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RADAR MEASURES OF STRUCTURES' CONFORMABILITY

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Abstract: In the following work a new method was proposed to study similarity of objects' structures. This method is an adaptation of radar methods of objects' ordering and cluster analysis, which are being developed by the authors. The value added by the authors is the construction of measures for conformability of structures of two objects. Those measures may also be used to define similarities between given objects. Proposed measures are independent of the order of features.

Key words: radar method, radar measure of conformability, measure of similarity, synthetic measures, classification, cluster analysis.

INTRODUCTION

Authors have for many years been researching into the problem of regional measurement of differentiation of agriculture in both dynamic and static aspects [Binderman, Borkowski, Szczesny 2008, 2009a, 2009b, 2010; Borkowski, Szczesny 2002]. In economic-agricultural research based on empirical data almost invariably there is a need of ordering, classification and clustering of homesteads, objects (units) of a multidimensional space of variables. Study of regional differentiation of agriculture is crucial now because of EU's politic of regionalization of allocation of funds. Presently, there are many methods used for classification and clustering of objects [Gatnar, Walesiak 2009, Hellwig 1968, Kukuła 2000, Malina 2004, Młodak 2006, Pociecha 2009, Strahl 1990, Zeliaś 2000].

Dynamic analysis of regional differentiation of agriculture based on a single feature was the common ground of those studies. Key differences in evaluation of similarity or spatial differentiation of agriculture formed by different authors using different measurement methods were apparent. Methods of measuring conformability (differentiation) of structures in a dynamic aspect were seldom used in economic-agricultural research. The basis of comparative analysis of structures is a set of *m* spatial units (in our scenario voivodeships) characterized by *n* features. The problem of examining structures' conformability is present in numerous scientific publications, e.g. [Binderman, Borkowski, Szczesny 2008, Binderman, Szczesny 2009, Ciok, Kowalczyk, Pleszczyńska, Szczesny 1995, Kukuła 2000, 2010, Ostasiewicz 1999]. In order to compare structures different methods are used, depending on the goals of research, possibility of evaluation, interpretation of analysis results and desired algebraic and statistical properties. Many methods are constructed intuitively, based on graphical analysis. Radar methods, which are used to display objects defined by a number of features, are an example of such methods. Synthetic index is constructed based on the area of a polygon which is used to illustrate objects in question. This method is simple and intuitive but has a serious flaw because the field value is dependent on the order of features. Our research is aimed at eliminating this flaw. Several proposed indices without this flaw are presented in [Binderman, Borkowski, Szczesny 2008] and [Binderman, Szczesny 2009]. In this work we present a manner in which the idea of measures based on the area of a polygon may be used to measure the conformability of two structures. This manner creates opportunities of using those measures to compare agricultural regions, which are characterized by many features. The use of methods given in this work is included in the article [Binderman, Borkowski, Szczesny 2010]. For entire collection of the conformability of two structures see [Grabiński, Wydymus, Zeliaś 1989, Kukuła 1989, 2010, Malina 2004, Strahl 1985, 1996, Walesiak 1983, 1984].

CONSTRUCTION OF RADAR MEASURES OF CONFORMABILITY

In their previous works authors used radar methods to order and classify objects [Binderman, Borkowski, Szczesny 2008, 2009, 2009a, 2010, Binderman, Szczesny 2009, Binderman 2009, 2009a]. Those methods are independent of the manner of ordering of features that describe a given object. In this work authors attempted to adapt radar methods to compare structures of given objects. Methods presented below may seem to be complicated in terms of calculations. However, with the beginning of the digital age that became inconsequential. Moreover, software to perform calculations for those methods is being developed.
Let Q and R be two objects described by sets of values of *n* ($n > 2$) features. We assume that objects Q,R are described by two vectors $\mathbf{x},\mathbf{y} \in \Re_+^n$, where

$$\mathbf{x} = (x_1, x_2, ..., x_n), \mathbf{y} = (y_1, y_2, ..., y_n); x_i, y_i \geq 0; i = 1, 2, ..., n \text{ and } \sum_{i=1}^{n} x_i = 1, \sum_{i=1}^{n} y_i = 1.$$

For a geometric representation of the method we inscribe a regular n-gon into a unit circle (with a radius of one) centered at the origin in the polar coordinate system and connect the vertices of the polygon with the origin. Obtained line-segments with a length of one will be named, in sequence, O1, O2, …, On, for definiteness, beginning with the line-segment covering the w axis. Let's assume that at least two coordinates of each of the vectors x, y are nonzero. As features of objects x and y take on a value between 0 and 1, meaning

$$\mathbf{0}\le\mathbf{x}\le\mathbf{1}\equiv 0\le x_i\le 1,\ \mathbf{0}\le\mathbf{y}\le\mathbf{1}\equiv 0\le y_i\le 1,\ i=1,2,...n,\ \text{where } \mathbf{0}=(0,0,...,0),\ \mathbf{1}=(1,1,...,1),$$

it is possible to represent those values on a radar chart. To do this let x_i (y_i) denote points of intersections of axes Oi with circles centered at the origin of the coordinate system with a radius of x_i (y_i), i=1,2,...,n. By connecting points x_1 with x_2, x_2 with x_3, ..., x_n with x_1 (y_1 with y_2, y_2 with y_3, ..., y_n with y_1) we obtain n-gons S_Q and S_R, which areas $|S_Q|$, $|S_R|$ are given by

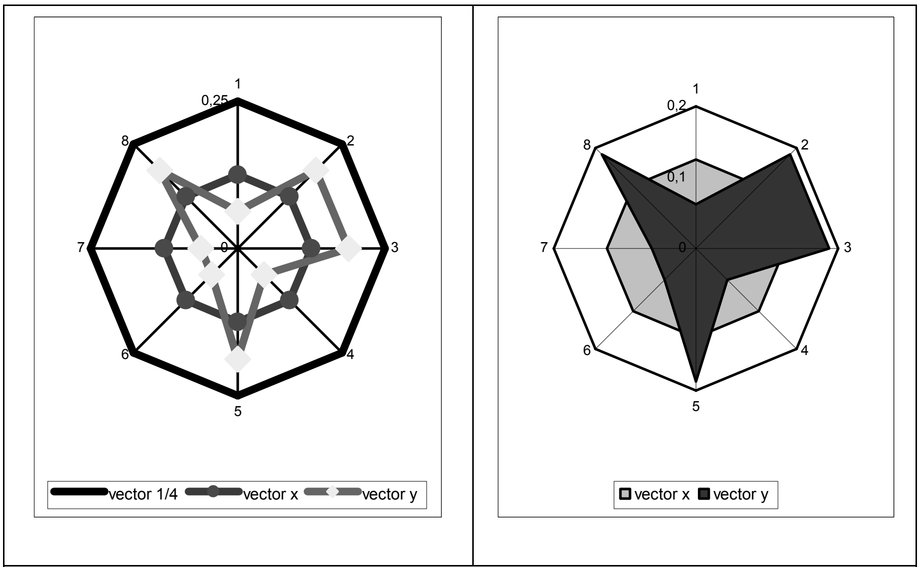
$$|S_Q| = |S_{\mathbf{x}}| = \sum_{i=1}^{n}\frac{1}{2}x_i x_{i+1}\sin\frac{2\pi}{n} = \frac{1}{2}\sin\frac{2\pi}{n}\sum_{i=1}^{n}x_i x_{i+1},\ \text{ where }\ x_{n+1} := x_1\ ,$$

$$|S_R| = |S_{\mathbf{y}}| = \sum_{i=1}^{n}\frac{1}{2}y_i y_{i+1}\sin\frac{2\pi}{n} = \frac{1}{2}\sin\frac{2\pi}{n}\sum_{i=1}^{n}y_i y_{i+1},\ \text{ where }\ y_{n+1} := y_1\ .$$

The following graph gives an illustration for vectors:

$$\mathbf{x} = \frac{\mathbf{1}}{\mathbf{8}} = (\frac{1}{8},\frac{1}{8},...,\frac{1}{8}),\quad \mathbf{y} = (\frac{1}{16},\frac{3}{16},\frac{3}{16},\frac{1}{16},\frac{3}{16},\frac{1}{16},\frac{1}{16},\frac{3}{16}),\ n = 8.$$

Illustration 1. Radar charts for vectors x and y



Source: our investigation

Given such a graphical illustration, each of the objects Q and R is defined by a polygon of vertices $Q_1,Q_2,\ldots,Q_n$ and $R_1, R_2,\ldots,R_n$, respectively. In a Cartesian coordinate system those points take on coordinates $Q_i(s_i,t_i)$, $R_i(w_i,z_i)$, $i=1,2,\ldots,n$; where

$$s_i = x_i \cos\varphi_i, \quad t_i = x_i \sin\varphi_i, \quad w_i = y_i \cos\varphi_i, \quad z_i = y_i \sin\varphi_i,$$

$$\varphi_i = (i-1)\frac{2\pi}{n}, \; i = 1,2,\ldots,n.$$

Let us denote the areas set by vectors **x** and **y** (describing objects Q and R) by S_x and S_y, respectively, and their intersection by:

$$S_{\mathbf{x}\cap\mathbf{y}} := S_{\mathbf{x}} \cap S_{\mathbf{y}}$$

Let us consider one segment of the area $S_{\mathbf{x}\cap\mathbf{y}} - \Pi_i$, contained within an angle

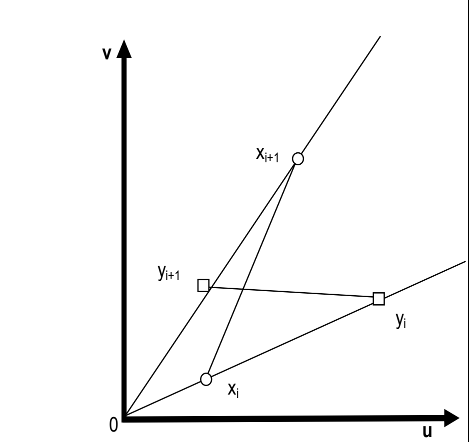
$[\frac{2\pi i}{n}, \frac{2\pi(i+1)}{n}]$. The following, mutually exclusive cases are possible:

1. $y_i > x_i > 0 \wedge x_{i+1} > y_{i+1} > 0 \quad \vee \quad x_i > y_i > 0 \wedge y_{i+1} > x_{i+1} > 0,$
2. $y_i = x_i > 0 \wedge x_{i+1} > y_{i+1} > 0 \quad \vee \quad x_i = y_i > 0 \wedge y_{i+1} > x_{i+1} > 0,$
3. $x_{i+1} = y_{i+1} > 0 \wedge x_i > y_i > 0 \quad \vee \quad x_{i+1} = y_{i+1} > 0 \wedge y_i > x_i > 0,$
4. $y_i = x_i > 0 \wedge x_{i+1} = y_{i+1} > 0,$
5. $x_i > y_i > 0 \wedge x_{i+1} > y_{i+1} > 0 \quad \vee \quad y_i > x_i > 0 \wedge y_{i+1} > x_{i+1} > 0,$

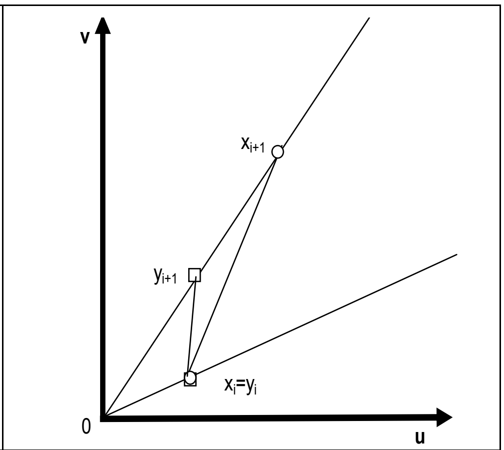
6. Product of coordinates $x_i y_i x_{i+1} y_{i+1} = 0$.

Below we provided representative cases of the above possible situations, linked with the manner of application of the equations.

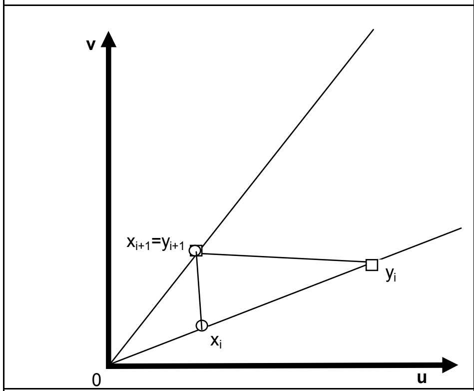
Illustration 2. Graphical representation of possible cases.



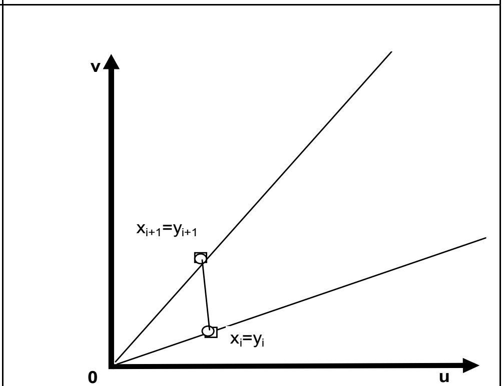
1. $y_i > x_i > 0 \wedge x_{i+1} > y_{i+1} > 0$



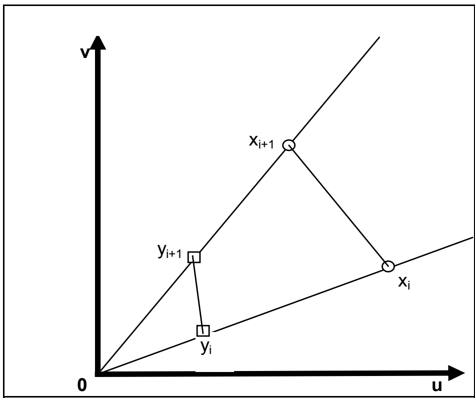
2. $y_i = x_i > 0 \wedge x_{i+1} > y_{i+1} > 0$



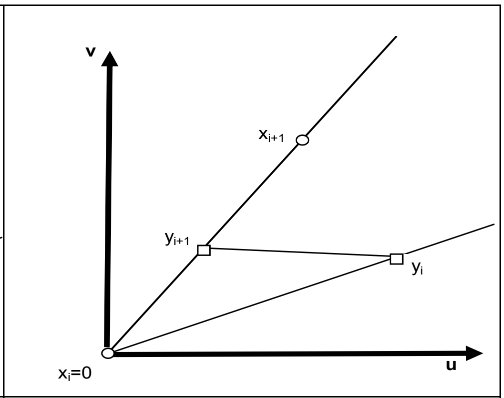
3. $x_{i+1} = y_{i+1} > 0 \wedge x_i > y_i > 0$



4. $y_i = x_i > 0 \wedge x_{i+1} = y_{i+1} > 0$,



5. $x_i > y_i > 0 \wedge x_{i+1} > y_{i+1} > 0$

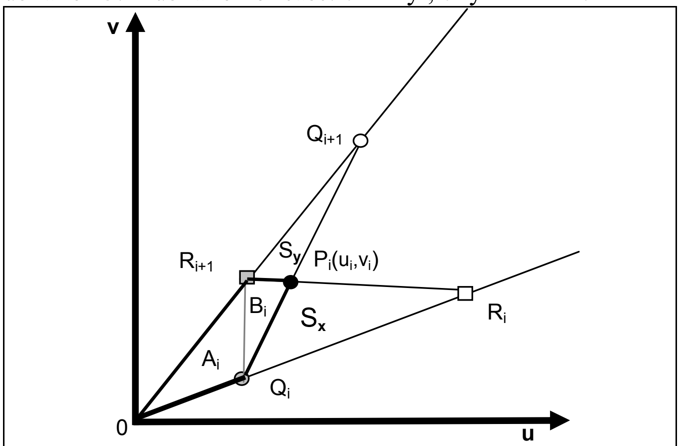


6. $x_i = 0$

Source: own investigation

Let us consider the first case when one of the segments of the area $S_{x \cap y} - P_i$, is a quadrilateral, given by the origin of the coordinate system O(0, 0) and points Q_i, R_{i+1}, which satisfies our assumption that $0<x_i<y_i$, $0<y_{i+1}<x_{i+1}$ (see illus. 3) $i \in \{1,2,\ldots,n\}$.

Illustration 3. Illustration of case: 0<xi<yi, 0<yi+1<xi+1.



Source: own investigation

Area P consists of two triangles A_i and B_i. Value of the area of triangle A_i is defined as $\left|A_i\right| = \frac{1}{2} x_i y_{i+1} \sin\frac{2\pi}{n}$. Thus, to calculate the area of the quadrilateral S_i:=$OQ_iP_iR_{i+1}$ we must find the value of the area of triangle B_i. In order to do this we notice that the line containing points $R_i(w_i,z_i)$ and $R_{i+1}(w_{i+1},z_{i+1})$ is described by the equation:

$$\left|\begin{pmatrix} u-w_i & v-z_i \\ w_{i+1}-w_i & z_{i+1}-z_i \end{pmatrix}\right| = 0$$

Whereas the line containing points $Q_i(s_i,t_i)$ i $Q_{i+1}(s_{i+1},t_{i+1})$ is described by the equality:

$$\left|\begin{pmatrix} u-s_i & v-t_i \\ s_{i+1}-s_i & t_{i+1}-t_i \end{pmatrix}\right| = 0.$$

Coordinates of point $P_i(u_i,v_i)$, which is the point of intersection between the above lines, are the solution to the following system of equations:

$$(z_{i+1}-z_i)u-(w_{i+1}-w_i)v = w_i z_{i+1} - z_i w_{i+1},$$
$$(t_{i+1}-t_i)u-(s_{i+1}-s_i)v = s_i t_{i+1} - s_{i+1} t_i.$$

The solutions can be described using Cramer's rule as follows:

$$u_i = \frac{W_u}{W},\ v_i = \frac{W_v}{W},\quad \text{where } W = \left|\begin{pmatrix} z_{i+1}-z_i & w_i-w_{i+1} \\ t_{i+1}-t_i & s_i-s_{i+1} \end{pmatrix}\right|,$$

$$W_u = \left|\begin{pmatrix} w_i z_{i+1}-z_i w_{i+1} & w_i-w_{i+1} \\ s_i t_{i+1}-s_{i+1}t_i & s_i-s_{i+1} \end{pmatrix}\right|, W_v = \left|\begin{pmatrix} z_{i+1}-z_i & w_i z_{i+1}-z_i w_{i+1} \\ t_{i+1}-t_i & s_i t_{i+1}-s_{i+1}t_i \end{pmatrix}\right|.$$

Let us notice that a line containing two points $Q_i(s_i,t_i)$ i $R_{i+1}(w_{i+1},z_{i+1})$ can be described by the equation:

$$\left|\begin{pmatrix} u-s_i & v-t_i \\ w_{i+1}-s_i & z_{i+1}-t_i \end{pmatrix}\right| = 0,$$

which is identical to $(z_{i+1}-t_i)u-(w_{i+1}-s_i)v - z_{i+1}s_i + w_{i+1}t_i = 0$.

The distance h between point $P_i(u_i,v_i)$ and line containing points $Q_i(s_i,t_i)$ and $R_{i+1}(w_{i+1},z_{i+1})$ is determined by the equality:

$$h_i = \frac{|(z_{i+1}-t_i)u_i-(w_{i+1}-s_i)v_i - z_{i+1}s_i + w_{i+1}t_i|}{\sqrt{(z_{i+1}-t_i)^2+(w_{i+1}-s_i)^2}} = \frac{|(z_{i+1}-t_i)u_i-(w_{i+1}-s_i)v_i - z_{i+1}s_i + w_{i+1}t_i|}{\sqrt{y_{i+1}^2+x_i^2-2x_i y_{i+1}\cos\frac{2\pi}{n}}}$$

We utilize the cosine rule to calculate the length of a_i, a line-segment between points $Q_i(s_i,t_i)$ and $R_{i+1}(w_{i+1},z_{i+1})$

$$a_i = \sqrt{y_{i+1}^2+x_i^2-2x_i y_{i+1}\cos\frac{2\pi}{n}}$$

Thus, we obtain that the area of the triangle B_i is equal to:

$$|B_i| = \frac{1}{2}a_i h_i = \frac{1}{2}\left|(z_{i+1} - t_i)u_i - (w_{i+1} - s_i)v_i - z_{i+1}s_i + w_{i+1}t_i\right|$$

and so the area of quadrilateral S_i is described by the equality:

$$|S_i| = |A_i| + |B_i| = \frac{1}{2}x_i y_{i+1}\sin\frac{2\pi}{n} + \frac{1}{2}\left|(z_{i+1} - t_i)u_i - (w_{i+1} - s_i)v_i - z_{i+1}s_i + w_{i+1}t_i\right|.$$

In a similar manner one can obtain the area of segment S_i which is a quadrilateral given by the origin of the coordinate system O(0, 0) and points Q_{i+1}, R_i, what corresponds with the assumption that $0<y_i<x_i$, $0<x_{i+1}<y_{i+1}$ $i\in\{1,2,\ldots,n\}$.

In the case of $x_i = y_i > 0$ oraz $x_{i+1}y_{i+1} > 0$ $\{x_{i+1} = y_{i+1} > 0$ oraz $x_i y_i > 0\}$ segment S_i is a triangle with an area described by the equality:

$$|S_i| = \frac{1}{2}\sin\frac{2\pi}{n}x_i\min(x_{i+1}, y_{i+1})\ \left\{|S_i| = \frac{1}{2}\sin\frac{2\pi}{n}x_{i+1}\min(x_i, y_i)\right\}.$$

In the case of $x_i > y_i > 0$ oraz $x_{i+1} > y_{i+1} > 0$ $\{y_i > x_i > 0$ oraz $y_{i+1} > x_{i+1} > 0\}$ Segment S_i is a triangle with an area described by the equality:

$$|S_i| = \frac{1}{2}\sin\frac{2\pi}{n}y_i y_{i+1}\ \left\{|S_i| = \frac{1}{2}\sin\frac{2\pi}{n}x_{i+1}x_i\right\}.$$

In the case of $x_i y_i x_{i+1} y_{i+1} = 0$ segment S_i is a line-segment or a point and its area is equal to 0: $|S_i| = 0$.

The area of the intersection of polygons $S_{\mathbf{x}}$ and $S_{\mathbf{y}}$ is described by the equality:

$$|S_{\mathbf{x}} \cap S_{\mathbf{y}}| = \sum_{i=1}^{n}|S_i|.$$

Let us assume $\boldsymbol{\mu}_{\mathbf{xy}}$ as a measure of conformability of structures of two objects Q and R induced by vectors **x** and **y**. thus:

$$\mu_{\mathbf{xy}} = \begin{cases} \sqrt{\dfrac{|S_{\mathbf{x}} \cap S_{\mathbf{y}}|}{\sigma_{\mathbf{xy}}}} & \text{for } n=3 \\ \sqrt{\dfrac{|S_{\mathbf{x}} \cap S_{\mathbf{y}}|}{\omega_{\mathbf{xy}}}} & \text{for } n\geq 4 \end{cases} \tag{1}$$

where $\sigma_{\mathbf{xy}} := \begin{cases} \min(|S_{\mathbf{x}}|, |S_{\mathbf{y}}|) & \text{gdy } |S_{\mathbf{x}}||S_{\mathbf{y}}|>0 \\ 1 & \text{gdy } |S_{\mathbf{x}}||S_{\mathbf{y}}|=0 \end{cases}$, $\omega_{\mathbf{xy}} := \begin{cases} \max(|S_{\mathbf{x}}|, |S_{\mathbf{y}}|) & \text{gdy } |S_{\mathbf{x}}||S_{\mathbf{y}}|>0 \\ 1 & \text{gdy } |S_{\mathbf{x}}||S_{\mathbf{y}}|=0 \end{cases}$.

Let us notice that the above measure of conformability satisfies $0\leq\mu_{x,y}\leq 1$ and is dependent on the order of features [Binderman, Borkowski, Szczesny 2008].

In order to define a measure of conformability which is independent of the order of features let us denote by p_j a j-th permutation of numbers 1, 2, …, n. There are *n!* such permutations. Each permutation corresponds to a permutation of coordinates of vectors **x** and **y**. Let $\mathbf{x}_j$, $\mathbf{y}_j$ denote the j-th permutation of coordinates of vectors **x** and **y** accordingly, where $\mathbf{x}_1=\mathbf{x}$ and $\mathbf{y}_1=\mathbf{y}$. E.g. if n=3, $\boldsymbol{x}=(x_1,x_2,x_3)$, $\boldsymbol{y}=(y_1,y_2,y_3)$ and $p_1=(1,2,3)$, $p_2=(1,3,2)$, $p_3=(2,1,3)$, $p_4=(2,3,1)$, $p_5=(3,1,2)$, $p_6=(3,2,1)$ then: $\boldsymbol{x}_1=(x_1,x_2,x_3)$, $\boldsymbol{y}_1=(y_1,y_2,y_3)$, $\boldsymbol{x}_2=(x_1,x_3,x_2)$, $\boldsymbol{y}_2=(y_1,y_3,y_2)$ $\boldsymbol{x}_3=(x_2,x_1,x_3)$, $\boldsymbol{y}_3=(y_2,y_1,y_3)$, $\boldsymbol{x}_4=(x_2,x_3,x_1)$, $\boldsymbol{y}_4=(y_2,y_3,y_1)$, $\boldsymbol{x}_5=(x_3,x_1,x_2)$, $\boldsymbol{y}_5=(y_3,y_1,y_2)$, $\boldsymbol{x}_6=(x_3,x_2,x_1)$, $\boldsymbol{y}_6=(y_3,y_2,y_1)$.
Based on our previous pondering we conclude that each j-th permutation $\mathbf{x}_j$, $\mathbf{y}_j$ of coordinates of vectors **x** and **y** corresponds to a measure of conformability of structures:

$$\mu^j_{Q,R} = \mu_{\mathbf{x}_j \mathbf{y}_j} \quad , \tag{2}$$

where, naturally, $\mu^1_{Q,R} = \mu_{\mathbf{xy}}$.

In accordance with the above let us define three different measures of conformability of considered objects Q and R:

$$\begin{aligned} \mathfrak{M}_{Q,R} &= \max_{1\le j\le n!} \mu^j_{Q,R}, \\ m_{Q,R} &= \min_{1\le j\le n!} \mu^j_{Q,R}, \\ s_{Q,R} &= \frac{1}{n!}\sum_{j=1}^{n!} \mu^j_{Q,R}. \end{aligned} \tag{3}$$

To compare structures of two objects

$$\mathbf{x} = (x_1, x_2, \ldots, x_n), \mathbf{y} = (y_1, y_2, \ldots, y_n): \; x_i, y_i \ge 0; \; i = 1, 2, \ldots, n \; ; \; \sum_{i=1}^{n} x_i = 1, \; \sum_{i=1}^{n} y_i = 1,$$

utilizing a popular and simple in use coefficient [Chomątowski, Sokołowski 1978]

$$W_{\mathbf{xy}} := \sum_{i=1}^{n} \min(x_i, y_i) \, . \tag{4}$$

In order to present the described above method of comparing structures we will consider three simple examples.

Example 1.

Let $Q=\mathbf{x}=\left(\frac{1}{2},\frac{1}{2},0\right)$, $R=\mathbf{y}=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$. Let the following take on values:

$$\mathbf{x}_1:=\mathbf{x}_4:=\mathbf{x},\ \mathbf{x}_2:=\mathbf{x}_5:=\left(\frac{1}{2},0,\frac{1}{2}\right),\ \mathbf{x}_3:=\mathbf{x}_6:=\left(0,\frac{1}{2},\frac{1}{2}\right),\ \mathbf{y}_1:=\mathbf{y}_2:=\mathbf{y}_3:=\mathbf{y}_4:=\mathbf{y}_5:=\mathbf{y}_6=\mathbf{y}$$

Thus, we receive:

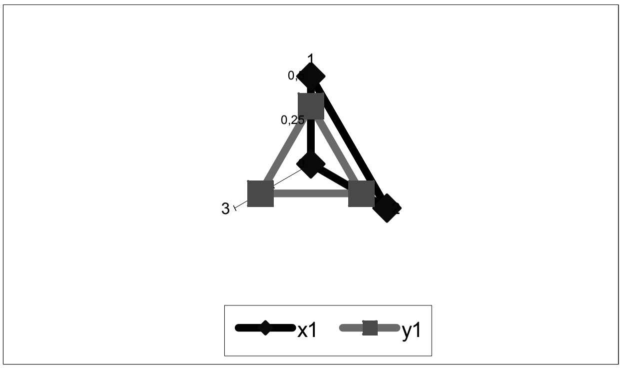
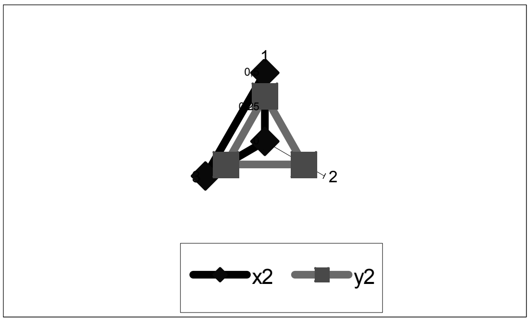
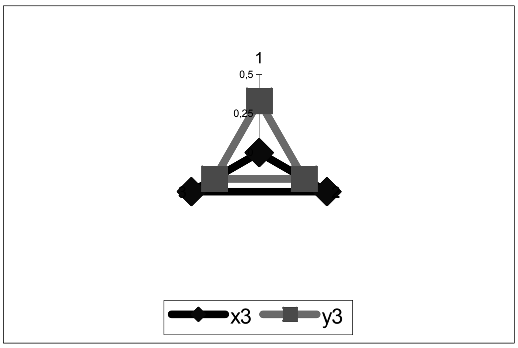
$$\left|S_{\mathbf{x}_i}\right|=\frac{1}{2}\sin\frac{2\pi}{3}\frac{1}{2}\frac{1}{2},\ \left|S_{\mathbf{y}_i}\right|=3\frac{1}{2}\sin\frac{2\pi}{3}\frac{1}{3}\frac{1}{3},\ \left|S_{\mathbf{x}_i}\cap S_{\mathbf{y}_i}\right|=\frac{1}{2}\sin\frac{2\pi}{3}\frac{1}{3}\frac{1}{3},$$

$$\mu_{\mathbf{x}_i\mathbf{y}_i}=\sqrt{\frac{\frac{1}{2}\sin\frac{2\pi}{3}\frac{1}{3}\frac{1}{3}}{\frac{1}{2}\sin\frac{2\pi}{3}\frac{1}{2}\frac{1}{2}}}=\frac{2}{3},\ \text{for }\ i=1,2,...,6.$$

And so $\mathfrak{M}_{Q,R}=m_{Q,R}=S_{Q,R}=\frac{2}{3}$, where coefficients $\mathfrak{M}_{Q,R}, m_{Q,R}, S_{Q,R}$ are described by equations (3). It is worth noting that when a coefficient is described by the equation (4) then $W_{\mathbf{xy}}=\frac{1}{3}+\frac{1}{3}+0=\frac{2}{3}$.

The following illustrations present the considered example.

Illustration 4. Graphical presentation of the method for vectors $\mathbf{x}=\left(\frac{1}{2},\frac{1}{2},0\right)$, $\mathbf{y}=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$.

$\mathbf{x}_1=\mathbf{x}_4=\left(\frac{1}{2},\frac{1}{2},0\right)$, $\mathbf{y}_1=\mathbf{y}_4=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	
$\mathbf{x}_2:=\mathbf{x}_5:=\left(\frac{1}{2},0,\frac{1}{2}\right)$ $\mathbf{y}_2=\mathbf{y}_5=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	
$\mathbf{x}_3:=\mathbf{x}_6:=\left(0,\frac{1}{2},\frac{1}{2}\right)$ $\mathbf{y}_3=\mathbf{y}_6=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	
Source: own investigation	

Example 2.

Let $\mathrm{Q} = \mathbf{x} = \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$, $\mathrm{R} = \mathbf{y} = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$. Let the following take on values: $\mathbf{x}_1 := \mathbf{x}_2 := \mathbf{x}_3 := \mathbf{x}_4 := \mathbf{x}$, $\mathbf{x}_5 := \mathbf{x}_6 := \mathbf{x}_7 := \mathbf{x}_8 := \left(\frac{1}{2}, 0, \frac{1}{2}, 0\right)$,

$\mathbf{x}_9 := \mathbf{x}_{10} := \mathbf{x}_{11} := \mathbf{x}_{12} := \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right), \mathbf{x}_{13} := \mathbf{x}_{14} := \mathbf{x}_{15} := \mathbf{x}_{16} := \left(0, \frac{1}{2}, \frac{1}{2}, 0\right), \mathbf{x}_{17} := \mathbf{x}_{18} := \mathbf{x}_{19} :=$ T

$:= \mathbf{x}_{20} := \left(0, 0, \frac{1}{2}, \frac{1}{2}\right), \mathbf{x}_{21} := \mathbf{x}_{22} := \mathbf{x}_{23} := \mathbf{x}_{24} := \left(0, \frac{1}{2}, 0, \frac{1}{2}\right)$, $\mathbf{y}_i := \mathbf{y}$ for $i = 1, 2, ..., 24$.

hus, we receive:

$$\left|S_{\mathbf{x}_j}\right| = \frac{1}{2}\sin\frac{\pi}{2}\cdot\frac{1}{2}\frac{1}{2},\ \left|S_{\mathbf{y}_i}\right| = 4\cdot\frac{1}{2}\sin\frac{\pi}{2}\cdot\frac{1}{4}\frac{1}{4},\ \left|S_{\mathbf{x}_j} \cap S_{\mathbf{y}_j}\right| = \frac{1}{2}\sin\frac{\pi}{2}\cdot\frac{1}{4}\frac{1}{4}, j = 1, ..., 4; i = 1, 2, ..., 24;$$

$$\mu_{\mathbf{x}_j\mathbf{y}_j} = \sqrt{\frac{\frac{1}{2}\sin\frac{\pi}{2}\cdot\frac{1}{4}\frac{1}{4}}{4\cdot\frac{1}{2}\sin\frac{\pi}{2}\frac{1}{4}\frac{1}{4}}} = \frac{1}{2},$$ for $j = 1, 2, 3, 4$; where $\mu_{\mathbf{x}_j\mathbf{y}_j}$ is defined by formula (2).

It can be easily verified that

$\mu_{\mathbf{x}_j\mathbf{y}_j} = \frac{1}{2}$, for $j = 8, 9, ..., 20$ and $\mu_{\mathbf{x}_j\mathbf{y}_j} = 0$ for $j = 5, 6, 7, 8, 21, 22, 23, 24$.

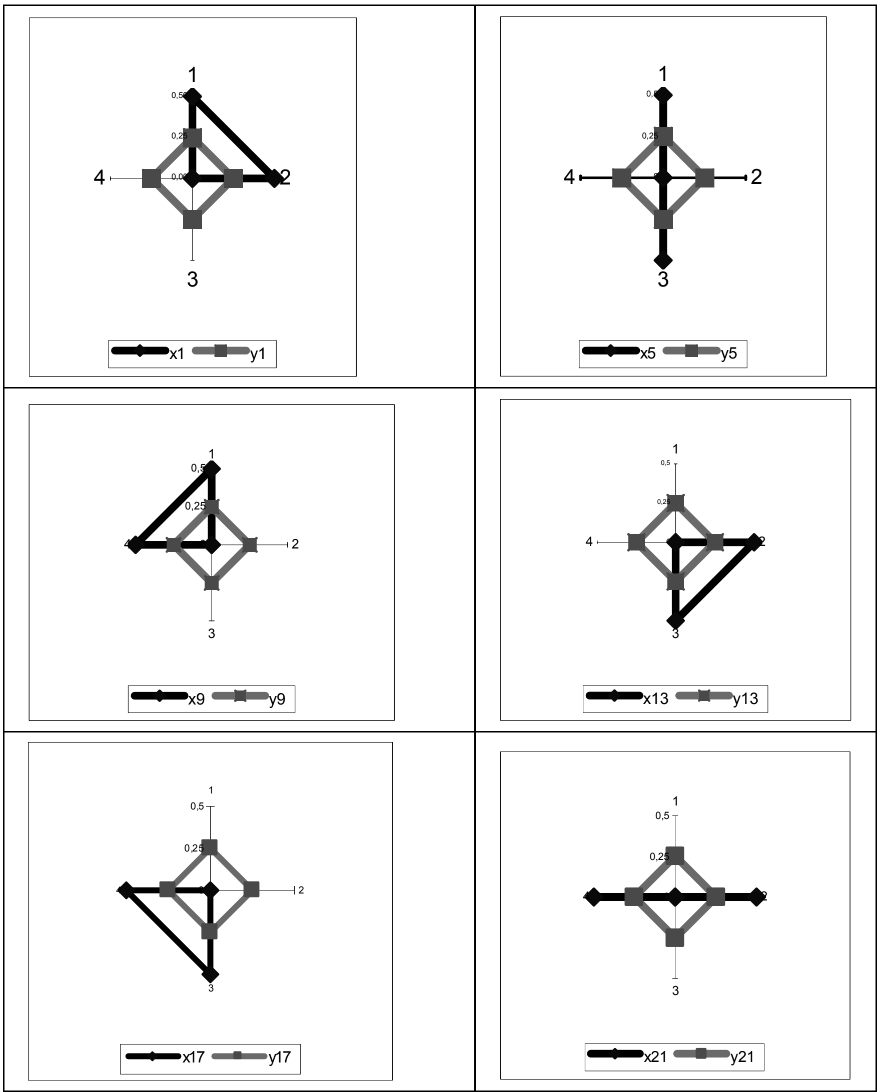
Hence $\mathfrak{M}_{Q,R} = \frac{1}{2}$, $m_{Q,R} = 0$, $S_{Q,R} = \frac{1}{3}$, where the coefficients $\mathfrak{M}_{Q,R}, m_{Q,R}, S_{Q,R}$ are described by equations (3). In this example the coefficient of structures conformability is equal to:

$$W_{\mathbf{xy}} = \frac{1}{4} + \frac{1}{4} + 0 + 0 = \frac{1}{2},$$

where $W_{\mathbf{xy}}$ is described by the formula (4).

The following illustrations present the considered example.

Illustration 5. Graphical presentation of the method for vectors $\mathbf{x} = \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$, $\mathbf{y} = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$.



Source: own investigation

Example 3.

Let $Q=\mathbf{x}=\left(\frac{1}{2},\frac{1}{2},0,0,...,0\right)$, $R=\mathbf{y}=\left(\frac{1}{n},\frac{1}{n},...,\frac{1}{4}\right)\in\Re^{n}, n\geq 4$. It can be shown that $\mathfrak{M}_{Q,R}=\frac{2}{n}$, $m_{Q,R}=0$, $S_{Q,R}=\frac{4}{n(n-1)}$, where coefficients $\mathfrak{M}_{Q,R}, m_{Q,R}, S_{Q,R}$

are described by equations (3). In this example the coefficient of structures conformability (described by the equation (4)) takes on the value:

$$W_{\mathbf{xy}} = \frac{1}{n} + \frac{1}{n} + 0 + \ldots + 0 = \frac{2}{n}.$$

SUMMARY

This work presents a mean to study conformability of structures but it is easily seen that introduced norms can also be used to analyze similarity of studied objects. Presented radar methods create a possibility of utilizing them in decision analysis on a local level. Radar methods are commonly used due to the ease of visualization of multidimensional data. However, some analyses incorrectly employ indices based solely on those illustrations, meaning they do not ensure the basic requirement of stability of the employed method – independence of the order of features [Jackson 1970]. The method presented by the authors does not have that flaw. As complicated as the presented methods may seem, in the digital age it remains largely inconsequential. Even more so with software for the presented methods is being developed.

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