QUANTITATIVE METHODS IN ECONOMICS
Volume XXII, No. 1, 2021, pp. 39 – 50

PRICING EUROPEAN OPTIONS IN THE HESTON AND THE DOUBLE HESTON MODELS

Arkadiusz Orzechowski[1] https://orcid.org/0000-0003-2872-189X
Institute of Risk and Financial Markets
Warsaw School of Economics – SGH, Poland
e-mail: aorzec@sgh.waw.pl

Abstract: Two models of pricing European options are presented and compared in this paper, i.e. the Heston model and the double Heston model. As the models belong to the class of stochastic volatility models, particular attention is paid to the way the characteristic functions and their inverse Fourier transforms are determined. The aim of the study is to investigate computational efficiency of pricing European calls. The method applied is based on the assumption that the prices of the derivatives are evaluated by means of Gauss-Kronrod quadrature.

Keywords: option pricing, the Heston model, the double Heston model, characteristic functions

JEL classification: C02, G13

INTRODUCTION

2021 has seen a significantly increased interest in the options market. According to Options Clearing Corporation (OCC), over 850 million option contracts were traded in December, 2021, a 12,4% growth as compared to December, 2020. Full year average daily cleared contract volume for 2021 was over 39 million, a 32,5% growth as compared to 2020[2]. In the period analyzed, equity options were the fastest to gain the market share (a 33,7% growth as compared to 2020), followed by index and ETF options (8,8% and 5,1% increase, respectively, compared to 2020).

[1] The views expressed in the article are the personal views of the author and do not express the official position of the institution in which he is employed.

[2] https://www.theocc.com/Newsroom/Press-Releases/2022/01-04-OCC-Clears-Record-Setting-9-93-Billion-Total [access: 04.01.2022].

https://doi.org/10.22630/MIBE.2021.22.1.4

In 2021 significant increase in trading options activity was reported among retail investors who were responsible for more than 25% of the total options trading volume. Easy access to commission-free online brokers was one of the main factors influencing the market structure. Such trading environment allowed retail investors to implement strategies based on execution of high-speed transactions.

As vast majority of retail option traders in 2021 were involved in basic call or put contracts, numerous different models of pricing options could be applied. Among the possible approaches to the valuation of options the most popular is the Black-Scholes model [Black & Scholes 1973]. The model has been widely used for years by both theoreticians and practitioners, because it enables fast obtainment of option prices. Unfortunately, this affects the accuracy of pricing, because the model is based on many unrealistic assumptions, e.g. constant variance of underlying asset's returns. As a result, many alternative approaches to the valuation of options have been proposed. They all can be divided into three categories, i.e. pure jump models, jump-diffusion models and stochastic volatility models (with all their variations).

In the pure jump models it is assumed that the price of the underlying asset changes in a discrete manner. As the discontinuities in the price movements of the underlying asset can be modelled in a number of ways many different methods of pricing options have been proposed, e.g. Madan et al. [1998], Madan et al. [1991], Carr et al. [2002], Eberlein et al. [1998].

An extension of the Black-Scholes model by possible discontinuities in the prices of the underlying asset allows for the valuation of options in the jump-diffusion models, e.g. Merton [1976], Kou [2002]. The construction of the models is based on the assumption stating that the continuous changes in the prices of the underlying asset can be occasionally disrupted by jumps. In these models both the frequency and the amplitude of the jumps can be modelled using different processes.

The stochastic volatility models assume the volatility of the underlying asset prices to be inconstant. The process responsible for the dynamics of the volatility can take different forms, depending on the model applied to the valuation of options [Heston 1993, Christoffersen et al. 2009]. It is worth noting that the stochastic volatility models are extended not only by changing the dynamics of the volatility, but also by introducing assumptions concerning the price dynamics of the underlying asset, e.g. Bates [2006].

The aim of the article is to compare the Heston model [Heston 1993] with the Christoffersen et al. model (further referred to as the double Heston model) [Christoffersen et al. 2009] in terms of computational speed, based on the example of pricing European calls. The article consists of several sections. In the first section two models of pricing European calls are formulated. In the second section the characteristic functions are applied to the process of option valuation. The third section includes the determination of the inverse Fourier transforms for the previously introduced characteristic functions. The speed of pricing European calls is also analyzed. Finally, the article has been summarized and major conclusions have been drawn.

THE HESTON AND THE DOUBLE HESTON MODELS

In this section, the Heston and the double Heston models are formulated and then applied to the valuation of the European calls. For this purpose the originally derived characteristic functions and their inverse Fourier transforms are applied.

The Heston model

The derivation procedure of the Heston model [Heston 1993] starts from two equations:

$$dS_t = \mu S_t dt + \sqrt{\sigma_t^2} S_t dW_{1,t}. \tag{1}$$

$$d\sigma_t^2 = \kappa(\theta - \sigma_t^2)dt + \upsilon\sqrt{\sigma_t^2} dW_{2,t}. \tag{2}$$

where: S_t denotes the spot price of the underlying asset at time t, σ_t^2 is the instantaneous variance, $\mu, \theta, \kappa, \upsilon$ are the constants associated with the drift, the long-term variance, the mean-reversion rate, and the volatility of the variance process, respectively. In the Heston model the Brownian motions W_1 and W_2 are correlated with a constant ρ.

Valuation of a European call is based on the following formula:

$$C^H(s_t, \sigma_t^2, t) = S_t P_1^H(s_t, \sigma_t^2, \tau) - e^{-r\tau} K P_2^H(s_t, \sigma_t^2, \tau). \tag{3}$$

where: $\tau = T - t$, r is the risk-free rate, K is the exercise price, $P_1^H(s_t, \sigma_t^2, \tau)$ and $P_2^H(s_t, \sigma_t^2, \tau)$ are unknown probabilities of expiring a European call in-the-money calculated as the inverse Fourier transform of characteristic function (for $j = 1,2$), i.e.:

$$P_j^H(s_t, \sigma_t^2, \tau) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \Re\left(\frac{e^{-\mathbb{I}\xi lnK}\phi^{j,H}(\xi, s_t, \sigma_t^2)}{\mathbb{I}\xi}\right) d\xi. \tag{4}$$

where: $\Re(.)$ is the real part of the subintegral function, $\mathbb{I}$ is the imaginary unit of the complex number, $\phi^{j,H}(\xi, s_t, \sigma_t^2)$ is the characteristic function of $s_t = lnS_t$ (corresponding to $P_j^H(s_t, \sigma_t^2, \tau)$). The remaining notation is the same as previously introduced.

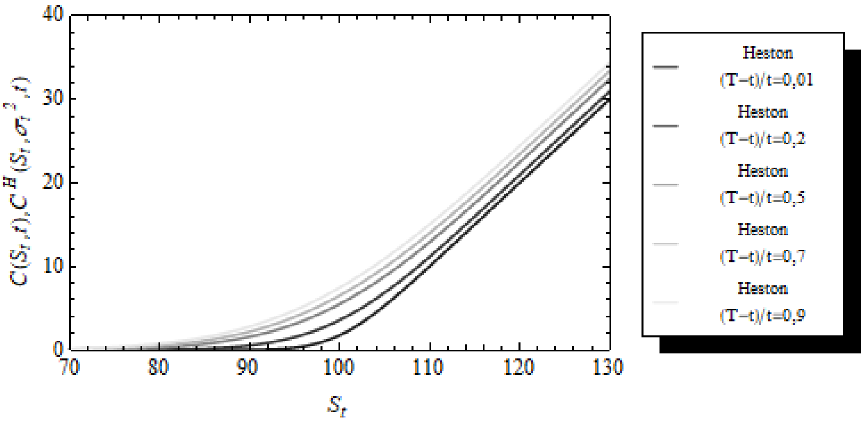
In the Heston model the general form of the characteristic function of s_t (corresponding to P_j^H, for $j = 1,2$) is expressed in the following form:

$$\phi^{j,H}(\xi, s_t, \sigma_t^2) = e^{C_j(\xi,\tau) + D_j(\xi,\tau)\sigma_t^2 + \mathbb{I}\xi s_t}. \tag{5}$$

where: $C_j(\xi,\tau) = r\mathbb{I}\xi\tau + \frac{a}{\upsilon^2}\left[(b_j - \upsilon\rho\mathbb{I}\xi + d_j)\tau - 2ln\left(\frac{1-g_j e^{d_j\tau}}{1-g_j}\right)\right]$, $D_j(\xi,\tau) = \frac{b_j - \upsilon\rho\mathbb{I}\xi + d_j}{\upsilon^2}\left(\frac{1-e^{d_j\tau}}{1-g_j e^{d_j\tau}}\right)$, $u_1 = \frac{1}{2}$, $u_2 = -\frac{1}{2}$, $a = \kappa\theta$, $b_1 = \kappa + \lambda - \upsilon\rho$, $b_2 = \kappa + \lambda$, $g_j = \frac{b_j - \upsilon\rho\mathbb{I}\xi + d_j}{b_j - \upsilon\rho\mathbb{I}\xi - d_j}$, $d_j = \sqrt{(\upsilon\rho\mathbb{I}\xi - b_j)^2 - \upsilon^2(2u_j\mathbb{I}\xi - \xi^2)}$.

The figure below presents the payoff functions of a European call in the Heston model ($C^H(S_t, \sigma_t^2, t)$) assuming that: $S_t \in [70, 130]$, $K = 100$, $\sigma_t = 0.2$, $r = 5\%$, $\upsilon = 0.3$, $\kappa = 1.5$, $\lambda = 3$, $\theta = 0.04$, $\rho = 0.8$ for different periods remaining to expiration, i.e. $\frac{T-t}{T} \in \{0.01; 0.2; 0.5; 0.7; 0.9\}$.

Figure 1. Payoff functions of a European call in the Heston model assuming that: $S_t \in [70, 130]$, $K = 100$, $\sigma_t = 0.2$, $r = 5\%$, $\frac{T-t}{T} \in \{0.01; 0.2; 0.5; 0.7; 0.9\}$, $\upsilon = 0.3$, $\kappa = 1.5$, $\lambda = 3$, $\theta = 0.04$, $\rho = 0.8$



Source: developed by the author

One of the Heston model features is its computational inefficiency. This is the result of the fact that in its original form two characteristic functions are used in the formula for the price of a European call. It makes the pricing process computationally more costly comparing to other approaches where only one characteristic function is implemented. This issue will be analyzed in more detail in the next section of this article.

The double Heston model

In the double Heston model [Christoffersen et al. 2009] three equations are used to describe the price process of the underlying asset, i.e.:

$$dS_t = \mu S_t dt + \sqrt{\sigma_{1,t}^2} S_t dW_{1,t} + \sqrt{\sigma_{2,t}^2} S_t dW_{2,t}. \tag{6}$$

$$d\sigma_{1,t}^2 = \kappa_1\left(\theta_1 - \sigma_{1,t}^2\right)dt + \upsilon_1\sqrt{\sigma_{1,t}^2}dW_{3,t}. \tag{7}$$

$$d\sigma_{2,t}^2 = \kappa_2\left(\theta_2 - \sigma_{2,t}^2\right)dt + \upsilon_2\sqrt{\sigma_{2,t}^2}dW_{4,t}. \tag{8}$$

where: S_t denotes the spot price of the underlying asset at time t, $\sigma_{1,t}^2$, $\sigma_{2,t}^2$ are two variance factors of the price process S_t, μ, θ_1, θ_2, κ_1, κ_2, υ_1, υ_2 are the constants associated with the drift, the long-term variance factors, the mean-reversion rates, and the volatilities of the variance factors processes, respectively. In the double Heston model the Brownian motions W_1 and W_2 are correlated with Brownian motions W_3 and W_4 with a constants ρ_1 and ρ_2.

The price of a European call in the double Heston model can be determined using the same formula as in the case of the Heston model except of the fact that the probabilities of expiring a European call in-the-money are calculated as follows:

$$P_1^{dH}\left(s_t,\sigma_{1,t}^2,\sigma_{2,t}^2,\tau\right)=\frac{1}{2}+\frac{1}{\pi}\int_0^{\infty}\Re\left(\frac{e^{-\mathbb{I}\xi lnK}\phi^{dH}(\xi-\mathbb{I},s_t,\sigma_{1,t}^2,\sigma_{2,t}^2)}{\mathbb{I}\xi s_t e^{r\tau}}\right)d\xi. \tag{9}$$

$$P_2^{dH}\left(s_t,\sigma_{1,t}^2,\sigma_{2,t}^2,\tau\right)=\frac{1}{2}+\frac{1}{\pi}\int_0^{\infty}\Re\left(\frac{e^{-\mathbb{I}\xi lnK}\phi^{dH}(\xi,s_t,\sigma_{1,t}^2,\sigma_{2,t}^2)}{\mathbb{I}\xi}\right)d\xi. \tag{10}$$

where: $\phi^{dH}\left(\xi,s_t,\sigma_{1,t}^2,\sigma_{2,t}^2\right)$ is the characteristic function of $s_t=lnS_t$. The remaining notation is the same as previously introduced.

In the double Heston model the general form of the characteristic function of s_t differs from the one appearing in the Heston model and takes the following form:

$$\phi^{dH}\left(\xi,s_t,\sigma_{1,t}^2,\sigma_{2,t}^2\right)=e^{A(\xi,\tau)+B_1(\xi,\tau)\sigma_{1,t}^2+B_2(\xi,\tau)\sigma_{2,t}^2+\mathbb{I}\xi s_t}. \tag{11}$$

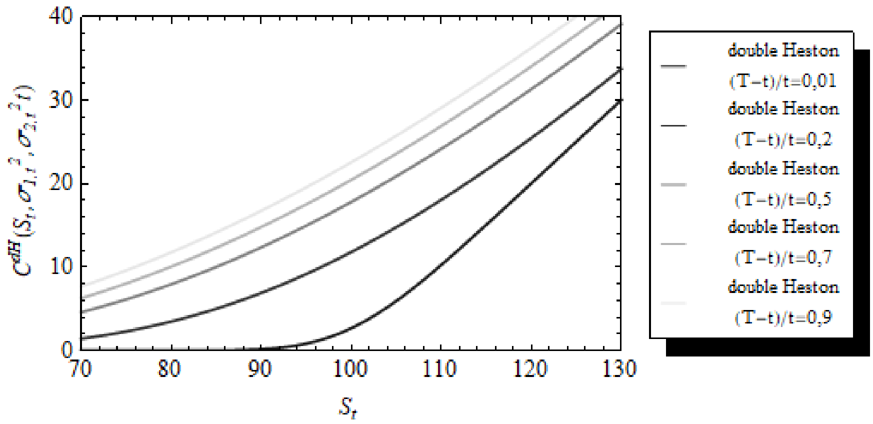
where: $A(\xi,\tau)=r\mathbb{I}\xi\tau+\sum_{j=1}^{2}\frac{\kappa_j\theta_j}{v_j^2}\left[\left(\kappa_j-\rho_j v_j\mathbb{I}\xi+d_j\right)\tau-2ln\left(\frac{1-g_j e^{d_j\tau}}{1-g_j}\right)\right]$,

$B_j(\xi,\tau)=\frac{\kappa_j-v_j\rho_j\mathbb{I}\xi+d_j}{v_j^2}\left(\frac{1-e^{d_j\tau}}{1-g_j e^{d_j\tau}}\right)$, $g_j=\frac{\kappa_j-v_j\rho_j\mathbb{I}\xi+d_j}{b_j-v_j\rho_j\mathbb{I}\xi-d_j}$, and

$d_j=\sqrt{\left(\kappa_j-v_j\rho_j\mathbb{I}\xi\right)^2+v_j^2\xi(\xi+\mathbb{I})}$. The remaining notation is the same as previously introduced.

The figure below presents the payoff functions of a European call in the double Heston model ($C^{dH}\left(S_t,\sigma_{1,t}^2,\sigma_{2,t}^2,t\right)$) assuming that: $S_t\in[70,130]$, $K=100$, $\sigma_{1,t}=0.2$, $\sigma_{2,t}=0.25$, $r=5\%$, $v_1=0.3$, $v_2=0.35$, $\kappa_1=1.5$, $\kappa_2=1.1$, $\lambda=3$, $\theta_1=0.04$, $\theta_2=0.06$, $\rho_1=0.8$, $\rho_2=0.2$ for different periods remaining to expiration, i.e. $\frac{T-t}{T}\in\{0.01;0.2;0.5;0.7;0.9\}$.

Figure 2. Payoff functions of a European call in the double Heston model assuming that: $S_t \in [70, 130]$, $K = 100$, $\sigma_{1,t} = 0.2$, $\sigma_{2,t} = 0.25$, $r = 5\%$, $\frac{T-t}{T} \in \{0.01; 0.2; 0.5; 0.7; 0.9\}$, $\upsilon_1 = 0.3$, $\upsilon_2 = 0.35$, $\kappa_1 = 1.5$, $\kappa_2 = 1.1$, $\lambda = 3$, $\theta_1 = 0.04$, $\theta_2 = 0.06$, $\rho_1 = 0.8$, $\rho_2 = 0.2$



Source: developed by the author

It is worth noting that the double Heston model shares the same drawbacks as the Heston model. It means that in its original form it is inefficient. Luckily there are some other methods of calculating inverse Fourier transforms of characteristic function $\phi^{dH}(\xi, s_t, \sigma_{1,t}^2, \sigma_{2,t}^2)$ which allow to lessen computational effort related to pricing European options.

CHARACTERISTIC FUNCTIONS

There are many approaches to determining characteristic function of s_t and calculating its inverse Fourier transform [Carr & Madan 1999, Attari 2004, Bates 2006 and Orzechowski 2018]. As some of the approaches has already been presented [Orzechowski 2020] in the later part of the article only formulas concerning the double Heston model are of interest. As before, for the purpose of the article, it is assumed that $t = 0$. The remaining notation remains consistent with the previously introduced.

The double Heston model

1. The Carr & Madan approach [Carr, Madan 1999] for $\alpha = 1$:

$$C^{dH}(S_0, \sigma_{1,0}^2, \sigma_{2,0}^2, 0) = \frac{e^{-\alpha k}}{\pi} \int_0^\infty \Re\left(e^{-\mathbb{I}\xi k} \frac{e^{-rT} \phi^{dH}(\xi - (\alpha+1)\mathbb{I}, s_0, \sigma_{1,0}^2, \sigma_{2,0}^2)}{\alpha^2 + \alpha - \xi^2 + \mathbb{I}(2\alpha+1)\xi}\right) d\xi. \quad (12)$$

2. The Attari approach [Attari 2004]:

$$C^{dH}\left(S_0,\sigma_{1,0}^2,\sigma_{2,0}^2,0\right)=S_0\left(1+\frac{e^l}{\pi}\int_0^\infty \Re\left(\frac{e^{-\mathbb{I}\xi l}}{\mathbb{I}(\xi+\mathbb{I})}\psi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)\right)d\xi\right)+$$
$$-e^{-rT}K\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \Re\left(\frac{e^{-\mathbb{I}\xi l}}{\mathbb{I}\xi}\psi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)\right)d\xi\right). \tag{13}$$

where: $l=ln\left(\frac{K}{S_0e^{rT}}\right)$.

3. The Bates approach [Bates 2006]:

$$C^{dH}\left(S_0,\sigma_{1,0}^2,\sigma_{2,0}^2,0\right)=$$
$$=S_0-e^{-rT}K\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \Re\left(\frac{e^{-\mathbb{I}\xi ln\left(\frac{K}{S_0}\right)}}{\mathbb{I}\xi(1-\mathbb{I}\xi)}\right)\varphi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)d\xi\right). \tag{14}$$

4. The Orzechowski approach [Orzechowski 2018]:

$$C^{dH}\left(S_0,\sigma_{1,0}^2,\sigma_{2,0}^2,0\right)=\frac{1}{2}S_o+e^{-rT}\frac{1}{\pi}\int_0^\infty \Re\left(e^{-\mathbb{I}\xi k}\frac{\phi^{dH}\left(\xi-\mathbb{I},s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)}{\mathbb{I}\xi(\mathbb{I}\xi+1)}\right)d\xi. \tag{15}$$

It is worth noting that: $\phi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)$, $\psi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)$ as well as $\varphi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)$ are characteristic functions determined by the following equations:

$$\phi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)=e^{A(\xi,\tau)+B_1(\xi,\tau)\sigma_{1,t}^2+B_2(\xi,\tau)\sigma_{2,t}^2+\mathbb{I}\xi s_t}. \tag{16}$$

$$\psi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)=\phi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)e^{-\mathbb{I}\xi s_0-\mathbb{I}\xi rT}. \tag{17}$$

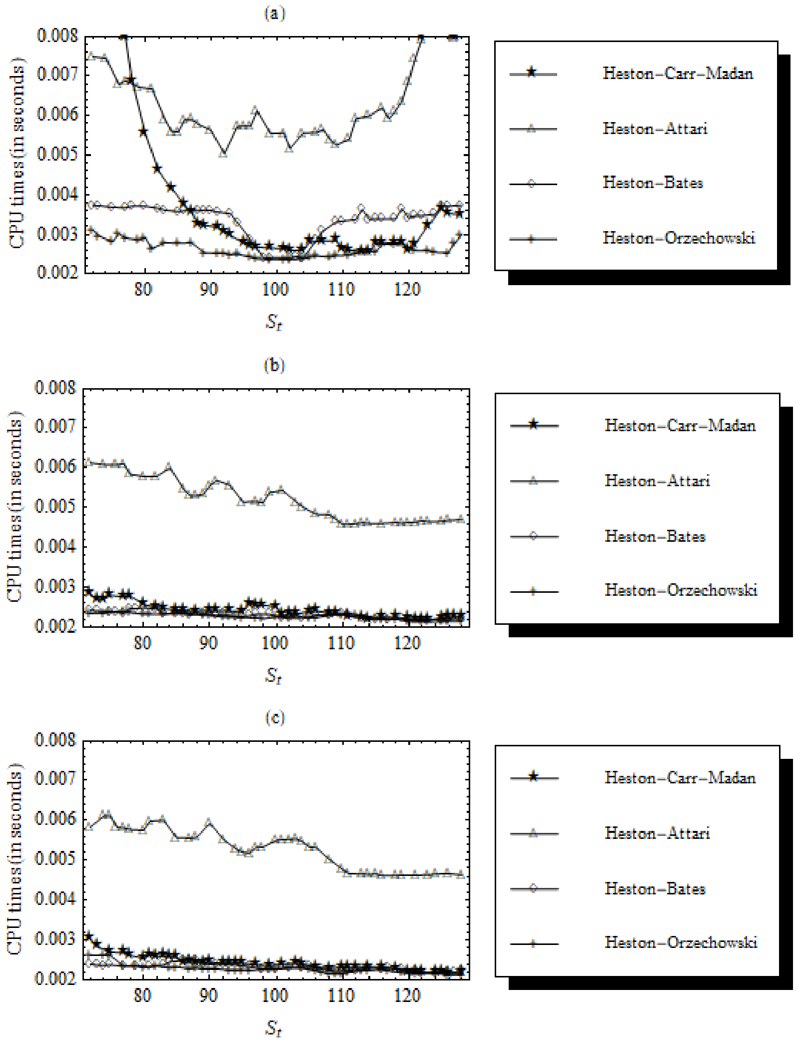
$$\varphi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)=\phi^{dH}\left(\xi,s_0,\sigma_{1,0}^2,\sigma_{2,0}^2\right)e^{-\mathbb{I}\xi s_0}. \tag{18}$$

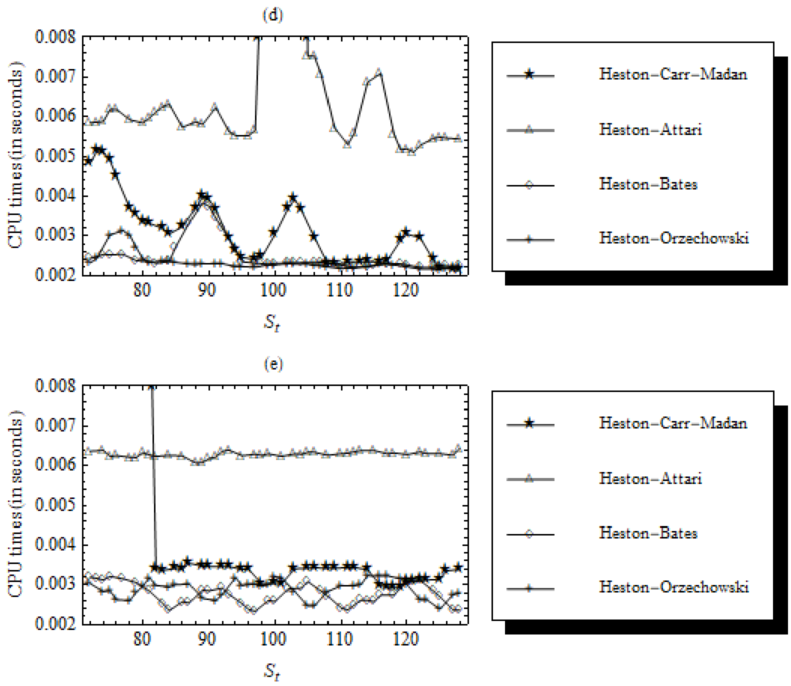
RESULTS

The determination of the most efficient approach both in the Heston and the double Heston models, is based on the results generated with the use of codes developed in Mathematica 10.2. The methodology proposed in the research is compatible with the approach applied previously [Orzechowski 2020]. It means that the theoretical prices of European calls have been evaluated numerically by means of the Gauss-Kronrod quadrature. Additionally the graphs have been smoothed by averaging runs of five elements. It is also worth noting that hardware with the same characteristics has been used for the computation purposes. Cache memory has been cleared before starting codes allowing for the valuation of options.

The results of the research carried out are shown in the graphs below - see Figures 3 and 4.

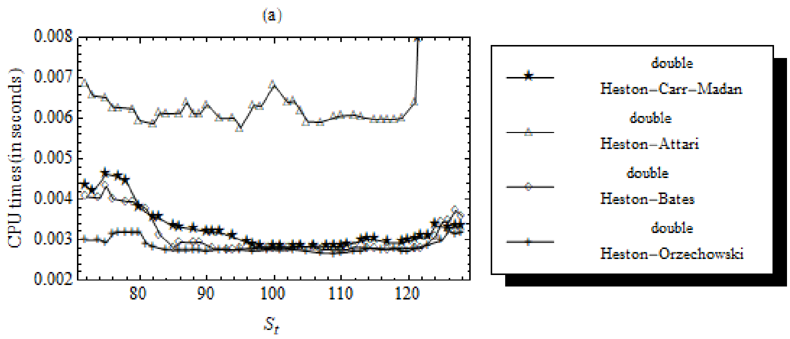
Figure 3. Computational speed in the Heston model assuming that: $S_t \in [70, 130]$, $K = 100$, $\sigma_t = 0.2$, $r = 5\%$, $\upsilon = 0.3$, $\kappa = 1.5$, $\lambda = 3$, $\theta = 0.04$, $\rho = 0.8$ for (a) $\frac{T-t}{T} = 0.01$, (b) $\frac{T-t}{T} = 0.2$, (c) $\frac{T-t}{T} = 0.5$, (d) $\frac{T-t}{T} = 0.7$ and (e) $\frac{T-t}{T} = 0.9$

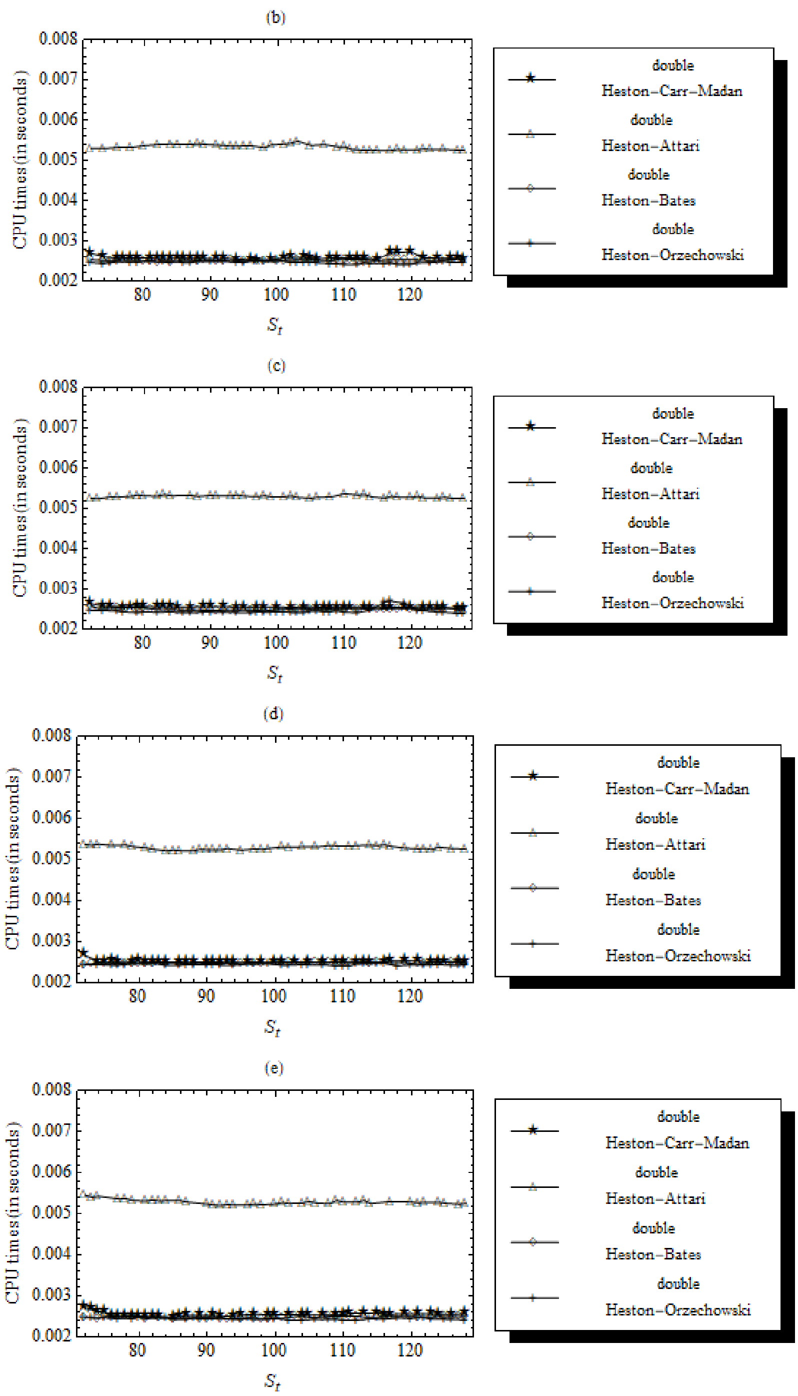




Source: developed by the author

Figure 4. Computational speed in the double Heston model assuming that: $S_t \in [70, 130]$, $K = 100$, $\sigma_{1,t} = 0.2$, $\sigma_{2,t} = 0.25$, $r = 5\%$, $\upsilon_1 = 0.3$, $\upsilon_2 = 0.35$, $\kappa_1 = 1.5$, $\kappa_2 = 1.1$, $\lambda = 3$, $\theta_1 = 0.04$, $\theta_2 = 0.06$, $\rho_1 = 0.8$, $\rho_2 = 0.2$ for (a) $\frac{T-t}{T} = 0.01$, (b) $\frac{T-t}{T} = 0.2$, (c) $\frac{T-t}{T} = 0.5$, (d) $\frac{T-t}{T} = 0.7$ and (e) $\frac{T-t}{T} = 0.9$





Source: developed by the author

The results obtained allow to state that the conclusions drawn previously [Orzechowski 2020] can be extended onto the double Heston model. It means that the computational efficiency of pricing European options depends on the way the characteristic functions and their inverse Fourier transforms are calculated. Such statement is correct not only for the Heston model, but also for the double Heston model used to pricing European options that are close to expiration. On the basis of Figures 3 and 4 it can be also easily concluded that the closer the time to expiration of the European options, the more computationally efficient is the method based on eq. 15. This is, however, not true for the European options close to the moment of their writing. In this case the results are more ambiguous.

SUMMARY

Two models of pricing European options, i.e. the Heston model and the double Heston model were analyzed in this article and then compared in terms of computational speed. Special attention in this regard was paid to the way the characteristic functions and their inverse Fourier transforms are calculated.

On the basis of the results obtained it can be concluded that the closer the time to expiration the greater is the advantage of the method based on eq. 15, regardless the model being considered. At the same time the closer the time to writing European options the more blurred become the differences in efficiency between the models. It is also important to note that these properties hold under the assumptions that the prices of the European calls are evaluated numerically by means of the Gauss-Kronrod quadrature.

REFERENCES

Attari M. (2004) Option Pricing Using Fourier Transform: A Numerically Efficient Simplification. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=520042, [access: 20 12.2021].

Bates D. S. (2006) Maximum Likelihood Estimation of Latent Affine Processes. Review of Financial Studies, 19(3), 909-965.

Black F., Scholes M. (1973) The Pricing of Options and Corporate Liabilities. Journal of Political Economy, 81(3), 637-654.

Carr P., Geman H., Madan D. B., Yor M. (2002) The Fine Structure of Asset Returns: An Empirical Investigation. Journal of Business, 75(2), 305-332.

Carr P., Madan D. B. (1999) Option Valuation Using the Fast Fourier Transform. Journal of Computational Finance, 2(4), 61-73.

Christoffersen P., Heston S. L., Jacob K. (2009) The Shape and Term Structure of the Index Option Smirk: Why Multifactor Stochastic Volatility Models Work So Well. Management Science, 55(12), 1914-1932.

Eberlein E., Keller U., Prause K. (1998) New Insights into Smile, Mispricing and Value at Risk: The Hyperbolic Model. Journal of Business, 71(3), 371-405.

Heston S. (1993) A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. The Review of Financial Studies, 6(2), 327-343.

Kou S. G. (2002) Jump - Diffusion Model for Option Pricing. Management Science, 48(8), 1086-1101.

Madan D. B., Carr P., Chang E. (1998) The Variance Gamma Process and Option Pricing Model. European Finance Review, 2(1), 79-105.

Madan D. B., Milne F. (1991) Option Pricing with VG Martingale Components, Mathematical Finance, 1(4), 39-55.

Merton R. C. (1976) Option Pricing When Underlying Stock Returns Are Discontinuous. Journal of Financial Economics, 3(1-2), 125-144.

Orzechowski A. (2018) Pricing Correlation Options: from the P. Carr and D. Madan Approach to the New Method Based on the Fourier Transform. Economics and Business Review, 4(1), 16-28.

Orzechowski A. (2020) Pricing European Options in Selected Stochastic Volatility Models. Quantitative Methods in Economics, 21(3), 145-156.