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FX-LINKED STRUCTURED TIME DEPOSITS VERSUS BARRIER AND STANDARD OPTIONS: A COMPARATIVE STUDY

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Abstract: The paper provides a short description of barrier options together with an analysis of their performance compared to the performance of standard options and structured time deposits that incorporate the element of barrier in their construction. The results obtained show that some of considered structured time deposits linked to the foreign exchange rates and standard options could bring some profits unlike the majority of coressponding barrier options. The disadavantage of barrier options is they can stay inactive or a "spike" in the underlying asset price can cause the option to be knocked-out.

Keywords: structured time deposits, barrier options, foreign exchange rates

JEL classification: G10, G13

INTRODUCTION

Options are popular instruments widely used for hedging in the commodity and financial markets. They give their holders the right to receive certain cash payoffs under certain conditions. For this privilage the holders pay premiums to the writers of the options. Traditional options (sometimes called vanilla options) have been traded for hundreds of years. The earliest recorded account of options can be traced back to the ancient Greek philosopher and mathematician – Tales. During winter, when there was little demand, he negotiated for the use of olivepresses for the following spring. The demand was contingent on having great harvest [Ong 1996]. In the first half of the 17th century, options were intensively used in Holland during the tulip bulb craze called also tulipomania (Dash [1999] presents an interesting study of this phenomenon). In the United States, options first appeared in the 1790s. Much newer innovations are non-standard options. For example, down-

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and-out call options, members of the barrier options family, have been sporadically available in the U.S. over-the-counter (OTC) market since 1960s. Initially, these non-standard instruments were called "boutique" or "designer" options, however since the publication of the working paper "Exotic options" by Mark Rubinstein and Eric Reiner [Rubinstein, Reiner 1992], they are commonly called exotic options.

The range of exotic options being offered in the market is very wide, but the interest in the options centres mainly on barrierr options, average rate options, basket options, digital options, and rainbow options. Their applicability may be found in the commodity, foreign exchange, equity, and interest rate markets. In addition, some elements of exotic options are used in construction of structured time deposits. These instruments are regularly offered to individual investors by numerous banks in Poland. The profit fom the structured time deposit is conditional and depends upon the performance of some underlying asset (equity, currency, commodity) during the investment period. The most often these deposits incorporate barriers that are the elements of barrier options and they are usually linked to the foreign exchange (FX) rates. Thus the aim of the paper is to present the description of barrier options, methods for their pricing and analysis of their performance in relation to the performance of structured time deposits with barrier mechanism. The empirical study is based on structured time deposits linked to the foreign exchange rates that the biggest banks in Poland have offered to their clients within last few years.

BARRIER OPTIONS DESCRIPTION

Standard barrier options

Barrier options are similar in some ways to ordinary options. There are puts and calls, as well as European and American varieties[1], but there is an additional element to barrier option, which is the barrier level set in the contract. In general, barrier options fall into two broad cathegories: "in" and "out" options. "In" options start their lives worthless and only become active in the event a predetermined knock-in barrier price is reached. "Out" options start their lives active and become null and void in the event a certain knock-out barrier price is breached [Chriss 1997].

Given the spot underlying asset price, the barrier can be placed either above or below it. If the barrier is below the spot price, the option is called a "down" option, if the barrier is above the spot price, the option is called an "up" option. Table 1 shows basic types of barrier options and their properties.

[1] Call options give to their holders the right to buy some underlying asset, whereas puts give the right to sell the underlying asset. European options can be exercised only on the day of expiration, whereas American options can be exercised anytime during their lives.

Table 1. Basic types of barrier options and their properties

	Down	Up
Out	Down-and-out dies if the underlying crosses the barrier coming down	Up-and-out dies if the underlying crosses the barrier coming up
In	Down-and-in becomes activated if the underlying crosses the barrier coming down	Up-and-in becomes activated if the underlying crosses the barrier coming up

Source: [Nelken 2000], p. 134

All this permits eight types of options [Kolb,Overdahl 2007]:
– down-and-in call,
– up-and-in call,
– down-and-in put,
– up-and-in put,
– down-and-out call,
– up-and-out call,
– down-and-out put,
– up-and-out put.

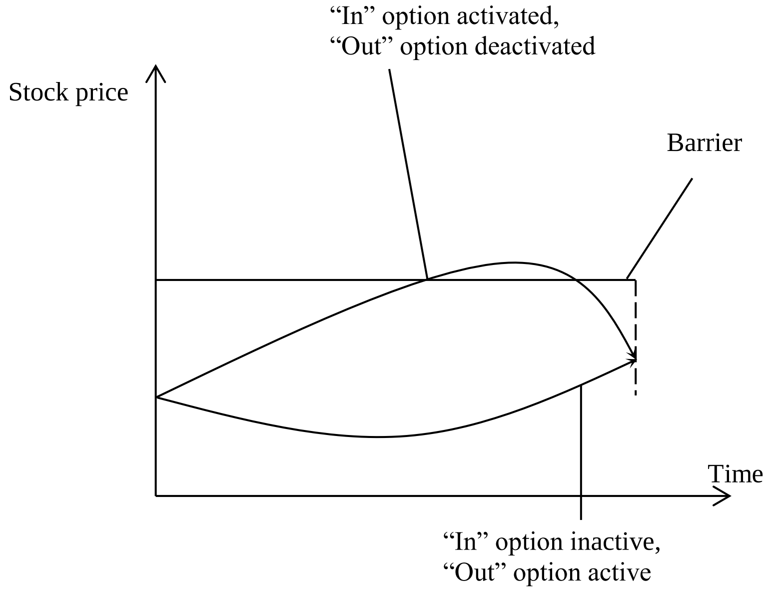
In the absence of rebate payments[2], the following decomposition always holds:

vanilla = knock-out + knock-in.

The idea is that simultaneously holding the "in" and the "out" options, guarantees that one and only one of the two will pay off. The argument, called "in-out" parity, only works for European options. In Figure 1, we can see an example, where a barrier is denoted by the heavy horizontal line, an expiration date by the vertical dashed line, and there are the two price paths (one that crosses the barrier and one that does not cross the barrier). The one that crosses the barrier simultaneously activates the knock-in option and deactivates the knock-out option. Conversly, the path that does not cross the barrier behaves in the opposite manner: the knock-in option is never activated, while the knock-out option is never deactivated. The expected payout of holding the "in" and "out" portfolio is therefore always the same: at expiration, the portfolio has exactly the same payout as holding a simple option [Chriss 1997].

[2] If a knock-out option gets knocked out or fails to materialize, the investor can receive a cash rebate. With a knock-out option, the rebate can be paid immediately upon being knocked out. With a knock-in option, we have to wait until expiration to know either or not the option was knocked in [Nelken 2000].

Figure 1. An “in-out” portfolio



Source: [Chriss 1997], p. 438

According to Kolb and Overdahl [2007], barrier options may be viewed as conditional plain vanilla options. “In” barrier options become plain vanilla options if the barrier is hit. “Out” barrier options are plain vanilla options, with the condition that they may pass out of existence if the barrier is hit. These conditions make barrier options inferior to unconditional plain vanilla options, so barrier options will be cheaper than otherwise identical plain vanilla options. This cheapness gives the barrier options a special usefulness in hedging applications.

Non-standard barrier options

Besides the standard barrier options, there are many variations of single barrier options, that are called exotic or non-standard barrier options. According to Zahng [2006], these are:

– floating barrier options, called also curvilinear barriers, where the barrier is no longer assumed to be constant throughout the life of the option, but may change with time in many applications (it may either increase or decrease with time, or follow some other deterministic paths);

– forward-start barrier options, where barriers are not effective immediately after the contracts are signed, but become effective at time t_1 ($t_0<t_1<T$), where t_0 and T represent current and maturity time, respectively;

– early-ending barrier options with barriers stopping to be effective at time t_e before the expiration of the option ($t_0<t_e<T$);

– window barrier options, where the barriers are effective only within one or more than one prespecified periods during the options lives (actually, the former two options – the forward-start and the early-ending barrier options are special window barrier options).

There are also multiple barrier options with two or more barrier levels H_i ($i = 2, 3, \dots, n$). The most commonly traded in the market are double-barrier options. The upper and lower tresholds can either be knock-in or knock-out or a combination of both. It makes no difference whether the up-barrier or the low-barrier is touched, or which is touched first and the direction of approaching the barrier is no longer a factor affecting the option value. Therefore, there are four basic types of double-barrier options:

– out calls,
– out puts,
– in calls,
– in puts.

METHODS FOR PRICING STANDARD BARRIER OPTIONS

Barrier options are options where the payoff depends on whether the underlying asset price reaches a certain level during a certain period of time. A down-and-out call is a regular call option that ceases to exist if the asset price reaches a certain barrier level H. The barrier level is below the initial asset price (S_0). The corresponding knock-in option is a down-and-in call. This is a regular call that comes into existence only if the asset price reaches the barrier level. If H is less than or equal to the strike price K, the value of a down-and-in call at time zero is:

$$c_{di} = S_0 e^{-qT} (H / S_0)^{2\lambda} N(y) - K e^{-rT} (H / S_0)^{2\lambda - 2} N(y - \sigma\sqrt{T}), \qquad (1)$$

where:

$$\lambda = \frac{r - q + \sigma^2 / 2}{\sigma^2}, \qquad (2)$$

$$y = \frac{\ln[H^2 / (S_0 K)]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \qquad (3)$$

and r is the continuously compounded risk-free rate, σ is the underlying asset price volatility, q is the dividend yield, and T is the time to maturity of the option. Because the value of regular call equals the value of a down-and-in call plus the value of a down-and-out call, the value of a down-and-out call is given by:

$$c_{do} = c - c_{di}. \qquad (4)$$

If $H >= K$, then:

$$c_{do} = S_0 N(x_1) e^{-qT} - Ke^{-rT} N(x_1 - \sigma\sqrt{T}) - S_0 e^{-qT} (H / S_0)^{2\lambda} N(y_1) + Ke^{-rT} (H / S_0)^{2\lambda-2} N(y_1 - \sigma\sqrt{T}) \quad (5)$$

and

$$c_{di} = c - c_{do} \ , \quad (6)$$

where

$$x_1 = \frac{\ln(S_0 / H)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \ , \quad (7)$$

$$y_1 = \frac{\ln(H / S_0)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \ . \quad (8)$$

An up-and-out call is a regular call option that ceases to exist if the asset price reaches a barrier level *H* that is higher than the current asset price. An up-and-in call is a regular call option that comes into existence only if the barrier is reached. When *H* is less than or equal to *K*, the value of the up-and-out call (c_{uo}) is zero and the value of the up-and-in call (c_{ui}) is *c*. When *H* is greater than *K*:

$$c_{ui} = S_0 N(x_1) e^{-qT} - Ke^{-rT} N(x_1 - \sigma\sqrt{T}) - S_0 e^{-qT} (H / S_0)^{2\lambda} [N(-y) - N(-y_1)] + Ke^{-rT} (H / S_0)^{2\lambda-2} [N(-y + \sigma\sqrt{T}) - N(-y_1 + \sigma\sqrt{T})] \quad (9)$$

and

$$c_{uo} = c - c_{ui} \ . \quad (10)$$

Put barrier options are defined similarly to call barrier options. An up-and-out put is a put option that ceases to exist when a barrier *H* that is greater than the current asset price is reached. An up-and-in put is a put that comes into existence only if the barrier is reached. When the barrier *H* is greater than or equal to the strike price *K*, their prices are:

$$p_{ui} = -S_0 e^{-qT} (H / S_0)^{2\lambda} N(-y) + Ke^{-rT} (H / S_0)^{2\lambda-2} N(-y + \sigma\sqrt{T}) \quad (11)$$

and

$$p_{uo} = p - p_{ui} \ . \quad (12)$$

When *H* is less than or equal to *K*:

$$p_{uo} = -S_0 N(-x_1) e^{-qT} + Ke^{-rT} N(-x_1 + \sigma\sqrt{T}) + S_0 e^{-qT} (H / S_0)^{2\lambda} N(-y_1) - Ke^{-rT} (H / S_0)^{2\lambda-2} N(-y_1 + \sigma\sqrt{T}) \quad (13)$$

and

$$p_{ui} = p - p_{uo} \ . \quad (14)$$

A down-and-out put is a put option that ceases to exist when a barrier less than the current asset price is reached. A down-and-in put is a put option that comes into existence only when the barrier is reached. When the barrier is greater than the strike price, p_{do}=0 and p_{di}=p. When the barrier is less than the strike price:

$$p_{di} = -S_0 N(-x_1)e^{-qT} + Ke^{-rT} N(-x_1 + \sigma\sqrt{T}) + S_0 e^{-qT} (H / S_0)^{2\lambda} [(N(y) - N(y_1)] - Ke^{-rT} (H / S_0)^{2\lambda-2} [N(y - \sigma\sqrt{T}) - N(y_1 - \sigma\sqrt{T})] \quad (15)$$

and

$$p_{do} = p - p_{di}. \quad (16)$$

All of these valuations make the usual Black-Scholes assumption that the probability distribution for the asset price at a future time is lognormal. Another important issue for barrier options is the frequency with which the asset price (S) is observed for purposes of determining whether the barrier has been reached. The analytic formulas given above assume that S is observed continuously [Hull 2012]. According to Ravindran [1998], barrier options can be also valued using multivariate integrals, binomial method, and Monte Carlo method. In recent years, numerous papers presenting alternative approaches to barrier options pricing have been published (see for example Chiarella et al. [2012], Hong et al. [2015], Rashidi Ranjbar and Seifi [2015], Kirkby et al. [2017], or Nouri and Abbasi [2017]). The majority of the papers focus on double-barrier options pricing.

COMPARATIVE ANALYSIS OF THE FX-LINKED STRUCTURED TIME DEPOSITS, BARRIER AND STANDARD OPTIONS

This section of the paper provides an analysis comparing the performance of structured time deposits with the performance of corresponding barrier and standard options on foreign exchange rates. The research is based on real market data on deposits that have been offered to individual investors in Poland within last few years. None of the time deposits guarantees final investment profit. They only offer the 100% payback of invested capital.

Case 1

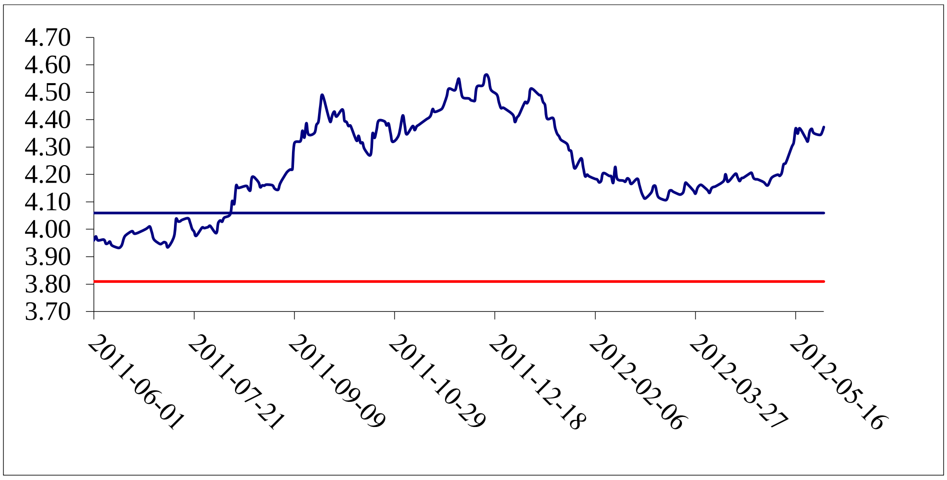
The investment starts on June 1, 2011 and lasts util May 30, 2012. It offers the conditional profit that depends upon the performance of EUR/PLN exchange rate. The contingent profit is calculated in the following manner:

$$8\% \times n / N,$$

where n – number of the days when the exchange rate ranges between two barriers, N – number of exchange rate observations.

A lower barrier (L) = EUR/PLN exchange rate on June 1, 2011 (3.9595) minus 0.15 PLN, which is 3.8095. An upper barrier (U) = EUR/PLN exchange rate on June 1, 2011 (3.9595) plus 0.10 PLN, which is 4.0595. Figure 2 shows the performance of the EUR/PLN exchange rate during the investment period. The detailed analysis of the data on the exchange rate enables us to detect the days when its values range between the barriers. We have n = 48 observations. Thus, the investment provides the profit of: $8\% \times 48 / 253 = 1.52\%$.

Figure 2. EUR/PLN exchange rate from June 1, 2011 to May 30, 2012



Source: own preparation

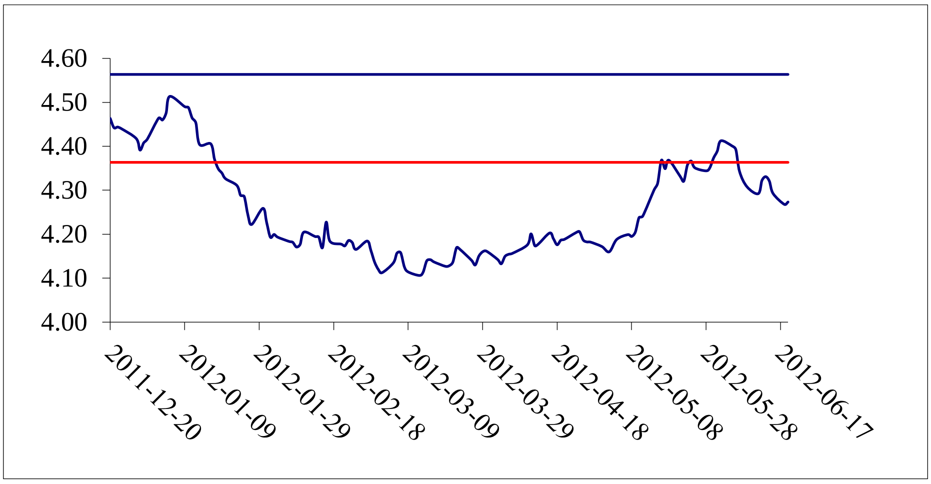
An alternative to the time deposit could be purchase of two barrier options: up-and-out call and down-and-out put with $K = S_0$ = 3.9595, time to maturity T = 1 (one year), and the barriers respectively: 4.0595 for a call, and 3.8095 for a put. On the base of formulas given in the previous section (with q replaced by r_f – the foreign risk-free rate), the up-and-out call premium c_{uo} = 0.0005, and the down-and out put premium p_{do} = 0.0021. Figure 2, enables us to find out that on the day of expiration of the options (May 30, 2012) the up-and-out call is not active. The down-and-out put is active, however taken into account the level of the exchange rate S_T = 4.3889, an investor should not exercise the contract, so his (her) total loss from the portfolio of the two options reaches 0.0026 PLN per 1 EURO[3]. Standard call and put options with analogous parameters would cost respectively: c = 0.1575 and p = 0.0569. On the day of expiration the call option pays off: $S_T - K$ = 4.3889 – 3.9595 = 0.4294 and brings the net profit of: 0.4294 – 0.1575 = 0.2719. It allows to cover the premium for the put and still gain 0.2150 PLN per 1 EURO.

[3] In practice, the currency options traded in the Polish market, usually cover the amounts of 10 000 or 50 000 Euros and 10 000 or 50 000 U.S. dollars.

Case 2

The time deposit starts on December 20, 2011 and lasts until June 19, 2012. It offers three scenarios. Each of them is dedicated to investors with different expectations about future behavior of the EUR/PLN exchange rate. Its initial value equals 4.4635 PLN per 1 EURO. Scenario A is dedicated to investors hoping that the exchange rate will increase. It guarantees a 5% profit if on the day of expiration the exchange rate is higher or equal to the level of 4.5635. Scenario B is dedicated to investors hoping that the exchange rate will decrease and guarantees a 5% profit if on the day of expiration the exchange rate is lower or equal to the level of 4.3635. Finally, scenario C is recommended to investors expecting stable exchange rates over time. It offers a 5% profit if on the day of expiration the exchange rate will range between 4.3635 and 4.5635. Figure 3 shows the performance of the exchange rate during the investment period.

Figure 3. EUR/PLN exchange rate from December 20, 2011 to June 19, 2012



Source: own preparation

Figure 3 enables us to find out that scenario B is the one to be realized, so after six months the investor who expected the exchange rate to fall down, is the winner with 5% profit from the time deposit. The three scenarios could be replaced by purchasing corresponding barrier options. The alternative to scenario A could be an up-and-in call, the alternative to scenario B could be a down-and-in put, and the alternative to scenario C would be a portfolio of one up-and-out call and one down-and-out put. All of them with the following input parameters: $K = S_0 = 4.4635$, $T = 0.5$, and barriers respectively: 4.3635 for "down" options, and 4.5635 for "up" options. Applying proper formulas presented in the previous section of the paper, we obtain:

– $c_{ui} = 0.1614$,
– $p_{di} = 0.0904$,

– c_{uo} = 0.00037,
– p_{do} = 0.00038.
On the day of expiration, the up-and-in call, substituting scenario A, is not active, so the investor's loss is equal to the premium paid (0.1614 PLN per 1 EURO). The down-and-in put, substituting scenario B, is active and pays off: $K - S_T = 4.4635 - 4.2733 = 0.1902$, providing the net profit of 0.0998 PLN per 1 EURO. The two options constituting the alternative for scenario C (up-and-out call and down-and-out put) together bring the total loss of 0.00075 (the former should not be exercised, the latter is not active). Standard call and put options with analogous parameters would cost respectively: $c = 0.1650$ and $p = 0.0919$. On the day of expiration only the put pays off 0.1902 and provides the net profit of 0.1902 – 0.0919 = 0.0983 which is less than the value of the premium for call. This is why the loss from the portfolio of two standard options equals 0.0667 PLN per 1 EURO.

Case 3

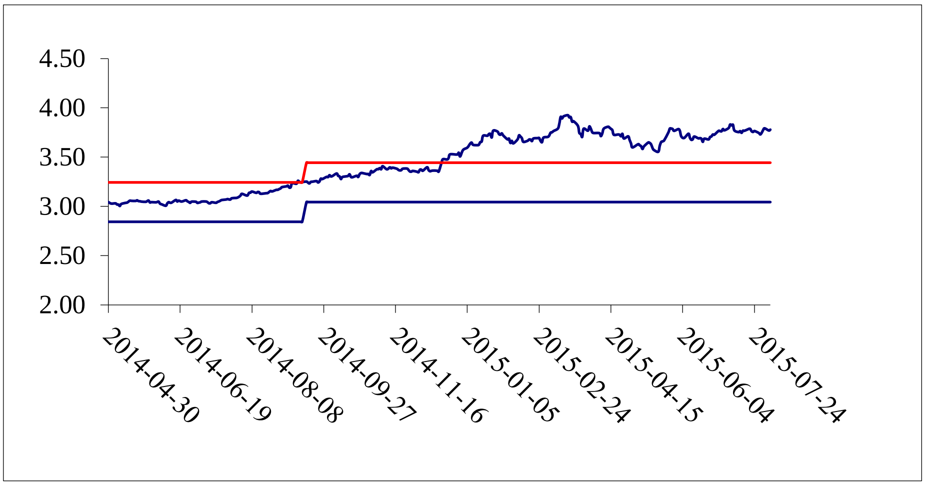
The time deposit is linked to the USD/PLN exchange rate. The investment starts on April 30, 2014 and ends on July 30, 2015. Here, the conditional profit is calculated in the following manner:

- If the exchange rate does not reach any of the two barriers: L_A = initial exchange rate minus 0.20 and U_A = initial exchange rate plus 0.20, the deposit provides 8% profit.
- If the exchange rate crosses the lower barrier (L_A), the new barriers are set. They are: L_B = initial exchange rate minus 0.40, U_B = initial exchange rate. If the exchange rate does not touch any of the new barriers, L_B and U_B, interest rate equals 8%.
- If the exchange rate crosses the upper barrier U_A, the new barriers are set to L_C = initial exchange rate, U_C = initial exchange rate plus 0.40. If none of the new barriers is touched, 8% interest rate is guaranteed.
- In all other cases, the time deposit generates 0% interest.

The initial exchange rate $S_0 = 3.0440$ determines $L_A = 2.8440$ and $U_A = 3.2440$. Figure 4 shows the USD/PLN exchange rate performance during the investment period. As on September 12, 2014, the upper barrier is crossed, the new barriers are set: $L_C = 3.0440$ and $U_C = 3.4440$ (the barriers shift is visible in Figure 4). Unfortunately, the upper barrier U_C is crossed again, which reslults in 0% profit from the investment.

Some alternative to the time deposit could be purchase of two barrier options: an up-and-out call and a down-and-out put with the following input parameters: $K = S_0 = 3.0440$, $T = 1.25$, and barriers: 3.2440, 2.8440 for the call and put respectively. Their premiums are: $c_{uo} = 0.0075$ and $p_{do} = 0.0073$. On the day of expiration, the call option is inactive. The put is active, however its exercise is unreasonable as the exchange rate level is $S_T = 3.7792$. So the portfolio of the two options generates the loss of 0.0148 PLN per 1 USD. Comparable standard call and put options would cost respectively: $c = 0.1507$ and $p = 0.0672$. On the day of expiration the call option pays off: $S_T - K = 3.7792 - 3.0440 = 0.7352$ and gives the net profit of $0.7352 - 0.1507 = 0.5845$. It is enough to cover the ammount paid for the put and to earn 0.5173 PLN per 1 USD from the portfolio of the two standard options.

Figure 4. USD/PLN exchange rate from April 30, 2014 to July 30, 2015

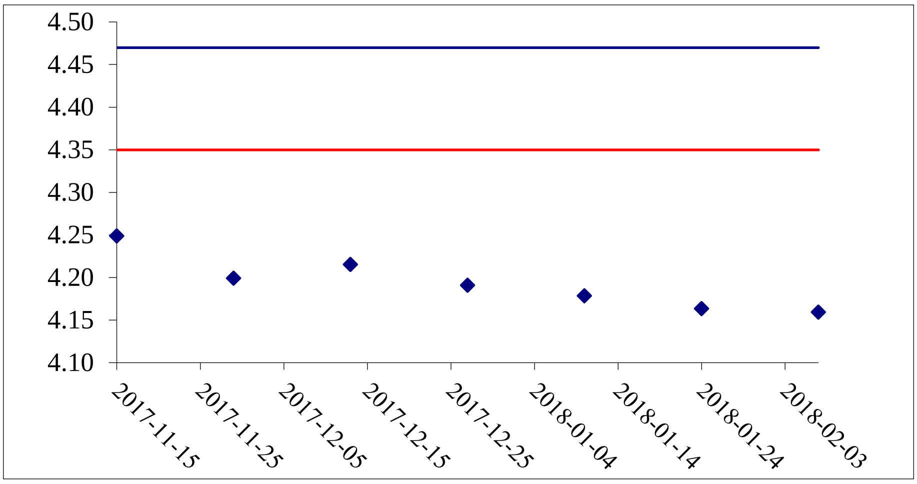


Source: own preparation

Case 4

The time deposit is linked to the EUR/PLN exchange rate and starts on November 12, 2016. The investment lasts until February 7, 2018. There are two barriers: a lower one L = the exchange rate on the reference day (November 15, 2016) minus 0.06 (4.4098 – 0.06 = 4.3498) and an upper barrier U = the exchange rate on the reference day (November 15, 2016) plus 0.06 (4.4098 + 0.06 = 4.4698). The investment profit is: $N \times 0.25\% + 1\%$, where N denotes the number of observations – days, when the exchange rate ranges between the two barriers. There are the following dates of observations: November 15, 2017, November 29, 2017, December 13, 2017, December 27, 2017, January 10, 2018, January 24, 2018, and February 7, 2018. Figure 5 shows the exchange rates on the days of interest together with the two barriers. In this figure, we can see that none of the observations is located between the barriers, so the investment profit equals: $0 \times 0.25\% + 1\% = 1\%$. An attempt to replace the time deposit with the portfolio of two barrier options: up-and-out call and down-and-out put with $K = S_0 = 4.4098$, $T = 1.25$, $L = 4.3498$, and $U = 4.4698$, provides the total loss of 0.00012. This is so, because on the day of expiration the put with the premium $p_{do} = 0.00006$ is not active. Moreover, the call with the similar premium (0.00006) that is active, expires worthless as the exchange rate $S_T = 4.1593$. Its owner should not exercise it. Analogous standard call and put options would cost respectively: $c = 0.1782$ and $p = 0.0736$. Exercising of the put option generates the net profit amounting 4.4098 – 4.1593 – 0.0736 = 0.1769. It is not enough to cover the already paid premium for the call. The total loss from the portfolio of the two standard options is 0.0013 PLN per 1 EURO.

Figure 5. EUR/PLN exchange rate from November 15, 2017 to February 7, 2018



Source: own preparation

CONCLUDING REMARKS

Barrier options are probabely the oldest of all exotic options. They have been traded sporadically in the U.S. market since 1967. That is six years before the publication of the seminal paper by Black and Scholes entitled “The pricing of options and corporate liabilities” [Black, Scholes 1973] and six years before the Chicago Board of Options Exchange came into being in 1973. Nowadays, barrier options are among the few most popular exotic options in the OTC marketplace because they are cheaper than vanilla options in general.

Besides the standard barrier options, other modifications of barrier options have been designed to increase the flexibility of vanilla barriers or to capture some more general features. These are, for example, floating barrier options, partial barrier options or double barrier options. What is more, barrier options are often combined with other exotic options, so we have for example Asian barrier options, look barrier options, digital barrier options, or two-asset barrier options (see [Haug 2007]).

The mechanism of barriers is often inconporated into financial products such as structured time deposits. According to Jagielnicki [2011], usually the terms set in these investment vehicles are hardly met in practice, so it is important that investors understand the nature of the transactions. That is why the paper provides a short description of barrier options and a comparative analysis of their performance against the performance of structured time deposits with barrier mechanism. Surprisingly, some of the time deposits linked to the foreign exchange rates, that have been analyzed in the paper, perform better than the barrier options that could be considered certain alternatives to the deposits. Vanilla options also performed better than their barrier counterparts, even though barrier options are cheaper than standard contracts. However, their disadvantage is they can stay inactive or a "spike" in the underlying asset price can cause the barrier option to be knocked-out. Thus, the analysis presented in the paper may be helpful in estimating possible chances for gaining profits from the investments of this kind.

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