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AN APPLICATION OF RADAR CHARTS TO GEOMETRICAL MEASURES OF STRUCTURES' OF CONFORMABILITY

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Abstract: In the following work we presented a method of using radar charts to calculate measures of conformability of two objects according to formulas given by, among others, Dice, Jaccard, Tanimoto and Tversky. This method incorporates another one presented by the authors of this study [Binderman, Borkowski, Szczesny 2010]. Presented methods can be also utilized to define similarities between given objects, as well as to order and group objects. Measures described in this work satisfy the condition of stability as they do not depend on the order of studied features.

Key words: radar method, radar measure of conformability, Dice's, Jaccard's measure of similarity, synthetic measures, classification, cluster analysis.

CONSTRUCTION OF RADAR MEASURES OF CONFORMABILITY

In previous works authors used methods that have a simple interpretation in the form of a radar chart to order, classify and measure similarity of objects [Binderman, Borkowski, Szczesny 2008, 2009, 2009a, 2010, 2010a, b, c, d, Binderman, Szczesny 2009, 2011, Binderman 2009, 2009a]. Those methods do not depend on the way the features of a given object are ordered. In the following work authors attempted to utilize those methods in other, widely known means of measuring similarity between two objects. Comparing structures of objects is chosen here as an example. Coefficients of Jaccard, Dice and Tanimoto, Tversky index and cosine similarity are all exemplary geometrical measures of similarity.

The methods presented here may seem numerically complicated but in the age of computers this problem is of little significance.

Numerous studies conducted in many different fields of science: economics, statistics, computer science, chemistry, biology, ecology, psychology, culture and tourism have proven the usefulness of those methods [Binderman 2009a, Binderman, Borkowski, Szczesny 2010b, c, Ciok, Kowalczyk, Pleszczyńska, Szczesny 1995, Deza E., Deza M.M. 2006, Duda, Hart, Stork 2000, Gordon 1999, Hubalek 1982, Kukuła 2000, 2010, Legendre P., Legendre L. 1998, Monev 2004, Szczesny 2002, Tan, Steinbach, Kumar 2006, Warrens 2008].

Let Q and P be two objects that are described by a set of n (n>2) features. Assume that objects Q and P are described by two vectors $\mathbf{x},\mathbf{y} \in \Re_+^n$, where:

$$\mathbf{x} = (x_1, x_2, \dots, x_n),\ \ \mathbf{y} = (y_1, y_2, \dots, y_n);\ \ x_i, y_i \geq 0;\ \ i = 1,2,\dots,n$$

and

$$\sum_{i=1}^{n} x_i = 1,\ \ \sum_{i=1}^{n} y_i = 1.$$

In order to graphically represent the methods we inscribe a regular n-gon into a unit circle (with a radius of 1) with a centre in the origin of a polar coordinate system 0uv and we will connect the vertices of this n-gon with the origin of the coordinate system. Thus, one constructs line segments of length 1, we will denote, in sequence, O1,O2,...,0n, starting, for definitiveness, with the line segment covering w axis. Assume that at least two coordinates of each of the vectors **x** and **y** are non-zero. Because features of objects **x** and **y** take on values from an interval <0,1>, that is $\mathbf{0} \leq \mathbf{x} \leq \mathbf{1} \equiv 0 \leq x_i \leq 1$, $\mathbf{0} \leq \mathbf{y} \leq \mathbf{1} \equiv 0 \leq y_i \leq 1$, i=1,2,...n, where $\mathbf{0}$:=(0,0,...,0), $\mathbf{1}$:=(1,1,...,1), we can represent the values of those features as a radar chart. To do so, let x_i (y_i) denote those points on the 0i axis that came into being by intersecting the 0i axis with a circle with the centre at the origin of the coordinate system and radius of x_i (y_i), i=1,2,...,n. By connecting the points: x_1 with x_2, x_2 with x_3, ..., x_n with x_1 (y_1 with y_2, y_2 with y_3, ..., y_n with y_1) we get n-gons S_Q and S_P, where its areas $|S_Q|$ and $|S_P|$, are given by formulas:

$$|S_Q| = |S_{\mathbf{x}}| = \sum_{i=1}^{n} \frac{1}{2} x_i x_{i+1} \sin\frac{2\pi}{n} = \frac{1}{2}\sin\frac{2\pi}{n}\sum_{i=1}^{n} x_i x_{i+1},\ \ \text{gdzie}\ \ x_{n+1} := x_1\ ,$$
$$|S_P| = |S_{\mathbf{y}}| = \sum_{i=1}^{n} \frac{1}{2} y_i y_{i+1} \sin\frac{2\pi}{n} = \frac{1}{2}\sin\frac{2\pi}{n}\sum_{i=1}^{n} y_i y_{i+1},\ \ \text{gdzie}\ \ y_{n+1} := y_1\ . \tag{1}$$

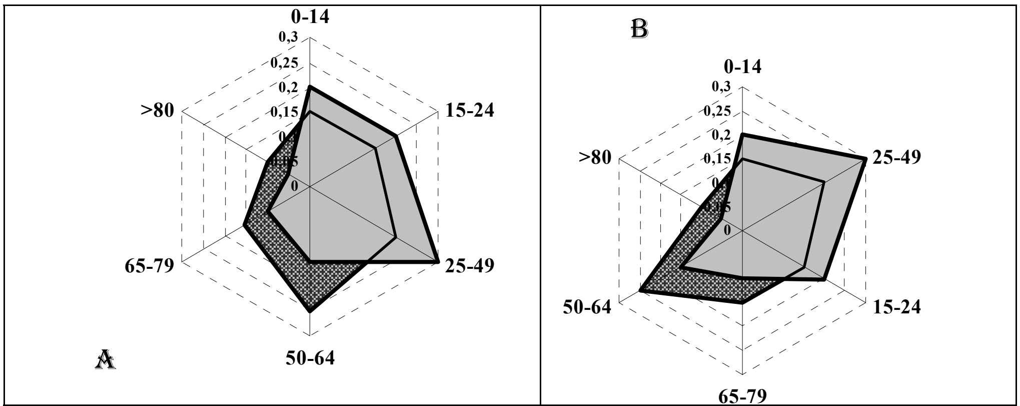
The formula for the area of the intersection of those n-gons, which we will denote by $S_{\mathbf{x}\cap\mathbf{y}} := S_{\mathbf{x}} \cap S_{\mathbf{y}}$ has a more complicated form. Its form and detailed determination can be found in [Binderman, Borkowski, Szczesny 2010]. Using those formulae we can denote the area of the union of n-gons S_x and S_y as

$$\left|S_x \cup S_y\right| = \left|S_x\right| + \left|S_y\right| - \left|S_x \cap S_y\right|, \tag{2}$$

where the areas $\left|S_x\right|, \left|S_y\right|$ are defined by formulae (1).

Figure 1 presents two graphical illustrations of vectors **x**=(0,2, 0,2, 0,3, 0,15, 0,1, 0,05) and **y** =(0,15, 0,15, 0,2, 0,25, 0,15, 0,1) that describe two exemplary demographical structures (for age ranges: 0-14, 15-24, 25-49, 50-64, 65-79, >80), while Fig. 1A and 1B differ only by the order of axes (meaning the permutation of the coordinates).

Fig. 1. Radar charts for vectors **x** and **y,** which coordinates present two exemplary demographical structures, by different ordering of axes.



Source: own work

From the figure it is clear that areas of n-gons $\left|S_x\right|, \left|S_y\right|$ and their unions on figures 1A and 1B differ in size. They are: 0,076; 0,074; 0,051 and 0,075; 0,069; 0,047, respectively.

In works [Binderman Borkowski, Szczesny 2008, 2010] authors proposed a measure of conformability of objects that uses a geometrical interpretation in the form of radar charts and is defined as follows:

$$R_{xy} = \begin{cases} \sqrt{\dfrac{\left|S_x \cap S_y\right|}{\sigma_{xy}}} & \text{for } n=3 \\ \sqrt{\dfrac{\left|S_x \cap S_y\right|}{\omega_{xy}}} & \text{for } n \geq 4 \end{cases}, \tag{3}$$

where

$$\sigma_{\mathbf{xy}} := \begin{cases} \min(|S_{\mathbf{x}}|,|S_{\mathbf{y}}|) \text{ gdy } |S_{\mathbf{x}}||S_{\mathbf{y}}|>0 \\ 1 \qquad\qquad \text{gdy } |S_{\mathbf{x}}||S_{\mathbf{y}}|=0 \end{cases}, \quad \omega_{\mathbf{xy}} := \begin{cases} \max(|S_{\mathbf{x}}|,|S_{\mathbf{y}}|) \text{ gdy } |S_{\mathbf{x}}||S_{\mathbf{y}}|>0 \\ 1 \qquad\qquad \text{gdy } |S_{\mathbf{x}}||S_{\mathbf{y}}|=0 \end{cases}.$$

Note that such a measure of conformability (similarity) has the property of: $0 \le \mu_{x,y} \le 1$ and depends on the ordering of features [cf. Binderman Borkowski, Szczesny 2008].

To define a measure of conformability of objects that does not depend on the ordering of features, let us denote by π_j – a j-th permutation of numbers 1,2,…,n. It is known that the number of all such permutations is equal to *n!* [Mostowski, Stark 1977]. Each permutation of coordinates of vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ corresponds to one permutation π_j. Let vectors $\boldsymbol{x}_j$, $\boldsymbol{y}_j$ denote the j-th permutation of coordinates of vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, assuming that $\boldsymbol{x}_1 := \boldsymbol{x}$, $\boldsymbol{y}_1 := \boldsymbol{y}$.

For example, if n=3, $\boldsymbol{x}=(x_1,x_2,x_3)$, $\boldsymbol{y}=(y_1,y_2,y_3)$ and $\pi_1=(1,2,3)$, $\pi_2=(1,3,2)$, $\pi_3=(2,1,3)$, $\pi_4=(2,3,1)$, $\pi_5=(3,1,2)$, $\pi_6=(3,2,1)$ then: $\boldsymbol{x}_1=(x_1,x_2,x_3)$, $\boldsymbol{y}_1=(y_1,y_2,y_3)$, $\boldsymbol{x}_2=(x_1,x_3,x_2)$, $\boldsymbol{y}_2=(y_1,y_3,y_2)$ $\boldsymbol{x}_3=(x_2,x_1,x_3)$, $\boldsymbol{y}_3=(y_2,y_1,y_3)$, $\boldsymbol{x}_4=(x_2,x_3,x_1)$, $\boldsymbol{y}_4=(y_2,y_3,y_1)$, $\boldsymbol{x}_5=(x_3,x_1,x_2)$, $\boldsymbol{y}_5=(y_3,y_1,y_2)$, $\boldsymbol{x}_6=(x_3,x_2,x_1)$, $\boldsymbol{y}_6=(y_3,y_2,y_1)$.

A result from our earlier works is that a coefficient of conformability of structures corresponds to each *j*-th permutation $\boldsymbol{x}_j$, $\boldsymbol{y}_j$ of coordinates of vectors **x** and **y**

$$R^j_{Q,P} = R_{\mathbf{x}_j \mathbf{y}_j}, \tag{4}$$

where naturally $R^1_{Q,P} = R_{\mathbf{xy}}$.

Therefore, we can assume that the following designations of three different measures of conformability of considered objects Q and P. Naturally, those measures are invariant under the ordering of coordinates for vectors **x** and **y.**

$$\begin{aligned} R^M_{Q,P} &= R^M_{\mathbf{xy}} = \max_{1\le j\le n!} R^j_{Q,P}, \\ R^m_{Q,P} &= R^m_{\mathbf{xy}} = \min_{1\le j\le n!} R^j_{Q,P}, \\ R^s_{Q,P} &= R^s_{\mathbf{xy}} = \frac{1}{n!}\sum_{j=1}^{n!} R^j_{Q,P}. \end{aligned} \tag{5}$$

Other well-known in literature techniques that use geometrical interpretations, such as radar charts, may be used to compare two structures $\mathbf{x} = (x_1, x_2, \ldots, x_n), \mathbf{y} = (y_1, y_2, \ldots, y_n)$. Most well known among them are:

cosine similarity [Deza, Deza 2006]

$$\widehat{\mathrm{c}}_{\mathbf{xy}} = \begin{cases} \dfrac{\left|\mathrm{S_x} \cap \mathrm{S_y}\right|}{\sqrt{\left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right|}} & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| > 0 \\ 0 & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| = 0 \end{cases}, \tag{6}$$

Jaccard coefficient [Jaccard 1901, 1902, 1908]

$$\widehat{\mathrm{J}}_{\mathbf{xy}} = \begin{cases} \dfrac{\left|\mathrm{S_x} \cap \mathrm{S_y}\right|}{\left|\mathrm{S_x} \cup \mathrm{S_y}\right|} & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| > 0 \\ 0 & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| = 0 \end{cases}, \tag{7}$$

Dice's coefficient [Dice 1945]

$$\widehat{\mathrm{D}}_{\mathbf{xy}} = \begin{cases} \dfrac{2\left|\mathrm{S_x} \cap \mathrm{S_y}\right|}{\left|\mathrm{S_x}\right| + \left|\mathrm{S_y}\right|} & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| > 0 \\ 0 & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| = 0 \end{cases}, \tag{8}$$

Tanimoto coefficient [Tanimoto 1957, 1959]

$$\widehat{\mathfrak{T}}_{\mathbf{xy}} = \begin{cases} \dfrac{\left|\mathrm{S_x} \cap \mathrm{S_y}\right|}{\left|\mathrm{S_x}\right| + \left|\mathrm{S_y}\right| - \left|\mathrm{S_x} \cap \mathrm{S_y}\right|} & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| > 0 \\ 0 & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| = 0 \end{cases}, \tag{9}$$

Tversky index [Tversky 1957]

$$\widehat{\mathrm{T}}_{\mathbf{xy}} = \begin{cases} \dfrac{\left|\mathrm{S_x} \cap \mathrm{S_y}\right|}{\left|\mathrm{S_x} \cap \mathrm{S_y}\right| + \alpha\left|\mathrm{S_x} \setminus \mathrm{S_y}\right| + \beta\left|\mathrm{S_y} \setminus \mathrm{S_x}\right|} & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| > 0 \\ 0 & \text{for } \left|\mathrm{S_x}\right|\left|\mathrm{S_y}\right| = 0 \end{cases} \quad \alpha, \beta \geq 0. \tag{10}$$

Let us note that if in the above formula the coefficients fulfill $\alpha=\beta=1$ then we get Tanimoto's formula and if $\alpha=\beta=\frac{1}{2}$ then we get Dice's formula. Here and in the sequel we shall assume that $\alpha=\beta=\frac{1}{4}$.

Note that the defined above measures of similarity, take a value between [0, 1], **are dependent on the ordering of features** in case once represents the object by a radar chart.

Another simple way of visualizing the structure $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a bar graph, in which each coordinate is represented as a rectangle of width 1 and height x_i (for i=1,…,n). The area of such graph is equal to 1 and one of the most popular indicators of similarity of two structures $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)$ is defined as [Malina 2004]:

$$W_{\mathbf{xy}} := \sum_{i=1}^{n} \min(x_i, y_i) \ , \tag{11}$$

It is clear that its value is independent of the ordering of features and, in the case of such graphical representation of structure, takes a value identical to the values of coefficients defined in (6) and (8).

In every situation when the indicator of similarity of two structures that uses a graphical interpretation is not invariant under the permutation of coordinates, we may modify its definition, in a way shown above (see formula (5)). Thus, to define a measure of conformability that would be independent of the ordering of features, let us denote by p_j – the j-th permutation of numbers 1,2,…,n. Naturally, each permutation of coordinates of vectors ***x*** and ***y*** corresponds to one permutation p_j. Let vectors $\boldsymbol{x}_j$, $\boldsymbol{y}_j$ denote j-th permutation of coordinates of vectors ***x*** and ***y***, respectively. Assume that $\boldsymbol{x}_1=\boldsymbol{x}$, $\boldsymbol{y}_1=\boldsymbol{y}$, for each j-th permutation $\boldsymbol{x}_j$, $\boldsymbol{y}_j$ of coordinates of vectors x and y corresponds a coefficient of conformability of structures

$$c_{Q,P}^{j} = \widehat{c}_{\mathbf{x}_j \mathbf{y}_j} \quad , \tag{12}$$

where naturally $c^{1}_{Q,P} = \overset{\frown}{c}_{\mathbf{xy}}$, and the cosine similarity $\overset{\frown}{c}_{\mathbf{xy}}$ is defined as in formula (6).

With regard to the above, let us assume the following definitions of three different measures of conformability for objects Q and P

$$\begin{aligned} c^{M}_{Q,P} &= c^{M}_{\mathbf{x},\mathbf{y}} = \max_{1\le j\le n!} c^{j}_{Q,P}, \\ c^{m}_{Q,P} &= c^{m}_{\mathbf{x},\mathbf{y}} = \min_{1\le j\le n!} c^{j}_{Q,P}, \\ c^{s}_{Q,P} &= c^{s}_{\mathbf{x},\mathbf{y}} = \frac{1}{n!}\sum_{j=1}^{n!} c^{j}_{Q,P}. \end{aligned} \tag{13}$$

In a similar manner we can define other coefficients $\mathrm{J}^{\mathrm{M}}_{\mathbf{xy}}, \mathrm{J}^{\mathrm{m}}_{\mathbf{xy}}, \mathrm{J}^{\mathrm{s}}_{\mathbf{xy}}; \mathrm{D}^{\mathrm{M}}_{\mathbf{xy}}, \mathrm{D}^{\mathrm{m}}_{\mathbf{xy}}, \mathrm{D}^{\mathrm{s}}_{\mathbf{xy}}; \mathfrak{J}^{\mathrm{M}}_{\mathbf{xy}}, \mathfrak{J}^{\mathrm{m}}_{\mathbf{xy}}, \mathfrak{J}^{\mathrm{s}}_{\mathbf{xy}}; \mathrm{T}^{\mathrm{M}}_{\mathbf{xy}}, \mathrm{T}^{\mathrm{m}}_{\mathbf{xy}} \mathrm{T}^{\mathrm{s}}_{\mathbf{xy}}$.

In order to demonstrate the presented above method of comparing structures, let us consider a simple example.

Example 1. Let $Q = \mathbf{x} = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$, $R = \mathbf{y} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. Let us assume the following denotations:

$$\mathbf{x}_1 := \mathbf{x}_4 := \mathbf{x},\ \mathbf{x}_2 := \mathbf{x}_5 := \left(\frac{1}{2}, 0, \frac{1}{2}\right),\ \mathbf{x}_3 := \mathbf{x}_6 := \left(0, \frac{1}{2}, \frac{1}{2}\right),$$

$$\mathbf{y}_1 := \mathbf{y}_2 := \mathbf{y}_3 := \mathbf{y}_4 := \mathbf{y}_5 := \mathbf{y}_6 = \mathbf{y}.$$

Thus we have:

$$\left|S_{\mathbf{x}_i}\right| = \frac{1}{2}\frac{1}{2}\frac{1}{2}\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{16}, \quad \left|S_{\mathbf{y}_i}\right| = 3\frac{1}{2}\frac{1}{3}\frac{1}{3}\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{12},$$

$$\left|S_{\mathbf{x}_i} \cap S_{\mathbf{y}_i}\right| = \frac{1}{2}\frac{1}{3}\frac{1}{3}\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{36}, \quad \left|S_{\mathbf{x}_i} \cup S_{\mathbf{y}_i}\right| = \frac{17\sqrt{3}}{144},$$

$$R_{\mathbf{x}_i\mathbf{y}_i} = \sqrt{\frac{\sqrt{3}}{36} \Big/ \frac{\sqrt{3}}{16}} = \frac{2}{3}, \text{ dla } i = 1, 2, \ldots, 6.$$

So $R^{M}_{\mathbf{x,y}} = R^{m}_{\mathbf{x,y}} = R^{s}_{\mathbf{x,y}} = \frac{2}{3} =\sim 0,666$, where coefficients $R^{M}_{Q,P}, R^{s}_{Q,P}, R^{m}_{Q,P}$ are defined as in formulae (5). It can be easily verified that coefficients of conformability of structures: cosine (formula (13)), Jaccard, Dice's are equal to:

$$c^{M}_{\mathbf{x,y}} = c^{m}_{\mathbf{x,y}} = c^{s}_{\mathbf{x,y}} = 0,385;\ J^{M}_{\mathbf{x,y}} = J^{m}_{\mathbf{x,y}} = J^{s}_{\mathbf{x,y}} = 0,236;\ D^{M}_{\mathbf{x,y}} = D^{m}_{\mathbf{x,y}} = D^{s}_{\mathbf{x,y}} = 0,381.$$

Note that

$$\sqrt{c^{M}_{\mathbf{x,y}}} = 0,620;\quad \sqrt{J^{M}_{\mathbf{x,y}}} = 0,486;\quad \sqrt{D^{M}_{\mathbf{x,y}}} = 0,617.$$

It is also noteworthy that in this case the coefficient of conformability of structures (defined by formula (11)) is $W_{\mathbf{xy}} = \frac{1}{3} + \frac{1}{3} + 0 = \frac{2}{3}$. The value of the coefficient define by formula (7) or (9) that uses an interpretation of the structure as a bar graph is equal to 0,5 =0,6(6)/1,3(3).

The above example shows that measures of similarity of two objects calculated by different methods (e.g. a method that includes the manner of the graphical representation of the structure or a method of normalizing, which, when applied, causes the measure of the area of the union of faces to take a value between [0, 1]), can be significantly different. A single measure of similarity of objects can be far from optimal in the understanding of a given expert. Furthermore, experts can disagree on the meanings of individual measures. Thus it is safer to use, in the analysis of structures, a measure that is, for example, an average of several different measures of similarity [see: Breiman 1994].

EMPIRICAL RESULTS

In order to verify the approach described in the previous section, we present an evaluation of the size of changes in demographical structures of European countries between the years 1999 and 2000, using the discussed above coefficients.

The following Tables 1 and 2 contain values of indicators evaluating the change of demographical structures for 27 countries between years 1999 and 2010; with an indication what position they occupied in the ranking of values of individual measures as well as two partitions of countries into 4 groups (columns **C1** and **C2**). The partition is made based on the values of indicator M (arithmetic mean of values of indicators **R**, **C**, **J**, **D** and **T**) and indicator W**,** while the thresholds were defined as: *A*-*d*, *A*, *A*+*d*, where *A* denotes an average and *d* – standard deviation.

Table 1. Values and rankings of indicators evaluating the similarity of demographical structures of 27 European countries in the years 1999 and 2010. Indicators are defined on the grounds of formulas: **R** - (3), **C** - (6), **J** - (7), **D** - (8), **T**-(10), **M**=(**R**+**C**+**J**+**D**+**T**)/5, **W** - (11) for the following ordering of age ranges: 0-14, 15-24, 25-49, 50-64, 65-79, >80. The last two columns contain information about the partition into 4 groups, according to values of indicators **M** and **W**, respectively.

No.	country	R	C	J	D	T	M	W	R	C	J	D	T	M	W	C1	C2
1	Austria	0,9676	0,9536	0,9110	0,9534	0,9762	0,9524	0,9630	5	5	5	5	5	5	6	**1**	**1**
2	Belgium	0,9616	0,9427	0,8913	0,9425	0,9704	0,9417	0,9590	9	7	7	7	7	7	8	**2**	**2**
3	Bulgaria	0,9455	0,9038	0,8243	0,9037	0,9494	0,9054	0,9480	14	15	15	15	15	15	11	**3**	**2**
4	Cyprus	0,9319	0,8967	0,8120	0,8963	0,9453	0,8964	0,9320	21	16	16	16	16	17	24	**3**	**4**
5	Czech Republic	0,9347	0,8776	0,7818	0,8776	0,9348	0,8813	0,9370	19	23	23	23	23	23	21	**3**	**3**
6	Denmark	0,9781	0,9606	0,9242	0,9606	0,9799	0,9607	0,9700	1	3	3	3	3	2	3	**1**	**1**
7	Estonia	0,9634	0,9438	0,8933	0,9437	0,9710	0,9430	0,9600	6	6	6	6	6	6	7	**2**	**2**
8	Finland	0,9469	0,9123	0,8385	0,9121	0,9540	0,9128	0,9400	13	14	14	14	14	14	19	**2**	**3**
9	France	0,9504	0,9180	0,8482	0,9178	0,9572	0,9183	0,9480	12	13	13	13	13	13	11	**2**	**2**
10	Germany	0,9624	0,9340	0,8762	0,9340	0,9659	0,9345	0,9470	7	10	10	10	10	10	15	**2**	**3**
11	Greece	0,9364	0,8900	0,8016	0,8899	0,9417	0,8919	0,9450	17	18	18	18	18	18	16	**3**	**3**
12	Hungary	0,9441	0,8950	0,8100	0,8950	0,9446	0,8978	0,9480	16	17	17	17	17	16	10	**3**	**2**
13	Ireland	0,9225	0,8754	0,7779	0,8751	0,9334	0,8769	0,9400	25	24	24	24	24	24	19	**4**	**3**
14	Italy	0,9620	0,9346	0,8771	0,9345	0,9662	0,9349	0,9710	8	9	9	9	9	9	2	**2**	**1**
15	Latvia	0,9587	0,9386	0,8839	0,9384	0,9682	0,9375	0,9520	10	8	8	8	8	8	9	**2**	**2**
16	Lithuania	0,9451	0,9239	0,8577	0,9234	0,9602	0,9220	0,9440	15	12	12	12	12	12	17	**2**	**3**
17	Luxembourg	0,9716	0,9650	0,9318	0,9647	0,9820	0,9630	0,9720	2	1	1	1	1	1	1	**1**	**1**
18	Malta	0,9263	0,8809	0,7867	0,8806	0,9365	0,8822	0,9260	24	22	22	22	22	22	25	**3**	**4**
19	Netherlands	0,9531	0,9276	0,8646	0,9274	0,9623	0,9270	0,9470	11	11	11	11	11	11	13	**2**	**3**
20	Poland	0,9061	0,8464	0,7331	0,8460	0,9166	0,8497	0,9220	27	27	27	27	27	27	26	**4**	**4**
21	Portugal	0,9357	0,8873	0,7973	0,8872	0,9402	0,8895	0,9470	18	19	19	19	19	19	13	**3**	**3**
22	Romania	0,9346	0,8820	0,7888	0,8820	0,9373	0,8849	0,9360	20	21	21	21	21	21	22	**3**	**3**
23	Slovakia	0,9131	0,8531	0,7435	0,8529	0,9206	0,8566	0,9220	26	26	26	26	26	26	27	**4**	**4**
24	Slovenia	0,9288	0,8667	0,7647	0,8667	0,9286	0,8711	0,9330	23	25	25	25	25	25	23	**4**	**4**
25	Spain	0,9299	0,8860	0,7949	0,8857	0,9394	0,8872	0,9440	22	20	20	20	20	20	17	**3**	**3**
26	Sweden	0,9689	0,9594	0,9215	0,9592	0,9792	0,9576	0,9670	4	4	4	4	4	4	5	**1**	**1**
27	United Kingdom	0,9700	0,9625	0,9272	0,9622	0,9808	0,9605	0,9690	3	2	2	2	2	3	4	**1**	**1**

Source: own work

Note that each of the first 5 indicators presented in Table 1, has an identical geometrical interpretation of similarity of structures, an intersection of two hexagons that represent those structures. They differ only by the method used to normalize that area, so that the value of the indicator of similarity is between [0, 1]. That is why all the indicators, with the exception of indicator **R**, they give the same ordering of European countries, according to the similarity of structures for the years 1999 and 2010. Small differences are visible only in the case of indicator **R**. The results do not change if we modify the indicator so that its value is independent of the ordering of coordinates of the vector representing the structure (see. Table 2). On the other hand, differences between the ordering by the value of indicator **W** (based on a different visualization of structures that the rest), and the ordering by the value of indicator **M** are noticeable. Even more so in the last two columns of Table 2, which represent the partition of countries into 4 groups, according to the similarity of structures for years 1999 and 2010. In case of Spain, we can observe a substantial difference in the assignment to a group depending on the used indicator.

Table 2. Description is similar to that of Table 1. The calculations of individual indicators where performed based on the first formulas (maximum) from (5), (13) and analogous modifications freeing the value of an indicator from the ordering of coordinates of a vector describing a given structure.

No.	country	R^M	C^M	J^M	D^M	T^M	M^M	W	R^M	C^M	J^M	D^M	T^M	M^M	W	C1	C2
1	Austria	0,9737	0,9572	0,9177	0,9571	0,9781	0,9567	0,9630	6	6	6	6	6	6	6	**1**	**1**
2	Belgium	0,9671	0,9427	0,8913	0,9425	0,9704	0,9428	0,9590	10	9	9	9	9	10	8	**1**	**2**
3	Bulgaria	0,9658	0,9360	0,8796	0,9360	0,9669	0,9369	0,9480	13	15	15	15	15	15	11	**2**	**2**
4	Cyprus	0,9592	0,9239	0,8575	0,9233	0,9601	0,9248	0,9320	19	20	21	21	21	19	24	**2**	**4**
5	Czech Republic	0,9582	0,9216	0,8545	0,9215	0,9592	0,9230	0,9370	21	22	22	22	22	22	21	**2**	**3**
6	Denmark	0,9812	0,9658	0,9338	0,9658	0,9826	0,9658	0,9700	2	2	2	2	2	2	3	**1**	**1**
7	Estonia	0,9702	0,9512	0,9066	0,9510	0,9749	0,9508	0,9600	8	7	7	7	7	7	7	**1**	**2**
8	Finland	0,9605	0,9294	0,8673	0,9290	0,9632	0,9299	0,9400	18	18	18	18	18	18	19	**2**	**3**
9	France	0,9590	0,9234	0,8577	0,9234	0,9602	0,9247	0,9480	20	21	20	20	20	20	11	**2**	**2**
10	Germany	0,9668	0,9386	0,8842	0,9385	0,9683	0,9393	0,9470	11	12	12	12	12	12	15	**1**	**3**
11	Greece	0,9629	0,9316	0,8718	0,9315	0,9646	0,9325	0,9450	17	17	17	17	17	17	16	**2**	**3**
12	Hungary	0,9635	0,9318	0,8722	0,9317	0,9647	0,9328	0,9480	15	16	16	16	16	16	10	**2**	**2**
13	Ireland	0,9665	0,9429	0,8918	0,9428	0,9705	0,9429	0,9400	12	8	8	8	8	9	19	**1**	**3**
14	Italy	0,9804	0,9669	0,9357	0,9668	0,9831	0,9666	0,9710	3	1	1	1	1	1	2	**1**	**1**
15	Latvia	0,9632	0,9416	0,8890	0,9412	0,9697	0,9410	0,9520	16	11	11	11	11	11	9	**1**	**2**
16	Lithuania	0,9527	0,9254	0,8598	0,9246	0,9608	0,9247	0,9440	24	19	19	19	19	21	17	**2**	**3**
17	Luxembourg	0,9791	0,9650	0,9318	0,9647	0,9820	0,9645	0,9720	4	3	4	4	4	4	1	**1**	**1**
18	Malta	0,9491	0,9097	0,8340	0,9095	0,9526	0,9110	0,9260	25	26	26	26	26	26	25	**3**	**4**
19	Netherlands	0,9653	0,9379	0,8831	0,9379	0,9680	0,9384	0,9470	14	13	13	13	13	14	13	**1**	**3**
20	Poland	0,9468	0,9031	0,8233	0,9031	0,9491	0,9051	0,9220	27	27	27	27	27	27	26	**3**	**4**
21	Portugal	0,9680	0,9376	0,8826	0,9376	0,9678	0,9387	0,9470	9	14	14	14	14	13	13	**1**	**3**
22	Romania	0,9548	0,9172	0,8470	0,9171	0,9568	0,9186	0,9360	22	23	23	23	23	23	22	**3**	**3**
23	Slovakia	0,9544	0,9118	0,8378	0,9118	0,9538	0,9139	0,9220	23	25	25	25	25	25	27	**3**	**4**
24	Slovenia	0,9482	0,9151	0,8416	0,9140	0,9551	0,9148	0,9330	26	24	24	24	24	24	23	**3**	**4**
25	Spain	0,9705	0,9425	0,8913	0,9425	0,9704	0,9434	0,9440	7	10	10	10	10	8	17	**1**	**3**
26	Sweden	0,9814	0,9649	0,9322	0,9649	0,9822	0,9651	0,9670	1	4	3	3	3	3	5	**1**	**1**
27	United Kingdom	0,9790	0,9634	0,9293	0,9634	0,9813	0,9633	0,9690	5	5	5	5	5	5	4	**1**	**1**

Source: own work

Tables 1 and 2 show that the greatest stability of the demographical structure between 1999 and 2010 was possessed by: Austria, Denmark, Luxembourg, Sweden and United Kingdom. On the other hand, the greatest changes were observed in: Cyprus, Malta, Poland, Slovakia and Slovenia. The greatest change occurred in Poland, and the smallest one in Luxembourg.

SUMMARY

Means for defining the values of indicators of similarity that use geometrical interpretations in the form of a value of an area and are described in this work can also be used in other geometrical ways of studying the similarity of structures as well as objects. These ways are an example of applying geometrical methods that are introduced by the authors using radar charts [Binderman, Borkowski, Szczesny 2008, 2010]. The empirical analysis shows that when structures are not subject to large changes then the values of individual indicators, based on the same geometrical interpretation, they order the structures similarly. However, if we change the way of visualizing the similarity (the geometrical interpretation) then we see changes in ordering. That is the reason why it is advisable to use several different indicators that use different means of visualization.

Furthermore, it is worth noting that by using geometrical interpretation as a basis to construct an indicator of similarity we can obtain an indicator that is very sensitive to changes in the ordering of coordinates of a vector that numerically represents a given structure. In practice there may be situations in which a researcher desires such quality in an indicator so it may visibly highlight even small differences between structures, but for a given ordering of their components. However, one needs to remember that methods of constructing indicators of similarity that use a geometrical interpretation are often applied mainly because of the ease of visualization of multidimensional data. Then an unseasoned researcher may misuse them. It must be highlighted that indicators based solely on those illustrations do not satisfy – often posed in the literature on this subject – the basic requirement of stability of the used method [see Jackson 1970], that means the independence of the ordering of features. Techniques presented by the authors show how a definition of an indicator must be modified (the method of measurement) to remove this flaw. Techniques that were pointed out may seem numerically complex; nevertheless, in the age of computers that problem became insignificant. On the other hand, this simple and stable empirical example shows that by applying modifications, that is making the measurement of similarity independent of the ordering of individual components of the structure, we obtain different results (see Tables 1 and 2, e.g., Spain).

The measurement of similarity of structures based on geometrical interpretation becomes even more complicated when a researcher is interested in changes that occurred in a given structures during the whole studied period and not only between the beginning and the end of the sample. Further works on this subject can be found in the work Binderman and Szczesny 2011.

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