

## STOCHASTIC FRONTIER ANALYSIS OF REGIONAL COMPETITIVENESS<sup>1</sup>

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**Abstract:** The regional competitiveness is the source of national competitiveness and the efficiency measuring and relative regional efficiency comparison are crucial questions for analysts as well as for economic policy creators. Regional competitiveness becomes a subject of evaluation due to increasing significance of regions in concept of European Union. This paper deals with the application of parametric benchmarking method – Stochastic Frontier Analysis (SFA) for measuring technical efficiency of NUTS2 regions of V4 countries within the time period of 8 years.

**Key words:** Stochastic Frontier Analysis, Technical Efficiency, NUTS2 regions, Competitiveness

### INTRODUCTION

During the last few years economic policy making and research have shown increasing interest for regional competitiveness evaluation. The increasing significance of regional competitiveness evaluation deserves more attention especially because of the economic efficiency of regions representing the basis of economic success for micro-economic level and also the competitiveness of the country. Competitiveness is a complex economic phenomenon, with many definitions and quantification methods upon which the specialists have not yet

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<sup>1</sup> This paper is supported by the Grant Agency of Slovak Republic - VEGA, grant no. 1/0595/11 "Analysis of Business Cycles in the Economies of the Euro Area (with Regards to the Specifics of the Slovak Economy) Using Econometric and Optimization Methods"

reached full consensus, but the need of competitiveness gaining is frequently discussed both in the economic literature and in the everyday practice. At the same time, the increasing importance of competitiveness issues may be explained by the deeper economic integration and increased globalization, which require a constant increase in the competitive power of every economic entity belonging to a certain country, as well as in the competitive power of the country itself. There is no general consensus about what regional competitiveness means and there are many different its definition. For example the European Commission interprets the term the following way: "Competitiveness is the ability to produce goods and services which meet the test of international markets, while at the same time maintaining high and sustainable levels of income or, more generally, the ability of companies, industries, regions, nations and supra-national regions to generate, while being exposed to international competition, relatively high income and employment levels" [European Commission 1999].

#### METHODOLOGY FOR ANALYZING REGIONAL COMPETITIVENESS

Approaches to evaluation of regional competitiveness went through many debates because of non existence of unique methodology for measuring and evaluation competitiveness. Even the definition of „region“ is problematic because the regional competitiveness is not a simple sum of competitiveness of the firms located in a given region. For this reason the regional competitiveness evaluation is determined by the selection of the regional unit. Growing interest in Europe for regional competitiveness may be explained by the strength of sub-national territorial EU units and the NUTS (Nomenclature of Units for Territorial Statistics) nomenclature can be used for region classification. The non existence of the unique method for regional competitiveness evaluation caused that there are various evaluation methodologies and approaches. One of these regional evaluation methods is the method according to Viturka [Nevima et al. 2009] and his method is oriented to a long term horizont. For short term horizont regional competitiveness evaluation could be used the group of specific economic indicators of efficiency [Nevima et al. 2009]. The basic idea is to identify the internal sources of regional competitiveness in detail. There are proposed following specific indicators: coefficient of employment, coefficient of efficiency of disposability, coefficient of efficiency in development, coefficient of efficiency of investment construction, coefficient of efficiency of revenues and coefficient of efficiency of building works. Each specific coefficient compares a concrete level of the value in the region with respect to its total level in the country. These specific economic indicators are the basis for further analysis and through the techniques of multicriteria decision making methods should be obtained the comparison and final ranking of the regions. From multicriteria decision making methods could be

exploit Ivanovic deviation (for more details see [Nevima et al. 2009]), variety of multicriteria alternative evaluation methods (e.g. TOPSIS, PROMETHEE, ELECRE) or models of Data Envelopment Analysis – DEA<sup>2</sup>. DEA models are able to identify the best performers (regions) and separate them from their inefficient counterparts and in addition to identify the sources of inefficiency of units (regions). DEA models are based on benchmarking, that is, measuring a unit's (region's) efficiency compared with a reference performance (so-called efficient frontier). Inefficiency of unit's can result from technological deficiencies or non-optimal allocation of resources into production. Both technical and allocative inefficiencies are included in cost inefficiency, which is by definition, the deviation from minimum costs to produce a given level of output with given input prices. The efficient frontier (cost or production) is unknown and must be empirically estimated from the real data set by parametric and nonparametric techniques (see [Coelli et al. 2005]). Due to the impossibility of the separation of the inefficiency effect from the statistical noise which is the shortcoming of DEA we decided to exploit parametric benchmarking method, namely Stochastic Frontier Analysis (SFA) for regional competitiveness evaluation. This methodology is based on econometric theory and pre-specified functional form is estimated and inefficiency is modeled as an additional stochastic term. In our analysis we tried to use SFA for measuring technical efficiency of NUTS2 regions of V4 countries within the time period of 8 years. The analysis is based on production function principle in macroeconomic context, evaluated regions are treated as producers of output given some inputs. In our analysis are applied various versions of stochastic frontier production function models only for panel data due to the fact that panel data provide information on the same units over several periods that is not possible with cross section data. We estimated levels of technical efficiency for each NUTS2 region and the differences in estimated scores, parameters and ranking of regions are compared across different panel data models.

#### STOCHASTIC FRONTIER ANALYSIS – PARAMETRIC BENCHMARKING TECHNIQUE

One of the simplest structures we can impose on the inefficiency effect is

$$u_{it} = u_i \quad i = 1, \dots, N \quad t = 1, \dots, T \quad (1)$$

where  $u_i$  is treated as either a fixed parameter or random variable. These models are known as the fixed effects model and random effects model respectively. Supposing that technical efficiency is time invariant, a Cobb-Douglas production frontier with time invariant technical efficiency can be written as:

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<sup>2</sup> DEA is non-parametric benchmarking method and originates from operations research and uses linear programming to calculate an efficient deterministic frontier against which units are compared.

$$\ln y_{it} = \beta_0 + \sum_n \beta_n \ln x_{nit} + v_{it} - u_i \quad i=1,\dots,N \quad t=1,\dots,T \quad (2)$$

where  $\ln y_{it}$  is logarithm of output quantities,  $\ln x_{nit}$  is logarithm of inputs quantities,  $\beta$  is unknown vector of parameters to be estimated,  $v_{it}$  is symmetric random variable,  $u_i$  is time invariant technical inefficiency term of compound error term  $\mathcal{E}_{it} = v_{it} - u_i$ .

In this specification the error term is composed of two uncorrelated parts. The first part  $u_i$  is capturing the effect of technical inefficiency and the second part  $v_{it}$  is reflecting effect of statistical noise. This random effect model can be estimated using Maximum Likelihood Estimation (MLE) method or Method of Moments. Using Maximum Likelihood Estimation method requires make distribution assumptions for stochastic terms. Usually we assume that  $v_{it}$  are random variables to be normally distributed ( $v_{it} \sim \text{iid } N(0, \sigma_v^2)$ ) and  $u_i$  are non negative time-invariant random variables to be half normal distributed ( $u_i \sim \text{iid } N^+(0, \sigma_u^2)$ ) or truncated normal distribution ( $u_i \sim \text{iid } N^+(\mu, \sigma_u^2)$ ) can be also considered. The next step is to obtain estimates of the technical efficiency of each unit. The problem is to extract the information that  $\mathcal{E}_i$  contains on  $u_i$  (we have estimates of  $\mathcal{E}_{it} = u_i + v_{it}$ , which obviously contain information on  $u_i$ ). A solution to the problem is obtained from the conditional distribution of  $u_i$  given  $\mathcal{E}_i$ , which contains whatever information  $\mathcal{E}_i$  contains concerning  $u_i$ . This procedure is known as JLMS decomposition (for more details see [Jondrow et al. 1982]). For separation the inefficiency effect from the statistical noise can be also used an alternative minimum squared error predictor estimator (for more details see [Kumbhakar et al. 2000]). Once the point estimates of  $u_i$  are obtained, estimates of the technical efficiency of each unit can be obtained by substituting them into equation (3). If the production frontier is specified as being stochastic, the appropriate measure of individual technical efficiency becomes:

$$TE_i = \frac{y_{it}}{f(x_{it}, \beta) \exp\{v_{it}\}} = \exp\{-u_i\} \quad i=1,\dots,N \quad t=1,\dots,T \quad (3)$$

which defines technical efficiency as the ratio of observed outputs quantities to the maximum outputs quantities attainable in an environment characterized by  $\exp\{v_{it}\}$ .

If we allow efficiency changes in time, inefficiency component will consist of two parts, namely cross-section component ( $u_i$ ) and time component ( $\beta_t$ ):

$$u_{it} = u_i + \beta_t \quad i=1,\dots,N \quad t=1,\dots,T \quad (4)$$

The time invariant production efficiency model given by equation (2), we reformulate as follows:

$$\ln y_{it} = \beta_{0t} + \sum_n \beta_n \ln x_{nit} + v_{it} - u_{it} \quad i=1,\dots,N \quad t=1,\dots,T \quad (5)$$

where  $\beta_{0t}$  is the production frontier intercept common to all units in period  $t$ ,  $\ln y_{it}$  is logarithm of output quantities,  $\ln x_{nit}$  is logarithm of inputs quantities,  $\beta$  is unknown vector of parameters to be estimated,  $v_{it}$  is symmetric random variable,  $u_{it}$  is time variant technical inefficiency term of compound error term  $\mathcal{E}_{it} = v_{it} - u_{it}$ . There are proposed various approaches to estimated time varying production frontier model given by equation (5) (for more details see [Kumbhakar et al. 2000]).

Battese and Coelli (see [Coelli et al. 2005] or [Kumbhakar et al. 2000]) presented a model where they model the inefficiency component in (4) according to following exponential time function:

$$u_{it} = \exp\{-\eta(t-T)\}u_i \quad i=1,\dots,N \quad t=1,\dots,T \quad (6)$$

where  $\eta$  is unknown parameter to be estimated. The function value is determined by value of parameter  $\eta$  and number of observations. The function is decreasing for  $\eta > 0$ , increasing for  $\eta < 0$  or constant for  $\eta = 0$ , i.e. if  $\eta > 0$  technical inefficiency will have decreasing effects through time (positive effect in technical efficiency over time) and  $\eta < 0$  inefficiency will be always increasing through time. This function does not allow a change in the rank ordering of unites over time, the unit that is ranked  $n$ -th at the first period is always ranked  $n$ -th. This fact is main shortcoming of this formulation. On the other hand this model requires additional estimation only of one parameter  $\eta$ . This model can be estimated by using the method of maximum likelihood. The likelihood function of this model is a generalization of the likelihood function for the conventional model (for more details see e.g. [Kumbhakar et al. 2000]). Estimates of the technical efficiency of each unit at time  $t$  can be obtained by substituting estimates of  $u_{it}$  into equation (7):

$$TE_{it} = \exp\{-u_{it}\} \quad i=1,\dots,N \quad t=1,\dots,T \quad (7)$$

## MODEL SPECIFICATION AND DATA

Above mentioned SFA models have been used for analyzing the process of regional competitiveness differentiation of V4 (Visegrad four countries) countries regions. The territorial unit of our analysis will be the sub-national territorial unit of public administration (NUTS2). Slovakia has 4 NUTS2 regions, Czech Republic has 8 NUTS2 regions, Hungary has 7 NUTS2 regions and Poland is divided into 16 NUTS2 regions. Our balanced panel data set of 35 NUTS2 regions observed over a period from 2001 to 2008 includes 280 observations in total. All data are based on information from regional statistics of OECD and Eurostat. The evaluated regions are treated as producers of output given some

inputs. The output and input selection is a crucial step in our analysis and must be done with respect to the competitiveness definition. We shall compare the competitiveness of regions through the estimated levels of technical efficiency as the efficiency we perceive as the “mirror” of the competitiveness. Overall performance of the regional economy affects employment in various sectors, therefore, we selected as the first input Employment Rate - *ER* (annually in %). Efficiency in our model should demonstrate the ability of the regions to transform its capital for its further development. For this reason as the second input was chosen Gross Fixed Capital Formation – *GFCF* (in % of GDP). This indicator includes investment activity of domestic companies and fixed assets of foreign companies and is largely influenced by the inflow of foreign investment. The third included input is the Net Disposable Income of Households - *NI* (per capita). In terms of competitiveness the disposable income plays an important role because it directly reflects the purchasing power of the region. Full picture of efficiency in regional competitiveness investigating might provide also input Income of Corporation. Our analysis could not be improved by adding this additional input data due to unavailability of this indicator in the structure of NUTS2 in regional OECD and Eurostat statistics. Previously mentioned inputs are used to produce one output, output is measured by Gross Domestic Product – *GDP* (in purchasing power parity standards per capita) the most important macroeconomic indicator.

The first part of analysis was based on the assumption of time invariant technical efficiency. We applied SFA panel data models with time invariant technical efficiency assumption (Model1 and Model2) and the analysis was based on the estimation of the model given by equation (8):

$$\ln(GDP_{it}) = \beta_0 + \beta_1 \ln(ER_{it}) + \beta_2 \ln(GFCF_{it}) + \beta_3 (\ln NI_{it}) + v_{it} - u_i \quad (8)$$

$$i = 1, \dots, N \quad t = 1, \dots, T$$

In order to estimate the model with time varying technical efficiency (Model3) we formulated model given by equation (9):

$$\ln(GDP_{it}) = \beta_0 + \beta_1 \ln(ER_{it}) + \beta_2 \ln(GFCF_{it}) + \beta_3 (\ln NI_{it}) + v_{it} - u_{it} \quad (9)$$

$$i = 1, \dots, N \quad t = 1, \dots, T$$

where

$$u_{it} = \exp\{-\eta(t-T)\}u_i$$

In all models  $v_{it}$  is reflecting effect of statistical noise and  $u_i$  or  $u_{it}$  are random variables reflecting time invariant or time varying technical inefficiency respectively. Remaining variables have been defined before. The traditional Cobb-Douglas production function has been considered and MLE method has been used in all three models. The MLE method requires making the distributional assumptions for stochastic terms. We made following distributional assumptions:

$$\text{Model1: } v_{it} \sim iid N(0, \sigma_v^2), u_i \sim iid N^+(0, \sigma_u^2),$$

$$\text{Model2: } v_{it} \sim iid N(0, \sigma_v^2), u_i \sim iid N^+(\mu, \sigma_u^2),$$

Model3:  $v_{it} \sim iid N(0, \sigma_v^2)$ ,  $u_{it} \sim iid N^+(\mu, \sigma_u^2)$ .

In all models for separation the inefficiency effect from the statistical noise was used Battese and Coelli point estimator (see [Coelli et al. 2005]). The individual technical efficiency estimates were obtained by substituting the inefficiency effects to the equations (3) and (7). The final estimates of the parameters all frontier models are listed in Table 1. Table 2 provides efficiency scores according to all models.

Table 1. Parameters of the Production Function

|            | Model1   |              | Model2   |              | Model3   |              |
|------------|----------|--------------|----------|--------------|----------|--------------|
|            | Estimate | Stand. error | Estimate | Stand. error | Estimate | Stand. error |
| $\beta_0$  | 1,6009*  | 0,1694       | -2,7287* | 0,4302       | -4,2246* | 0,9951       |
| $\beta_1$  | 0,1795*  | 0,0426       | 0,8061*  | 0,0957       | 0,4728   | 0,9104       |
| $\beta_2$  | -0,0567* | 0,0171       | -0,1654* | 0,0403       | -0,0611  | 0,4835       |
| $\beta_3$  | 0,8249*  | 0,0099       | 1,0046*  | 0,0343       | 1,3508*  | 0,9587       |
| $\sigma^2$ | 0,0771*  | 0,0089       | 0,0208*  | 0,0042       | 0,0377   | 0,1070       |
| $\gamma$   | 0,9890*  | 0,0016       | 0,8951*  | 0,0203       | 0,8502   | 0,9376       |
| $\mu$      |          |              | 0,2728*  | 0,0812       | 0,1151   | 0,3277       |
| $\eta$     |          |              |          |              | -0,0961  | 0,1808       |
| logLF      | 399,1864 |              | 305,3365 |              | 298,4673 |              |

Source: own calculations (Frontier 4.1)

\* significant at  $\alpha = 0,05$

## CONCLUSION

Inefficiency effects were estimated by using three models SFA. As for Model1 and Model2 we can see (Table1) that all the parameters are significant and the parameters are mildly different from one model to another. All estimated parameters have expected positive signs besides surprisingly negative sign of parameter  $\beta_2$  corresponding to variable Gross Fixed Capital Formation. Model1 and Model2 are both based on the assumption time invariant technical efficiency but were estimated under two different distributional assumptions for inefficiency term, the first is the half normal distribution (Model1) and the second is truncated normal distributional assumption (Model2). Therefore additional estimated parameter  $\mu$  is listed for Model2 in Table 1. Although the mean of truncated normal distribution  $\mu$  is found to be significant, half normal specification is preferred for the distribution of  $u$  following results of LR test. If the model has been estimated by the method of maximum likelihood, hypothesis concerning individual coefficients or more than one coefficient can be tested using LR test. Our null hypotheses of special interest was set as  $H_0 : \mu = \eta = 0$ , which implies time

invariant half normal inefficiency effects and this hypotheses was confirmed by this LR test.

Table 2. Efficiency scores – Model1, Model2

| Name of region      | Code of region | Model1 | Model2 |
|---------------------|----------------|--------|--------|
| Bratislavský kraj   | SK01           | 1,0000 | 0,9123 |
| Západné Slovensko   | SK02           | 0,7427 | 0,6904 |
| Stredné Slovensko   | SK03           | 0,6688 | 0,6612 |
| Východné Slovensko  | SK04           | 0,6499 | 0,6764 |
| Average Efficiency  |                | 0,7653 | 0,7351 |
| Praha               | CZ01           | 1,0000 | 0,9948 |
| Střední Čechy       | CZ02           | 0,7781 | 0,6464 |
| Jihozápad           | CZ03           | 0,8106 | 0,6871 |
| Severozápad         | CZ04           | 0,7735 | 0,7009 |
| Severovýchod        | CZ05           | 0,7762 | 0,6597 |
| Jihovýchod          | CZ06           | 0,8163 | 0,7062 |
| Střední Morava      | CZ07           | 0,7412 | 0,6565 |
| Moravskoslezsko     | CZ08           | 0,7869 | 0,7406 |
| Average Efficiency  |                | 0,8104 | 0,7240 |
| Köszep-Magyarország | HU10           | 0,9758 | 0,8077 |
| Köszep-Dunántul     | HU21           | 0,7688 | 0,7047 |
| Nyugat-Dunántul     | HU22           | 0,8469 | 0,7600 |
| Del-Dunántul        | HU23           | 0,6769 | 0,6901 |
| Eszak-Magyarország  | HU31           | 0,6479 | 0,6921 |
| Eszak-Alfold        | HU32           | 0,6839 | 0,7322 |
| Del-Alfold          | HU33           | 0,6619 | 0,6537 |
| Average Efficiency  |                | 0,7517 | 0,7201 |

| Name of region      | Code of region | Model1 | Model2 |
|---------------------|----------------|--------|--------|
| Lódzkie             | PL11           | 0,6266 | 0,5489 |
| Mazowieckie         | PL12           | 0,9048 | 0,8313 |
| Malopolskie         | PL21           | 0,6546 | 0,6296 |
| Slaskie             | PL22           | 0,7104 | 0,7312 |
| Lubelskie           | PL31           | 0,5639 | 0,5132 |
| Podkarpackie        | PL32           | 0,6122 | 0,6283 |
| Swietokrzyskie      | PL33           | 0,6007 | 0,5682 |
| Podlaskie           | PL34           | 0,6088 | 0,6007 |
| Wielkopolskie       | PL41           | 0,7206 | 0,6877 |
| Zachodnio-pomorskie | PL42           | 0,6773 | 0,7184 |
| Lubelskie           | PL43           | 0,6741 | 0,6712 |
| Dolnoslaskie        | PL51           | 0,7339 | 0,7675 |
| Opolskie            | PL52           | 0,6948 | 0,7601 |
| Kujawsko-Pomorskie  | PL61           | 0,6602 | 0,6437 |
| Warminsko-Mazurskie | PL62           | 0,6303 | 0,6637 |
| Pomorskie           | PL63           | 0,7428 | 0,7985 |
| Average Efficiency  |                | 0,6760 | 0,6726 |

Source: own calculations (Frontier 4.1)

The time invariant technical efficiency assumption was relaxing in Model3 following model defined in equation (5) and (6). This model is preferred in applied work due to its simplicity and flexibility. Modeling the variation in efficiency across time requires estimation of only one extra parameter, namely parameter  $\eta$ . In this model, the inefficiency term is modeled as a truncated normal random variable multiplied by a specific function of time. However, this simplicity comes at a cost. This specification does not allow for efficiency to increase or decrease at a decreasing rate. The time invariant technical assumption relaxation has led to non signification almost all estimated parameters. Moreover negative sign of insignificant parameter  $\eta$  means that inefficiency will be always increasing through time or technical efficiency of regions will have decreasing effects through time.

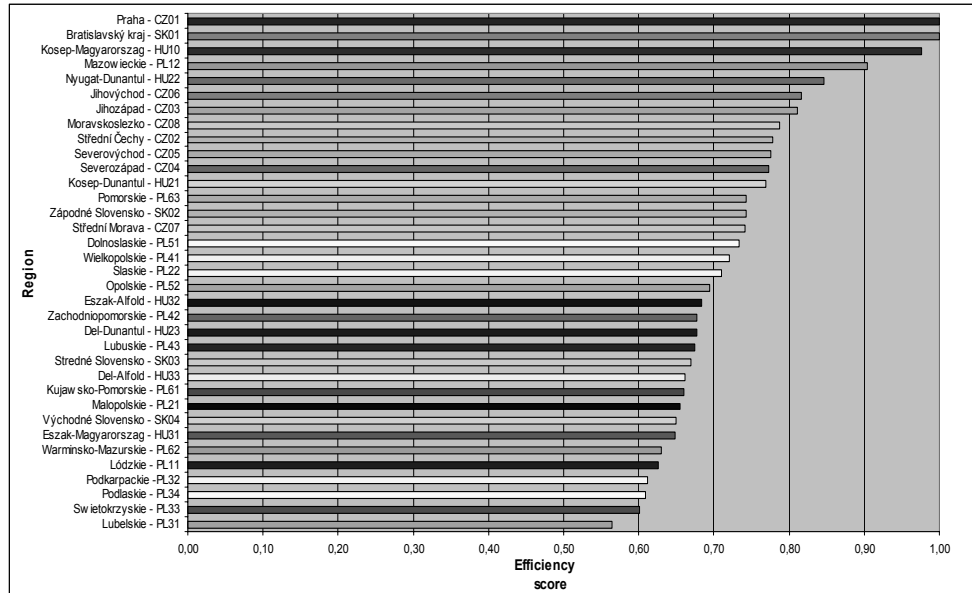


According to results of LR test applied before we decided to prefer Model1 to Model2 and Model3. In our data set is not necessary apply time varying efficiency model and half normal distribution assumption for inefficiency term is preferred.

Figure 1 provides efficiency estimates and ordering of V4 NUTS2 regions according to Model1. The scores can move between 0 and 1, where the highest value implies a perfectly efficient region. The highest competitive regions (see Table2 or Graph1) are regions CZ01 – Praha and SK01 – Bratislavský kraj (perfectly technically efficient regions, the efficiency scores equals 1). Next positions belong to regions HU10 – Kosep-Magyarország (0,9758) and PL12 – Mazowieckie (0,9048). We can notice that these “best regions” are capital cities regions and usually have better economic positions than other regions of country with weaker economic positions. As low competitive regions according to our analysis were evaluated polish regions and the lowest competitive region was region PL31 – Lubelskie and its technical efficiency level was only 56,39 % . The highest average efficiency achieved Czech regions (81,04 %) and the lowest average efficiency had Polish regions (67,60 %).

The individual efficiency estimates are relatively stable across Model1 and Model2 even despite of different distribution assumption about inefficiency component. In consequence of relaxation time invariant efficiency assumption we obtained efficiency scores for each observed year in Model3. In spite of this advantage and for reason of non signification of  $\eta$  and following results of LR test mentioned before we prefer Model1 in our data set (by this reason and insufficient space we present efficiency scores results only according to Model1). Due to the absence of the methodological approach to regional competitiveness mainstream we presented SFA methodology as a contribution to the discussion about quantitative measurement of competitiveness at the regional level.

Figure 1. Efficiency scores for NUTS 2 regions – Modell



Source: own calculations

## REFERENCES

- Battese, G. E., Coelli, T. J. (1992) Frontier production functions, technical efficiency and panel data: with application to Paddy farmers in India, *Journal of Productivity Analysis*, Volume 3, pp. 153 – 169, ISSN 0895-562X.
- Coelli, T. J., Rao Prasad, D., Battese, G. (2005) *An introduction to efficiency and productivity analysis*, Springer, ISBN 0387242661.
- Enyedi, G. (2009): Competitiveness of the Hungarian regions, *Hungarian Geographical Bulletin*. Volume 58, No 1, pp. 33–48.
- European Commission 1999 (1999) *Sixth periodic report on the social and economic situation of regions in the EU*, European Commission, Brussels.
- Jondrow, J., Lovell, C. A. K., Materov, I. S, Schmidt, P. (1982) On the estimation of technical inefficiency in the stochastic frontier production function model, *Journal of Econometrics*, Volume 19, pp. 233 – 238, ISSN 0304-4076.
- Kumbhakar, S. C., Lovell, C. A. K. (2000) *Stochastic frontier analysis*. Cambridge University Press, ISBN 05216666635.
- Nevima, J., Ramík, J. (2009) Application of multicriteria decision making for evaluation of regional competitiveness, *Mathematical Methods in Economics*, 27th International Conference, Praha, ISBN 978-80-213-1963-9.
- Ramík, J. (2010) Multicriteria approaches to competitiveness, *Quantitative Methods in Economics Multiple Criteria Decision Making XV – International Scientific Conference*, Smolenice, ISBN 978-80-8078-364-8.