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SOME REMARKS ON APLICATIONS OF ALGEBRAIC ANALYSIS TO ECONOMICS

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Abstract: In this paper, the author continues the investigations started in his earlier work [Binderman 2009]. Here, problems of linear equation Dx=y with the difference operator D are studied. The work is an introduction to applications of the theory of right invertible operators to economics. As an example, quotations of KGHM on Warsaw Stock Exchange are considered.

Key words: algebraic analysis, right invertible operator, difference operator, quotations of Stock Exchange, Jacobian matrix

In memory of Professor Krystyna Twardowska

INTRODUCTION

In mathematics the term Algebraic Analysis is used in two completely different senses [cf. Przeworska - Rolewicz 2000]. Here, meaning of Algebraic Analysis is closely connected with theory of right invertible operators [cf. Przeworska - Rolewicz 1988]. In the earlier work of the author [Binderman 2009] a new definition of elasticity operators in algebras with right invertible operators was proposed. The definition uses logarithmic mappings of algebraic analysis [cf. Przeworska-Rolewicz 1998]. The obtained results were applied to economics in order to find a function if elasticity of this function is given.

Here, possibilities of applications of algebraic analysis to economics, on the simple example of the difference operator D and the linear equation Dx=y are presented. The paper is an introduction in this range.

Throughout this work $\mathbb{F}$ will denote either the real field, $\mathbb{R}$, or the complex field, $\mathbb{C}$. Let X and Y be a linear space over $\mathbb{F}$. The set of all linear operators domains contained in X and ranges contained in Y will be denoted by $\mathfrak{L}(X,Y)$. We shall write:

$$\mathfrak{L}_0(X,Y) := \{A \in \mathfrak{L}(X,Y) : \mathrm{dom}\, A = X\}, \quad \mathfrak{L}(X) := \mathfrak{L}(X,X),$$

$$\mathfrak{L}_0(X) := \mathfrak{L}_0(X,X), \quad \ker A : \{x \in \mathrm{dom}\, A : Ax = 0\} \text{ for } A \in \mathfrak{L}(X,Y).$$

Following D. Przeworska - Rolewicz [c.f. Przeworska - Rolewicz 1988], an operator $D \in L(X)$ is said to be *right invertible* if there is an operator $R \in \mathfrak{L}\, L_0(X)$ such that $RX \subset \mathrm{dom}\, D$ and DR=I. The operator R is called a *right inverse* of D.

We shall consider in $L(X)$ the following sets:

- the set $\mathcal{R}(X)$ of all right invertible operators belonging to $L(X)$;
- the set $\mathcal{R}_D := \{ R \in \mathfrak{L}_0(X) : DRx = x \text{ for all } x \in X \}$;
- the set $\mathfrak{F}_D := \{ F \in \mathfrak{L}_0(X) : F^2 = F, FX = \ker D \text{ and } \exists R \in R_D : FR=0\}$ of all initial operators for a $D \in \mathcal{R}(X)$.

We note, if $D \in \mathcal{R}(X)$, $R \in \mathcal{R}_D$ and $\ker D \neq \{0\}$, then the operator D is right invertible, but not invertible. We have

$$DRx = x \text{ for all } x \in X \quad \text{and} \quad \exists\, x \in \mathrm{dom}\, D: RDx \neq x.$$

Here, the invertibility of an operator $A \in \mathfrak{L}(X)$ means that the equation Ax=y has the unique solution for every $y \in X$. If $D \in \mathcal{R}(X)$ and $0 \neq z \in \ker D$ and x_1 is a solution of the equation DX=y then the element $x_1 + z$ is also the solution of this equation.

If F is an initial operator for D corresponding to R then

$$Fx = x - RDx = (I-RD)x \text{ for } x \in \mathrm{dom}\, D \text{ and } Fz = z \text{ for } z \in \ker D. \tag{1}$$

We note, a different approach to the definition of right invertible linear operators is presented in the work [Binderman 2009].

In the sequel we shall assume that $D \in \mathcal{R}(X)$, $R \in \mathcal{R}_D$, $F \in \mathfrak{F}_D$ is an initial operator for D corresponding to R and dim ker D>0, i.e. D is right invertible but not invertible.

We observe, that if we know one right inverse of D then the sets [c.f. Przeworska - Rolewicz 1988]

$$\mathcal{R}_D = \{R + FA : A \in \mathfrak{L}_0(X)\}; \tag{2}$$

$$\mathfrak{F}_D = \{F(I-AD) : A \in \mathfrak{L}_0(X)\}. \tag{3}$$

We shall need the two following theorems [c.f. Przeworska - Rolewicz 1988].

Theorem 1. The general solution of the equation

$$Dx = y, \quad y \in X, \tag{4}$$

is given by the formula

$$x=z + Wy, \tag{5}$$

where $z\in \ker D$ is arbitrary and $W\in \mathcal{R}_D$ is arbitrarily fixed.

Theorem 2. The initial value problem

$$Dx = y, \quad y \in X, \tag{4}$$

$$Fx= z_0, \quad z_0\in \ker D, \tag{6}$$

has the unique solution of the form

$$x=z_0 +Ry, \tag{7}$$

where $R\in \mathcal{R}_D$ is the right inverse of operator D, corresponding to F.

We consider the several examples of operators which are often present in economics.

EXAMPLE 1. [cf. Przeworska – Rolewicz 1988, Bittner 1974, Levy, Lessman 1959, Gelfond 1957]. We suppose that X is the set of all sequences $\mathbf{x}=\{x_n\}$, where $x_n\in\mathbb{R}$, $n\in\mathbb{N}=\{1,2,\ldots\}$ with addition and multiplication by scalars defined as follows: if $\mathbf{x}=\{x_n\}$, $\mathbf{y}=\{y_n\}$, $\lambda\in\mathbb{R}$ then $\mathbf{x}+\mathbf{y}==\{x_n+y_n\}$, $\lambda\mathbf{x}=\{\lambda x_n\}$. Define the difference operators by the equalities:

$$D\mathbf{x}=\{x_{n+1}-x_n\},\ x=\{x_n\},$$

$$R\mathbf{x}=\{0, x_1, x_1+x_2, x_1+x_2+x_3,\ldots\}.$$

For $\mathbf{x}=\{x_n\}\in X$ we have:

$$DR\mathbf{x} = D\{0, x_1, x_1+x_2, x_1+x_2+x_3,\ldots\}=\{x_1-0, x_1+x_2-x_1, x_1+x_2+x_3-x_2-x_1,\ldots\}= =\{x_1, x_2, x_3 ,\ldots\}=\mathbf{x}.$$

$$RD\mathbf{x} = R\{x_2-x_1, x_3-x_2, x_4-x_3,\ldots\}=\{0, x_2-x_1, x_2- x_1+x_3-x_2, x_2-x_1+x_3-x_2, x_2-x_1+x_3-x_2+x_4-x_3,\ldots \}= \mathbf{x}-x_1\mathbf{e} \neq \mathbf{x} \text{ for } x_1\neq 0,$$

where $\mathbf{e}:=\{1,1,1,1,\ldots\}$.

The above shows that D is not invertible. The kernel of the operator D has the form:

$$\ker D = \{\mathbf{z}=\{z_n\}: z_n=c,\ n\in\mathbb{N},\ c\in\mathbb{R}\}.$$

By Theorem 1, the general solution of the equation (4) is of the form

$$\mathbf{x}=\{x_n\}==\{c, c+x_1, c+x_1+x_2, c+x_1+x_2+x_3,\ldots\}, \tag{8}$$

where $c \in \mathbb{R}$ is arbitrarily fixed.

We observe that the initial operator F for D corresponding to R is defined by the formula:

$$F\mathbf{x}=(I-RD)\mathbf{x} = \mathbf{x} - (\mathbf{x} - x_1\mathbf{e})= x_1\mathbf{e} \in \ker D,\ \mathbf{x}=\{x_n\}\in X.$$

EXAMPLE 2. [cf. Przeworska-Rolewicz 1988]. We denote by X=C[a,b] the set of all real-valued function defined and continuous on a closed interval [a,b]. The set X is a linear space over the field of real numbers $\mathbb{R}$ if the addition and multiplication by a number are defined as follows: (x+y)(t)=x(t)+y(t); (αx)(t)= αx(t) for x,y$\in$X, t$\in$[a,b], $\alpha\in\mathbb{R}$. Similarly properties has the set $C^1[a,b]\subset X$ of all real-valued functions defined on a closed interval [a,b] and having continuous derivative in (a,b). Suppose that we are given a point $t_0\in[a,b]$ and c is an arbitrary fixed real number. We define operators as follows:

$$(Dx)(t):= x'(t) \quad \text{for } x\in C^1[a,b]\subset X;\ t\in[a,b],$$

$$(Rx)(t)=\int_{t_0}^{t} x(\tau)d\tau \quad \text{for } x\in X;\ t\in[a,b].$$

The definition of D and R implies that R$\in \mathcal{R}_D$ and

$$(Fx)(t)=[(I-RD)x](t) =x(t_0) \quad \text{on dom } D= C^1[a,b].$$

EXAMPLE 3. [cf. Binderman 1992, 1993, 2000] Suppose that X is defined as in Example 2, where 0$\in$(a,b). Let $C_0^1[a,b]\subset X$ denotes the set of all real-valued function defined on a closed interval [a,b] and having continuous derivative in the point 0. We define operators as follows:

$$(Dx)(t)=\begin{cases}\dfrac{x(t)-x(0)}{t} & \text{for } t\in[a,b]\\ x'(0) & \text{for } t=0\end{cases},\quad x\in C_0^1[a,b].$$

$$(Rx)(t)=tx(t),\ x\in X;\ t\in[a, b]$$

The operator D is called a Pommiez operator or a backward shift operator [Dimovski 1990, Dimovski 2005, Douglas, Shapiro, Shields 1970, Fage, Nagnibida 1987, Linchuk 1988].The definition of D, R implies that R$\in \mathcal{R}_D$ and

$$(Fx)(t) =[(I-RD)x](t) =x(0).$$

EXAMPLE 4. [cf. Binderman 2009]. In similar way as in Example 2 we denote by X=C[a,b], where a>0, the set of all real-valued function defined and continuous on a closed interval [a,b]. Suppose that we are given a point $t_0\in[a,b]$. We define the operators D as follows:

$$(Dx)(t):=tx'(t) \quad \text{for } x\in C^1[a,b]\subset X;\; t\in[a,b].$$

$$(Rx)(t)=\int_{t_0}^{t}\frac{x(\tau)}{\tau}d\tau \quad \text{for } x\in X;\; t\in[a,b].$$

The operator R is well-defined for all continuous functions. The definition of R implies that $RX\subset \operatorname{dom} D= C^1[a,b]$ and

$$(DRx)(t)=D(Rx(t))=t\frac{d}{dt}\left[\int_{t_0}^{t}\frac{x(\tau)}{\tau}d\tau\right]=t\,\frac{x(t)}{t}=x(t), \quad \text{for all } x\in X;\; t\in[a,b].$$

The operator D is right invertible but not invertible since

$$(RDx)(t)=\int_{t_0}^{t}\frac{(Dx)(\tau)}{\tau}\,d\tau=\int_{t_0}^{t}\frac{\tau x'(\tau)}{\tau}\,d\tau= x(t)-x(t_0) \quad \text{for } x\in \operatorname{dom} D.$$

In this case, the operator defined as follows

$$(Fx)(t):= x(t)-(RDx)(t)=((I-RD)x)(t)=x(t_0) \text{ for } x\in C^1[a,b];\; t\in[a,b]$$

is an initial operator of D corresponding to the right inverse R of D.

We note, in the work of the author [Binderman 2009] the operator D was used to construct an operator of elasticity.

<u>EXAMPLE 5.</u> [cf. Binderman 2000**]**. Suppose that X is defined as in Example 2. We define the family of operators D_h as follows:

$$\left(D_h x\right)(t):=\begin{cases} h\dfrac{x(t)-x(h)}{t-h} & \text{for } t\neq h, \\ & \qquad h\in(a,b), x\in X, \\ hx'(h) & \text{for } t=h, \end{cases}$$

and

$$\left(R_h x\right)(t):=\frac{t-h}{h}, 0\neq h\in(a,b).$$

We can prove, the operators D_h are right invertible, $R_h \in \Re_{D_h}$ and

$$(F_h x)(t) = [(I - R_h D_h)x](t) = x(h) \text{ for all } 0 \neq h \in (a,b),\ x \in X.$$

DIFFERENCE EQUATIONS

PROBLEM. Suppose that X, D, R is defined as in Example 1. Let a sequence $\mathbf{x}=\{x_n\}\in X$ denotes a point series or time series. We set the problem of finding elements of **x** if we know the members: $y_1, y_2, \ldots, y_m$ of the sequence $\mathbf{y}=\{y_n\}$, under the condition $D\mathbf{x}=\mathbf{y}$.

If x_1 is given then by the definition of the operator D, only we receive:

$$x_2=y_1+x_1,\ x_3=y_2+y_1+x_1, \ldots, x_{m+1} = x_1 + \sum_{i=1}^{m} y_i .$$

Clearly, by Theorem 2 we obtain the same result

$$\mathbf{x} = x_1\mathbf{e} + R\mathbf{y} = \{x_1, x_1, \ldots, x_1, \ldots\} + \{0, y_1, y_1+y_2, y_1+y_2+y_3, \ldots\}.$$

Let us consider the following problem: find a solution of the equation

$$D\mathbf{x}=\mathbf{y}, \tag{4}$$

which satisfies the linear condition

$$(\mathbf{a},\mathbf{x})=b, \tag{9}$$

where $(\mathbf{a},\mathbf{x}) := \sum_{i=1} a_i x_i$, $\mathbf{a},\mathbf{x},\mathbf{y}\in X$; $b\in\mathbb{R}$.

In order to determine the members $x_1,x_2,\ldots,x_m$ of a sequence **x**, we assume that elements $a_1,a_2,\ldots,a_m$; $y_1,y_2,\ldots y_{m-1}$ of the sequences **a**, **y**, respectively are known and

$$\sum_{i=1}^{m} a_i \neq 0;\ a_j = 0 \text{ for } j > m \in \mathbb{N}.$$

We can prove that the problem is equivalent to the initial value problem

$$D\mathbf{x} = \mathbf{y}, \tag{4}$$

with the condition

$$F\mathbf{x} = \mathbf{z}_0, \text{ where } \mathbf{z}_0 := \frac{b}{\sum_{i=1}^{m} a_i}\mathbf{e} \in \ker D,\ \mathbf{e}=\{1,1,\ldots\}. \tag{10}$$

It is easy to check that F is the initial operator for D corresponding to the operator

$$\mathrm{R}\mathbf{y} = \mathrm{R}\{y_n\} = \{u_n\} := \begin{cases} u_1 = \dfrac{-1}{\sum_{i=1}^{m} a_i} \sum_{i=2}^{m}\left(\sum_{j=i}^{m} a_j\right) y_{i-1} \\ u_n = u_1 + \sum_{j=1}^{n-1} y_j \text{ for } n > 1 \end{cases}. \tag{11}$$

By the Theorem 2 we receive that $\mathbf{x} = \mathbf{z}_0 + \mathrm{R}\mathbf{y}$. Hence we receive the following theorem.

Theorem 3. The unique solution of the problem (4), (9) is determined by the following formula:

$$x_1 = \frac{b - \sum_{i=2}^{m}\left(\sum_{j=i}^{m} a_j\right) y_{i-1}}{\sum_{i=1}^{m} a_i}, \; x_n = x_1 + \sum_{j=1}^{n-1} y_j \text{ for } n = 2,3,\ldots,m. \tag{12}$$

We consider special cases of the problem (4), (9). The above result implies the following conclusions.

Remark 1. [see also Przeworska-Rolewicz 1988] Let the condition (9) has the form: $a_p x_p = b, \; a_p \neq 0, \; p \in [1,m] \cap \mathbb{N}$. By formulas (10), (11) we obtain that the operator $\mathrm{F}_{(p)}\mathbf{x} := \{b/a_p\}$; $\mathbf{x} \in \mathrm{X}$ is the initial operator for D corresponding to the operator

$$\mathrm{R}_{(p)}\mathbf{x} = \mathrm{R}_{(p)}\{\mathrm{x}_n\} := \begin{cases} -\sum_{i=n}^{p-1} \mathrm{x}_i & \text{for } n < p \\ 0 & \text{for } n = p, \\ \sum_{i=1}^{n-p} \mathrm{x}_{p+i-1} & \text{for } n > p \end{cases} \quad \mathbf{x} \in \mathrm{X}.$$

By Theorem 3 we obtain that the solution of the considering problem is determined by the formula

$$\mathbf{x} = \frac{b}{a_p}\mathbf{e} + \mathrm{R}_{(p)}\mathbf{y}.$$

Hence, the first m members of the sequence $\mathbf{x} = \{x_n\} \in \mathrm{X}$ is determined by the following formula:

$$x_n = \begin{cases} \dfrac{b}{a_p} - \sum_{i=n}^{p-1} y_i & \text{for } n = 1,2...,p-1, \\ \dfrac{b}{a_p} & \text{for } n = p, \\ \dfrac{b}{a_p} + \sum_{i=1}^{n-p} y_{p+i-1} & \text{for } n = p+1, p+2, ..., m \end{cases}$$

Remark 2. [see also Przeworska - Rolewicz 1988] Let the condition (9) has the form: $\sum_{i=1}^{k} x_i = k\bar{x}_k, k \in [2,m]$, where $\bar{x}_k \in \mathbb{R}$ is given. As it was pointed out in the introduction, if D∈ $\mathfrak{R}$(X), then there is the set of right inverses, determined by Formula (2). Let **x**={x$_n$}∈X, we define the operator

$$\mathrm{R}_k\mathbf{x} := \mathbf{y} = \{\mathrm{y_n}\} = \begin{cases} \dfrac{-1}{k}\sum_{i=1}^{k}(k-i)x_i, & \text{for } n = 1, \\ y_1 + \sum_{i=1}^{n-1} x_i & \text{for } n > 1, \end{cases}.$$

It is easy to check that R$_k$∈ $\mathfrak{R}_D$ and the operator

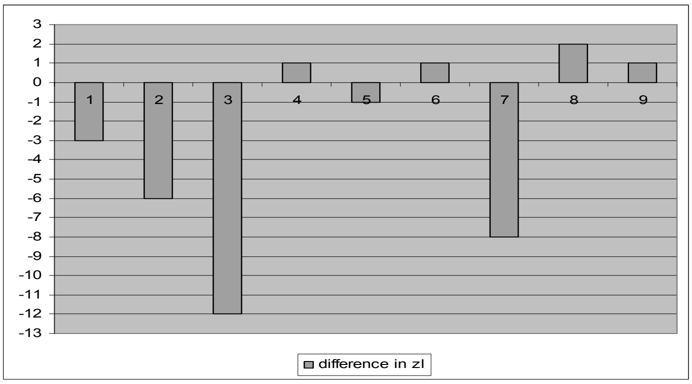
$$\mathrm{F}_k\mathbf{x} := \left\{\frac{1}{k}\sum_{i=1}^{k} x_i\right\} = \bar{x}_k \mathbf{e} \in \ker D,$$

is the initial operator for D corresponding to the operator R$_k$. Hence, the first k members of the sequence **x**={x$_n$}∈X are determined by the following formula:

$$x_n = \begin{cases} \bar{x}_k - \dfrac{1}{k}\sum_{i=1}^{k}(k-i)y_i & \text{for } n = 1, \\ x_1 + \sum_{i=1}^{n-1} y_i & \text{for } n = 2,3,...,k, \end{cases}. \tag{13}$$

EXAMPLE 6. We consider the quotations (in pln) of KGHM (KGHM Polska Miedź S.A.) on Warsaw Stock Exchange for the period 1 August 2011 – 12 August 2011. Let $x_1, x_2, ..., x_{10}$ be unknown daily quotations of KGHM in 1, 2, 3, 4, 5, 8, 9, 10, 11, 12 of August 2011, respectively. We can check that the average of the quotations $\bar{x}_{10} := \frac{1}{10}\sum_{i=1}^{10} x_i$ =170,7; the differences between the quotations $y_i=x_{i+1} - x_i$, i=1,2,…,9 are presented in Figure 1.

Figure 1. Differences between the daily quotations of KGHM on 2,…, 12.08.2011 (in pln)



Source: own calculations

Using Theorem 3, we receive by Formula 12 the daily quotations of KGHM in August 2011, which is presented in the last row of Table 1.

Table 1. Daily quotations of KGHM on 1, 2,…, 12.08.2011 (in pln)

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	1.08	2.08	3.08	4.08	5.08	8.08	9.08	10.08	11.08	12.08.
y		-3	-6	-12	1	-1	1	-8	2	1
x	188	185	179	167	168	167	168	160	162	163

Source: own calculations

Remark 3. We consider the equation (4) D**x**=**y** with the condition

$$\varphi(\mathbf{x}) = \varphi(x_1, x_2, \dots, x_m) = 0, \tag{14}$$

where the differentiable mapping $\varphi\colon \mathbb{R}^m \to \mathbb{R}, m \in \mathbb{N}$ and $y_1, y_2, \dots, y_{m-1} \in \mathbb{R}$ are given. It is easy to observe that the considered problem is equivalent to the following system of equations

$$\begin{cases} \varphi(x_1, x_2, \ldots, x_m) = 0 \\ x_2 - x_1 = y_1 \\ ----- \\ ----- \\ x_m - x_{m-1} = y_{m-1} \end{cases} . \tag{15}$$

The Jacobian matrix of the above system is of the form

$$J(x_1, x_2, \ldots, x_m) = \begin{bmatrix} \frac{\partial\varphi}{\partial x_1} & \frac{\partial\varphi}{\partial x_2} & - & - & - & \frac{\partial\varphi}{\partial x_m} \\ -1 & 1 & 0 & - & - & 0 \\ 0 & -1 & 1 & 0 & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & - & - & -1 & 1 \end{bmatrix}.$$

We can check that the determinant of J - Jacobian is equal

$$|J| = \sum_{i=1}^{m} \frac{\partial\varphi}{\partial x_i}.$$

Under some additional conditions, we can prove [cf. Sikorski 1969] the theorem.
If the Jacobian $|J|$ does not vanish at a point $\mathbf{x}_0 \in \mathbb{R}^m$, then in a neighborhood of the point $\mathbf{x}_0$ there exists an unique solution of the system (15).

We observe that in special case, when

$$\varphi(x_1, x_2, \ldots, x_m) \equiv m\sigma^2 - \sum_{i=1}^{m} \left(x - \bar{x}_m\right)^2 = 0,$$

where $\sigma > 0$, $\bar{x}_m = \frac{1}{m}\sum_{j=1}^{m} x_j$ are given, then the problem has not a solution or has the solutions determined by the formula:

$$x_n = \begin{cases} c & \text{for } n = 1, \\ c + \sum_{i=1}^{n-1} y_i & \text{for } n = 2, 3, \ldots, m, \end{cases} \tag{16}$$

where $c \in \mathbb{R}$ is arbitrarily fixed.
Indeed, the Jacobian

$$|J| = \sum_{i=1}^{m} \frac{\partial\varphi}{\partial x_i} = \sum_{i=1}^{m} 2\left(x_i - \bar{x}_m\right)\left(1 - \frac{1}{m}\right) = \frac{2(m-1)}{m}\left(\sum_{i=1}^{m} x_i - m\bar{x}_m\right) = 2(m-1)\left(\frac{1}{m}\sum_{i=1}^{m} x_i - \bar{x}_m\right) = 0.$$

On the other hand, the members $y_1, y_2, \ldots, y_{m-1}$ determine the standard deviation σ of the elements $x_1, x_2, \ldots, x_m$. Indeed, by the formula (8) we have

$$x_1 - \bar{x}_m = x_1 - \frac{1}{m}\sum_{i=1}^{m} x_i = x_1 - \frac{1}{m}x_1 - \frac{1}{m}\left(\sum_{i=2}^{m}\left(x_1 + \sum_{j=1}^{i-1} y_j\right)\right) = \frac{-1}{m}\sum_{i=2}^{m}\sum_{j=1}^{i-1} y_j,$$

$$x_i - \bar{x}_m = x_1 + \sum_{j=1}^{i-1} y_j - \frac{1}{m}\sum_{j=1}^{m} x_j = \sum_{j=1}^{i-1} y_j - \frac{1}{m}\sum_{j=2}^{m}\sum_{k=1}^{j-1} y_k .$$

The above shows that standard deviation σ is entirely determined by the known elements $y_1, y_2, \ldots, y_{m-1}$.

We observe, the minor of the (m-1)th order of the element $\partial\varphi / \partial x_1$ of the determinant $|J|$

$$\begin{vmatrix} 1 & 0 & 0 & - & - & 0 \\ -1 & 1 & 0 & 0 & - & 0 \\ 0 & -1 & 1 & 0 & - & - \\ 0 & 0 & -1 & 1 & - & 0 \\ - & - & - & - & - & - \\ 0 & 0 & - & 0 & -1 & 1 \end{vmatrix} = 1 \neq 0 .$$

By the Kronecker–Cappelli theorem we receive that considered problem has not a solution or the problem has infinite number of solutions, determined by Formula (16).

Remark 4. If for the considered problem the members $y_1, y_2, \ldots, y_{m-1}$ of a sequence **y**=$\{y_n\}$ are known only. For example, next members of the **y** we can determine by the one of the simplest extrapolation formulas [Ralston 1965]:

$$y_n = 2y_{n-1} - y_{n-2} \text{ - linear,}$$

$$y_n = \frac{1}{2}\left(5y_{n-1} + y_{n-3} - 4y_{n-2}\right) \text{ - quadratic,}$$

where $n=m, m+1, \ldots,$.

Clearly, we can use the Newton interpolation formula [Ralston 1965].

CONCLUSION

It seems to the author that presented here considerations will permit us to use methods of Algebraic Analysis in much more complicated cases. The author intends to show other applications to economics with another right invertible operators, equations and conditions. Mathematical theory of right invertible operators provides good tools for solving these problems.

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