# THE RISK PREMIUM IN THE FOREIGN EXCHANGE MARKET. THE APPLICATION OF ARCH-IN-MEAN MODEL

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**Abstract:** Forward premium anomaly is one of the most popular puzzles in the theory of international finance. The phenomenon is explained by, among others, the existence of non-zero risk premium in the foreign exchange market. The paper applies ARCH-in-mean models to assess whether there exists a time-varying risk premium in the USD/PLN and AUD/JPY foreign exchange markets. The results indicate the existence of a non-zero risk premium in the analyzed markets. As far as the USD/PLN is concerned, the risk premium takes negative values when the risk measured by conditional variance rises. The results suggest that when there is a surge in risk, the US dollar's appreciation and Polish zloty depreciation increases. The results confirm the US dollar as a safe-haven currency that tends to appreciate during high-volatility and crisis periods. Moreover, the study shows that the risk premium in the AUD/JPY market takes positive values when the risk measured by conditional variance rises. It implies that when there is a mount in risk, the appreciation of Japanese yen increases. Furthermore, research results reveal the positive and significant relationship between stock market uncertainty and exchange rates conditional volatility.

**Keywords:** foreign exchange market, risk premium, ARCH-M model, forward premium puzzle, VIX

**JEL classification:** C22, D84, E44, F31, G15

## INTRODUCTION

Uncovered interest rate parity (UIP) states that interest rate differential is equal to the expected change in exchange rates. One of the most puzzling features of the foreign exchange market is the tendency of low interest-yielding currencies to

depreciate rather than appreciate as UIP suggests. The UIP puzzle is generally known as the forward premium puzzle. Literature provides several explanations of the phenomenon. One possible reason is the existence of a risk premium. Other explanations involve invalidity of the rational expectations hypothesis, peso problems and market inefficiency. The paper is focused on risk premium inherent in the uncovered interest rate parity condition. The time-varying risk premium is one of the most frequently cited reasons for the existence of UIP failure [e.g. Froot, Thaler 1990; McCallum 1994; Chinn, Meredith 2004; Li et al. 2012; Kumar 2019].

In the paper, we presume the rationality of market participants expectations. We assume that the forward premium puzzle results from the existence of a non-zero, time-varying risk premium. The research is carried out for two currency pairs, i.e. AUD/JPY and USD/PLN from June 2006 to November 2020. The paper aims to assess whether there exists a significant and time-varying risk premium in the foreign exchange market. The article applies ARCH-in-mean (ARCH-M) models. Engle, Lilien and Robins [1987] are among the first who described ARCH-M models' use in explaining risk premiums in the financial market. Berk and Knot [2001] applied ARCH-M models to analyse the risk premium in the currency market.

The remainder of the paper is organised as follows. Section 2 reviews the relevant literature. The subsequent one presents methodology and data. The empirical results are described in section 4. The last section provides concluding remarks.

## MODELING THE RISK PREMIUM IN THE FOREIGN EXCHANGE MARKET

Uncovered interest rate parity (UIP) states that interest rate differential equals to the expected change in exchange rates:

$$E_t(s_{t+k}|\Omega_t) - s_t = i_t - i_t^* \tag{1}$$

where $E_t(s_{t+k}|\Omega_t)$ denotes the expectation of natural logarithm of the spot exchange rate at time $t + k$, based on information known at time $t$; $s_t$ denotes the natural logarithm of the spot exchange rate at time $t$ (quote currency units per unit of base currency); $i_t$ and $i_t^*$ are nominal interest rates of a quote and base currency, respectively. Assuming Covered Interest Parity (CIP) holds ($f_t^{(k)} - s_t = i_t - i_t^*$), the UIP can be expressed as follows:

$$E_t(s_{t+k}|\Omega_t) - s_t = f_t^{(k)} - s_t \tag{2}$$

where $f_t^{(k)}$ is the natural logarithm of the $k$-period forward exchange rate.

Market expectations of future spot exchange rates are hardly observable. Therefore, the UIP hypothesis is tested jointly with the assumption of rational expectations in the exchange rate market. Under the assumption of rational expectations, the future value of spot exchange rate ($s_{t+k}$) is equal the expected spot

exchange rate at time $t+k$ ($E_t(s_{t+k}|\Omega_t)$) plus a white-noise error term ($\eta_{t+k}$) which is uncorrelated with information available at time $t$:

$$s_{t+k} = E_t(s_{t+k}|\Omega_t) + \eta_{t+k} \tag{3}$$

Many researchers have tested UIP by using the equation (4):

$$s_{t+k} - s_t = \alpha + \beta\left(f_t^{(k)} - s_t\right) + \varepsilon_{t+k} \tag{4}$$

where $s_{t+k}$ and $s_t$ are natural logarithms of spot exchange rate at time $t + k$ and time $t$ respectively; $f_t^{(k)}$ is the logarithm of the $k$-period forward exchange rate and $\varepsilon_{t+k}$ is a disturbance term which is uncorrelated with information available at time $t$. If an agent is endowed with rational expectations and risk-neutral, we should expect the slope parameter $\beta$ to be equal to unity ($\beta = 1$) and the coefficient α to be equal to zero (α = 0). A well-known empirical regularity is that $\beta$ in equation (4) is significantly less than one, and very often closer to minus unity than plus unity [Froot, Thaler 1990]. According to Fama [1984], the negativity of the $\beta$ parameter results from the risk premium required by risk-averse market participants. They demand a higher profit than the interest rate differential in return for the risk of holding foreign currency.

If market participants are risk-averse, then the forward rate will differ from the expected spot exchange rate by a risk premium. Froot and Frankel [1989] define a risk premium ($p_{t+k}$) as the difference between the forward exchange rate at time $t$ for $k$ periods ahead ($f_t^{(k)}$) and the expected spot exchange rate at time $t+k$ ($E_t(s_{t+k}|\Omega_t)$):

$$p_{t+k} = f_t^{(k)} - E_t(s_{t+k}|\Omega_t) \tag{5}$$

Under risk neutrality, forward exchange rate at time $t$ for $k$ periods ahead ($f_t^{(k)}$) should equal to the expected spot exchange rate at time $t+k$ ($E_t(s_{t+k}|\Omega_t)$). If $f_t^{(k)} \neq E_t(s_{t+k}|\Omega_t)$) then the investors incur a premium from buying the currency forward at time $t$ for $k$ periods ahead relative to its expected spot price at time $t+k$ [Engel, 1996]. Under the assumption of rational expectations (3) we define the risk premium ($p_{t+k}$) as the difference between the forward exchange rate at time $t$ for $k$ periods ahead ($f_t^{(k)}$) and the future value of spot exchange rate ($s_{t+k}$) [Czech 2016]:

$$p_{t+k} = f_t^{(k)} - s_{t+k} \tag{6}$$

Although the risk premium in the foreign exchange market has been examined in many papers before, the article focuses mainly on works that apply autoregressive conditional heteroscedasticity models (ARCH). According to Hodrick [1987, p. 67] "modelling the conditional variance may be a fruitful direction to pursue to understand the nature of the rejection of the unbiasedness hypothesis and to determine whether the rejection is due to a time-varying risk premium". Domowitz and Hakkio [1985] are the first who modelled the risk premium based on the ARCH specification of Engle [1982]. They assume that risk premium has a constant

component ($\alpha$) and a time-varying component, i.e. the conditional variance of the error term ($h_{t+k}$):

$$p_{t+k} = \alpha + \delta h_{t+k} \tag{7}$$

There is no risk premium if $\alpha$ and $\delta$ are insignificantly different from zero. There is a constant risk premium if $\alpha \neq 0$ and $\delta = 0$. There is a time-varying risk premium when $\alpha \neq 0$ and $\delta \neq 0$. Domowitz and Hakkio [1985] provide evidence of non-zero constant risk premium for the United Kingdom and Japan's currencies. However, they cannot reject the null hypothesis of no risk premium for the currencies of Germany, France and Switzerland.

In 1987 Engle et al. [1987] introduced ARCH-in-mean (ARCH-M) models were the conditional variance is a determinant of the risk premium. These models allow conditional variance to affect the mean. By applying ARCH-M, they modelled the interest rate time series. Berk and Knot [2001] apply the ARCH-M model to estimate the UIP relationship with a time-varying risk premium. Malliaropulos [1997] uses multivariate generalised autoregressive conditional heteroscedasticity in mean model (GARCH-M) to explain the risk premium as a function of cross-currency conditional covariance. Following Bollerslev [1990] and Baillie and Bollerslev [1990], he employs a particular parametrisation of the multivariate GARCH process to model the conditional covariance matrix of unforecastable components of deviation from uncovered interest rate parity. Malliaropulos [1997] provides evidence of a time-varying risk premium for all currencies studied. His model suggests that time-variation in risk premium results from the time-varying conditional second moments determining conditional betas in the capital asset pricing model and fluctuations in expected excess market returns.

## METHODOLOGY AND DATA

The forward premium puzzle can be reflected by assuming that the forward exchange rate does not provide an unbiased forecast of the future spot rate. One possible explanation for the forward discount bias is the existence of a non-zero risk premium. In the paper, the GARCH-M($q$,$p$) model is applied to test the risk premium in the USD/PLN and AUD/JPY exchange rate markets. The class of GARCH-M are often used to test for the risk premium in financial time series. Based on the equations (6) and (7) the applied GARCH-M model is described as follows:

$$\begin{cases} f_t^{(k)} - s_{t+k} = \alpha + \delta h_{t+k} + \varepsilon_{t+k} \\ \varepsilon_{t+k} = \sqrt{h_{t+k}}\vartheta_{t+k} \\ h_{t+k} = \varphi_0 + \sum_{i=1}^{q} \varphi_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \varphi_j h_{t-j} + \gamma VIX_t \\ \vartheta_{t+k} \sim GED(0,1,\omega) \end{cases} \tag{8}$$

The difference between logarithmic values of forward exchange rate at time $t$ for $k$ periods ahead ($f_t^{(k)}$) and the logarithmic values of spot exchange rate in time $t+k$ ($s_{t+k}$) reflects the risk premium ($p_{t+k}$). The risk premium is described by the linear function of conditional variance with a constant component (α) and a time-varying component ($\delta h_{t+k}$). The conditional variance is parameterised as a function of the information set available to investors [Engle et al. 1987]. The variance equation contains previous innovations or surprises ($\varepsilon_{t-i}^2$) and the VIX Index (S&P 500 option-implied volatility index) that is treated as a benchmark for the uncertainty level in the financial markets. The VIX Index, created by Whaley [2009], is perceived as the popular measure of investors' attitude towards risk [Coudert, Gex 2008] and is widely used as a barometer for financial market uncertainty. The increase in the VIX Index might be associated with an appreciation of safe haven (e.g. US dollar) or low-yielding (e.g. Japanese yen) currencies [Clarida et al. 2009]. The model captures the conditional distribution's fat tails by applying the generalised error distribution (GED). The series stationarity is checked based on the augmented Dickey-Fuller (ADF) test [1979]. The orders $q$ and $p$ in GARCH-M ($q$,$p$) are selected based on the Akaike Information Criterion (AIC) [1979]. The models are estimated with the maximum likelihood method. The log-likelihood function was maximised using Marquardt's algorithm [1963].

Data set consists of weekly AUD/JPY and USD/PLN spot exchange rates, weekly AUD/JPY and USD/PLN 1-week forward exchange rates and weekly VIX Index logarithmic changes. Non-overlapping weekly data with 1-week forward exchange rates are applied to avoid possible estimation biases in standard errors arising from overlapping data. The study covers the period from 13 June 2006 to 24 November 2020. The time range results from the availability of data on forward exchange rates. Data are obtained from Refinitiv Datastream.

## EMPIRICAL RESULTS

ARCH-M model (8) is built based on the stationarity time series, i.e. the difference between logarithmic values of forward exchange rate at time $t$ for $k$ periods ahead ($f_t^{(k)}$) and the logarithmic values of spot exchange rate in time $t+k$ ($s_{t+k}$), and the logarithmic changes of the VIX Index ($\ln(\frac{VIX_{t+k}}{VIX_t})$). The model is built for $k$ equals 1 week. The Augmented Dickey-Fuller tests indicate that the null hypothesis of a unit root is rejected for all considered time series, at the 1% significance level. Based on the Akaike information criterion (AIC), the ARCH-M(1) model is selected. The sign and bias test for asymmetric GARCH effects does not reveal any asymmetry; thus, the application of asymmetric GARCH models such as threshold GARCH or exponential GARCH is not justified.

Table 1 presents the results of the ARCH(1)-M models for USD/PLN and AUD/JPY exchange rates. The intercept $\alpha$ is significant at the 1% level, which means

that there is a constant risk premium both in the USD/PLN and AUD/JPY markets (table 1). The coefficient of the time-varying risk premium $\delta$ is significant at the 1% level for both analysed exchange rates; however, it takes negative values for USD/PLN exchange rate and positive values for AUD/JPY exchange rate. The USD/PLN represents the exchange rates where base currency, i.e. the US dollar is a lower-yielding currency and Polish zloty a higher-yielding currency. When there is an increase of risk in the market, the US dollar appreciates against Polish zloty and the USD/PLN exchange rate increases. It is worth emphasising the US dollar is an example of a safe-haven currency that tends to appreciate during high-volatility, crisis periods [Hossfeld, MacDonald 2015; Wen, Cheng 2018]. Thus, the risk premium $f_t^{(k)} - s_{t+k}$ in the USD/PLN market takes negative values when the risk measured by conditional variance increases. Estimated negative coefficient $\delta$ implies that when there is an increase in risk, the appreciation of base currency (US dollar) and depreciation of quote currency (PLN) increases. The results confirm the US dollar as a safe-haven currency that tends to appreciate during high-volatility and crisis periods.

Table 1. ARCH(1)-M model results

| Exchange rates | Parameters | Estimated Coefficients | P-values |
|---|---|---|---|
| USD/PLN | $\alpha$ | 0.004 | 0.002 |
| | $\delta$ | -10.209 | 0.007 |
| | $\varphi_0$ | 0.001 | <0.001 |
| | $\varphi_1$ | 0.296 | <0.001 |
| | $\gamma$ | 0.001 | <0.001 |
| | $\omega$ | 1.289 | <0.001 |
| AUD/JPY | $\alpha$ | -0.027 | <0.001 |
| | $\delta$ | 75.666 | <0.001 |
| | $\varphi_0$ | 0.001 | <0.001 |
| | $\varphi_1$ | 0.022 | 0.003 |
| | $\gamma$ | 0.001 | <0.001 |
| | $\omega$ | 1.029 | <0.001 |

Source: own calculation based on data from Refinitiv Datastream

The AUD/JPY represents the exchange rates where base currency, i.e. the Australian dollar is a higher-yielding currency and Japanese yen a lower-yielding currency. When there is an increase of risk in the market, the Japanese yen appreciates against the Australian dollar and the AUD/JPY exchange rate decreases. The Japanese yen is an example of funding currency in the well-known currency strategies carry trade and tends to appreciate during high-volatility, crisis periods

[Liu et al. 2012; Czech 2020]. Thus, the risk premium $f_t^{(k)} - s_{t+k}$ in the AUD/JPY market takes positive values when the risk measured by conditional variance increases. Estimated positive coefficient $\delta$ implies that when there is an increase in risk, the appreciation of quote currency (Japanese yen) increases. The significantly different than zero coefficient $\delta$ suggests that there are risk-averse market participants in the market, and they require more return when they face a higher risk. The research results indicate the existence of a statistically significant, non-zero risk premium in the analyzed currency markets. The results are in line with Froot and Thaler [1990], Li et al. [2012], Kumar [2019], among other.

Variance equation coefficients are positive and significant in both estimated ARCH-M(1) models (table 1). The variance equation includes an additional independent variable, i.e. S&P 500 option-implied volatility index VIX, that reflects the stock market uncertainty. The results show that the VIX Index has a positive and significant impact on the USD/PLN and AUD/JPY conditional variance. It means that when the stock market uncertainty rises the volatility in the analysed foreign exchange markets increases.

## SUMMARY

One of the most puzzling features of the foreign exchange market is the tendency of low interest-yielding currencies to depreciate rather than appreciate as the uncovered interest rate parity suggests. One possible reason for the UIP failure is the existence of a risk premium. The paper is focused on risk premium inherent in the uncovered interest rate parity condition. It aims to assess whether there exists a significant and time-varying risk premium in the foreign exchange market. The research is carried out for two currency pairs, i.e. AUD/JPY and USD/PLN from June 2006 to November 2020. The paper applies ARCH-M models.

The results reveal a significantly different than zero risk premium in the USD/PLN and AUD/JPY foreign exchange markets. The time-varying risk premium coefficient is significant at the 1% level for both analysed exchange rates; however, it takes negative values for USD/PLN exchange rate and positive values for AUD/JPY exchange rate. The USD/PLN market's risk premium takes negative values when the risk measured by conditional variance increases. The estimated negative coefficient of the time-varying risk premium implies that when there is a surge in risk, the appreciation of the US dollar and depreciation of Polish zloty increases. The results confirm the US dollar as a safe-haven currency that tends to appreciate during high-volatility and crisis periods. The risk premium in the AUD/JPY market takes positive values when the risk measured by conditional variance increases. The estimated positive coefficient of the time-varying risk premium implies that when there is a mount in risk, the appreciation of lower-yielding Japanese yen increases. The Japanese yen is an example of funding currency

in the well-known currency strategies carry trade and tends to appreciate during high-volatility periods.

Moreover, the study shows that that the VIX Index has a positive and significant impact on the conditional variance of USD/PLN and AUD/JPY. It means that when the stock market uncertainty rises the volatility in the analysed foreign exchange markets increases. The results suggest that there exist non-zero risk premium in the foreign exchange market, and it might contribute to the existence of UIP failure.

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