QUANTITATIVE METHODS IN ECONOMICS
Volume XIX, No. 1, 2018, pp. 38 – 47

Received: 25.01.2018
Accepted: 05.04.2018

SIMULATION RESEARCH IN THE PROCESS OF DETERMINATION OF STOCK PRICES USING A MODIFIED GORDON GROWTH MODEL

Marcin Halicki (ORCID: 0000-0002-5343-0093)
Faculty of Biology and Agriculture, University of Rzeszow, Poland
e-mail: mhalicki@ur.edu.pl
Tadeusz Kwater (ORCID: 0000-0001-5095-613X)
Faculty of Mathematics and Natural Sciences, University of Rzeszow, Poland
e-mail: tkwater@univ.rzeszow.pl

Abstract: In the latest literature there is a lack of broader discussion of the Gordon Model, with a desire to point out that its assumptions do not fit the expectations of modern investors and the nature of capital markets. The issue prompts us to ask whether this model can still be used by modern scientists and investors. The aim of the article is to present a classic Gordon model and show the direction of its modification. An attempt was also made to use simulation studies in a graphical interpretation of a selected share, which incorporates the Gordon's modified and classic models.

Keywords: risk, stock, investments, model, simulations

JEL classification: G11, G17

INTRODUCTION

The Gordon Growth Model is one of the most well known in the finance area, which, alongside Markowitz's theory, has undoubtedly contributed to the development of the field of investment management and portfolios of financial instruments. Nevertheless, one must not forget that capital markets, by their very nature, undergo constant transformation. They are facing new challenges, which are mainly attributable to the pressure of customers, other participants and regulators [PWC - Capital Markets 2020]. They form a tight corset that limits and encourages, among other things, the search for new or improved model solutions. And these seem to be desirable from the perspective of developing capital markets,

DOI: 10.22630/MIBE.2018.19.1.4

which are important mainly because they are an important part of any mature economy [Ouandlous 2010], positively influencing the functioning of its mechanisms. This corset of high demands does not allow for the fact that it will be loosened, so it has become an inspiration to consider the Gordon model. In addition, it was found on the basis of literature research that there was a lack of broader discussion of the model, and that brought the desire to point out that its assumptions did not fit the expectations of modern investors and the nature of capital markets. All this prompts us to ask whether this model can still be used by modern scientists and investors.

The purpose of this article is to present the Gordon model, characterizing its most important features and disadvantages, and showing the direction of its exemplary modification to fit the needs of both today's capital markets as well as investors and theorists. The study also attempted to use simulation studies in the process of graphical interpretation for theoretical stock based on selected models. For the purposes of research, the predicted values of the various parameters are assumed for specific probabilities for obtaining the price, which was calculated using the classical and modified Gordon model, taking the form different formulas. Literary research and scientific reflection were used in the discussion. It has been shown that the Gordon model continues to be an attractive research base that can be used to create a modified model that will be able to meet the expectations of modern economists.

GENERAL CHARACTERISTICS OF THE GORDON MODEL

The natural consequence of the development of capital markets and the development of capitalist states, was the development of numerous financial models in the 1950s and 1960s. One of the most important of these is the Gordon Growth Model [Gordon & Shapiro 1956], but the problem lies in the fact that the model seems to have been forgotten by researchers. This conclusion can be deduced from the literature. The starting point for the Gordon Growth Model characterisation is a approach of Williams [Williams 1938], who first introduced the classic stock valuation model, known as the dividend discount model (DDM). It assumed that the price of a company's shares depended on the sum of future dividends paid to shareholders whose value should be adjusted to determine their current value:

$$\mathrm{P_t} = \mathbb{E}_{\boldsymbol{t}}[\textstyle\sum_{\mathrm{n=1}}^{\infty} \frac{\mathrm{D_{t+n}}}{(1+\mathrm{R_{t+n}})^{\mathrm{n}}}], \tag{1}$$

where $\mathrm{P_t}$ is the stock price at period t, $\mathbb{E}_{\boldsymbol{t}}[\,]$ - expected value, $\mathrm{D_t}$ is the value of future dividend paid at time t, and $\mathrm{R_t}$ is the rate used to discount cash flows or the rate of return required by investors. The Williams approach has contributed significantly to the development of fundamental analysis of listed companies and

has become the basis for the development of a stock valuation model that ultimately adopts the following form:

$$P_0 = \frac{D_1}{(R-g)} = \frac{D_0(1+g)}{(R-g)}, \tag{2}$$

where P_0 is the price of the stock in the current period, D_1 is an expected dividends of one year from now, and g is the annual dividend growth rate, therefore $D_1 = D_0(1+g)$, but it should be assumed that $R > g$ and stock company will pay out dividends. Thus, the calculated share price in the current period takes into account continuous and uninterrupted growth, while the equation (2), which is presented above and which allows the stock to be valued in the current period, is known as the Gordon Growth Model and nowadays it seems to be losing popularity, so it is worth considering the way it evolved. In the first place, it is worth answering why the Gordon Growth Model is less and less used in research around the capital markets and around portfolio management. This is due to its main assumptions, which are considered unrealistic [Investopedia.com]. These assumptions are following:

- Sensitivity to the required rate of return and to the increase in the value of the dividend, which can consequently lead to very extreme stock valuations, making the analyzed model unsuitable,
- The rate of dividend growth must always be the same, irrespective of the company's financial situation and stock market conditions, and may not exceed the required rate of return.

Considering the above, one should also pay close attention to the clash of economic realities and assumptions on which the Gordon Growth Model is based. There is no need to further explain that they cannot be fulfilled in a real world that is variable and unpredictable. Moreover, the sensitivity of the stock price, calculated on the basis of this model to the variability of its variables, is extremely high, which is why economists have attempted to modify it to eliminate certain elements of the model or to add new, which are beneficial for equity investments.

MODIFICATION OF THE GORDON GROWTH MODEL

The Gordon model should be modified mainly for two reasons. First and foremost, the stock price calculated on its classical form is very sensitive to changes in the value of parameters that may change during the course of the investment, thereby reducing the theoretical and calculated price. Secondly, other and equally important parameters (such as book value of a company) are not included in such price calculations. The modification of the Gordon's model in literature [Chapados 2011] may be based on the classic RIM-Model (residual income valuation model), which takes into account the relationship between the book value and the net earnings per share and dividends per share:

$$B_t = B_{t-1} + E_t - D_t, \tag{3}$$

where B_t is the book value per share at time t, E_t is the earnings per share in the period t. Then, taking into account the Edwards' approach, Bella and Ohlson [Edwards & Bell 1961, Ohlson 1995], the relationship between shareholder payments and the so-called "abnormal" earnings can be determined, but including the interest rate in the form of their model, which is used to discount the cash flows. The model described here assumes the following form:

$$E_t^{\propto} \triangleq E_t - R * B_{t-1}, \tag{4}$$

where $E_t^{\propto}$ determines "abnormal" earnings, B_{t-1} is the book value per share at time $t-1$, R is also a discount factor. By analyzing the aforementioned shots, one can build an equation allowing to calculate a dividend per share at time, but at a given book value per share and "abnormal" earnings, because:

$$D_t = E_t^{\propto} - B_t + (1 + R) * B_{t-1}, \tag{5}$$

This dividend calculation allows us to develop two equations that represent the modified Gordon model, which does not include dividends per share, or g, which most changes the price of shares in the classic model. Thus, the price of shares under the new approach can be calculated in two ways:

$$P_t = B_t + \mathbb{E}_t\left[\sum_{\tau=1}^{\infty} \frac{E_{t+\tau}^{\propto}}{(1+R)^{\tau}}\right], \tag{6}$$

and

$$P_t = B_t + \mathbb{E}_t\left[\sum_{\tau=1}^{\infty} \frac{E_{t+\tau} - R * B_{t+\tau-1}}{(1+R)^{\tau}}\right], \tag{7}$$

The modification of the Gordon's model is that it does not take into account the values g, so the price of the stock is not dependent on them. In contrast, it is sensitive to the change of the book value per share and the change of the "abnormal" earnings or of the earnings per share. The presented R is similar in role to the classical model, although the book value per share may lose its share of the stock price at low R values. Graphical interpretation of the presented formulas is unprecedented in the literature and may therefore be desirable for the use in the process of stock assessment and portfolio construction.

SIMULATION RESEARCH OF STOCK PRICE DETERMINATION USING THE CLASSIC AND MODIFIED GORDON MODEL

The MATLAB software was used for the simulation study and was analyzed for graphical analysis. They were divided into three parts to highlight the differences in the formulas analyzed. The first one was based on the classic Gordon model (2), the second on the modified model with the "abnormal" earnings (6), and the third model on the modified but without "abnormal" earnings (7). In-depth analysis of formulas indicates that, in order to obtain the desired dependency of the

parameters, their graphical interpretation requires a series of values. The simulation study is therefore based on a theoretical value of the stock, with its parameters matched to the three variants corresponding to the models being analyzed. Table 1 presents the value of the parameters for calculating this share value for each of the analyzed models, and it is assumed that the purpose is to calculate stock prices in the current period (P_0).

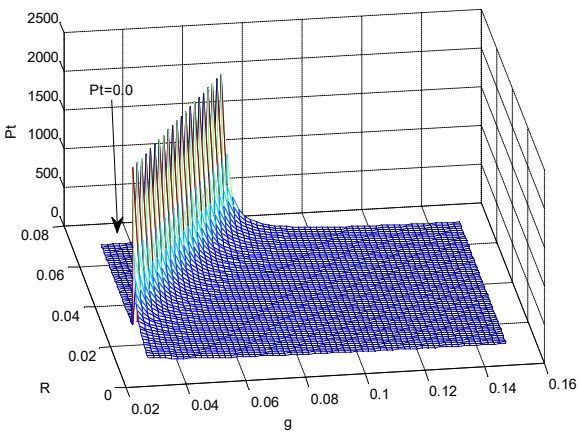
Table 1. Values of sample stock price simulations

Parameter	*Value (or changes)*
Classic model - equation (2)	
R (changes with step 0.002)	(0.03; 0.15)
g (changes with step 0.001)	(0.07; 0.01)
D_t $(t = 1)$	2.0
Figure number	1; 2
Modified model – equation (6)	
$E_t^{\propto}$ (changes with step 0.0048)	(3.0; 5.4)
R (changes with step 0.00024)	(0.03; 0.15)
B_t	3.0
τ	500
Probability (changes with step 0.002)	(0.8; 1)
Figure number	3; 4; 5
Modified model - equation (7)	
$E_{t+\tau}$ (changes with step 0.0048)	(3.0;5.4)
R (changes with step 0.00024)	(0.03;0.15)
B_t (changes with step 0.06)	(3.0;9.0)
τ	100 and 500
Probability (changes with step 0.002)	(0.8;1)
Figure number	6; 7

Source: own calculations

One can mention that for modified models, a value of $\tau = 500$ was chosen, because the smaller value of this indicator changes the stock valuation, and above this, the stock price is independent of τ. In addition, the most important distinguishing features of the presented formulas were expressed by two-dimensional and three-dimensional charts. Only the first numerical experiment for stocks, based on the classical model (2), is shown in Figures 1 and 2. The rest concerns the modified models.

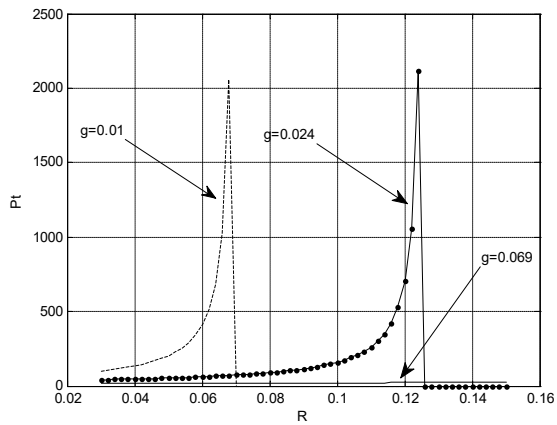
Figure 1. A three-dimensional graph of the relationship between the parameters R and g for the classical model (2)



Source: own study

Figure 1 provides that in certain cases it was assumed that for $R < g$ this price was "0". To enrich the presentation, this solution was highlighted not only on the presented three-dimensional graph, but also on the next two-dimensional graph (Figure 2). It presents the fragments of the solutions of Figure 1 for the selected values of the parameter g.

Figure 2. Two-dimensional graph of the relationship between the parameters R and P_t for the classical model (2) with the three values g

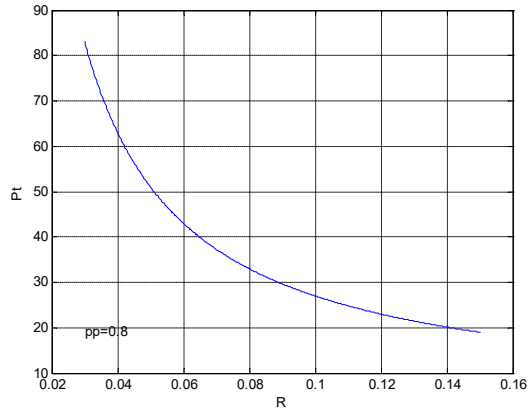


Source: own study

Under certain values g, the price of shares, is more sensitive and therefore change R can very quickly raise the price. It is easy enough to see that some g

values (e.g. Figure 2 with g = 0.069) cause low sensitivity of stock prices to changes in R.

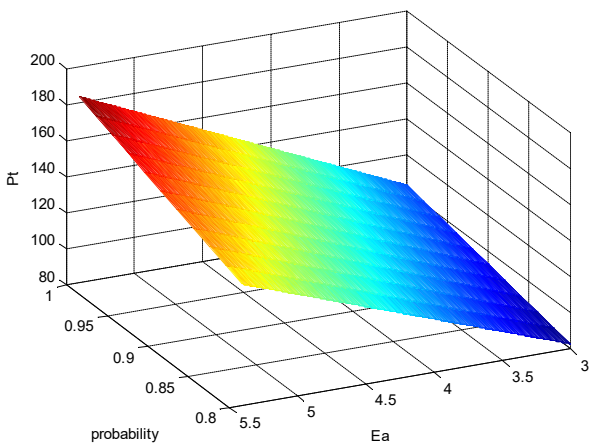
Figure 3. Relationships between R and P_t parameters for the modified model (6) at probability of 0.8 and variable abnormal earnings



Source: own study

Figure 3 shows that the increase in the required rate of return on stock, lowers its price. In Figure 4 the abnormal earnings is denoted as Ea.

Figure 4. Three-dimensional graph of the relationship between the parameters P_t and Ea for the modified model (6) with increasing probability and R of 3%

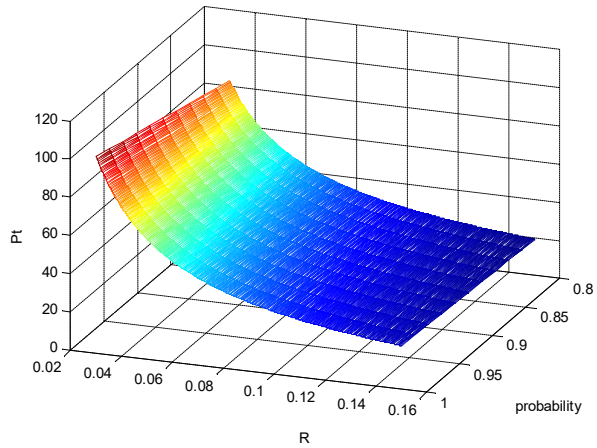


Source: own study

By analyzing the above chart, it can be easily concluded that the probability of large abnormal earnings in companies significantly influences the stock price,

but if the relatively high level of abnormal earnings is comparatively unlikely, then still the stock price is high.

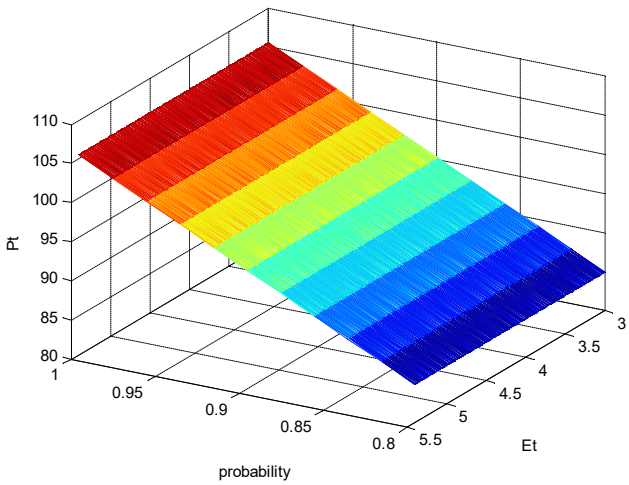
Figure 5. Relationship of P_t to R and probability for the modified model (6) at constant abnormal earnings of 3.0



Source: own study

The graph in Figure 5 confirms that the high required rate of return from stock, significantly decreases its price, even if the probability of abnormal earnings is high.

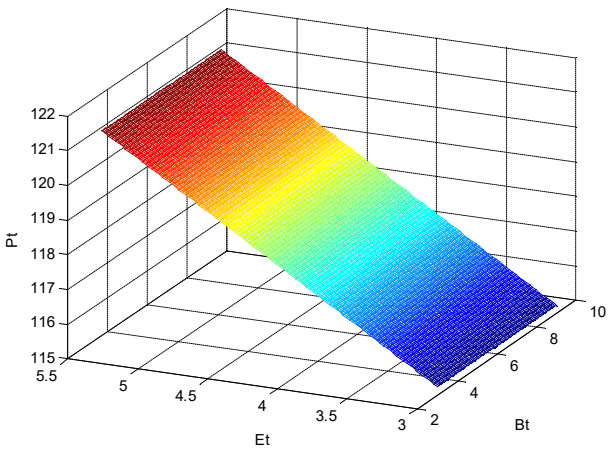
Figure 6. Price dependence of P_t on E_t and probability for the modified model (7) with a constant R of 3% and B_t of 3.0



Source: own study

The Figure 6 shows that the increase in earning per share at low probability, at constant R, does not raise the stock price to the extent that the probability increases. For any investor it means that, in the modified Gordon model, low returns but achievable are more important than high but unlikely.

Figure 7. Relationships between the parameters P_t, E_t and B_t for the modified model (7) with a constant R of 3% and a constant probability of 0.8



Source: own study

Figure 7 shows the effect of book value on stock price at different earning per share levels. Such appreciation can be considered cognitively interesting because the modified model (7) is the only one shown, which in its formula contains the book value multiplied in each period by the rate of return. Figure 7 reflects that with a given book value level, the stock price rises with the increase in earning per share. Conversely, the reverse relationship is different, because large book value changes, at constant earning per share, do not affect the price of the stock.

SUMMARY

To present the essence of the paper models, numerical experiments were conducted and the graphs were made. On the basis of the research, the following general conclusions were made. Firstly, the Gordon model in classical terms (2) should be treated as a theoretical tool that should be carefully used in practice, for at least the reason that, it allows analysis of shares only if R is higher than g and in the defined area the value of the g (g comparable to R) price of the stock price is very sensitive to the change in R. Secondly, the modified model (6) is an interesting analytical alternative to the classical solution, primarily because it does

not take into account the g parameter but it is based on the abnormal earnings level, which most significantly influences the stock price, even with the probability change. Thirdly, the modified model in the second (7) spin also represents an alternative to the classical solution, since it is not based on the g parameter but primarily on the earning per share and book value. It has been shown that the share price is very sensitive to a given book value level with the simultaneous earning per share changes.

The final conclusions cannot be overlooked that the R parameter is also a factor influencing the price of shares in each of the models presented, but it is interesting that the modified model in each figure does not take into account either the parameter g or the level of dividends, making it an instrument of interest. for investors. But do not forget that recent empirical studies have shown [Khanal & Mishra 2017] that stock prices are rising in dividends announcements, so the classic Gordon model seems to be based on robust parameters.

REFERENCES

Chapados N. (2011) Portfolio Choice Problems - An Introductory Survey of Single and Multiperiod Models. Springer-Verlag New York, 27-29.

Edwards E. O., Bell P. W. (1961) The Theory and Measurement of Business Income. University of California Press, Berkeley, Los Angeles, London, 7 th edition.

Gordon M. J., Shapiro E. (1956) Capital Equipment Analysis: The Required Rate of Profit. Management Science, 3 (1), 102-110.

Investopedia.com (http://www.investopedia.com/ask/answers/032415/why-gordon-growth-model-not-more-widely-used.asp) [Access: 29 June 2017].

Khanal A. R., Mishra A. K. (2017) Stock Price Reactions to Stock Dividend Announcements: A Case from a Sluggish Economic Period. The North American Journal of Economics and Finance, 42, 338-345.

Ohlson J. A. (1995) Earnings, Book Values, and Dividends in Equity Valuation. Contemporary Accounting Research, 11(2), 661-687.

Ouandlous A. (2010) Capital Markets And Economic Development: A Framework For Newly Liberalized Economies. Journal of Business & Economics Research, 8(6), 9-16.

PWC-Capital Markets 2020, 5 (https://www.pwc.com/sg/en/banking-capital-markets/assets/capital-markets-2020.pdf) [Access: 29 June 2017].

Williams J. B. (1938) Theory of Investment Value. Harvard University Press, Cambridge, MA.