IDENTIFICATION OF AN AGGREGATE PRODUCTION FUNCTION FOR POLISH ECONOMY

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Abstract: This paper explores an aggregated production function, built on the distribution of production capacity with a limited age. Technologies are determined at the time of capacity creation. With increasing age, production capacity is decreasing, keeping the number of workplaces. The lowest labour input and the coefficient of capital intensity are reduced due to scientific and technological progress. The parameters of this production function were identified by parallel calculations according to the data of the Polish economy 1970-2017. The economic interpretation of the obtained results is given.

Keywords: aggregated production function, production capacity, parameter identificatin, parallel calculations, Polish economy

JEL classification: L11, M11, E23, C4

INTRODUCTION

The aggregation problem for productive opportunites of production units of an industry was first formulated in [Houthakker 1955]. The approach to derive propeties of the standard production function in macroeconomics from microfoundations is widespread. For example, if the distribution of ideas is Pareto, then the global production function is Cobb-Douglas, and technical change in the long run is labour-augmenting [Jones 2005].

A review of the literature on vintage capital growth models that have been in the heart of growth theory in the 60s, the reasons for its collapse in the late 60s and the reasons for its revival in the 90s are presented in [Boucekkine et al. 2011].

Unlike these works the paper uses the consept of production capacity instead of capital. The increases in capital and in production capacity are related by the incremental capital-intensity ratio, which is assumed here to be variable over time.

The concept of production capacity was introduced in [Johansen 1968]. The production capacity is the maximal potential output a producing unit could produce in a given period of time, given technology, and fixed factors of production. The concept of capacity was used for construction of a production function represented by the distribution of production capacities by putty-clay technology [Johansen 1972]. This description of production functions arose from practical needs in the analysis of specific sectors of the economy. The mathematical study of production functions constructed by locally surnmable distributions of capacity among technologies is consided in [Shananin 1984].

The paper presents an evaluation of an original aggregate production function with limited age of production capacities [Olenev 2017] for recent Poland economy based on the vintage capacity model with putty-clay technology [Olenev et al. 1986]. Production capacity is determined as a maximum of possible output in a year. Gross domestic product (GDP) of Polish economy at constant 2010 prices measured in PLN is used here as the output. At a given capital intensity and a given depreciation rate one can evaluate age structure of production capacities by the past real investments. This two unknown parameters (the capital intensity, the depreciation rate) along with unknown parameters of a production function can be determined in an indirect way by comparison of pairs of time series for each macroeconomic index calculated by the model and taken from statistical data.

MODEL DESCRIPTION

Micro description of production capacity dynamics

Production capacity $m(t, t)$ created in the year t is determined by gross fixed capital formation $\Phi(t)$ of this year t divided by a coefficient b of capital intensity.

$$m(t, t) = \Phi(t)/b. \tag{1}$$

The value $\Phi(t)$ is the increases in total capital at time t. The value $m(t, t)$ is incrice of the total production capacity at time t. The capital intensity b is the incremental capital-intensity ratio at time t which is used to move the description from the capital to the production capacity. The value of the capital intensity b depends on a present technological structure of Polish economy. Let's use here the form have used for economies of Greece [Olenev 2016] and Russia [Olenev 2017]:

$$b(t) = b(0) \exp(-\beta t). \tag{2}$$

Production capacity created in the year $\tau \leq t$ decreases with increasing of its age $t - \tau$ by specified rate $\mu > 0$.

$$m(\tau, t) = m(\tau, \tau)\exp(-\mu(t - \tau)). \tag{3}$$

It is supposed that a number r of workplaces in the firm remains constant through out the life period from its creation up to its dismantling. If a labour input of firm created in year τ at time $t \geq \tau$ is denoted by $\lambda(\tau, t)$ then the number of workplaces $r(\tau, t) = \lambda(\tau, t)m(\tau, t)$ and therefore the labour input increases (labour productivity decreases)

$$\lambda(\tau, t) = \lambda(\tau, \tau)\exp(\mu(t - \tau)). \tag{4}$$

Paper [Olenev et al. 1986] shows that if one switches the variables (τ, t) to the variables (λ, t) in the description of capacity dynamics, then the dynamics for a distribution density of production capacities $m(\lambda, t)$ satisfies a partial differential equation of the first order:

$$\frac{\partial m(\lambda, t)}{\partial t} = j(\lambda, t) - 2\mu m(\lambda, t) - \mu\lambda\frac{\partial m(\lambda, t)}{\partial \lambda}, \tag{5}$$

where $j(\lambda, t)$ is an investment in technology with labour intensity λ. The switch in variables is similar to the switch-over of the Lagrangian description to the Eulerian description for dynamics of each particle of the body in continuum mechanics [Mase et al. 2010]. The functions of $m(\tau, t)$ and $m(\lambda, t)$ have different economic meanings and are represented by completely different dependencies. The Lagrangian function $m(\tau, t)$ shows the dependence of capacity on time, and the Eulerian function of capacity density $m(\lambda, t)$ at each fixed t shows the dependence of capacity on labor intensity. They should not be confused and one can denotes them by different symbols. In the calculations we will use the Lagrangian notation.

Equation (5) completely determines the density $m(\lambda, t)$ if an initial condition $m(\lambda, 0) = n(\lambda)$ is specified. If as it is supposed in equation (1) all investments $J(t) = \frac{\Phi(t)}{b}$ come in a new technology with labour input $v(t)$ then $j(\lambda, t) = J(t)\delta(t - v(t))$ and we can find (see [Olenev et al. 1986]) by integrating (5) an equation for a total capacity of an industry or an economy

$$M(t) = \int m(\lambda, t)d\lambda. \tag{6}$$

The equation obtained here from microeconomic description is usually used in macroeconomic models:

$$\frac{dM(t)}{dt} = J(t) - \mu M(t). \tag{7}$$

If share of new capacities in total capacity of an economy $\sigma = J(t)/M(t)$ is constant then this microeconomic description allows to build an analytical expression for the production function of the economy, that is, the dependence of the output $Y(t)$ on the production factors: total capacity $M(t)$ and the total labour $L(t)$:

$$Y(t) = M(t)f(t, x), \tag{8}$$

where $x = L(t)/M(t)$. One only needs to define function of scientific and technical progress which in this model [Olenev et al. 1986] is reflected in a dynamics of the best and the lowest labour input $v(t) = \lambda(t, t)$.

$$\frac{1}{v(t)}\frac{dv(t)}{dt} = -\varepsilon\sigma(t), \tag{9}$$

where parameter $\varepsilon > 0$ is a rate of scientific and technical progress .

In general case when the share $\sigma(t)$ is not constant one can construct a production function numerically using relations (1)-(4),(9).

Aggregate production function with limited age A of production capacities

An analitical expression for aggregate production function was obtained in [Olenev 2017]. GDP, $Y(t)$, is determined by total capacity $M(t)$, labour $L(t)$, and production function (8).

The total capacity is determined from

$$\frac{dM(t)}{dt} = J(t) - \mu M(t) - \left(1 - \frac{dA(t)}{dt}\right)J(t - A(t))e^{-\mu A(t)}, \tag{10}$$

where A(t) is a maximal age of capacities [Olenev 2017].

If share of new capacities in total capacity is constant,

$$\sigma(t) = J(t)/M(t) = \sigma = \text{const}, \tag{11}$$

maximal age of capacities is fixed, $A(t) = A = const$, the total capacity and output increase exponentially,

$$M(t) = M_0 e^{\gamma t}, \; Y(t) = Y_0 e^{\gamma t}, \tag{12}$$

then production function has the form [Olenev 2017, p.431, formula (14)]

$$f(t, x) = \frac{\sigma}{\gamma + \mu}\left\{1 - \left[1 - \frac{(\gamma - \varepsilon\sigma)}{\sigma}\frac{x}{v(t)}\right]^{(\gamma + \mu)/(\gamma - \varepsilon\sigma)}\right\} \tag{13}$$

where growth rate $\gamma = \gamma(\mu, \sigma, A)$ is determined from

$$\gamma + \mu = \sigma\left(1 - e^{-(\gamma + \mu)A}\right). \tag{14}$$

So let's begin numerical estimations of the model parameters by statistical data for Polish economy and on the base of microfoundations of the model. The use of the model microdescription in the process of identification makes it possible to estimate parameters even with varying $\sigma(t)$.

INDENTIFICATION OF PARAMETERS

We use UN statistical data [National Accounts Main Aggregates Database] for GDP and gross fixed capital formation and official data of Polish statistical agency [Statistics Poland] for employments data. Recall that here GDP at constant 2010 prices in PLN is used as the output Y(t) of Poland economy. The following obvious notation for τ, t is used here in the calculations and presentation of graphical results. If the current year $c \in [1970, 2017]$, then the model year $t = c - 1970$, the model year τ of capacity creation $\tau \leq t$. So that $t \in [0, 47]$, $m(\tau, t) = m(a, c)$, where $a = \tau + 1970$ is the factual year of vintage capacity creation.

Let us evaluate the parameters b(0), β, μ, ν(0), ε, m(0) of the Poland economy 1970-2017 by the model described above from some natural conditions. One of the condition is that the production capacities are utilized an average on 70% approximately, implying an existence of a normal reserve of capacities on the level of approximately 30%.

Let use for estimation fitting of time series for labour L(t) and output Y(t). Since this model has only two macroeconomic indicators compared with statistical data, we can choose one of them equal to its statistical time series (we chose Y(t) in this algorithm), and adjust the other macro indicator (here L(t)) by selecting the desired parameters using the Theil inequality index T_L.

$$T_L = \sqrt{\frac{\sum_{t=t_0}^{t_n}(L(t)-L^{stat}(t))^2}{\sum_{t=t_0}^{t_n}(L(t))^2+(L^{stat}(t))^2}} \rightarrow min.$$

Figure 1. Vintage production capacity in 2000 in constant prices of 2010, PLN billions
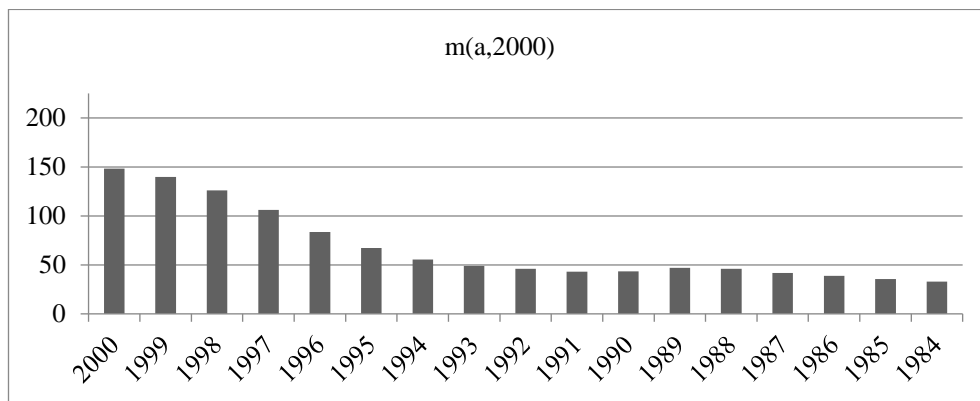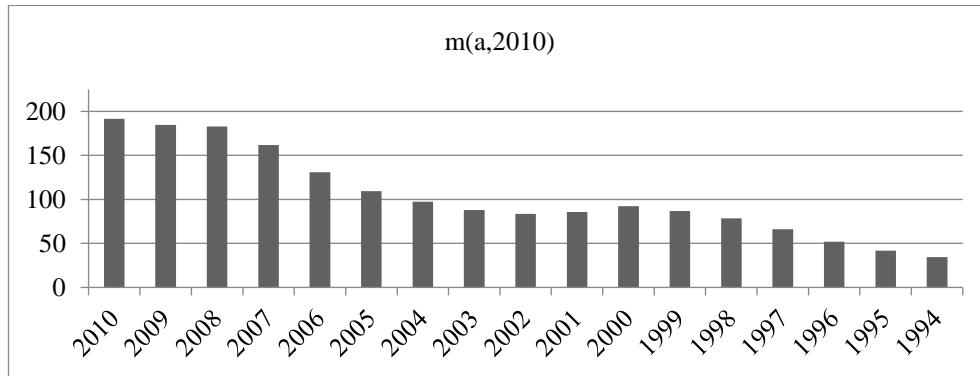

Source: own preparation

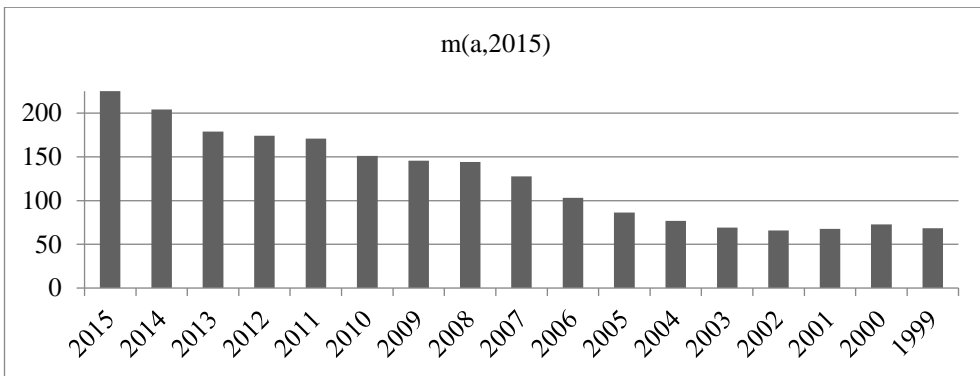Figure 2. Vintage production capacity in 2005 in constant prices of 2010, PLN billions
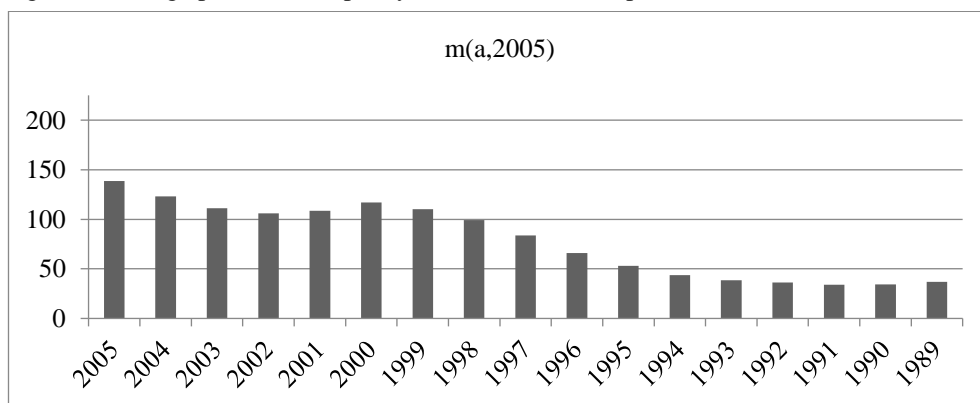
Source: own preparation

Figure 3. Vintage production capacity in 2010 in constant prices of 2010, PLN billions
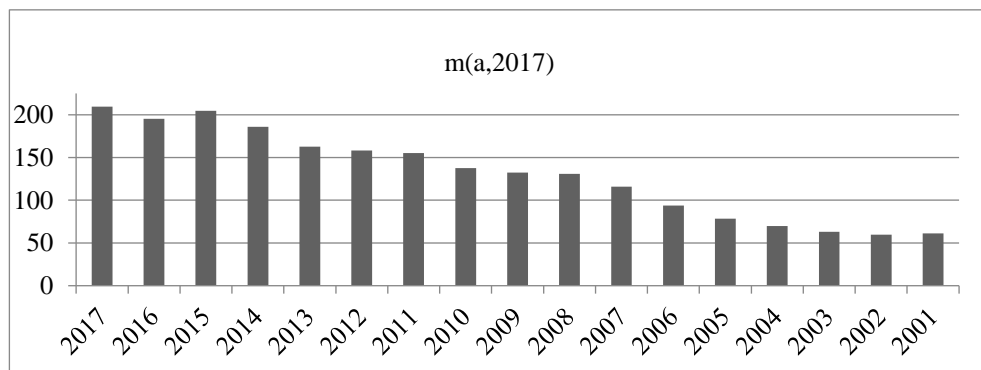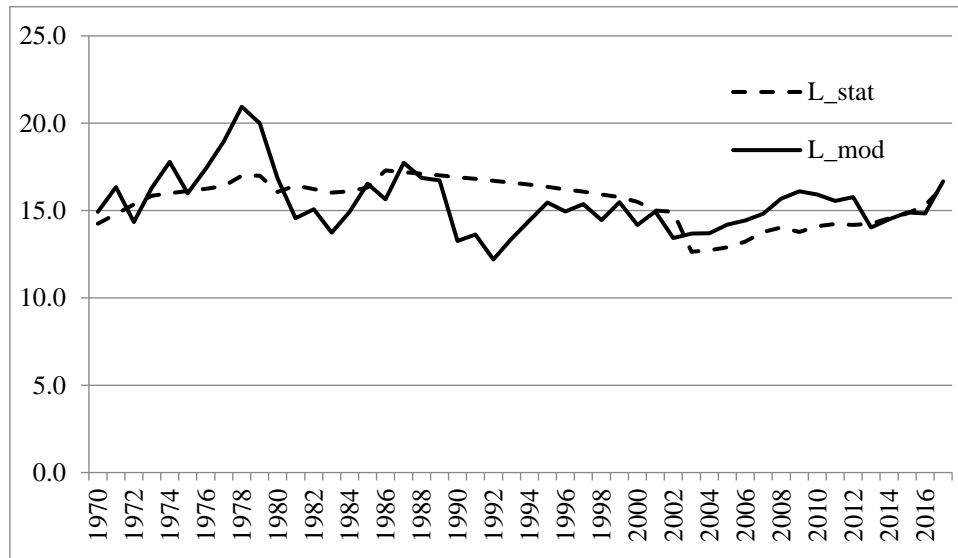

Source: own preparation

Figure 4. Vintage production capacity in 2015 in constant prices of 2010, PLN billions

Source: own preparation

Figure 5. Vintage production capacity in 2017 in constant prices of 2010, PLN billions

Source: own preparation

Figure 6. Identification of the model of Polish economy by fitting of the employment. Time series for employment: L_mod – estimation by vintage capacity model, L_stat – statistical data, millions people
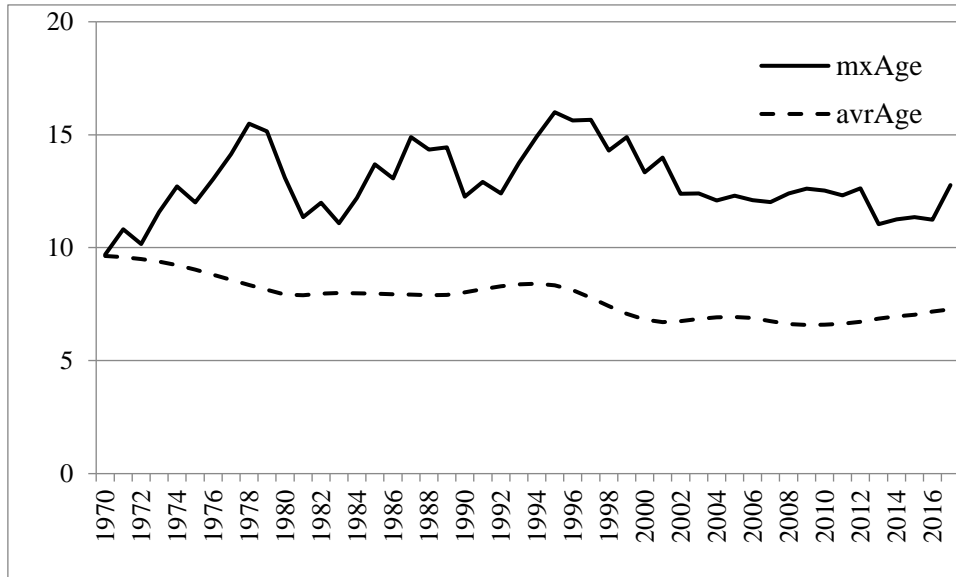

Source: own preparation

Figure 7.Time series for macroeconomic indices: f(x) – capacity utilization, bJ/Y – ratio of investment product to GDP
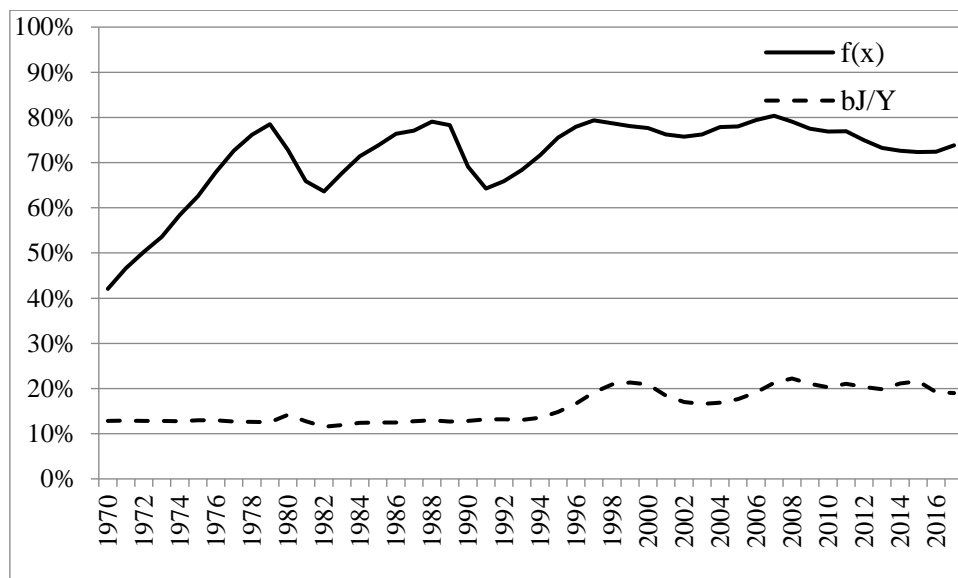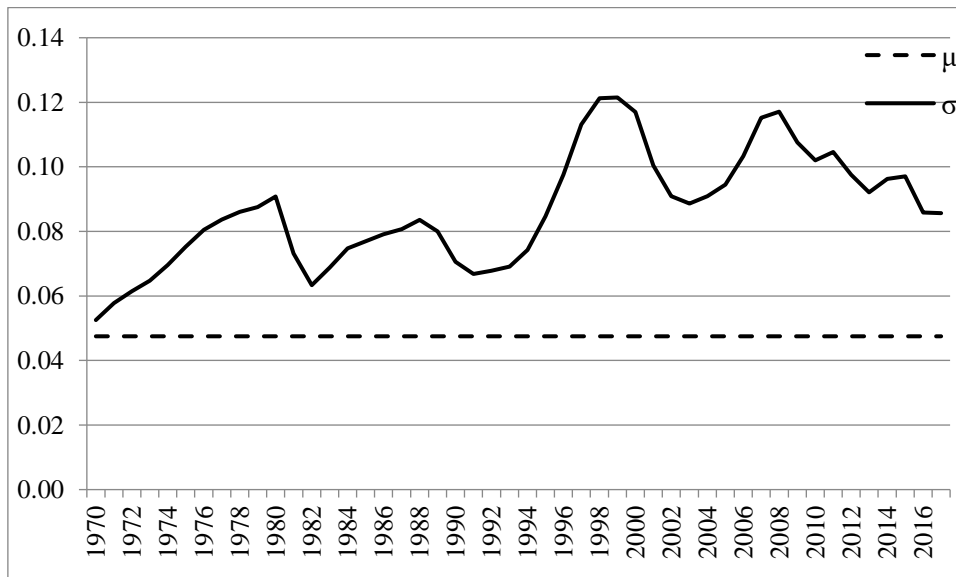

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Figure 8. Time series for macroeconomic indices: mxAge – maximal age of capacities which are used in production of output, avrAge – average age of production capacities
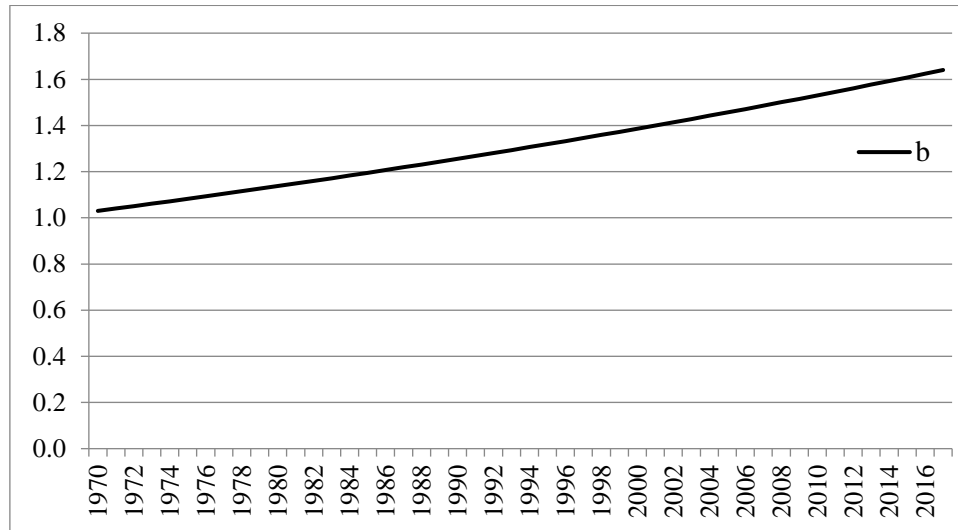

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Figure 9. Time series for macroeconomic indices: σ(t) share of investments in total capacity, μ - rate of depreciation

Source: own preparation

Figure 10. Time series for macroeconomic indices: b(t) – capital intensity

Source: own preparation

RESULTS

Distribution of production capacity in 2000-2017 by age (vintage capacity) in constant prices of 2010, PLN billions are presented in the Figures 1-5. It is seen that in 2000-2017, the distribution structure of production capacity has improved. The share of new, more productive production capacity has increased. The quality of identification can be found in the Figure 6. Time series for some macroeconomic indices are presented in the Figures 7-10. It is interesting to note that the capital intensity in Poland is growing (see Figure 10) in contrast to Greece [Olenev 2016] and Russia [Olenev 2017]. The share of new capacities in total capacity has two periods: oscillations near 0.08 in 1970-1996, and oscillations near 0.10 in 1997-2017. So, this is an attempt of Polish economy to move to a faster growth rate.

SUMMARY

In the paper we present an estimation of an original aggregate production function for Polish economy obtained by the micro model identification on the base of official statistical data. The values of parameters are the next: the limit age for capacities of the Polish economy $A = 17$ years. the capital intensity coefficient $b(0) = 1.03$ years in 1970, $\beta = -0.00995$, $\mu = 0.0475$, the best labour input in 1970 was $v(0) = 0.0205$ in millions employed peoples necessary to produce one billion PLN in constant prices of 2010, $\varepsilon = 0.290$, $m(\tau, 0) = m(0,0)e^{0.010\tau}$, where $\tau \leq 0$; $t = 0$ corresponds to the year 1970. All parameters of the model are found by a complete search usung hight performance computations.

Note that the parameter $\beta < 0$ for the Polish economy in contrast to the Greek and Russian economies. This means that the technological structure of the Polish economy becomes more complicated, and production becames more capital-intensive. The model of economy, techniques of its identification and especially its application for Polish economy require further research. For examp;e, the identification set method [Kamenev, Olenev 2015] can allow to study the forecast stability of the model.

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