic to determine whether this effect is present in the data set. One of the popular methods for examining the trading-day effects is the so-called "visual test" (implemented for example in X-12-ARIMA). This method is based on a graphical observation of the usual periodogram (statistics). Note that a detailed interpretation of the periodogram depends on the assumptions. First, it is an inconsistent estimator of spectral density function (under zero mean assumption; see: Priestley 1981; Hamilton 1994), and second, any point from the interval $[0,2\pi)$ can be interpreted as the estimator of the magnitude of the Fourier coefficient (for the Fourier representation of an almost periodic mean function [Lenart et al. 2013a, Lenart et al. 2013b, Lenart 2013, Lenart 2015, and Lenart et al. 2016b].

In this paper, we consider the autoregressive model with an almost periodic mean function, introduced in [Lenart et al. 2016a]. Under standard prior distributions for the parameters, the posterior distribution for vectors of Fourier frequencies in Fourier representation of the mean function can be explicitly evaluated (by simple integration). Based on this distribution, we evaluate the mass of the probability concentrated around main calendar frequencies. The closer these masses are to one around some frequency, the stronger the data support this frequency. We find that, for a large majority of year over year (in short YOY) production in industry (monthly data) in the period 2001–2014, the frequency 2.18733 is predominant over other calendar frequencies in the case of gross data. The corresponding data adjusted by working days was also examined. We found that for some economies in these data sets, the frequency 2.18733 is still predominant. We use the datasets published by Eurostat.

METHODOLOGY

Almost periodic in mean time series - basics

The class of the almost periodic function on an integer line is well known in the literature [Corduneanu 1989]. Economic applications of almost periodic functions in the first or second moments of time series have been included in [Mazur et al. 2012, Lenart et al. 2013a, Lenart et al. 2013b, Lenart 2013, Lenart 2015 and others]. Generally, in the case of the Almost Periodically Correlated (APC) time series, the mean function and the autocovariance function have Fourier representation:

$$\mu(t) \sim \sum_{\psi\in\Psi} m(\psi)e^{i\psi t},\ B(t,\tau) \sim \sum_{\lambda\in\Lambda_\tau} a(\lambda,\tau)e^{i\lambda t}, \text{ for any } \tau \in Z,$$

where the Fourier coefficients $m(\psi)$ and $a(\lambda,\tau)$ are given by the limits:

$$m(\psi) = \lim_{n\to\infty}\frac{1}{n}\sum_{t=1}^{n}\mu(t)e^{-i\psi t}, \qquad a(\lambda,\tau) = \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}B(j,\tau)e^{-i\lambda j}$$

[Hurd 1989, Hurd 1991, Dehay et al. 1994].

The sets $\Psi = \{\psi \in [0,2\pi): m_X(\psi) \neq 0\}$ and $\Lambda_\tau = \{\lambda \in [0,2\pi): a(\lambda,\tau) \neq 0\}$ are countable. Note that, for any vector of frequencies $\boldsymbol{\psi} = (\psi_1,\psi_2,\dots,\psi_F) \in (0,\pi]^F$ there is a corresponding vector of Fourier coefficients $m = (m(\psi_2), m(\psi_2),\dots,m(\psi_F)) \in \mathbb{C}^F$. Note that $|m(\psi)| \neq 0 \Leftrightarrow \psi \in \Psi$, which means that statistical inference concerning the frequencies in the set Ψ can be based equivalently on statistical inference for $|m(\psi)|$.

"Visual test" for calendar frequencies

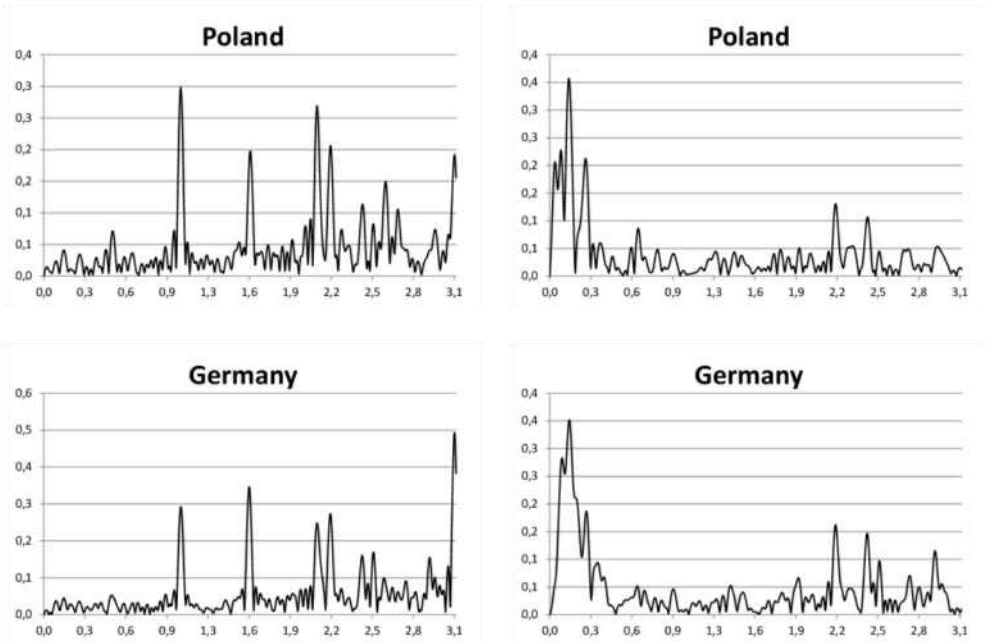
In a non-parametric approach, the natural estimator of the magnitude of Fourier coefficients $|m(\psi)|$based on sample $\{X_1,X_2,\dots,X_n\}$ has the following form

$$|\widehat{m}_n(\psi)| = |\tfrac{1}{n}\textstyle\sum_{j=1}^{n} X_j e^{-ij\psi}|$$

where $\psi \in [0,2\pi)$. As shown in [Lenart 2013], this estimator (after appropriate normalizing) is asymptotically normally distributed with known asymptotic variance-covariance matrix that depends on a spectral density function. Note that the statistic $|\widehat{m}_n(\psi)|$ is a usual periodogram function used in practical applications to examine the existence of calendar effects. The peak on the periodogram at frequency ψ_0 means the data support a periodic phenomenon connected with this frequency (in first or second moment). This simple tool is often used in applications. Let us consider an illustrative example of the properties of the periodogram.

We analyze the periodograms for percentage change over the previous period (MOM; gross data) and percentage change compared to the same period of the previous year (YOY; gross data) for production in industry (B-D: mining and quarrying; manufacturing; electricity, gas, steam and air conditioning supply) in Poland and Germany from Jan. 2000 to Dec. 2014. We use gross data in this example.

Figure 1. Estimate of the magnitude of Fourier transform (or periodogram): $|\widehat{m}_n(\psi)|$. The case of mining and quarrying; manufacturing; electricity, gas, steam and air conditioning supply (from Jan. 2000 to Dec. 2014) for MOM (on the left) and YOY (on the right).



Source: own preparation

In the case of MOM data, the frequency corresponding to seasonal fluctuations is clearly observed, whereas in the case of YOY, the mass on the periodograms mainly concentrates near the interval that corresponds to business cycle fluctuations. Note that for both cases, the mass is also concentrated near the frequency of approximately 2.19, which corresponds to predominant trading-day effect frequency.

Bayesian inference for the frequencies

Following [Lenart et al. 2016a], we consider a usual autoregressive model of order p:

$$L(B)(y_t - \mu(t)) = \varepsilon_t,$$

where $L(B) = 1 - \eta_1 B - \eta_2 B^2 - \cdots - \eta_p B^p$ is a lag polynomial,

$$\varepsilon_t \sim N(0, \tau^{-1}),$$

$$\mu(t) = \delta_0 + \sum_{f=1}^{F} \left[a_f \sin(t\psi_f) + b_f \cos(t\psi_f)\right].$$

For the parameters, we assume: $\delta_0 \in \mathbb{R}$, $\mathbf{a} = (a_1, a_2, \dots, a_F) \in \mathbb{R}^F$, $\mathbf{b} = (b_1, b_2, \dots, b_F) \in \mathbb{R}^F$, and $\boldsymbol{\psi} = (\psi_1, \psi_2, \dots, \psi_F) \in (0, \pi]^F$. We assume the following prior structure:

$$p(\mathbf{a}, \mathbf{b}, \tau, \boldsymbol{\psi}) = p(\mathbf{a}, \mathbf{b}, \tau) p(\boldsymbol{\varphi}) = p(\mathbf{a}, \mathbf{b} | \tau) p(\tau) p(\boldsymbol{\varphi}),$$

with uniform distribution on $(0, \pi]^F$ for frequency vector $\boldsymbol{\psi}$, $(\mathbf{a}, \mathbf{b}) | \, \tau \sim N(\mathbf{0}, (\tau\mathbf{B})^{-1})$ and $\tau \sim G\left(\frac{n_0}{2}, \frac{s_0}{2}\right)$, where $N(\mathbf{0}, (\tau\mathbf{B})^{-1})$ denotes the Normal distribution (with hyperparameter $\mathbf{B}$) and $G\left(\frac{n_0}{2}, \frac{s_0}{2}\right)$ denotes the Gamma distribution with hyperparameters s_0 and n_0. Under such standard prior distribution we obtain the following form of the posterior distribution for frequency vector $\boldsymbol{\psi}$:

$$p(\boldsymbol{\psi} | \mathbf{y}) \propto (\det(\mathbf{X}'\mathbf{X} + \mathbf{B}))^{-1/2} (\mathbf{y}'[\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X} + \mathbf{B})^{-1}\mathbf{X}']\mathbf{y} + s_0)^{-\frac{n+n_0}{2}},$$

where $\mathbf{y}$ is a vector of observations and $\mathbf{X}$ is a matrix that depends on vector $\boldsymbol{\psi}$ (see details in [Lenart et al. 2016a]).

Posterior distribution in examination of calendar frequency

Using the above posterior distribution, the mass concentration for frequencies can be examined (for different orders of an autoregressive part). Note that under assumption $\boldsymbol{\psi} \in (0, \pi]^F$, the data may strongly support the frequencies that correspond to business cycle fluctuations (see illustrative periodograms in previous section). Therefore, we restrict the support for frequencies by considering only the set $(\frac{2\pi}{T1.5}, \pi]^F$ (where T is a number of observation during the year), which excludes the fluctuations that correspond to fluctuations longer than 1.5 years. Summing up, we consider the mass location for the distribution related to the following kernel:

$$p(\boldsymbol{\psi}|\mathbf{y})1\{\boldsymbol{\psi} \in (2\pi/(T1.5), \pi]^F\}$$

where $1\{A\}$ is the indicator of the event A. Note that under this restriction, the only autoregressive part (in considered model) is allowed to model fluctuations identified with business fluctuations and other fluctuations with longer period.

In this paper, we propose to observe the mass concentration in the posterior distribution around the dominant calendar frequency, approximately 2.18733. Now we assume that $F = 1$, which means that we consider only one frequency (in theoretical specification). The more general case $F > 1$ can also be taken into consideration.

We propose the following methodology and interpretation strategy. For a fixed p (order of autoregressive part), we calculate the posterior probability on the interval (ball)

$$S_\gamma = [2.18733 - \gamma; 2.18733 + \gamma].$$

The mass concentration on this interval means that the data support the existence of trading-day effects related to this frequency. If this probability is comparative with fraction $2\gamma/(\pi - \frac{2\pi}{T1.5})$ it means that the data does not strongly support fluctuations connected with this frequency. In such a case (for gross data), another frequency can be supported more strongly.

EMPIRICAL ANALYSIS

Data description and existing empirical results

We consider production in industry (*mining and quarrying; manufacturing; electricity, gas, steam and air conditioning supply*, percentage change compared to the same period of previous year, YOY) for thirty European countries (Belgium, Bulgaria, Czech Republic, Denmark, Germany (until 1990 former territory of the FRG), Estonia, Greece, Spain, France, Croatia, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Malta, Netherlands, Austria, Poland, Portugal, Romania, Slovenia, Slovakia, Finland, Sweden, United Kingdom, Norway, Former Yugoslav Republic of Macedonia, and Serbia) from Jan. 2001 to Dec. 2014. In this case $T = 12$. The same data set was analyzed in [Lenart's 2015] paper using different methodologies, and we formulate the main thesis concerning calendar frequencies based on this paper. As a first step, the usual periodogram (related to "visual test") for both the gross data and the data adjusted by working day was examined on the interval $(0, \pi]$. The majority of the periodograms clearly show peaks near the frequency 2.18733 for gross data. In addition, based on the nonparametric test used in [Lenart 2015], the nonexistence of this frequency in the true set of frequencies was rejected (at significance level 5%) for most of the data sets. This frequency was also estimated using the contraction method (CM) proposed by [Li et al. 2002]. Note that for data adjusted by working days, the mass concentration around the frequency of approximately 2.19 was not observed in the periodograms.

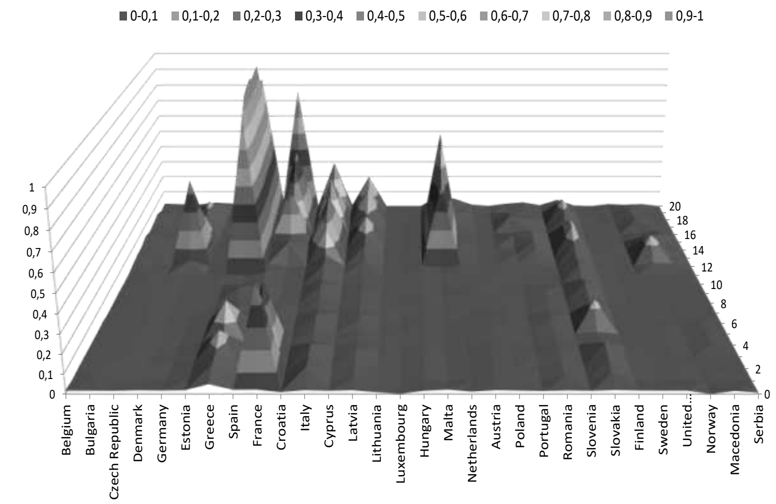
Results obtained in the proposed Bayesian approach

We take $S_\gamma = [2.18733 - \gamma; 2.18733 + \gamma]$ with $\gamma = 0.005$, such that the interval S_γ covers approximately $\frac{1}{50}$ of the length of whole support $\left(\frac{2\pi}{12*1.5}, \pi\right]$. We take $\gamma = 0.005$ by simple observation (graphical) of the posterior mass concentration. For each $p = 0,1,2, \dots 20$ (the order of the autoregressive model), the posterior probability of the set S_γ was individually calculated. Table 1 contains this posterior probability for the gross data, while Table 2 (see also Figure 2) shows the posterior probability for the data adjusted by working days.

The results concerning the gross data confirm that the frequency 2.18733 is predominant in the trading-day effect in the analyzed set of data. Only in the case of Lithuania (maximum posteriori probability on the set S_γ equals 0.045), Malta (maximum posteriori probability on the set S_γ equals 0.05) and Macedonia (maximum posteriori probability on the set S_γ equals 0.054) this frequency was not supported by the data. For other countries, this maximum probability exceeds levels of 0.3 (the lowest probability for Slovakia); 0.8 (the lowest probability for Luxemburg); and 0.9 for other countries.

In the case of the data adjusted by working-days, the results show that in a few cases the frequency 2.18733 is still supported by the data for some orders p. In the cases of Greece, France and Malta, calculated posterior probability exceeds 0.5 at least one time (see Table 2 or the same results on Figure 2). Hence, it can be concluded that, for these countries, the data strongly support the frequency 2.18733. However, the amplitude connected with this frequency is not high enough to say that there is a problem with the adjustment procedure. It means only that on the interval $\left(\frac{2\pi}{12*1.5}, \pi\right]$, the frequency 2.18733 is still strongly supported by the data.

Figure 2. Posterior probability on the set $S_{0.005}$ for considered countries (vertical axis) and different AR orders (horizontal axis)



Source: own preparation

SUMMARY

This paper proposed a new method for detecting calendar frequencies. The method is based on a Bayesian framework and an autoregressive model with an almost periodic mean function and an unknown set of frequencies. The mass location for posterior distribution for the frequency is analyzed in the proposed methodology for a single frequency. However, this methodology can be easily generalized to any set of frequencies (for example: two or more calendar frequencies simultaneously), and this is a topic of the author's future research.

Empirical analysis shows that this tool can clearly detect predominant calendar frequency (2.18733) in gross data for production in industry. In the case of data adjusted by working-days, this method also detects the same predominant frequency (2.18733) in a few cases, but the amplitude of this fluctuation was not analyzed in detail. This is a second point of the author's intensive research.

Table 1. Posterior probability of the set $S_{0.005}$ (in columns order p)

Order p →	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Belgium	0.72	1.00	1.00	0.96	0.43	0.47	0.99	0.66	0.07	0.13	0.14	0.25	1.00	1.00	0.99	0.99	0.69	0.51	0.47	0.02	0.13
Bulgaria	0.04	0.94	0.86	0.61	0.16	0.15	0.24	0.18	0.38	0.54	0.17	0.29	0.45	0.94	0.96	0.98	0.44	0.61	0.61	0.31	0.37
Czech Republic	0.42	1.00	1.00	1.00	0.82	0.36	0.98	0.87	0.84	0.91	0.36	0.38	0.80	0.99	0.98	1.00	0.83	0.81	0.99	0.73	0.77
Denmark	0.87	1.00	1.00	0.31	0.05	0.57	0.84	0.66	0.46	0.65	0.33	0.43	1.00	1.00	1.00	0.79	0.23	0.26	0.04	0.03	0.03
Germany	0.31	0.70	1.00	0.98	0.54	0.16	0.64	0.83	0.61	0.76	0.28	0.30	1.00	1.00	1.00	1.00	0.85	0.90	0.92	0.57	0.18
Estonia	0.06	0.99	0.98	0.97	0.25	0.07	0.72	0.15	0.01	0.02	0.02	0.05	0.71	0.56	0.66	0.62	0.20	0.25	0.81	0.15	0.17
Greece	0.06	0.01	0.28	0.04	0.04	0.33	0.09	0.13	0.15	0.04	0.01	0.00	0.94	0.99	0.99	0.99	0.93	0.91	0.91	0.84	0.74
Spain	0.71	1.00	1.00	0.70	0.63	0.28	0.84	0.77	0.44	0.47	0.18	0.17	0.87	0.78	0.54	0.46	0.51	0.43	0.33	0.21	0.21
France	0.91	1.00	1.00	1.00	0.95	0.65	1.00	1.00	0.99	0.99	0.91	0.96	1.00	1.00	1.00	1.00	0.98	0.98	0.98	0.61	0.72
Croatia	0.12	0.00	0.43	0.09	0.05	0.84	0.86	0.83	0.83	0.26	0.02	0.04	0.57	0.99	0.87	0.75	0.69	0.25	0.18	0.02	0.01
Italy	0.62	1.00	1.00	1.00	0.96	0.80	1.00	0.99	0.99	0.99	0.91	0.87	0.94	0.98	0.97	0.99	0.69	0.36	0.27	0.04	0.13
Cyprus	0.07	0.90	0.81	0.16	0.11	0.02	0.12	0.16	0.11	0.08	0.03	0.44	0.94	1.00	0.99	0.99	0.98	0.95	0.98	0.96	0.97
Latvia	0.07	0.98	0.98	0.88	0.19	0.18	0.59	0.13	0.06	0.05	0.06	0.06	0.30	0.69	0.76	0.83	0.52	0.40	0.89	0.54	0.50
Lithuania	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.04	0.05	0.03
Luxembourg	0.01	0.75	0.80	0.47	0.10	0.12	0.13	0.08	0.08	0.08	0.08	0.12	0.10	0.86	0.53	0.32	0.01	0.00	0.06	0.01	0.01
Hungary	0.10	1.00	0.99	0.99	0.81	0.72	0.96	0.32	0.40	0.56	0.14	0.32	0.67	0.94	0.88	0.85	0.22	0.23	0.17	0.01	0.06
Malta	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.04	0.05	0.01	0.01	0.01	0.01	0.01
Netherlands	0.51	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.68	0.59	1.00	1.00	1.00	1.00	0.97	0.97	0.99	0.92	0.93
Austria	0.97	1.00	1.00	1.00	0.99	0.98	1.00	0.98	0.89	0.96	0.07	0.31	0.96	0.99	0.99	0.97	0.72	0.61	0.76	0.18	0.40
Poland	0.24	0.90	0.97	0.85	0.35	0.22	0.59	0.19	0.03	0.12	0.06	0.10	0.84	0.52	0.57	0.53	0.21	0.33	0.33	0.04	0.09
Portugal	0.64	0.99	1.00	0.99	0.95	0.98	0.98	0.97	0.97	0.92	0.25	0.20	0.97	0.99	0.99	1.00	0.95	0.95	0.95	0.48	0.94
Romania	0.28	1.00	1.00	0.95	0.51	0.34	0.91	0.16	0.13	0.12	0.05	0.15	0.65	0.86	0.84	0.77	0.04	0.05	0.03	0.01	0.02
Slovenia	0.33	1.00	1.00	0.99	0.42	0.51	0.91	0.01	0.02	0.05	0.04	0.07	0.04	0.94	0.93	0.94	0.46	0.45	0.37	0.17	0.35
Slovakia	0.02	0.07	0.10	0.12	0.02	0.02	0.23	0.07	0.08	0.12	0.05	0.11	0.10	0.19	0.31	0.29	0.08	0.15	0.22	0.05	0.05
Finland	0.13	1.00	1.00	0.98	0.48	0.57	0.98	0.84	0.81	0.75	0.78	0.93	0.96	0.98	0.87	0.50	0.30	0.23	0.51	0.22	0.36
Sweden	0.25	1.00	1.00	1.00	0.79	0.45	0.79	0.72	0.53	0.67	0.26	0.47	0.86	1.00	0.98	0.96	0.71	0.54	0.50	0.25	0.25
United Kingdom	1.00	1.00	1.00	1.00	0.99	0.91	0.99	0.94	0.75	0.73	0.01	0.00	0.98	0.93	1.00	0.84	0.99	0.96	1.00	0.83	0.66
Norway	0.01	0.07	0.05	0.02	0.01	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.93	0.99	0.97	0.46	0.26	0.63	0.71	0.79	0.66
Macedonia	0.02	0.00	0.00	0.00	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.04	0.05	0.03	0.02	0.01	0.01	0.01
Serbia	0.01	0.08	0.07	0.04	0.04	0.05	0.05	0.03	0.03	0.01	0.00	0.00	0.01	0.80	0.91	0.90	0.94	0.93	0.89	0.81	0.93

Source: own calculations

Table 2. Posterior probability of the set $S_{0.005}$ (in columns order p)

Order $p \rightarrow$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Belgium	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.02	0.02	0.04	0.02	0.03	0.04	0.01	0.02
Bulgaria	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01
Czech Republic	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01
Denmark	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.47	0.32	0.15	0.07	0.08	0.12	0.06	0.02
Germany	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
Estonia	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.00	0.00	0.00	0.01	0.02	0.02	0.02	0.02	0.01	0.02	0.02	0.02	0.02
Greece	0.05	0.09	0.09	0.14	0.06	0.19	0.04	0.03	0.02	0.04	0.00	0.00	0.86	0.97	0.99	0.99	0.97	0.98	0.98	0.86	0.84
Spain	0.02	0.01	0.02	0.02	0.02	0.01	0.02	0.01	0.02	0.03	0.02	0.02	0.02	0.04	0.07	0.06	0.03	0.04	0.03	0.02	0.02
France	0.02	0.50	0.29	0.40	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.10	0.09	0.31	0.18	0.38	0.45	0.15	0.76
Croatia	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.06	0.02	0.00	0.00	0.00	0.00	0.00
Italy	0.02	0.08	0.06	0.07	0.02	0.04	0.05	0.03	0.03	0.05	0.04	0.07	0.07	0.18	0.26	0.29	0.14	0.31	0.22	0.09	0.29
Cyprus	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.01	0.02	0.04	0.02	0.01
Latvia	0.02	0.03	0.03	0.03	0.02	0.02	0.03	0.02	0.02	0.02	0.01	0.01	0.04	0.08	0.10	0.16	0.07	0.11	0.20	0.12	0.20
Lithuania	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Luxembourg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hungary	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Malta	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.50	0.67	0.11	0.12	0.12	0.05	0.07
Netherlands	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.01	0.02	0.03	0.03	0.03	0.02	0.04	0.06	0.05	0.05
Austria	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Poland	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.00	0.00	0.00
Portugal	0.02	0.00	0.01	0.02	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.01	0.05	0.01	0.07	0.05	0.00	0.02
Romania	0.02	0.03	0.03	0.03	0.02	0.02	0.03	0.01	0.02	0.02	0.00	0.00	0.01	0.05	0.06	0.08	0.02	0.06	0.02	0.01	0.03
Slovenia	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Slovakia	0.02	0.05	0.06	0.08	0.02	0.01	0.13	0.04	0.05	0.08	0.04	0.07	0.07	0.09	0.15	0.15	0.06	0.08	0.14	0.04	0.04
Finland	0.02	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Sweden	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
United Kingdom	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01
Norway	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.14	0.09	0.03	0.01	0.05	0.04	0.04	0.01
Macedonia	0.02	0.00	0.00	0.00	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.04	0.05	0.03	0.02	0.01	0.01	0.01
Serbia	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00

Source: own calculations

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