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TRADING VOLUME AND VOLATILITY OF STOCK RETURNS: EVIDENCE FROM SOME EUROPEAN AND ASIAN STOCK MARKETS♦

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Abstract: This paper analyses the relationship between the daily volatility of stock returns and the trading volume using the TGARCH models for selected European and Asian stock markets. The leverage effect has been proved in all analysed cases. The logarithm of the trading volume was included into the conditional volatility equation as a proxy for information arrival time. Although in case of all analysed Asian stock returns the inclusion of the trading volume led to the moderate decline of the conditional volatility persistence, the results in case of European stock returns were not so unambiguous.

Key words: volatility, TGARCH model, trading volume, stock returns

INTRODUCTION

The analysis of the stock returns volatility has attracted the interest of investors for a long time. One of the characteristic features of stock returns is that their volatility changes over time. Although this feature has long been recognized (see e.g. [Franses et al. 2000]), the pioneering work in the area of modelling volatility was presented by Engle [Engle 1982] who introduced the autoregressive conditional heteroscedasticity model ARCH. The generalized version of this model, the GARCH model, was first published by Bollerslev [Bollerslev 1986]. The main aim of the ARCH and GARCH models is to capture the time-varying

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volatility. In order to capture some other typical features of financial time series such as asymmetric effect (i.e. the different impact of the positive and negative shocks on the conditional volatility) or long – memory (i.e. variances generated by fractionally integrated processes) a large number of modifications of the standard ARCH and GARCH models has been developed and it is almost impossible to mention all of them (see e.g. [Franses et al. 2000], [Poon et al. 2003], [Rachev et al. 2007]). To capture the asymmetric behaviour of the stock returns e.g. the Engle's [Engle 1990] asymmetric GARCH (AGARCH) model, Nelson's [Nelson 1991] exponential GARCH (EGARCH) model or Zakoian's [Zakoian 1994] threshold GARCH (TGARCH) model[1] can be used. From the long – memory models, the most popular and well known is the fractional integrated GARCH (FIGARCH) model of Baillie, Bollerslev and Mikkelsen [Baillie et al. 1996].

Though the ARCH/GARCH – class models allow the volatility shocks to persist over time, they didn't provide the economic explanation for this phenomenon. The paper [Lamoureux et al. 1990] published in the Journal of Finance offers the explanation for volatility persistence. The authors proved that the daily trading volume, used as a proxy for information flow, has a significant explanatory power regarding the variance of daily returns. For a sample of 20 US actively traded stocks they found out that the GARCH effects disappeared when the trading volume was included into the conditional variance equation. The number of studies documenting the relationship between the stock returns and trading volume is constantly growing. The survey of some empirical studies dealing with this relationship can be found e.g. in [Ghysels et al. 2000], [Girard et al. 2007], [Gursoy et al. 2008], [Poon et al. 2003]. The above mentioned approach presented in [Lamoureux et al. 1990] has been applied in various studies to both individual stocks (stock-level analysis) and stock market indices (market-level analysis). Since the conclusions of the studies applying the approach of Lamoureux and Lastrapes on individual stocks are mostly in coincidence with those presented in [Lamoureux et al. 1990], the results of the market-level analysis are not so unambiguous (see e.g. [Girard et al. 2007], [Sharma et al. 1996]). There are also some papers which proved that the inclusion of the trading volume in conditional variance equation eliminates the ARCH effect for both the individual stocks and the stock index (see e.g. [Miyakoshi 2002]). The analyses in the studies [Girard et al. 2007] and [Gursoy et al. 2008] were done also for the decomposed total volume (into its predictable and unpredictable components) to examine the role of differing trading systems on the relationship between the conditional volatility and the trading volume.

The aim of this paper is to analyse the relationship between the trading volume and the daily volatility of eight European and five Asian stock returns data (i.e. market-level analysis) using the TGARCH models and applying the approach

[1] TGARCH model is equivalent to the GJR – GARCH model independently presented by Glosten, Jagannathan and Runkle [Glosten et al. 1993].

of Lamoureux and Lastrapes. The rest of the paper is organized as follows. The second section discusses the data and the methodology used in the paper. The third section contains the empirical results of this investigation and the final section provides the concluding remarks.

DATA AND METHODOLOGY

The paper investigates the daily close values of the eight European indices – Austrian ATX, Belgian BEL20, British FTSE100, Dutch AEX, French CAC40, German DAX, Spanish SMSI, Swiss SSMI and five Asian indices – Hong Kong HSI, Indian BSE SENSEX, Indonesian JKSE, Japanese NIKKEI225, Taiwanese TSEC and also the corresponding trading volumes for the period from October, 18 2004 to April, 28 2011[2]. The source of data is Yahoo! Finance [http://finance.yahoo.com] and the number of observations is in individual cases different and spans from 656 in case of DAX to 1671 for AEX. Trading volume is the number of shares traded on a particular day.

The whole analysis was done on logarithmic transformation of daily index returns and daily trading volume. The logarithmic stock returns are calculated as the logarithmic first difference of the daily closing values of the stock indices, i.e.

$$r_t = d(\ln(P_t)) = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (1)$$

where P_t is the closing value of the stock index at time t and r_t denotes logarithm of the corresponding stock return. The descriptive statistics for the logarithmic stock returns together with values of the Jarque-Bera statistics (J-B) testing the normality, values of the Ljung-Box Q(k) and Q^2(k) statistics testing the uncorrelatedness of the return series and the squared return series till the lag k respectively, and the Augmented Dickey – Fuller (ADF) test statistics testing the existence of the unit root are in Table 1.

Table 1 shows that the mean of logarithmic daily returns ranges between $-8{,}9.10^{-5}$ (SMSI) and $9{,}6.10^{-4}$ (JKSE), and the standard deviation between 1,2% (SSMI) and 1,9% (DAX). The calculated Jarque-Bera statistics (taking into account the skewness and the kurtosis of the tested distribution) reject the null hypothesis of normality at the 1% significance level for all analysed return series. The values of the skewness and kurtosis statistics furthermore indicate that the underlying data are leptokurtic, or fat-tailed and sharply peaked about the mean when compared with the normal distribution. The Ljung-Box Q-statistics Q(12) show (for majority of analysed return series) the existence of the serial correlation. Much higher

[2] The data for trading volume of SMSI and DAX, respectively, were not available from the start of the above mentioned period and therefore in these cases the analysis was done from June, 29 2006 and from September, 24 2008, respectively.

values of the Ljung-Box statistics for squared return series, $Q^2(12)$, than those for the corresponding return series (substantially higher than the corresponding critical values of the χ^2- distribution) indicate the presence of the conditional heteroscedasticity. From the ADF test results it seems to be clear that the hypothesis of a unit root is strongly rejected for the all analysed stock returns (for more information about unit root tests see e.g. [Franses et al. 2000]), i.e. the return series are also stationary.

Table 1. Descriptive statistics of the logarithmic return series and some test results

	Mean	Std. Dev.	Skew-ness	Kurt-osis	J-B	Obs.	Q(12)	$Q^2(12)$	ADF
ATX	$1{,}9\cdot10^{-4}$	0,018	-0,256	8,913	2365,5 ***	1612	9,033	1872,8 ***	-38,255 ***
BEL20	$3\cdot10^{-6}$	0,014	-0,160	10,389	3803,5 ***	1669	32,629 ***	1588,4 ***	-39,798 ***
FTSE100	$1{,}7\cdot10^{-4}$	0,013	-0,120	11,671	5167,1 ***	1648	68,255 ***	1396,5 ***	-20,044 ***
AEX	$5{,}7\cdot10^{-5}$	0,015	-0,175	12,629	6459,7 ***	1670	43,838 ***	1668,2 ***	-42,124 ***
CAC40	$6{,}9\cdot10^{-5}$	0,015	0,126	11,323	4822,2 ***	1669	53,283 ***	1071,2 ***	-43,893 ***
DAX	$3{,}2\cdot10^{-4}$	0,019	0,235	9,313	1093,9 ***	655	24,783 ***	355,44 ***	-25,425 ***
SMSI	$-8{,}9\cdot10^{-5}$	0,017	0,297	10,856	3168,4 ***	1225	19,180 *	400,76 ***	-35,101 ***
SSMI	$1{,}1\cdot10^{-4}$	0,012	0,061	11,873	5394,4 ***	1644	64,426 ***	1709,3 ***	-20,017 ***
HSI	$3{,}8\cdot10^{-4}$	0,018	0,076	11,835	5218,8 ***	1604	28,687 ***	1501,3 ***	-41,420 ***
BSE SENSEX	$7{,}6\cdot10^{-4}$	0,018	0,076	9,742	3048,8 ***	1609	25,601 **	512,38 ***	-36,943 ***
JKSE	$9{,}6\cdot10^{-4}$	0,016	-0,636	9,038	2472,9 ***	1559	33,637 ***	633,16 ***	-35,111 ***
NIKKEI 225	$-6{,}7\cdot10^{-5}$	0,017	-0,579	12,124	5614,0 ***	1593	16,037 ***	1814,2 ***	-41,479 ***
TSEC	$2{,}8\cdot10^{-4}$	0,014	-0,410	6,052	670,3 ***	1611	24,439 **	571,19 ***	-37,956 ***

Note: The symbols *, ** and *** denote the rejection of the null hypothesis at the 10, 5 and 1 % significance levels respectively.

Source: own calculations in EViews 5.1

In order to capture the above mentioned characteristics of the analysed stock returns, the appropriate model from the ARCH-class models can be used. To model the asymmetric characteristics, such as a leverage effect, in which the negative shocks increase volatility more than positive shocks of an equal magnitude, e.g. the TGARCH or GJR-GARCH models are used. So, besides the conditional mean equations we have to specify also the conditional variance equations.

The logarithmic stock returns equation, i.e. the conditional mean equation, can be in general written as a Box-Jenkins ARMA(m,n) model[3] of the form:

$$r_t = \omega_0 + \sum_{j=1}^{m} \phi_j r_{t-j} + \sum_{k=1}^{n} \theta_k \varepsilon_{t-k} + \varepsilon_t \qquad (2)$$

where ω_0 is unknown constant, $\phi_j \ (j = 1,2,...m)$ and $\theta_k \ (k = 1,2,...n)$ are the parameters of the appropriate ARMA(m,n) model, ε_t is a disturbance term.

The conditional variance equation h_t in case of a TGARCH(p,q) model can be specified as:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i}^2 I_{t-i}^- \qquad (3)$$

where from $I_{t-i}^- = \begin{cases} 1, if \ \varepsilon_{t-i} < 0 \\ 0, if \ \varepsilon_{t-i} > 0 \end{cases}$, it is clear the different impact of the positive shocks $\varepsilon_{t-i} > 0$ and negative shocks $\varepsilon_{t-i} < 0$ on the conditional variance. The impact of positive shocks is given by the value of α_i, the impact of negative shocks by the value $\alpha_i + \gamma_i$. If $\gamma_i > 0$, it means that the negative shocks increase volatility which confirms the presence of the leverage effect of i-th order. If $\gamma_i \neq 0$, we speak about the asymmetric impact of shocks.

To examine the effect of trading volume on stock returns volatility, the following modification of the conditional variance equation (3) is used:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i}^2 I_{t-i}^- + \delta V_t \qquad (4)$$

where V_t is the logarithm of the trading volume. Meaning of the remaining symbols is the same as in equation (3). According to the [Lamoureux et al. 1990] the parameter δ should be positive and the volatility persistence should become negligible.

[3] ARMA model = Autoregressive Moving Average model

EMPIRICAL RESULTS

The analysis was done in two steps. In the first step the parameters of conditional mean equation (2) together with the conditional volatility equation without trading volume (3) were estimated and in the second step we estimated parameters of the conditional mean equation (2) together with the parameters of conditional volatility equation with included trading volume (4)[4]. The whole analysis was done in econometrical software EViews 5.1.

The appropriate ARMA(m,n) models for logarithmic stock returns were as follows: ATX – AR(1), BEL20 – ARMA((3,4),4), FTSE100 – AR(1), AEX – AR(3,4,5,7), CAC40 – ARMA(1,1), DAX – AR(2), SMSI – ARMA(1,(1,3)), SSMI – AR(2), HSI – MA(9,10), BSE SENSEX – AR(1), JKSE – AR(1), NIKKEI225 – ARMA(1,1), TSEC – AR(1,6,8). The estimation results of conditional variance equations (together with the information about the values p and q in TGARCH model) for both model 1 and model 2 are presented in table 2.

The results summarized in table 2 show quite high degree of the volatility persistence, since the sum $\sum_{i=1}^{q}\hat{\alpha}_i + \sum_{i=1}^{p}\hat{\beta}_i$ is high in all analysed cases. In model 1 (i.e. model without trading volume variable) it takes values from 0,798 (JKSE) to 0,939 (TSEC), and besides also the existence of the leverage effect ($\gamma_i > 0$) was proved in all analysed cases. This means confirmation of the fact that the negative shocks increase volatility more than positive shocks of an equal magnitude.

For model 2 (i.e. model with trading volume variable) the volatility persistence varies between 0,793 (JKSE) and 0,930 (TSEC), but it is (similarly as for model 1) less than 1, proving the stationary persistence. As it is furthermore clear from the table 2, after inclusion of the trading volume variable, the volatility persistence declined for all analysed Asian stock returns and parameter δ corresponding to the trading volume variable was statistically significant at 1% significance level. For the Japanese NIKKEI225 the surprisingly negative relationship between trading volume and conditional volatility was proved. The impact of the trading volume on the European stock returns was different – the volatility persistence declined in five cases (ATX, BEL20, FTSE100, CAC40, SSMI), for AEX remains almost the same and in two cases even rose (DAX, SMSI). The significant negative relationships between trading volume and conditional volatility were confirmed for the British FTSE100 and German DAX. For two stock returns (AEX, SMSI) the parameter estimates of trading volume were insignificant. Regarding the presence of the leverage effect in model 2 we received the similar results as for model 1.

[4] The equations estimated in the first step we denote as model 1 and equations estimated in the second step we denote as model 2.

Table 2. Summary of the estimated parameters from the conditional variance equations (3) and (4)

	TGARCH (p,q)	Without trading volume, i.e. Model 1		With trading volume, i.e. Model 2		
		$\sum_{i=1}^{q}\hat{\alpha}_i+\sum_{i=1}^{p}\hat{\beta}_i$	$\hat{\gamma}_1$	$\sum_{i=1}^{q}\hat{\alpha}_i+\sum_{i=1}^{p}\hat{\beta}_i$	$\hat{\gamma}_1$	δ
ATX	(1,2)	0,870 ***	0,209 ***	0,850 **	0,209 ***	$6\cdot10^{-6}$ ***
BEL20	(1,1)	0,884 #	0,195 ***	0,850 #	0,210 ***	$3\cdot10^{-6}$ ***
FTSE100	(1,2)	0,895 ***	0,181 ***	0,881 ***	0,199 ***	$-3\cdot10^{-6}$ ***
AEX	(1,1)	0,896 ***	0,183 ***	0,896 ***	0,184 ***	$-5\cdot10^{-7}$
CAC40	(2,2)	0,893 ***	0,182 ***	0,859 #	0,224 ***	$5\cdot10^{-6}$ ***
DAX	(1,2)	0,846 ***	0,203 ***	0,857 **	0,243 ***	$-2\cdot10^{-5}$ ***
SMSI	(3,2)	0,923 ***	0,129 ***	0,928 ***	0,120 ***	$1\cdot10^{-6}$
SSMI	(1,2)	0,878 **	0,202 ***	0,862 **	0,203 ***	$3\cdot10^{-6}$ ***
HSI	(1,2)	0,934 ***	0,107 ***	0,873 ***	0,156 ***	$5\cdot10^{-6}$ ***
BSE SENSEX	(1,1)	0,914 ***	0,137 ***	0,843 *	0,198 ***	$2\cdot10^{-5}$ ***
JKSE	(1,1)	0,798 *	0,257 ***	0,793 *	0,258 ***	$2\cdot10^{-6}$ ***
NIKKEI 225	(1,2)	0,863 **	0,208 ***	0,830 ***	0,247 ***	$-6\cdot10^{-6}$ ***
TSEC	(1,1)	0,939 **	0,081 ***	0,930 #	0,094 ***	$6\cdot10^{-6}$ ***

Note: The symbols *, ** and *** indicate that the corresponding parameter is statistically significant at the 10, 5 and 1 % significance level respectively. In case of the sum of coefficients alpha and beta the mentioned symbols refer to the statistical significance of all the parameters. The symbol # means that at least one parameter alpha or beta was not statistically significant at any of the mentioned significance levels.

Source: own calculations in EViews 5.1

Finally, in order to have the information about adequacy of the estimates presented in table 2, we tested the standardized residuals. The uncorrelatedness of the standardized residuals and squared standardized residuals was tested using the

Ljung – Box Q – statistics and Q^2 – statistics, respectively till the lag 12 and the Jarque – Bera test was used to test the normality (see table 3).

Table 3 The diagnostic check statistics of the standardized residuals

	Without trading volume, i.e. Model 1			With trading volume, i.e. Model 2		
	Q(12)	Q^2(12)	J-B	Q(12)	Q^2(12)	J-B
ATX	9,935	12,497	36,868 ***	9,799	11,271	25,946 ***
BEL20	4,018	5,108	83,864 ***	4,787	5,157	55,058 ***
FTSE100	7,014	20,510 **	66,672 ***	7,557	20,402 **	65,103 ***
AEX	8,805	9,669	33,330 ***	8,953	9,774	34,226 ***
CAC40	5,226	15,127	56,880 ***	5,153	17,756 *	44,424 ***
DAX	5,104	9,706	14,322 ***	5,469	6,070	23,800 ***
SMSI	2,756	14,521	35,013 ***	2,668	12,476	30,000 ***
SSMI	12,617	15,607	62,558 ***	12,387	13,448	56,443 ***
HSI	12,765	9,712	24,851 ***	9,141	8,368	14,817 ***
BSE SENSEX	16,724	9,144	540,76 ***	16,765	7,108	784,91 ***
JKSE	5,784	5,080	471,56 ***	6,386	5,716	455,95 ***
NIKKEI 225	13,808	7,696	44,420 ***	15,008	9,940	51,398 ***
TSEC	6,192	9,106	172,56 ***	5,421	8,630	131,78 ***

Note: The symbols *, ** and *** denote the rejection of the null hypothesis at the 10, 5 and 1 % significance levels respectively.

Source: own calculations in EViews 5.1

From the results presented in table 3 it is clear that no first- and second-order dependence in standardized residual series at the significance level 1% was detected. It can be also concluded that there is no remaining heteroscedasticity in any case, i.e. the estimated TGARCH models have successfully accounted for all linear and nonlinear dependencies in the analysed return series[5]. The normality condition was violated in all analysed cases, which means that the estimates are consistent only as quasi-maximum likelihood estimates (see e.g. [Franses et al. 2000], [Poon et al. 2003], [Rachev et al. 2007]).

CONCLUDING REMARKS

The presented paper analyses the relationship between the trading volume and the daily volatility of eight European and five Asian stock returns data. To capture the conditional volatility it uses the TGARCH models and the relationship between the trading volume and the conditional volatility is tested following the approach of [Lamoureux et al. 1990].

In coincidence with this approach, the logarithm of the trading volume was included into the conditional volatility equation in order to confirm or to reject that it is a good proxy for information arrival. The results for European and Asian stock return volatilities are different. Since the inclusion of the trading volume led in case of all analysed Asian stock returns to the moderate decline of the conditional volatility persistence, the results for European stock returns were not so unambiguous. The volatility persistence declined only in five cases (ATX, BEL20, FTSE100, CAC40, SSMI), for AEX remains almost the same and in two cases even rose (DAX, SMSI). For AEX and SMSI the logarithmic variable trading volume was even not statistically significant. Taking into account some other papers (e.g. [Girard et al. 2007], [Gursoy et al. 2008], [Sharma et al. 1996]), the results of our analysis coincide with theirs, i.e. that we can join the conclusions that the trading volume can be in general considered (in case of the market-level analysis) to be only a poor proxy for information flow.

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