

INTRADAY VOLATILITY MODELING: THE EXAMPLE OF THE WARSAW STOCK EXCHANGE

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Abstract: We present an intraday volatility model for equally spaced data and apply it for the WIG Index- a broad market index of the Warsaw Stock Exchange. The current study is an application and extension of the model proposed by Engle and Sokalska [2010]. We decompose the conditional variance of intraday returns into components that have a natural interpretation and can be easily estimated.

Keywords: Volatility, ARCH, Intra-day Returns.

INTRODUCTION

As recent developments on world's stocks markets have shown, intraday asset price movements can be very dramatic. During the crisis of 2008-2009, it was not unusual to observe spectacular stock market rallies and sudden plunges of Dow Jones Industrial Index worth several hundred of points on a single day. Therefore there is great value in being able to forecast volatility on intraday basis. A paper by Andersen and Bollerslev [1997] documents that an application of standard ARCH-type volatility models [Engle, 1982] to intraday data gives unsatisfactory results. This is because there are pronounced periodic patterns in volatility throughout a day.

A number of papers have presented work on intraday returns related the current study. Andersen and Bollerslev [1997, 1998] propose models for 5-minute returns on Deutschmark-dollar exchange rate and the S&P500 index. They build a multiplicative model of daily and diurnal volatility. Andersen and Bollerslev [1998] add an additional component which takes account of the influence of macro-economic announcements on the foreign exchange volatility. For most of their models, the intra-daily volatility components are deterministic. The model applied in this paper contains stochastic intraday variance component.

THE MODEL

Our paper uses the volatility model for high frequency intraday financial returns proposed by Engle and Sokalska [2010]. We employ the following notation. Days in the sample are indexed by t ($t=1, \dots, T$). Each day is divided into 10 minute intervals referred to as bins and indexed by i ($i=1, \dots, N$). The current period is $\{t, i\}$. Price of an asset at day t and bin i is denoted by $P_{t,i}$. The continuously compounded return $r_{t,i}$ is modeled as: $r_{t,i} = \ln\left(\frac{P_{t,i}}{P_{t,i-1}}\right)$ for $i \geq 2$, whereas the return for the first bin of the day $i=1$ is $r_{t,i} = \ln\left(\frac{P_{t,i}}{P_{t-1,N}}\right)$. In our model, the conditional variance is a multiplicative product of daily, diurnal and stochastic intraday variance components. Intraday equity returns are described by the following process:

$$r_{t,i} = \sqrt{h_t s_i q_{t,i}} \varepsilon_{t,i} \quad \text{and} \quad \varepsilon_{t,i} \sim N(0,1) \quad (1)$$

where:

- h_t is the daily variance component,
- s_i is the diurnal (periodic) variance pattern,
- $q_{t,i}$ is the intraday volatility component, and
- $\varepsilon_{t,i}$ is an error term.

The model is estimated in several steps. Similarly to Andersen and Bollerslev [1997, 1998], daily variance component will be estimated from a daily ARCH –type specification for a longer sample, going back a number of years.

Empirical analysis indicates that GARCH (1,1) process [Bollerslev, 1986] proved to be the most successful daily volatility model.

$$r_t = \sqrt{h_t} \zeta_t \quad \zeta_t \sim N(0,1) \quad (2)$$

$$h_t = w_0 + \alpha_d r_{t-1}^2 + \beta_d h_{t-1}^2$$

Where ζ_t is an error term for daily returns r_t , whereas w_0 , α_d and β_d are parameters of the variance equation.

The diurnal component is calculated as the variance of returns in each bin after deflating by the daily variance component.

$$E\left(\frac{r_{t,i}^2}{h_t}\right) = s_i E(q_{t,i}) = s_i \quad (3)$$

Then we model the residual volatility as a GARCH(1,1) process ²³:

$$q_{\{t,i\}} = \omega + \alpha (r_{\{t,i-1\}} / \sqrt{h_{t,s_{i-1}}})^2 + \beta q_{\{t,i-1\}} \quad (4)$$

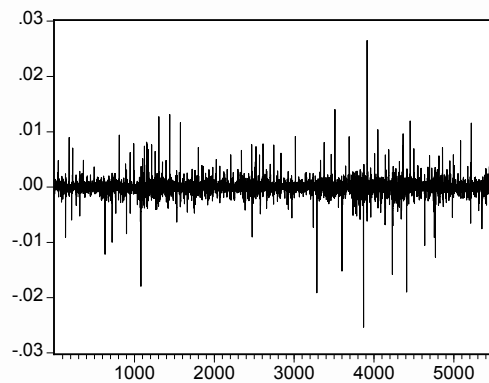
Engle and Sokalska [2010] show that this multistep estimator is consistent and asymptotically normal.

EMPIRICAL ANALYSIS

Our data consists of 10 minute logarithmic returns on the WIG index for the period 4 January 2010 – 31 May 2010. We estimated daily variance component using daily data on the WIG index between October 1994 and May 2010. Both intraday and daily series come from the Bloomberg database.

We exclude overnight returns from our analysis. Figure 1 and 2 depict intraday logarithmic returns of WIG including and excluding overnight returns, respectively. As can be seen from Figure 1 intraday returns are dominated by substantially negative and positive overnight changes; all of the returns that exceed +/- 1% are in fact overnight. Although the inclusion of overnight returns could yield a more complete analysis, it would require a far more complex model. Such a complete model would need to concentrate more on economic or global factors and this approach was not followed in this paper.

Figure 1. Intraday WIG 10-min logarithmic returns

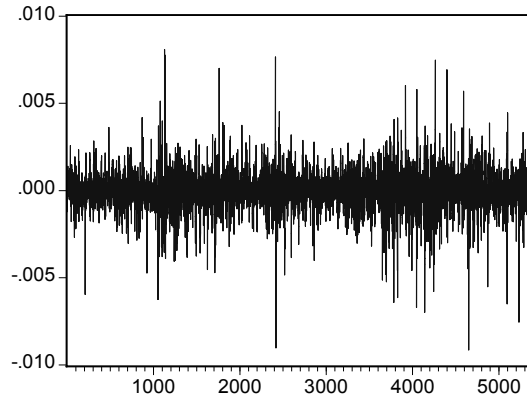


Labels of the horizontal axis denote observation number.

Source: own calculations

²³ Higher order GARCH models were fitted as part of the specification search for scaled intraday returns but GARCH(1,1) performed best.

Figure 2. Intraday WIG 10-min logarithmic returns excluding the overnight period



Labels of the horizontal axis denote observation number.
Source: own calculations

Table 1 presents estimation results of the daily GARCH (1,1) model (2). Attempts to fit higher order models yielded statistically insignificant coefficients for the respective parameters. The sum of coefficients $\alpha_d + \beta_d$ (0.9896) is close to but smaller than one. This indicates high persistence (high degree of volatility clustering).

Table 1. GARCH Results for Daily Data

GARCH Results for Daily WIG Index			
	Coefficient	Std. Error	t-Statistic
ω_d	3.52E-06	5.09E-07	6.925
α_d	0.0845	0.00498	16.984
β_d	0.9051	0.00479	188.941

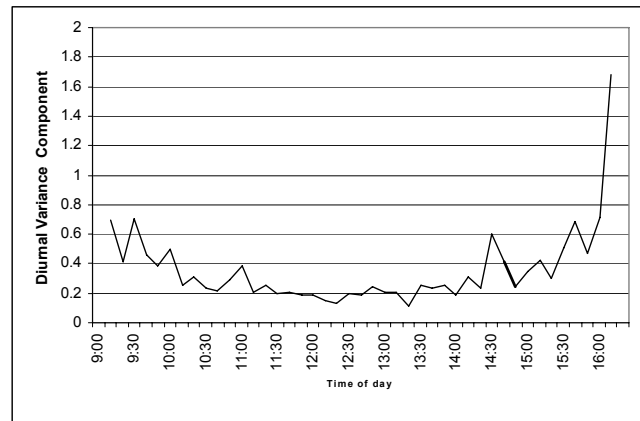
Notes: This table presents estimation results for GARCH(1,1) model for the WIG index. Sample period for daily data: October 1994 -May 2010. Symbols α_d , β_d and ω_d denote GARCH parameters from the variance equation (2).

Source: own calculation

Figure 3 depicts the estimates of diurnal variance component (3). We may observe a typical U-shaped pattern in volatility - an increased variability at the beginning of the day followed by a calm period in the middle and then a rise in variation at the close of the session. A small spike around 14.30 coincides with the time of macroeconomic announcements in the US. The volatility of the overnight

period, which is excluded from our analysis, would actually much exceed the volatility at the close.

Figure 3. Diurnal Variance Component Throughout a Day (Excluding Overnight Period)



Source: own calculations

Finally Table 2 presents results of estimation of the parameters of the stochastic intraday variance component (4). The persistence measure, $\alpha + \beta$, equals to 0.8884 and is smaller than $\alpha_d + \beta_d$ (0.9896) obtained for the daily component. The degree of volatility clustering for the intraday component is smaller than for the daily component because we estimated the intraday GARCH on scaled returns (compare equation (4)). In other words we have already taken account of some of the volatility persistence present in the data by scaling squared returns by the daily and diurnal variance components.

Table 2. GARCH Results for Intraday Data

GARCH Results for Intraday Daily WIG Index			
	Coefficient	Std. Error	t-Statistic
ω	1.61E-07	9.16E-09	17.555
α	0.1984	0.0108	18.354
β	0.6900	0.0136	50.886

Notes: This table presents estimation results for GARCH(1,1) model for WIG returns that have been previously scaled by the squared root of the daily and diurnal variance components. Sample period January-May 2010. Symbols α , β and ω denote GARCH parameters from the variance equation (4).

Source: own calculation

CONCLUSION

We have estimated an intraday volatility model for equally spaced logarithmic returns on the WIG index. In our specification, the conditional variance is a multiplicative product of daily, diurnal and stochastic intraday variance components. As the next step of our analysis, we plan to conduct forecasts comparison between our model and a selection of alternative benchmarks.

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CHANGES OF EXCHANGE RATE BEHAVIOR DURING AND AFTER CRISIS

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Abstract: This study extends earlier analysis, in which behavior of daily exchange rates during the global crisis was compared to that before crisis. We repeat similar comparison for data set extended until the end of April 2010, use ARMA/ARMAX and GARCH models with stock indices as additional regressors, for volatility and returns of EURPLN, EURUSD, USDPLN exchange rates. Marked increase in volatility during crisis, negatively affected quality of models. After crisis volatility and returns seem to stabilize, hence exchange rate risk seems to decline gradually. There is a slight improvement in quality of models after the crisis.

Key words: Exchange rates, stock indices, crisis, risk, autoregressive and conditional heteroskedasticity models, Granger causality.

INTRODUCTION

The aim of research presented is to study the effects of the current crisis on exchange rate behavior, and on quality of exchange rate models. We use daily data of exchange rates USDPLN, EURPLN and EURUSD, and stock indices S&P500 and WIG20, since 4th January 2000 until 30th April 2010. We note stabilization of exchange rate behavior since spring 2009 in comparison to previous period (2007-2008), and assume that this may ease exchange rate modeling and improve quality of models.

We study the behavior of variance and volatility of models, test Granger-causality from stock indices towards exchange rate variances and returns. Next we estimate ARMA and ARMAX models for exchange rate volatility, and ARMA and GARCH models for logarithmic returns. We use S&P500 and WIG20 volatility or

returns as additional variables in models describing respectively volatility or returns of the exchange rates.²⁴

In [Syczewska 2010] we compared behavior of rates and quality of modeling for two subperiods: before (up to September 2008) and during crisis (up to end of July 2009). We have shown that²⁵

- Volatility of returns, hence errors of forecasts from the ARMA and GARCH models of returns, hugely increased during the crisis.
- Introduction of corresponding stock indices returns led to a slight improvement of models and forecast performance.

There are several symptoms showing improvement of economic performance in current period. The Polish economy in particular during the crisis did well in comparison to other European economies. Whether the crisis ended can of course be argued, but let us treat the year (Spring 2009 – Spring 2010) as period “after crisis” and check if behavior of series modeled had stabilized enough to improve the quality of models.

QUALITATIVE DESCRIPTION OF THE DATA

We use daily data from 4th January 2000 until 30th April 2010 for the indices and exchange rates.²⁶ The typical measure of returns is:

$$(1) \quad z_t = 100 * (\ln y_t - \ln y_{t-1})$$

where y_t – closing values of an instrument; we use also logarithm of proportion of daily maximum and minimum

$$(2) \quad \sigma_t^2 = \ln(y_{\max,t} / y_{\min,t})$$

as a measure of variance/volatility (see [Brooks 2008]). Fig. 1 shows a typical behavior of stock index returns, an increase of volatility during both 2001 dotcom crisis and even higher increase during the last crisis. Fig. 2 shows a similar picture for exchange rate returns. Fig. 3 shows volatility defined by equation (2) for the corresponding exchange rate.

We compare both the whole sample and two equal subsamples: 16.10.2007 – 23.01.2009 as “crisis period” and 27.01.2009 – 30.04.2010 as “post-crisis period” (each consists of 312 observations). Choice of “crisis” period is to some extent arbitrary, but we follow [Reinhart and Rogoff 2008] as to characteristic symptoms of crisis.

²⁴ This was suggested by specification of models for daily returns of Norwegian krona [Bauwens, Rime and Succarat 2008].

²⁵ [Syczewska 2010] paper was presented at the International Conference “Zagadnienia aktuarialne – teoria i praktyka” in Warsaw, 2nd-4th September 2009.

²⁶ <http://stooq.pl> database, opening, closing, minimum and maximum daily quotes. We use only the dates, for which all quotes (Polish and American) were available.

Table 1. Comparison of variance in the two samples

Variable	Quote	Mean		Variance		Proportion of variance 2 nd to 1 st
		1st sample	2nd sample	1st sample	2nd sample	
S&P500	open	1244.6	1000.8	45552.4	17524.5	38.5%
	max	1257.7	1145.5	43986.7	16848.0	38.3%
	min	1227.3	1117.7	47624.3	18068.7	37.9%
	close	1242.2	1131.6	45817.4	17540.4	38.3%
WIG20	open	2695.8	2413.1	396358.4	117889.2	29.7%
	max	2723.8	2440.6	394610.1	115613.6	29.3%
	min	2657.3	2378.3	395741.6	119639.9	30.2%
	close	2690.0	2409.0	393756.3	117703.9	29.9%
USDPLN	open	2.4547	2.7100	0.09736	0.07607	78.1%
	max	2.4786	2.7390	0.10827	0.08185	75.6%
	min	2.4370	2.6860	0.09174	0.06955	75.8%
	close	2.4582	2.7120	0.10051	0.07587	75.5%
EURUSD	open	1.4608	1.4270	0.00953	0.00435	45.7%
	max	1.4697	1.4360	0.00909	0.00421	46.3%
	min	1.4519	1.4180	0.00999	0.00444	44.4%
	close	1.2970	1.4270	0.00959	0.00435	45.3%

Source: stooq.pl

Table 1 shows *decreases in variances* of the indices (between 30–40 percent) and exchange rates (between 75–80 percent for USD, approximately 45% for EURO); in contrast to comparison of “crisis” and “pre-crisis period” [Syczewska 2010] when it increased 2.6 times for S&P500, 2 times for WIG20, 2.8 times for USDPLN exchange rate, 1.7 times for EURUSD.

Volatility defined as in (2), i.e., log difference between maximum and minimum daily quotes for both stock indices has decreased to 70-90% of previous value, for USD exchange rate increased by 14 percent, for EURUSD – decreased to 89% of “crisis” volatility. For USDPLN, EURUSD exchange rates and for both stock indices difference in means is significant.

Table 2. Comparison of volatility during and after the crisis

	USDPLN	EURUSD	EURPLN	SP500	WIG20
Mean during crisis	.01598	.01248	.02846	.02674	.0264597
Mean after crisis	.01815	.01114	.02929	.01799	.0235659
Proportion	1.14	0.89	1.03	0.67	0.89
t statistic for difference in the means	-2.63	2.74	-0.67	6.66	2.53
Median during crisis	.01086	.01016	.02101	.01955	.02265
Median after crisis	.01630	.01038	0.02716	.01514	.02051
Proportion	1.50	1.02	1.29	0.77	0.91

Source: computations based on stooq.pl data.

GRANGER CAUSALITY TESTS

Variable x is defined as Granger-cause for another variable y , if lagged values of x used as additional regressors in a model describing y can improve quality of modeling/forecasting. There are several tests of this property. The Granger test of Granger causality is performed in the following way: we estimate VAR – type equation and check joint significance of lagged x parameters:

$$(3) \quad y_t = a_{11}y_{t-1} + \dots + a_{1k}y_{t-k} + b_{11}x_{t-1} + b_{12}x_{t-2} + \dots + b_{1k}x_{t-k} + \varepsilon_t$$

The null $H_0: b_{11} = b_{12} = \dots = b_{1k} = 0$ means that the x *does not* Granger-cause the y variable. This test is reported to work well for stationary variables, for non-stationary series, it should be used with caution. The Augmented Dickey-Fuller (in short, ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests show stationarity of logarithmic returns for the whole sample. Results for subsamples (“crisis”, “after crisis”) are slightly different (see Table 3).

Table 3. The ADF test results for returns and volatility of variables

ADF	S&P500	WIG20	USDPLN	EURUSD
Returns	-12.22 (.00)	-21.11 (.00)	-9.89 (.00)	-10.72 (.00)
Volatility				
whole sample	-4.48 (.0001)	-3.4889 (.008)	-4.9505 (.00)	-3.4384 (.0097)
- crisis	-3.1005 (.03)	-2.6481 (.08)	-0.8928 (.79)	-1.3654 (.60)
- after crisis	-2.4920 (.12)	-2.5312 (.11)	-2.0222 (.28)	-3.0360 (.03)

Source: own computations; bold denotes insignificant values.

To check whether stock indices volatility/returns Granger-cause respective measures for the exchange rates, we estimate VAR(5) models.²⁷ For bilateral USDPLN exchange rate we check whether stock indices of respective economies, i.e. American and Polish indices, Granger-cause the exchange rate. Joint significance test statistic for lagged values of S&P500 index volatility $F(5,2508) = 3.8661$ with p-value 0.0017. Hence we reject the null of lack of causality: the US stock index Granger-causes volatility in USDPLN exchange rate, as expected. There is no causality from the exchange rate towards volatility of index. Full results of the Granger test of Granger causality are given in Table 4.

²⁷ Number of lags chosen by reduction of insignificant lags.

Table 4. Granger-causality tests from stock indices towards exchange rates

Causal relationship	Whole sample	Crisis	After crisis
WIG20→USDPLN	2.3771 [.0367]	2.1216 [.0628]	1.5200 [.1833]
SP500→USDPLN	3.8661 [.0017]	1.8947 [.0951]	1.8937 [.0952]
WIG20→EURUSD	303.41 [.0000]	2.0457 [.0722]	1.0351 [.3970]
SP500→EURUSD	7.1983 [.0000]	40.887 [.0000]	4.4195 [.0007]

Source: own computations; p-values in brackets, greater than 0.05 marked in bold.

FRACTIONAL INTEGRATION, PERSISTENCY AND LONG MEMORY MEASURES

As shown by the ADF test for the whole sample, log returns of exchange rates are stationary. Fractional integration parameter is perhaps more accurate indicator of time series behavior, either stationary or nonstationary. It generalizes the Engle and Granger [1987] definition of integrated series (Hosking [1981], Granger and Joyeux [1980]), it can take any real values (not only integer, as in the ADF test), and is defined with use of binomial series expansion or the Gamma function. The fractional integration parameter is often estimated with periodogram regression (one of the variants was introduced by [Geweke and Porter-Hudak 1983]). Another, semiparametric, method is the Whittle local estimator, introduced by [Künsch 1987] and [Robinson 1995] (see e.g. [Phillips and Shimotsu 2000]).

Classification of d values shows whether series in question is stationary or not, but more accurately, if it is persistent or antipersistent, whether results of external shocks diminish in time etc.

- For $d = 1$ a series is nonstationary, with infinite variance.
- For $1 \leq d$ variance is infinite, moreover results of a shock increase with time;
- If $0.5 \leq d < 1$, the process is nonstationary, but in a long-term reverts to its mean [Hosking 1981];
- For $0 < d < 0.5$, the process is stationary, with finite variance, and is mean reverting;
- For $d = 0$ it is mean-reverting in the short term, has finite variance and effects of shocks diminishes quickly;
- For $-0.5 < d < 0$ it is stationary, but mean-averting (antipersistent).

Quite similar classification can be done with use of the Hurst exponent [Hurst 1951]: if $H=0.5$ we have a random walk, if H is in $(0; 0.5)$ – a mean-reverting process; for H in $(0.5; 1)$ – mean-averting process with a trend.

Table 5. Hurst exponents for logarithmic returns

Logarithmic returns of:	Hurst exponent				
	SP500	WIG20	USDPLN	EURUSD	EURPLN
Whole sample	0,553 [3,10]	0,578 [4,84]	0,574 [10,89]	0,567 [9,24]	0,547 [3,77]
First subsample	0,513 [0,75]	0,547 [1,69]	0,636 [2,93]	0,608 [4,69]	0,544 [0,62]
Second subsample	0,578 [5,15]	0,520 [1,99]	0,511 [0,28]	0,609 [3,06]	0,459 [-3,22]

Source: own computations; t statistics in brackets

The Hurst exponent value of 0.5 corresponds to a white noise process, values greater than 0.5 but less than 1 suggest persistency and stationarity of a series. Table 5 shows computed values of the Hurst exponent for logarithmic returns of stock indices and exchange rates, computed for the whole sample and for two subsamples – during crisis and after crisis. Critical value of the Student t statistics is 2.44 for the whole sample, 2.22 for both shorter subsamples. We test the null of $H = 0.5$. Computed values of the t statistics show that the null cannot be rejected for stock indices and the log returns of EURPLN exchange rate in the first sample, and for WIG20 and USDPLN in the second sample. In other cases H is slightly greater than 0.5. Hence the Hurst exponents suggest stationarity and persistency of all logarithmic returns.

Table 6. Estimates of fractional integration parameter for logarithmic returns²⁸

Returns of:	Method:	Whole sample	Subsample	
			First	Second
SP500	GPH	0,0013 [0,98]	-0,0374 [0,76]	0,1222 [0,69]
	Whittle	-0,0118 [0,81]	-0,0603 [0,51]	0,0913 [0,94]
WIG20	GPH	0,1208 [0,07]	0,0976 [0,49]	0,1393 [0,48]
	Whittle	0,0499 [0,30]	-0,0871 [0,34]	0,0913 [0,08]
USDPLN	GPH	0,0636 [0,32]	0,0396 [0,80]	0,1557 [0,79]
	Whittle	0,0861 [0,07]	0,0014 [0,99]	0,0913 [0,40]
EURUSD	GPH	0,0493 [0,42]	0,2026 [0,16]	0,1390 [0,73]
	Whittle	0,0560 [0,24]	0,0886 [0,33]	0,0913 [0,51]
EURPLN	GPH	0,0754 [0,23]	-0,2265 [0,06]	0,1170 [0,25]
	Whittle	0,0611 [0,20]	-0,0398 [0,66]	0,0913 [0,05]

Source: own computations

²⁸ In brackets there are p-values of t statistics in case of Geweke and Porter-Hudak method, z statistics in case of the Whittle estimator, both for a null of insignificance.

More accurate are results of the GPH and Whittle methods, allowing for tests of significance of the fractional integration parameter. Results presented in table 6, with p-values of statistics for null of $d = 0$ in brackets, show that all returns series are stationary. Estimates of the fractional integration parameter are in most cases insignificant, as shown by the t or z-statistics p-values. Only in few cases the null of insignificance is rejected – note in particular difference between results for EURPLN, with positive insignificant values for the whole sample, negative significant value for the first subsample, suggesting stationary antipersistent behavior, and positive significant value for the second subsample, suggesting persistent stationary behavior. For stationary series and ARMA model with finite number of parameters can be estimated.

FRACTIONAL INTEGRATION PARAMETERS FOR VOLATILITY

We estimate in a similar way fractional integration parameters for volatility defined as in (2). The results are as follows (see table 7); estimates of the fractional integration parameter are in interval (0.5; 1), suggesting nonstationarity of a series. The Hurst exponents, H , are close to 1. These may pose problems with the choice of number of lags for the ARMA model. Fig. 4 and 5 show periodogram for log returns and for volatility (2) for the USDPLN exchange rate. The first is similar to spectrum of a stationary series, the second has relatively high values for lower frequencies, which is closer to a behavior for nonstationary series.

We test whether $H = 1$ and whether the fractional integration parameter, d is equal first, to 0.5 and second, to 1.²⁹ The null hypothesis $H = 1$ is rejected for WIG20 and USDPLN. The null hypothesis $d = 0.5$ cannot be rejected for WIG20 and EURUSD in case of the GPH estimation, and for WIG20, USDPLN and EURUSD in case of the Whittle method. The null hypothesis $d = 1$ is rejected in all cases. Hence volatility is in all cases at least mean-reverting in long term, nonstationary if $d > 0.5$, stationary if $d = 0.5$.

Table 7. Hurst exponents, the Geweke-Porter-Hudak and Whittle estimators of fractional integration parameter for volatility

	SP500	WIG20	USDPLN	EURUSD	EURPLN
Hurst exponent	.9702 (.020)	.8845 (.015)	.9405 (.025)	.9741 (.038)	.9702(.031)
GPH estimator	.7031 (.058)	.4904 (.066)	.6248 (.050)	.5809 (.059)	.6894 (.055)
Whittle estimator	.6548 (.048)	.5240 (.048)	.5867 (.048)	.5878 (.048)	.6333(.048)

Source: own computations; standard errors in parentheses

²⁹ The Hurst exponent was estimated with use of R/S regression with 9 degrees of freedom, hence critical value $t^* = 2.26$. The GPH and Whittle estimators are based on regressions with 109 degrees of freedom, hence critical value $t^* = 1.98$.

ARMA/ARMAX MODEL

We expect that stock indices can improve quality of ARMA models for volatility (2). We apply ARMA formulation, with 4 lags as starting point, then try and reduce the model on the basis of significance tests and Schwarz Bayesian Information Criterion (procedure of reduction if similar, e.g., to [Matuszewska, Witkowska 2007], albeit their starting point is an autoregressive distributed lags model).

Model estimated for pre-crisis period has been reduced to ARMA(1,1) [Syczewska 2010]. For USDPLN exchange rate volatility 3rd lags are significant. We use S&P500 and WIG20 volatility (2) as additional regressors; both are significant. Similar results are obtained for other exchange rates. The roots of AR and MA polynomials have absolute values greater than 1, hence all ARMA/ARMAX models estimated are stable. For all exchange rates models with the stock indices variables have lower values of Akaike, Schwartz and Hannan-Quinn information criteria.

GARCH MODELS FOR LOGARITHMIC RETURNS

We estimate next the ARMA and ARMAX models for the logarithmic returns of exchange rates, starting first with an ARMA(4,4) model, and then adding logarithmic returns of both indices. The added variable parameters prove to be significant. For both models we could not reject the ARCH effect:

The Engle test of the ARCH effect is based on the regression

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \dots + \alpha_k e_{t-k}^2 + u_t$$

where e are error terms of the model in question. We check whether lagged error squares are jointly significant: the null $H_0 : \alpha_1 = \alpha_2 = \dots = \alpha_k = 0$ corresponds to lack of the ARCH effect. Under the null, the test statistic is asymptotically distributed as $\chi^2(k)$.

For USDPLN, computed values of the test statistics in case of ARMA and ARMAX models estimated for the whole sample, are equal respectively to $TR^2 = 396.238$ and 322.041 , respectively. Hence the null hypothesis of no ARCH effect is rejected.

As a result we estimate GARCH model for logarithmic returns of exchange rates, with and without log returns of stock indices as additional variables: starting with the GARCH model with 10 lags in autoregressive equation of mean value of the USDPLN log returns, and reducing insignificant lags, we reduce this model to one with AR(6) equation for expected value and GARCH(1,1) for variance.³⁰

³⁰ GARCH(1,1) is in most cases well suited for stock indices and exchange rates modeling, see e.g. [Brzeszczyński, Kelm 2002].

Starting with the GARCH model, 10 lags in autoregressive equation for the mean and log returns of the S&P500 and the WIG20 indices as additional variables, we reduce the model to only AR(1) with SP500 and WIG20 as additional variables, and GARCH(1,1) for conditional variance.

To compare results of forecasting exercise for both versions (with and without additional variables), we decide to use one lag for expected value equation. We reestimate the models for shorter sample, up to 2009/01/26, and compute forecasts for end of the whole sample (up to the end of April 2010). Forecasts quality is still not impressive, according to mean absolute error MAE, mean squared error MSE, mean absolute percentage error MAPE, and the Theil U indicator, given by

$$U = \frac{\sqrt{\frac{1}{m} \sum_{t=1}^m (y_t - y_t^p)^2}}{\sqrt{\frac{1}{m} \sum_{t=1}^m y_t^2 + \frac{1}{m} \sum_{t=1}^m (y_t^p)^2}}$$

(where y_t, y_t^p denote observation of the series and value of forecast in period t , and m denotes forecast horizon).³¹ Models with additional regressors perform slightly better.

CONCLUSIONS

We perform analysis for logarithmic returns and for volatility (defined as log difference between maximum and minimum of daily quotes) for daily values of exchange rates. We compare their behavior in period 2007-2010 (during crisis) and since January 2009 until April 2010. We note that the series have slightly stabilized, although volatility is still quite high in comparison to the period before the crisis.

Long memory of the series and values of the fractional integration parameter indicating nonstationarity for the crisis period result in greater number of lags in the ARMA model in comparison to the earlier period (where the ARMA model specification has been chosen using significance tests and information criteria).

Granger-causality tests show that the corresponding measures of stock indices Granger-cause returns or volatility of exchange rates. The Engle test shows presence of ARCH effect.

Indeed, results of estimation and of in-sample forecasting exercise, show that S&P500 and WIG20 stock indices measures used as additional regressors in mean equation, improve slightly the quality of ARMAX and GARCH models for either

³¹ See M. Gruszczyński and M. Podgórska (eds.), „Ekonometria”, Warsaw School of Economics, Warsaw 2004, p. 117.

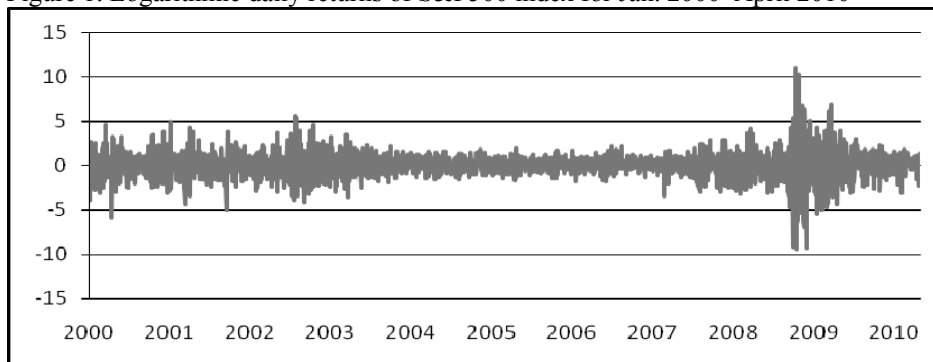
returns or volatility of the exchange rates. The volatility of exchange rates until the end of April 2010 is quite high, hence to improve quality of modeling we should wait for hopefully further stabilization of the series and work towards improvement of specification of the estimated econometric models.

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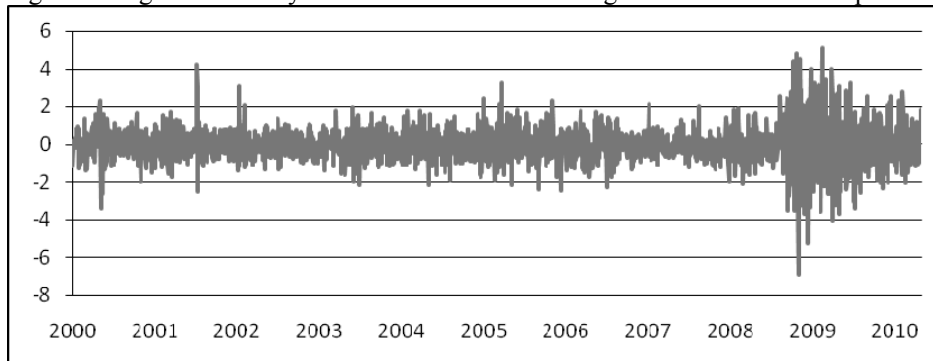
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Figure 1. Logarithmic daily returns of S&P500 index for Jan. 2000–April 2010



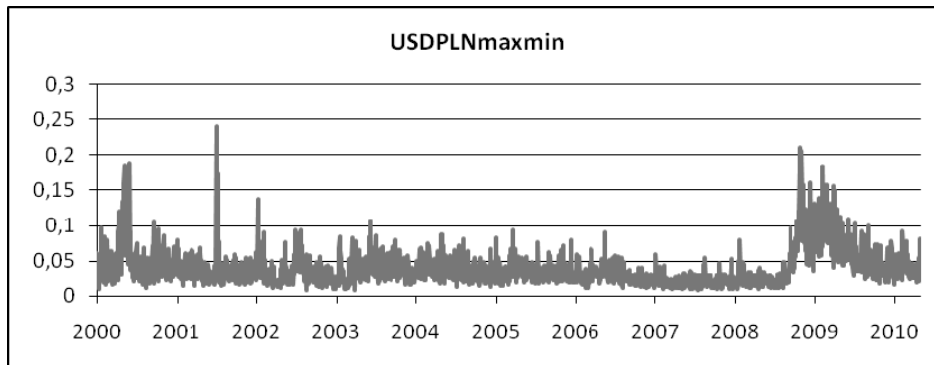
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Figure 2. Logarithmic daily returns of USDPLN exchange rate for Jan. 2000–April 2010



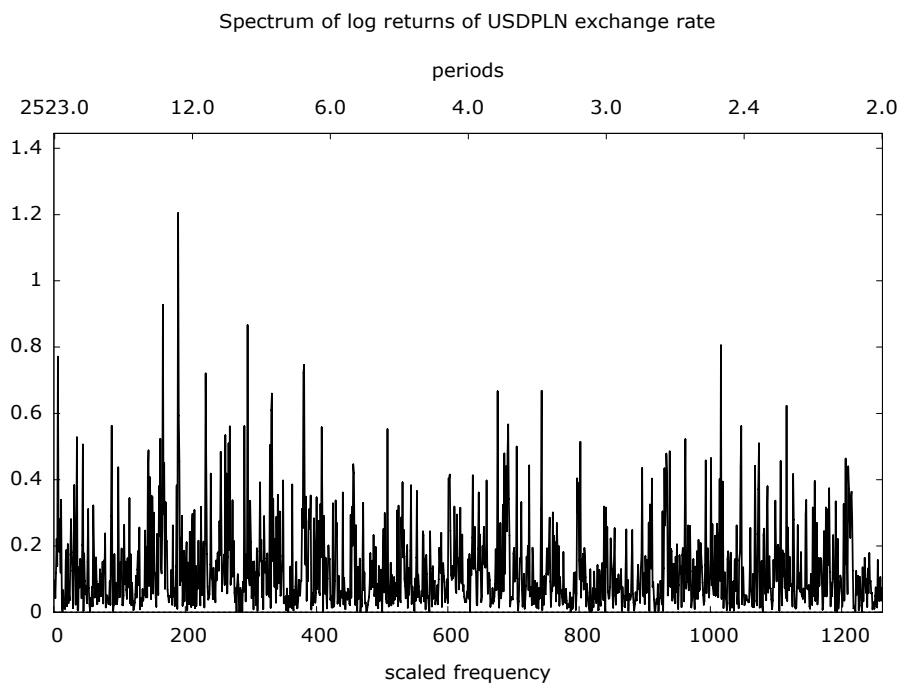
Source: <http://stoq.pl>

Figure 3. Same day max-min volatility of USDPLN exchange rate for Jan. 2000–April 2010



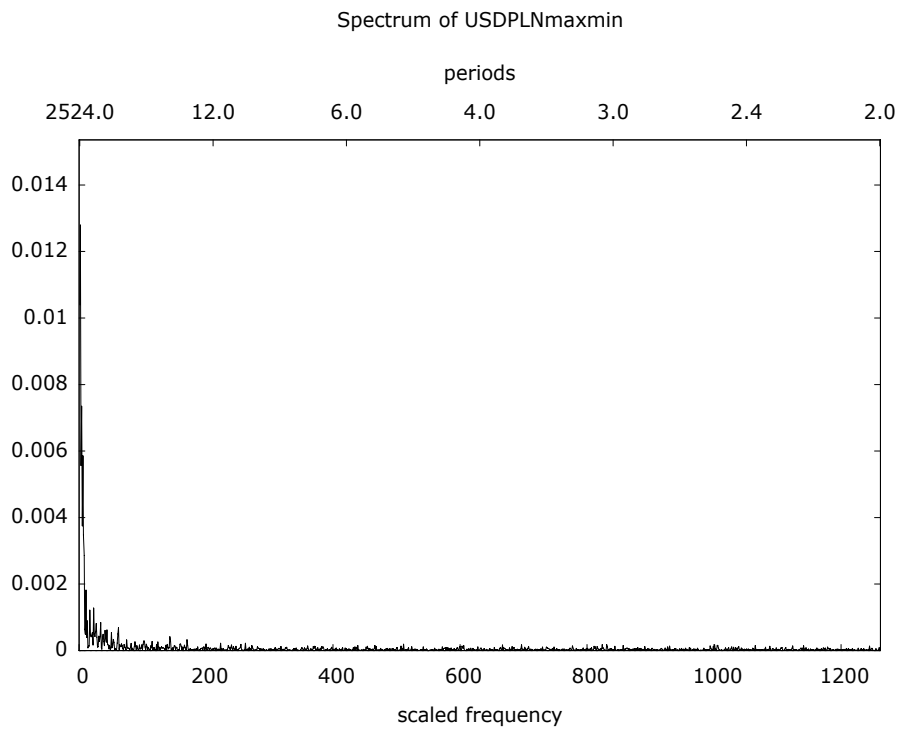
Source: own computations

Figure 4. Periodogram for logarithmic returns of USDPLN exchange rate for Jan. 2000–April 2010



Source: own computations

Figure 5. Periodogram for volatility of USDPLN exchange rate for Jan. 2000–April 2010



Source: own computations