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AN APPLICATION OF BRANCHING PROCESSES IN STOCHASTIC MODELING OF ECONOMIC DEVELOPMENT

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Abstract. In our paper, a stochastic model of forecasting of the number of firms of a given type, acting on the market in a given year, is proposed. The model uses the probabilistic tools of the theory of branching processes. Our approach is an alternative method to the forecasting methods proposed so far, including those based on time series. The theoretical results presented in the paper may be applied in the forecasting of the market position of the firms of a given sector.

Key words: branching processes, moment generating function, forecasting of financial positions of firms.

INTRODUCTION

Forecasting of the number of firms - preliminaries

The forecasting of economic events belongs to the most important tasks of contemporary econometrics. Accurate econometric predictions help the company management and governmental authorities in taking good finacial decisions and solutions. A lot of methods of financial forecasting have been worked out so far (see [Cieślak 2001], [Gajek and Kałuszka 1999] and [Harvey 1989] among others). In our paper, we propose a new approach to this topic. It is based on the probabilistic model, while the starting point for the earlier proposed methods was an appropriate econometric function or some time series model. A probabilistic treatment of the problem of forecasting of economic phenomena is possible, if, except for the realizations of the random process, something is also known about the elements, which generate this process. In some stages of the construction of our forecasting model we also use some econometric models, but we avoid the situation, when some equalities appear a priori, without any explanation. The important features of our approach are the following: 1) it gives the possibility of economic interpretation of the obtained parameters, 2) it enables to look at the problems of economic forecasting from a different point of view than the methods proposed so far.

Analysis of the dynamics of the number of firms may be useful in evaluation of the current situation on the labour market, as well as in forecasting of its development in the future. The decrease in the dynamics of the number of firms may be caused by: too high taxes, strong market competition, bureaucracy, unclear law or financial regulations, lack of development plan or financial liquidity of firms, unsettled political situation. On the other hand, an increase in the dynamics of the number of firms may be caused by: the growing share of the private sector in the economy structure, an increase of export, an increase of activity of local communities. Such a variety of factors, which slow down or stimulate the process of creation of a new firm makes the forecasting of the number of firms fairly difficult.

There are not many publications concerning the issue of forecasting of the number of firms. One of a few exceptions is the paper of [Chybalski], which is devoted to the forecasting of the number of small and medium-sized enterprises in Poland.

In our work, we propose the model based on the generalization of the so-called branching processes. The definitions of the branching process and their certain generalization are given in section 2 of our paper. This section contains also the definition of generating function, as some properties of generating functions will be applied in constructing of our model. In section 3, we present the main goals of our investigations, as well as the proposed model of forecasting of the number of firms. In section 4, we describe the estimation procedure applied in the estimation of the parameters of our model and present the results of the forecasts obtained with the use of this model. In section 5, we compare these results with the results of predictions received by applying of some other time series models; our final conclusions are also included here.

BRANCHING PROCESSES AND GENERATING FUNCTIONS

Let us consider the following population. Suppose that at the beginning (time 0) it has c elements (individuals) and each element changes into the new element (or elements), so at time n we have the nth generation of elements. We assume that, for every n, the individuals from the nth generation change independently into the new individuals - called descendants - and these new individuals form the generation numbered by $n+1$. In addition, we assume that the individual disappears, if it has no offspring in the subsequent generation. Let us define the random variables (r.v.'s) Y_n and ξ_{in} as follows: Y_n - the number of individuals in the n th generation, $Y_0 = c$, and ξ_{in} - the number of descendants of the i th individual from the n th generation, $i = 1,2,...,Y_n$. We assume that $\{\xi_{in}$: $i = 1,2,...,Y_n$; $n = 1,2,...\}$ are independent, identically distributed (i.i.d.) r.v.'s, and $\forall i,n \ P(\xi_{in} = j) = b_j, j = 0,1,...,$ where b_j - the probability that the individual will change into j descendants (in particular, b_0 denotes the probability that the individual will have no offspring and disappear). Then, the number of individuals in the generation $n+1$ is given by $Y_{n+1} = \begin{cases} \sum_{i=1}^{Y_n} \xi_{in}, & \text{if } Y_n > 0, \\ 0, & \text{if } Y_n = 0. \end{cases}$

We call $(Y_n)_{n=0}^{\infty}$ the standard Bienayme-Galton-Watson process. It describes the development of descendents of c ancestors. We assume here that the individuals change into the new ones independently of each other and, for any n, i, the r.v.'s ξ_{in} are independent r.v.'s from a certain common distribution.

It seems that branching processes have not been widely used in economic studies so far. We partly try to fill this gap and show that a certain generalization of the Bienayme-Galton-Watson branching process may be useful in economic forecasting.

We now introduce some generalization of the metioned branching process. Let us allow the situation, when the individual may exist longer than one unit of time and that it can have descendants at the different moments in time. Then, Y_n - the number of individuals at the moment n (in the n th generation) - is given by $Y_n = \sum_{k=0}^{n} f_n^k$, where f_n^k - the number of individuals in the n th year existing from k years. Furthermore, the number of new individuals in the generation numbered by $n+1$ is described by $f_{n+1}^0 = \sum_{i=1}^{Y_n} \xi_{in}$, and the number of all individuals in this generation is expressed by $Y_{n+1} = \sum_{k=1}^{n+1} f_{n+1}^k + \sum_{i=1}^{Y_n} \xi_{in}$. In order to find the distribution of the random variable Y_n, the so-called generating function may be used. It is defined as follows: Suppose that X is a discrete random variable with the values in the set of natural numbers. Then, the funcion T, defined on the interval $[-1;1]$, by $T(s) = \sum_{r=0}^{\infty} P(X = r)s^r$ for $|s| \le 1$ is called the generating function of X. Clearly,

$$T(1) = \sum_{r=0}^{\infty} P(X = r) = 1 \text{ and } T'(1) = \sum_{r=0}^{\infty} rP(X = r) = EX. \quad (1)$$

The properties in (1) will be used in further parts of our paper. For some more informations on the issue of branching processes and their applications, we refer to [Dawidowicz et al. 1995], [Epps 1996] and [Haccou et al. 2005].

FORMULATION OF THE PROBLEM AND THE PROPOSED MODEL

Our goals and empirical data

Our main purpose was to propose the model, which enables to forecast the number of firms. In our considerations we restricted ourselves to the firms of the building sector from the area of the Masovia Province (the Masovian Voivodeship) in Poland. We had the following two reasons for making such a choice: 1) the dynamics of development of the firms from the building sector is sensitive to the changes of both economical and political situation, 2) the Masovia Province belongs to the regions in Poland with the largest number of building companies. We carried out our forecasts for the years 2008 and 2009.

Our database consisted of the informations concerning the number of building companies from the Masovia Province, registered in the National Court Register (*Krajowy Rejestr Sądowy* - KRS) in the period 2001-2009. These informations included the date of registration of the firm in the KRS and the date of declaring bankruptcy. The empirical data are collected in the following table:

Table 1. The numbers of firms and their bankruptcies in the period 2001-2009 (the data marked by * include the number of firms established before 2001)

The year of registration of the firm	The number of firms at the end of 2009	The number of firms, which declared bankruptcy before the end of 2009	The total number of firms
2001	205*	24	229
2002	252*	28	280
2003	129	14	143
2004	144	9	153
2005	37	1	38
2006	24	0	24
2007	45	1	46
2008	44	0	44
2009	36	0	36

More formally, the purposes of our paper may by described as follows:
Let Y_n - the number of building firms from the area of the Masovia Province in the n th year. *Our main goal was to estimate* $E(Y_n)$ *for* $n > t$, *by the use of the historical data from the years* $0,1,...,t$, *where* $E(Y_n)$ *- the expected number of building companies in the Masovia Province in the* n th *year.* We carried out the

forecasts for the years 2008 and 2009 by applying of the historical data from the period 2001-2007. Thus, in our considerations: $n = 0$ denoted the year 2001, $n = 1$ the year 2002, ..., $n = t = 6$ the year 2007, and $n > t$ (i.e., $n = 7{,}8$) denoted the forecasting years 2008, 2009. After calculating (by means of our model) the forecasts of the number of firms for the years 2008 and 2009, we compared these forecasts with the real numbers of firms in those years and with the forecasts obtained by means of some other time series models. We also calculated the relative errors of our forecasts and the forecasts obtained for the other models.

The proposed model of the number of firms

Let: Y_n - the number of firms on the market in the n th year, f_n^k - the number of firms in the n th year existing from k years, $k \geq 0$ (in particular, f_n^0 - the number of new firms in the n th year). Obviously, we have $Y_n = \sum_{k=0}^{n} f_n^k$ and

$$E(Y_n) = \sum_{k=0}^{n} E(f_n^k) = E(f_n^0) + \sum_{k=1}^{n} E(f_n^k). \tag{2}$$

We made the following two complementary assumptions in our investigations: 1) establishing of a new firm may be connected with the existence of some other stable firms in the past (by stable firms we mean the firms existing from at least two years), 2) establishing of a new firm may be caused by some other reasons than the existence of the other firms in the previous years (the reasons for the creation of a new firm may be connected, for example, with the growth of demand for services of a certain type, with the development of high technologies, with the environmental changes, with the changes in law, etc.).

The assumptions above may be described by introducing the following notations: g_n^k ($k = 2,\dots,n$) - the number of new firms in the n th year, the creation of which was connected with the existence of firms existing from $k-1$ years in the year $n-1$, φ_n - the number of new firms in the n th year, the creation of which was not connected with the existence of firms in the year $n-1$. Clearly, we have $f_n^0 = \sum_{k=2}^{n} g_n^k + \varphi_n$, and

$$E(f_n^0) = \sum_{k=2}^{n} E(g_n^k) + E(\varphi_n). \tag{3}$$

Our goal now is to derive the formula for $E(f_n^0)$ in terms of f_{n-1}^{k-1}, $k = 1,\dots,n$. Due to (3), in order to do it, we need to obtain the formulas for $E(g_n^k)$, $E(\varphi_n)$.

Denote by ξ_{ikn} the number of new firms in the n th year, the creation of which was connected with the existence of the i th firm among the firms existing from $k-1$ years in the year $n-1$. We assume that $\{\xi_{ikn}\}$ are i.i.d. and $\forall i,k,n\ P(\xi_{ikn}=j)=b_j, j=0,1,...,$ where b_j - the probability that the number of new firms in the n th year, the creation of which was connected with the existence of the i th firm among the firms existing from $k-1$ years in the year $n-1$ is equal to j. It is clear that $g_n^k=\sum_{i=1}^{f_{n-1}^{k-1}}\xi_{ikn}$. Let: $\hat{B}$ - the generating function of the r.v.'s ξ_{ikn}, G_n^k - the generating function of the r.v. g_n^k. Then, the conditional generating function of the r.v. g_n^k given the event $f_{n-1}^{k-1}=r$ is given by $G_n^k\left(x \mid f_{n-1}^{k-1}=r\right)=\sum_{i=0}^{\infty}P\left(g_n^k=i \mid f_{n-1}^{k-1}=r\right)x^i=\left[\hat{B}(x)\right]^r$, since, under such conditioning, g_n^k is the sum of r i.i.d. r.v.'s ξ_{ikn} with a common generating function $\hat{B}$. The derived formula for $G_n^k\left(x \mid f_{n-1}^{k-1}=r\right)$ implies that

$$G_n^k(x)=\sum_{r=0}^{\infty}G_n^k\left(x \mid f_{n-1}^{k-1}=r\right)P\left(f_{n-1}^{k-1}=r\right)=\sum_{r=0}^{\infty}\left[\hat{B}(x)\right]^r P\left(f_{n-1}^{k-1}=r\right)=F_{n-1}^{k-1}\left(\hat{B}(x)\right), \quad (4)$$

where F_{n-1}^{k-1} - the generating function of f_{n-1}^{k-1}.

In view of (4) and the properties of generating functions in (1), we obtain

$$E(g_n^k)=\left(G_n^k(x)\right)'_{x=1}=\left(F_{n-1}^{k-1}\left(\hat{B}(x)\right)\right)'_{x=1}=\left(\hat{B}'(1)\right)\left(\left(F_{n-1}^{k-1}\right)'\left(\hat{B}(1)\right)\right)=\left(\hat{B}'(1)\right)\left(\left(F_{n-1}^{k-1}\right)'(1)\right)$$

$$=\hat{B}'(1)E(f_{n-1}^{k-1})=bE(f_{n-1}^{k-1}),\ \text{ where } b=E\left(\xi_{ikn}\right)=\sum_{j=0}^{\infty}jb_j. \quad (5),$$

Our next task is to derive the formula for $E\left(\varphi_n\right)$. Observe that assuming

$$\forall n\ \ P\left(\varphi_n=r \mid f_{n-1}^0=q\right)=e^{-(\alpha q)}\frac{(\alpha q)^r}{r!}, \text{where } \alpha \text{ is a certain parameter,} \quad (6)$$

we have $E\left(\varphi_n \mid f_{n-1}^0=q\right)=\alpha q$, which implies that

$$E\left(\varphi_n\right)=\sum_{q=0}^{\infty}E\left(\varphi_n \mid f_{n-1}^0=q\right)P\left(f_{n-1}^0=q\right)=\alpha\sum_{q=0}^{\infty}qP\left(f_{n-1}^0=q\right)=\alpha E(f_{n-1}^0). \quad (7)$$

The relations in (3), (5) and (7) yield

$$E(f_n^0)=\sum_{k=2}^{n}bE\left(f_{n-1}^{k-1}\right)+\alpha E(f_{n-1}^0). \quad (8)$$

Thus, in view of (2), in order to find the recursive formula for $E(Y_n)$, we need to derive the recursive formula for $E(f_n^k)$, where $k \geq 1$ Let us introduce the following notations:
h_n^k - the number of firms that declared bankruptcy in the n th year, for which it was the k th year of activity, p_{k-1} - the probability that the firm existing from $k-1$ years will declare bankruptcy in the k th year of its activity.
Assuming that the probabilities of bankruptcy of the firms in the k th year of activity are identical for all these firms and the events of bankruptcy of the firms are independent, we may write that

$$\forall n \;\; P(h_n^k = r \mid f_{n-1}^{k-1} = q) = \binom{q}{r} p_{k-1}^r (1-p_{k-1})^{q-r}. \tag{9}$$

Therefore, $E\left(h_n^k \mid f_{n-1}^{k-1} = q\right) = qp_{k-1}$ and

$$E\left(h_n^k\right) = \sum_{q=0}^{\infty} E(h_n^k \mid f_{n-1}^{k-1} = q) P\left(f_{n-1}^{k-1} = q\right) = p_{k-1} \sum_{q=0}^{\infty} q P\left(f_{n-1}^{k-1} = q\right) = p_{k-1} E\left(f_{n-1}^{k-1}\right). \tag{10}$$

Due to the identity $f_n^k = f_{n-1}^{k-1} - h_n^k$, for $k \geq 1$, and the relation in (10), we have

$$E(f_n^k) = E(f_{n-1}^{k-1}) - E(h_n^k) = E(f_{n-1}^{k-1}) - p_{k-1} E\left(f_{n-1}^{k-1}\right) = (1-p_{k-1}) E\left(f_{n-1}^{k-1}\right), \text{ if } k \geq 1. \tag{11}$$

By (2), (8) and (11), we conclude that

$$E(Y_n) = \sum_{k=2}^{n} (b + 1 - p_{k-1}) E\left(f_{n-1}^{k-1}\right) + (\alpha + 1 - p_0) E(f_{n-1}^0), \tag{12}$$

where $b = E(\xi_{ikn}) = \sum_{j=0}^{\infty} jb_j$ and α is such as in (6).

The recursive formula in (12), obtained with the use of (6), (9), establishes our model, which enables to estimate the expected number of firms in the n th year.

In the next part of our paper, we will estimate (by the use of the data from Table 1) the parameters of the model in (12), as well as we will present the results of the predictions of the number of building firms in the Masovia Province for the years of 2008 and 2009.

ESTIMATION OF THE MODEL PARAMETERS AND THE RESULTS OF FORECASTS

The table below presents the numbers of firms registered in the KRS, which declared bankruptcy in the subsequent years:

Table 2. The numbers of firms, which declared bankruptcy

The year of registration	'01	'02	'03	'04	'05	'06	'07	'08	'09
The year of bankruptcy									
'10	0	1	0	0	0	0	0	0	0
'09	0	3	0	1	0	0	1	0	0
'08	1	7	3	0	1	0	0	0	0
'07	4	2	2	3	0	0	0	0	0
'06	1	3	1	2	0	0	0	0	0
'05	4	3	2	0	0	0	0	0	0
'04	2	1	2	3	0	0	0	0	0
'03	5	6	4	0	0	0	0	0	0
'02	7	2	0	0	0	0	0	0	0

As we have already mentioned, in order to estimate our model (12), we used as the historical data ($n = 0,1,..., t$, t is the current time) the data from the period 2001-2007 (thus, $n = 0,..., t, t = 6$). After we had estimated the parameters of our model, we calculated the forecasts of the number of firms for the years 2008 and 2009 ($n = 7,8$). At the beginning, we put f_n^k for $E(f_n^k)$ for our historical data. The values of f_n^k for our historical data ($n = 0,...,6$) were as follows:

Table 3. The values of f_n^k for the historical data from the period 2001-2007

$f_0^0 = 229$	-	-	-	-	-	-
$f_1^0 = 278$	$f_1^1 = 222$	-	-	-	-	-
$f_2^0 = 139$	$f_2^1 = 272$	$f_2^2 = 217$	-	-	-	-
$f_3^0 = 150$	$f_3^1 = 137$	$f_3^2 = 271$	$f_3^3 = 215$	-	-	-
$f_4^0 = 38$	$f_4^1 = 150$	$f_4^2 = 135$	$f_4^3 = 268$	$f_4^4 = 211$	-	-
$f_5^0 = 24$	$f_5^1 = 38$	$f_5^2 = 148$	$f_5^3 = 134$	$f_5^4 = 265$	$f_5^5 = 210$	-
$f_6^0 = 46$	$f_6^1 = 24$	$f_6^2 = 38$	$f_6^3 = 145$	$f_6^4 = 132$	$f_6^5 = 263$	$f_6^6 = 206$

We used the data in Table 3 to estimate p_{k-1} - the probability that the firm existing from $k-1$ years will declare bankruptcy in the kth year of its activity. Since, for any $i = k,..., t$; $t = 6$, each quantity h_i^k / f_{i-1}^{k-1} is (on the condition that the kth year of the firm activity is the ith year among the considered years) a natural estimate of p_{k-1}, we put for $\hat{p}_{k-1}$ the average value of these quantities, i.e.,

$$\hat{p}_{k-1} = \frac{1}{6-k+1} \sum_{i=k}^{6} h_i^k / f_{i-1}^{k-1} = \frac{1}{6-k+1} \sum_{i=k}^{6} \left(1 - f_i^k / f_{i-1}^{k-1}\right) \qquad (13)$$

The values of $\hat{p}_{k-1}$, obtained from (13) for $k = 1,2,\ldots,6$, were as follows: 0,01109; 0,01083; 0,01199; 0,01491; 0,00614; 0,01905, respectively. It is obvious that, if we estimate p_{k-1} according to the scheme given above, we can estimate p_{k-1} only for $k \le t$, where $t = 6$ is the current time. There is no method, which enables to estimate p_{k-1} for $k \ge t+1$ only with the use of the historical data. For this reason, we assumed that the position of the firm existsting for $t = 6$ years is so stable that it seemed reasonable to make the additional condition $\hat{p}_{k-1} = \hat{p}_t$ for $k-1 \ge t = 6$. Thus, in our considerations, we made the assumption $\hat{p}_{k-1} = \hat{p}_5$ for $k-1 \ge 6$. After we had estimated p_{k-1}, we estimated the parameters b, α of the model (12). Observe that b, α are also the parameters in the equation (8): $E(f_n^0) = b\sum_{k=2}^{n} E\left(f_{n-1}^{k-1}\right) + \alpha E(f_{n-1}^0)$. Therefore, in order to estimate b and α, we applied the following model of multiple regression

$$f_n^0 = bx_{1,n} + \alpha x_{2,n} + \varepsilon_n \text{, where : } x_{1,n} = \sum_{k=2}^{n} f_{n-1}^{k-1},\ x_{2,n} = f_{n-1}^0. \tag{14}$$

By using of the historical data, we obtained the following input data for the estimation of the parameters b and α of the regression function (14)

Table 4. The empirical data for estimation of the regression function (14)

Rok	f_n^0	$x_{1,n} = \sum_{k=2}^{n} f_{n-1}^{k-1}$	$x_{2,n} = f_{n-1}^0$
'03 ($n = 2$)	$139 = f_2^0$	$222 = f_1^1$	$278 = f_1^0$
'04 ($n = 3$)	$150 = f_3^0$	$489 = f_2^1 + f_2^2$	$139 = f_2^0$
'05 ($n = 4$)	$38 = f_4^0$	$623 = f_3^1 + f_3^2 + f_3^3$	$150 = f_3^0$
'06 ($n = 5$)	$24 = f_5^0$	$764 = f_4^1 + f_4^2 + f_4^3 + f_4^4$	$38 = f_4^0$
'07 ($n = 6$)	$46 = f_6^0$	$795 = f_5^1 + f_5^2 + f_5^3 + f_5^4 + f_5^5$	$24 = f_5^0$

By applying of the data from Table 4, we obtained (by the method of least squares) the following estimates for the parameters b and α of the regression function in (14): $\hat{b} = 0{,}025$, $\hat{\alpha} = 0{,}4986$, where the coefficient of determination and the adjusted coefficient of determination were equal to: $R^2 = 0{,}83$, $R^2 adj = 0{,}72$.

Thus, since $\hat{p}_{k-1} = \hat{p}_5$ for $k-1 \ge 6$, we obtained - by the model in (12) - the following predicted value of the number of firms in 2008 ($n = 7$)

$$E(Y_7) = \sum_{k=2}^{7} \left(\hat{b} + 1 - \hat{p}_{k-1}\right) f_6^{k-1} + \left(\hat{\alpha} + 1 - \hat{p}_0\right) f_6^0 = 884{,}11.$$

In order to estimate the number of firms in 2009 ($n = 8$), we needed to obtain the estimates for f_7^0. By (14), we calculated that $\hat{f}_7^0 = \hat{b}\sum_{k=1}^{6} f_6^k + \hat{\alpha} f_6^0 = 43{,}2406$. Next, we calculated $\hat{f}_7^1$ - $\hat{f}_7^7$, from (11). The results were as follows: The values of $\hat{f}_7^k$ for k = 1,2,…,7 were: 45,49; 23,74;

37,54; 142,84; 131,18; 257,99; 202,08. By applying of the estimates for f_7^k , and the assumption $\hat{p}_{k-1} = \hat{p}_5$, for $k-1 \geq 6$, we received the following predicted values of the number of firms in the year 2009 ($n = 8$)

$$E(Y_8) = \sum_{k=2}^{8} \left(\hat{b} + 1 - \hat{p}_{k-1}\right)\hat{f}_7^{k-1} + \left(\hat{\alpha} + 1 - \hat{p}_0\right)\hat{f}_7^0 = 912{,}84.$$

MODEL ASSESSMENT AND FINAL CONCLUSIONS

Below, we present the predicted numbers of building firms from the Masovia Province, obtained with the use of our model and the five chosen time series models (*the real numbers of firms are given in parantheses*), together with the relative errors of all the forecasts (for the details concerning the chosen models see [Gajek and Kałuszka 1999] and [Harvey 1989]):

Table 5. The predicted numbers of firms in 2008 and 2009

Model	Forecasts'08 and '09	Relative errors of forecasts
The proposed model	884,11 (868), 912,84 (916)	1,86%, -0,34%
Linear trend	1022,3 (868), 1129,4 (916)	17,78%, 23,3%
Quadrating trend	793,89 (868), 729,19 (916)	-8,54%, -20,39%
Moving average of rank 2	804,5 (868), 814,75 (916)	-7,32%, -11,05%
Holt's model	920,63 (868), 1015,48 (916)	6,06%, 10,86%
Structural TS model	866,88 (868), 908,76 (916)	-0,13%, -0,79%

We conclude that: 1) in the case of forecasts for the year 2008, the only model, for which the relative error of prediction was smaller than the relative error of forecast obtained with the use of the proposed model was the Structural TS model, 2) the results of forecasts for the year 2009 show that in this case, the relative error of forecast obtained with the use of our model was the smallest of all the calculated errors. The established model may be applied not only to forecast the number of firms. For example, if we add to our data concerning the firms, the informations on the numbers of employees or the paid tax amounts, we may construct models, which enable to forecast the rate of employment or the potential tax revenues to the budget. The results of our predictions seem to be promising and show that the models based on a certain generalization of branching processes may become an interesing alternative for the models so far applied in economic forecasting.

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