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COINTEGRATION SINCE GRANGER: EVOLUTION AND DEVELOPMENT

Ewa Marta Syczewska
Institute of Econometrics, Warsaw School of Economics
e-mail: Ewa.Syczewska@sgh.waw.pl

Abstract: This paper is an attempt to give a subjective overview of evolution and development of cointegration concept since the first paper by C.W.J. Granger in 1991, Johansen's reduced rank method of 1987 and Engle and Granger 1987 paper. Various generalizations are rather diversified and find many applications in macroeconomics and financial econometrics. After 30 years the concept is still quite important in theory and in applied work.

Key words: cointegration, fractional cointegration, nonstationarity, Engle-Granger metod; Johansen method; seasonal cointegration; nonlinear cointegration

INTRODUCTION

The aim of this paper is to present a (subjective) overview of evolution of cointegration[1]. The reason of this is double anniversary of two important papers – one by C.W.J. Granger, published in 1981 and introducing a concept of cointegration, the other by S. Johansen, published in 1991 and introducing now well-known reduced rank method. Those and first historical method for cointegration estimation and testing [Engle and Granger 1987], are now the usual scope of general lectures on econometrics.

But the cointegration concept in 30 years since the seminal paper by Granger underwent an impressive evolution. Many a new method and concepts have evolved, such as stochastic and deterministic integration and cointegration, polynomial cointegration, with time-varying parameters, for fractional integrated

[1] First version of this paper was presented on the 12th International Conference on Quantitative Methods in Economics, Warsaw, 30th June – 1st July 2011. Helpful comments from conference participants and from the referee are gratefully acknowledged.

and cointegrated time series, in nonlinear framework, hidden cointegration [Granger and Yoon 2002], Bayesian version[2] of those concepts, and so on and so forth.

GRANGER AND COINTEGRATION IN ECONOMETRICS

[Gonzalo 2010, p.174] starts his paper reminiscing about earlier work on cointegration with words: „The eighties were very good for music as well as econometrics. In time-series econometrics, the first half of that decade was dominated by research on unit roots while cointegration was the queen of the second half.

Indeed it seems so. [Granger 1981] is still widely quoted nowadays as the first published paper introducing the concept of cointegration. In his Nobel lecture[3] Granger emphasized connection between error correction model ECM (by Sargan) and cointegration analysis: "I am often asked how the idea of cointegration came about; was it the result of logical deduction or a flash of inspiration? In fact, it was rather more prosaic. A colleague, David Hendry, stated that the difference between a pair of integrated series could be stationary. My response was that it could be proved that he was wrong, but in attempting to do so, I showed that he was correct, and generalized it to cointegration, and proved the consequences such as the error-correction representation."

The next famous publication [Engle Granger 1987] have had a story of its own, which has been described by Granger himself in the following way[4]. The first version of the paper, submitted by Granger to *Econometrica*, was rejected for several reasons: lack of empirical application, need of rewriting proof of theorem, etc. Granger then started to work on improved version of his Representation Theorem, and accepted help of Robert Engle in empirical work. New version "first became Granger and Engle, next Engle and Granger" during his leave on a sabbatical. Second version was again rejected by *Econometrica* so they contemplated sending it to other publishers when *Econometrica* asked them to publish it because "they get so many papers on cointegration that they needed this one for a reference".

Soon after the first Granger and Engle papers appeared applications of the concept, e.g. [Bossaerts 1988] to stock prices, which seems to be quite natural field of applications. [Stock Watson 1988] described a variant of cointegration testing.

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[2] For Bayesian approach to integration and cointegration see e.g. [Koop et al. 1997] and papers listed on Prof. Koop web page http://personal.strath.ac.uk/gary.koop/research.htm. For multivariate cointegration applications in economics see e.g. [Majsterek 2008].

[3] http://ideas.repec.org/p/ris/nobelp/2003_007.html

[4] See Granger, Clive W.J. (2010) Some thoughts on the development of cointegration, Journal of Econometrics, 158(1), pp. 3–6.

Next decade, 1990's, also seems to be not so bad for the concept. [Johansen 1991] gave a strong impulse for further development, with his reduced rank method and cointegration tests which give all elements of the cointegration space in multivariate case. There are several important papers by Katarina Juselius and Søren Johansen, e.g. [Juselius Johansen 1990, 1992], with application to long-run equilibrium relationships of purchasing power parity and uncovered interest parity. Such parities are often a subject of cointegration tests – see e.g. [Corbae, Ouliaris 1988][5]; or [Cheung Lai 1993] test of PPP with use of fractional cointegration. There were also other methods of cointegration testing. [Bewley and Yang 1995] introduced a method a bit similar to Johansen's but perhaps less known.

Those methods were usually applied to series such as annual values of macroeconomic variables or daily quotes of financial instruments. But quite early cointegration methods for seasonal variables were introduced – e.g. [Hylleberg, Engle, Granger, Yoo 1990].

Start of the new century was marked by a Prize in Economic Sciences in memory of Alfred Nobel, awarded to R.F.Engle and C.W.J. Granger by the Royal Swedish Academy in 2003. In his Nobel lecture Granger gave an interesting overview of concept of cointegration and other topics of time series econometrics[6].

COINTEGRATION – FORMAL DEFINITIONS

We shall remind several important definitions and methods. First of all, we need concept of stationarity and nonstationarity of a series. Time series is stationary if its expected value and variance are constant in time, and covariance

$$C_\tau = \tfrac{1}{n-\tau}\sum_{t=1}^{n-\tau}(x_t - \bar{x})(x_{t-\tau} - \bar{x})$$

depends only on τ.

Engle and Granger method

Definition of cointegration [Engle, Granger 1987] is the following. Assume that x, y are nonstationary, but integrated, i.e. with stationary first differences. If there is a linear combination which is stationary, then they are cointegrated, and more general: if there is a set of integrated variables $y_t, x_{1t}, x_{2t}, \ldots, x_{kt}$ ~I(d), but

$$y_t - \beta_1 x_{1t} - \beta_2 x_{2t} - \ldots - \beta_k x_{kt} \text{~I(d-b)},$$

where $y_t, x_{1t}, x_{2t}, \ldots, x_{kt}$ – variables of interest, $[1, -\beta_1, -\beta_2 \ldots - \beta_k]$ – cointegrating vector, we say that they are cointegrated.

[5] Corbae and Ouliaris are the authors of the COINT addition to the GAUSS programming language.

[6] See http://nobelprize.org/nobel_prizes/economics/laureates/2003/granger-lecture.pdf

Granger has shown[7] that cointegration is equivalent to an error correction mechanism (ECM in short), i.e. that for cointegrated variables one can build the model capturing both short- and long-run features:

$$\Delta y_t = c_0 + c_1 \Delta x_t + \alpha(y_{t-1} - \beta x_{t-1}) + u_t$$

where $e_{t-1} = y_{t-1} - \beta x_{t-1}$ denotes the error from the previous period, $[1, -\beta]$ is a cointegrating vector. The error correction mechanism holds if an estimate of α has negative sign.

The Engle-Granger (1987) method of testing and estimating a cointegration relationship consists of OLS estimation of regression

$$y_t = \beta x_t + u_t$$

and checking whether u_t is stationary (or, in general case, of order of integration lower than the variables). Note that if errors are stationary, then variables are cointegrated. If errors are nonstationary, then we conclude only that the OLS estimates $[1 - \hat{\beta}]$ are not a cointegrating vector. [Maddala and Kim 1998] gave an excellent overview on literature concerning properties of cointegrating vectors:

1) For two variables cointegrating vector is unique; this does not hold for more variables.
2) [Stock 1987] proved that the OLS estimator of β is superconsistent, i.e., converges to β at the rate T instead of $\sqrt{T}$
3) Engle-Granger two-step procedure, i.e. estimating β by OLS, substituting into ECM and estimating ECM by OLS gives the same asymptotic distribution for ECM parameters as if β were known[8].

Triangular system and Phillips-Hansen method

Another way of formulating cointegrating relationship is with triangular system (see [Maddala and Kim 1998]):

$$y_{1t} = \beta' y_{2t} + u_{1t}$$
$$\Delta y_{2t} = u_{2t}$$

where y_{2t} denotes all I(1) variables other than y_{1t}, by assumption have one unit root and are not cointegrated. Vector $u_t = [u_{1t} \quad u_{2t}]^T$ is strictly stationary with zero mean and covariance matrix $\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$; $\sum_{t=0}^{n} u_t$ is a multivariate random walk with limiting Wiener process $W(r) = [W_1(r) \quad W_2(r)]^T$. Its covariance

[7] This is the Granger Representation Theorem.

[8] [Maddala, Kim 1998], p. 156.

matrix is $\Omega = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{t=0}^{n} E(u_t\, u'_s) = \Sigma + \Lambda + \Lambda'$ where $\Sigma = E(u_0\, u'_0)$, $\Lambda = \sum_{t=1}^{\infty} E(u_0\, u'_t),\ \Lambda = \sum_{t=1}^{\infty} E(u'_t\, u_0)$.

Denote $\Omega = \begin{bmatrix} \omega_{11} & \omega'_{21} \\ \omega_{21} & \Omega_{22} \end{bmatrix}$ and $\delta = \sum_{t=0}^{\infty} E(u_{1t}\, u_{20})$ then OLS estimator $\hat{\beta} = (Y'_2\, Y)^{-1} Y'_2\, y_1$ has asymptotic distribution:

$$T(\beta - \hat{\beta}) \Rightarrow (\int_0^1 W_2 W_2^{'})(\int_0^1 W_2 dW_{1.2} + \omega_{12}\Omega_{22}^{-1} W_2 + \delta)$$

where $W_1, W_2, W_{1\cdot 2}$ denote particular Wiener processes.

[Phillips, Hansen 1990] gave a semiparametric estimator[9] with correction for endogeneity:

$$\hat{y}_t^{+} = y_{1t} - \hat{\omega}_{12}\hat{\Omega}_{22}\Delta y_{2t}$$
$$\hat{u}_{1t}^{+} = u_{1t} - \hat{\omega}_{12}\hat{\Omega}_{22}\Delta y_{2t}$$

and serial correlation correction term $\hat{\delta}^{+}$ as a consistent estimator of $\sum_{k=0}^{\infty} (u_{1t}^{+} u_{21}^{+})$.

Hence the fully modified estimator, FM–OLS, by Phillips and Hansen has the following form: $\tilde{\beta} = (Y'_2\, Y_2)^{-1} (Y'_2\, \hat{y}_1^{+} - T\hat{\delta}^{+})$; $\hat{\Omega}$ is estimated with use of Newey-West estimator.

System estimation – Johansen method

The Johansen reduced rank method[10] is based on maximum likelihood approach applied to VAR model, assuming that errors are Gaussian: in a way similar to ECM, from a VAR model for variables $\mathbf{Y}_t = [Y_{1t}, Y_{2t}, ..., Y_{mt}]'$:

$$\mathbf{Y}_t = \mathbf{A}_1 \mathbf{Y}_{t-1} + ... + \mathbf{A}_k \mathbf{Y}_{t-k} + \mathbf{u}_t, \quad t = 1,2,...,T$$

a VECM model can be built:

$$\Delta\mathbf{Y}_t = \Pi \mathbf{Y}_{t-1} + \mathbf{B}_2 \Delta\mathbf{Y}_{t-1} + ... + \mathbf{B}_k \Delta\mathbf{Y}_{t-k+1} + \mathbf{u}_t, \quad t = 1,2,...,T$$

where

$$\Pi = -\mathbf{I} + \sum\nolimits_{i=1}^{k} \mathbf{A}_i, \quad \mathbf{B}_j = -\sum\nolimits_{i=j}^{k} \mathbf{A}_i, \quad j = 2,...,k$$

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[9] P.C.B.Phillips, B.E.Hansen (1990), Statistical inference in Instrumental Variables Regression with I(1) Process, *Review of Economic Studies*, 57, 99–125.

[10] For description see e.g. [Maddala Kim 1998] or original texts by[Johansen 1991, 1995].

Π has rank $r < k$ (is not a full rank) and can be represented as $\Pi = \alpha\beta'$, where $r = rank(\Pi)$, matrices are $m \times r$, $\beta'\mathbf{Y}_{t-1}$ constitute r cointegrating vectors, α is a matrix of error-correction terms (adjustment matrix). Construction of the Johansen test is as follows. Regress $\Delta\mathbf{Y}_t$ on its lags; $\mathbf{R}_{0t}$ – residuals; regress $\mathbf{Y}_{t-1}$ on $\Delta\mathbf{Y}_{t-1},\ldots,\Delta\mathbf{Y}_{t-k}$; $\mathbf{R}_{1t}$ – residuals: $\mathbf{R}_{0t} = \alpha\beta'\mathbf{R}_{1t} + \mathbf{u}_t$

Let $\begin{bmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{bmatrix}$ – matrix of sums of squares and sums of products of $\mathbf{R}_{0t}$ and $\mathbf{R}_{1t}$.

Then the asymptotic variance of $\mathbf{R}_{0t}$ is Σ_{00}, of $\beta'\mathbf{R}_{1t}$ is $\beta'\Sigma_{11}\beta$; asymptotic covariance of $\beta'\mathbf{R}_{1t}$ and $\mathbf{R}_{0t}$ is $\beta'\Sigma_{10}$. Maximize the likelihood function with respect to α holding β constant, then maximize with respect to β in second step: $\hat{\alpha}' = (\beta' S_{11}\beta)^{-1}\beta' S_{10}$; conditional maximum is $| S_{00} - S_{01}\beta(\beta' S_{11}\beta)^{-1}\beta' S_{10} |$; solve $| S_{10}S_{00}^{-1}S_{01} - \lambda S_{11} | = 0$ or $| S_{10}S_{00}^{-1}S_{01} - \lambda S_{11} |$ for maximum eigenvalue.

The Johansen tests statistics are based on properties of eigenvalues, e.g.

- determinant of the matrix = product of its eigenvalues;
- rank of the matrix = number of non-zero eigenvalues:

$$\Pi_{i=1}^{n}(1-\lambda_i) = | \mathbf{I} - S_{11}^{-1}S_{10}S_{00}^{-1}S_{01} |, \quad L_{\max}^{-2/T} = | S_{00} | \cdot \Pi_{i=1}^{n}(1-\lambda_i)$$

If there are r cointegrating vectors, then the m–r smallest eigenvalues are zero. Hence one has to find all eigenvalues, order them by size, then the eigenvectors corresponding to r greatest eigenvalues are cointegrating vectors and form columns of the matrix β.

There is a distinction between stochastic and deterministic cointegration: A vector $\mathbf{Y}_t$ of I(1) variables is said to be stochastically cointegrated with cointegrating rank r, if there are r linearly independent combinations of the variables that are I(0). These combinations may have nonzero deterministic trends.

Variables $\mathbf{Y}_t$ are said to be deterministically cointegrated, with cointegrating rank r, if there are combinations of $\mathbf{Y}_t$ that are I(0) are stationary *without* deterministic trends. The original definition of Engle and Granger excluded deterministic trends.

Interpretation of cointegration is as follows. Cointegrating relationships represent long-term equilibria between nonstationary variables:

If for $y_{1t}, y_{2t},\ldots, y_{mt}$ there are r cointegrating relationships, then there are m–r integrated time series, $u_{j,t}$ called common trends, such that every series $y_{it} = \sum_{j=1}^{m-r}\gamma_j u_{j,t} + \varepsilon_{it}$, with stationary ε_{it}.

[Johansen 1995], p. 39, notes: "One motivation for the [ECM] model is to consider [cointegrating] relation as defining the underlying economic relations, and assume that the agents react to the disequilibrium error through the adjustment coefficient α, to bring back the variables of the right track, that is, such that they satisfy the economic relations"[11].

"The process is pushed along the attractor set by the common trends and pulled towards it by the adjustment coefficients" (ibidem, p. 41).

Polynomial (dynamic) cointegration

Classical cointegrating relationships are static; polynomial cointegration introduces a small number of lags into a cointegrating relationship: "In other words, cointegration reduces the order of integration by applying linear regressions between variables; dynamic cointegration reduces the order of integration by applying autoregressive modeling. A VAR model with *n* lags

$$\mathbf{Y}_t = \mathbf{A}_1\mathbf{Y}_{t-1} + \mathbf{A}_2\mathbf{Y}_{t-2} + ... + \mathbf{A}_n\mathbf{Y}_{t-n} + \varepsilon_t$$

exhibits dynamic cointegration if there exists a stationary autoregressive combination of the variables of the type $\alpha'\mathbf{Y}_t + \Delta\mathbf{Y}_t$" ([Focardi, Fabozzi 2004, p. 541)[12]. [Johansen 1995, p. 39] defines the I(2) process $\mathbf{Y}_t$ as polynomially cointegrated, if there exist β_0, β_1 such that $\beta'_0\,\mathbf{Y}_t + \beta'_1\,\Delta\mathbf{Y}_t$ are stationary. First term reduces I(2) variables to I(1) linear combination, and second term ensures that in turn its combination with I(1) differences is stationary.

[Focardi, Fabozzi 2004] explains that variables can be cointegrated *and* dynamically cointegrated: e.g., log prices of assets are nonstationary, log returns are stationary – factor models for returns and cointegrating models for prices can coexist; in addition, linear combination of prices and returns can also be stationary.

Cointegration and canonical correlation analysis

[Bossaerts 1998] applied canonical correlation analysis to tests of cointegration: used a model $\Delta\mathbf{Y}_t = \mathbf{HCY}_t + \varepsilon_t$, applied canonical correlation analysis for $\Delta\mathbf{Y}_t$ and $\mathbf{Y}_t$, and checked if the canonical variates are nonstationary. [Bewley and Yang 1995] improved the method, including deterministic trends, developed asymptotic theory, new tests and computed critical values for number of cointegrating vectors. The LCCA (*level canonical correlation analysis*) is similar in spirit to the Johansen method:

[11] Soren Johansen (1995) *Likelihood-based inference in cointegrated vector autoregressive models*, Oxford University Press, Oxford.

[12] Sergio Focardi, Frank J. Fabozzi (2004) *The mathematics of financial modeling and investment management*, Wiley, Hoboken, New Jersey.

1. eliminate additional variables by regression of $\mathbf{Y}_t$ and $\mathbf{Y}_{t-1}$ on those variables. For residuals $\mathbf{R}_{0t}, \mathbf{R}_{1t}$ of these regressions form $\mathbf{R}_{0t} = \mathbf{B}\,\mathbf{R}_{1t} + \mathbf{u}_t$.
2. determine the canonical correlation between $\mathbf{R}_{0t}, \mathbf{R}_{1t}$, solve the eigenvalue problem: $|\mathbf{S}_{10}\,\mathbf{S}_{00}^{-1}\mathbf{S}_{01} - \lambda\mathbf{S}_{11}| = 0 \qquad \mathbf{S}_{ij} = \dfrac{\mathbf{R}_i\mathbf{R}_j}{T}, i, j = 1,2$

   Interpretation is different: here canonical correlations are for levels only, in the Johansen method –for levels and differences.

Cointegration and state-space models

Another path of development is formulating cointegration theory for state-space models. According to [Fabozzi et al. 2006], who give an overview of Bauer and Wagner methods[13], D. Bauer and M. Wagner provided systematically developed theory of cointegration in this framework, tests for order and cointegrating rank of state-space models. An extensive description of Bauer and Wagner results can be found in [Wagner 2010][14].

Seasonal cointegration

One of seasonal cointegration tests, known as HEGY test, was introduced by [Hylleberg, Engle, Granger, Yoo 1990]. Unit root, causing nonstationarity of a series, corresponds to a root (of modulus one) of autoregressive polynomial A(L), where $\mathrm{A(L)}y_t = c_0 + \varepsilon_t$ is a representation of a series in question. Difference of the series is stationary. For quarterly data, seasonal differencing operator may be necessary to obtain stationarity: this operator (defined with use of lag operator L)

$$\Delta_4 y_t = (1-\mathrm{L}^4)y_t; (1-\mathrm{L}^4) = (1-\mathrm{L})(1+\mathrm{L})(1+\mathrm{L}^2) = (1-\mathrm{L})(1+\mathrm{L})(1+i\mathrm{L})(1-i\mathrm{L})$$

has four roots – one corresponds to zero frequency (a trend), one to two cycles per year, and a pair of complex roots – to one cycle per year. If a pair of series has unit root at frequency ω and their linear combination does not have a unit root at this frequency, they are called cointegrated with frequency ω. [Hylleberg et al. 1990] give tests for seasonal cointegration, provide (Monte Carlo) critical values for seasonal integration and cointegration, and give an example for consumption function of the UK[15].

[13] For description of state space model estimation, see [Fabozzi et al. 2006], p. 539-544.
[14] Available at http://www.ihs.ac.at/publications/eco/es-248.pdf
[15] For example of seasonal cointegration applied to Polish economy see e.g. Kotłowski, J. *Money and Prices in the Polish Economy, Seasonal Cointegration Approach* (in Polish), Warsaw School of Economics, or shorter version – Working Paper in English: http://ideas.repec.org/p/wse/wpaper/20.html

Fractional cointegration

Distinction between I(1) and I(0) series is an important one, but this is too crude a description for actual economic series. More accurate tool is a fractional differencing, see e.g. [Granger and Joyeux 1980], [Hosking 1981], and fractional integration and cointegration. Fractional difference or fractional filter is defined as:

$$(1\text{-}L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-L)^k = \sum_{k=0}^{\infty} \frac{\Pi_{a=0}^{k-1} (d-a)(-L)^k}{k!} = 1 - dL + \frac{d(d-1)}{2} L^2 + ...$$

where d denotes a fractional integration parameter, L – the lag operator.

Fractional integrated processes are intermediate between integrated of order 1 and 0. They exhibit long memory, i.e., their autocorrelation function decays very slowly. There are several methods of estimating fractional integration parameters: [Geweke and Porter-Hudak 1993] periodogram regression method, appropriate for $-1/2 < d < 1/2$ processes; generalization by P.C.B. Phillips[16] and [Robinson 1994, 1995]; algorithms using fast Fourier transform or wavelet transform; direct maximum likelihood method by [Sowell 1990, 1992], see extensive paper by [Baillie 1996] and also his later papers.

Fractional cointegration, suggested by [Engle, Granger 1987] and developed by [Cheung, Lai 1993] and [Marinucci, Robinson 2001], deals with linear common long-memory persistence feature (see [Marinucci, Robinson 2001], [Dittmann 2004]). This framework is routinely applied to tests of long-run equilibrium relationships, such as purchasing power or interest rate parities. It can be used as a tool of financial econometrics, e.g. in portfolio building.

Nonlinear cointegration

Nonlinear cointegration requires more sophisticate definitions and multistep procedures compared to the two-step Engle-Granger procedure or even the Johansen's procedure. Definition of nonlinear cointegration is not unique, as well as formulation of error correction model; empirical procedure requires testing several features of variables and of deviations from an equilibrium (i.e., an attractor).

Detailed overview of nonlinear cointegration is [Dufrénot, Mignon 2002]. They comment on unit root and stationarity tests adequacy in nonlinear case, describe several versions of nonlinear series and models, nonlinear measures of persistency, define equilibration, nonlinear cointegration, nonlinear error correction mechanism, and nonlinear cotrending. Last part contains asymmetric and threshold nonlinear cointegration models – those are important for applied work, where adjustment process starts only when the distance from equilibrium is greater than a particular threshold, and also some asymmetries of behaviour may occur.

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[16] See [Phillips 199a,b] and [Shimotsu, Phillips 2002].

An example of nonlinear cointegration and nonlinear error-correction models (NECM) applied for oil and stock markets is research by [Arouri, Jawadi, Nguyen 2010]. They first give an overview of nonlinear cointegration and NECM. To an equilibrium state for linear cointegration here corresponds an attractor representing long-run equilibrium (attractor for nonlinear dynamical systems). The authors present three definitions of nonlinear cointegration, based on concept of a mixing series: long-range dependence corresponds to non-mixing, short-range dependence to mixing.

They also give a description of empirical strategy of a single equation NEC estimation (subsection 9.2.3, pp. 180 – 181). First step is to check integration of series, second: to estimate an equation and test residuals of the long-run attractor for stationarity. Absence of unit root suggests linear cointegration, in this case mixing properties of the series should be checked. Third step consists of testing null of linear cointegration against nonlinear. Four: if linear cointegration is not rejected in step 2 and rejected in step 4, this may mean nonlinearity in the short-run dynamics (p. 181). The fifth step is testing for dynamic nonlinearity and estimation of short-run models. Rejection of linear cointegration in step 3 leads to Step 6: estimate a NECM by nonlinear least squares (NLS), and estimate NECM allowing for nonlinear mean-reversion process. Step 7: check the mixing hypotheses for residuals of the NECM, apply several diagnostic and misspecification tests.

Their example of application is to the relationship between world stock market index and world oil market index, based on monthly data since May 1987 until January 2009. Both indices are not stationary in levels, their differences are stationary. Log returns of the indices seem "to show some linkages between variables" and returns have downward trends and negative values during crisis (p. 183). The Dickey and Fuller test and the Zivot and Andrews test lead to rejection of linear cointegration hypothesis. Next they apply the KPSS test and modified R/S test [Lo 1991], thus show that there is a nonlinear cointegration relationship between oil and equity markets, which is confirmed by [Keenan 1985] and LM3 test [Van Dijk et al. 2002]. They use three variants of NEC model, and show that "lagged oil market returns significantly affect the dynamics of world market returns" (p. 187). Next they build the smooth transition Error Correction Model with logistic transition function and show that adjustment towards long-run equilibrium is activated when a shock (e.g., "the 1987 stock crash, the 1991 first Gulf War, the 2003 second Gulf War and the 2007-09 financial crisis", p. 190) affects one of the markets.

[Bruzda 2006] applies nonlinear cointegration analysis to money demand models. She also uses the KPSS test and the modified R/S analysis to check integration of series, and next (instead of NLS mentioned above) applies the Phillips-Hansen FM-OLS estimator to several versions of the money demand models. The KPSS test and [Breitung 2001] cointegration tests are then applied, checking not only cointegration in levels, but also long memory in information for residual series. Specification of models is based on a general equilibrium model with maximization of multiperiod utility function of a representative household, using budget constraint. A CES form of utility function leads to the double-logarithmic functional form; if liquidity preferences of agents depend on a monetary regime (levels of inflation and nominal interest rate), then preferred is a semilogarithmic function in which "the interest rate elasticity is an increasing function of the interest rate spread" [Bruzda 2006, p. 115]. Third form, a log-inverse specification, reflects a discontinuity of money demand of a household for certain nominal interest rates. Another three versions of models contain a linear trend in long-run relationship. Results of estimation of the first three models are consistent with economic theory, for models with trend signs are not interpretable, but the parameters are also stable. The Breitung's test is "in favour of the presence of (possibly non-linear) cointegration without the trend component" (p. 120), "the Hansen's test and tests for significance of mutual information coefficients distinguished the semilogarithmic model, while the KPSS test and the modified R/S analysis singled out the double-logarithmic functional form" (p. 121).

An example of nonlinear fractional cointegration with financial application is Andersen, Bollerslev, Diebold and Wu, concerning persistence of realized beta[17]. The authors study stock-systematic risk to check whether it is time-varying ("time varying betas from the CAPM model"). They explain that use of quarterly data did not allow for direct estimation of fractional integration parameters, hence they have adopted estimates from other papers, based on higher frequency data. Their estimates of quarterly betas are based on nonparametric realized quarterly market variances and individual equity covariances with the market.

[Andersen et al. 2006] are interested in nonlinear copersistence function – nonlinearity caused by the fact that the betas are ratios. They are not able to estimate and test fractional integration parameter for a function of betas, due to small number of observations. Instead, they check behaviour of autocorrelation function of fractional differences of betas (for a stationary function, the ACF should decay at higher rate than for fractionally integrated series) and are able to formulate conclusions about financial properties of studied instruments.

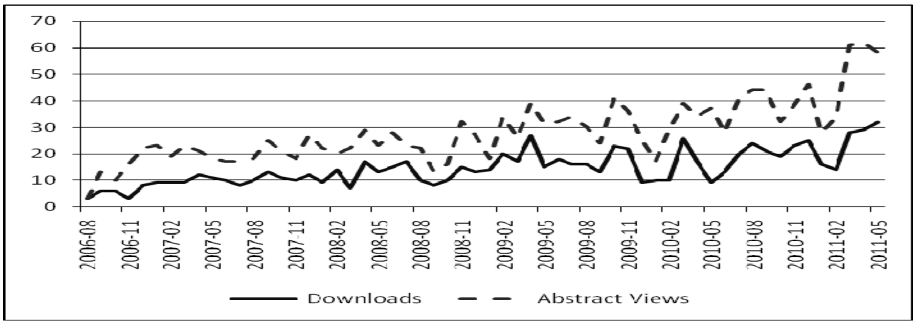
IMPORTANCE OF COINTEGRATION

We can visualize the influence of [Granger 1981] and [Johansen 1991] with use of diagrams from the http://ideas.repec.org website, which provides access to and statistics for economic and econometric publications. Both authors (and R. F. Engle) are among 5% most cited economists. [Granger 1981], [Johansen 1991] and [Engle Granger 1987] papers are among 1‰ most cited papers in economics.

[17] T.G. Andersen, T. Bollerslev, F.X. Diebold, G. Wu (2006) Realized beta: persistence and predictability, in: *Econometric Analysis of Financial and Economic Time Series, Part B. Advances in Econometrics*, Vol. 20, p. 1–39, Elsevier. The paper is dedicated to Clive W.J. Granger.

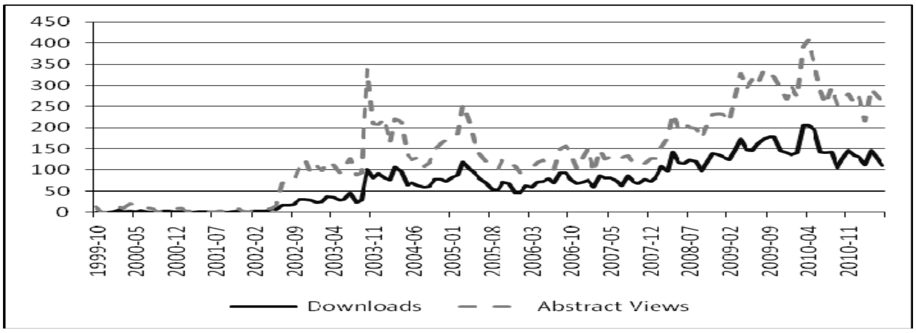
It seems that the interest in the papers is still growing, with some local maxima18 (see Fig. 1a,b,c). General interest in the concept of cointegration is reflected in number of Google search for reference in Internet (see Fig. 2), which is growing according to a increase in search diagram.

Figure 1a. Abstract and paper downloads of [Granger 1981]



Source: http://ideas.repec.org

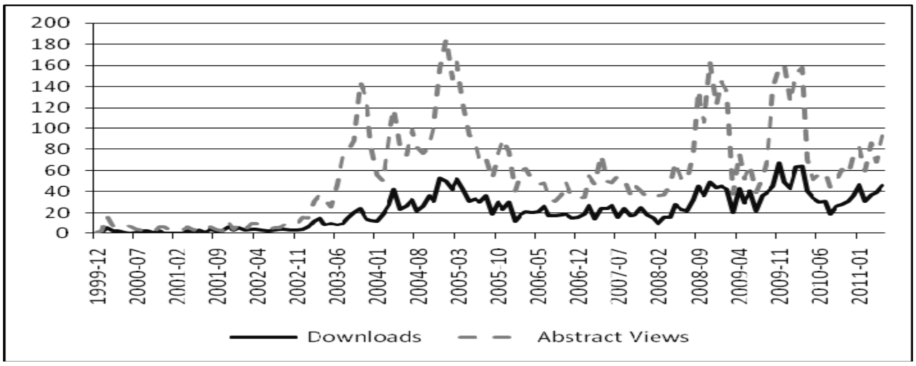
Figure 1b. Abstract and paper downloads of [Engle, Granger 1987]



Source: http://ideas.repec.org

[18] One of them around the timing of Prize in Economics award.

Figure 1c. Abstract and paper downloads of [Johansen 1991]

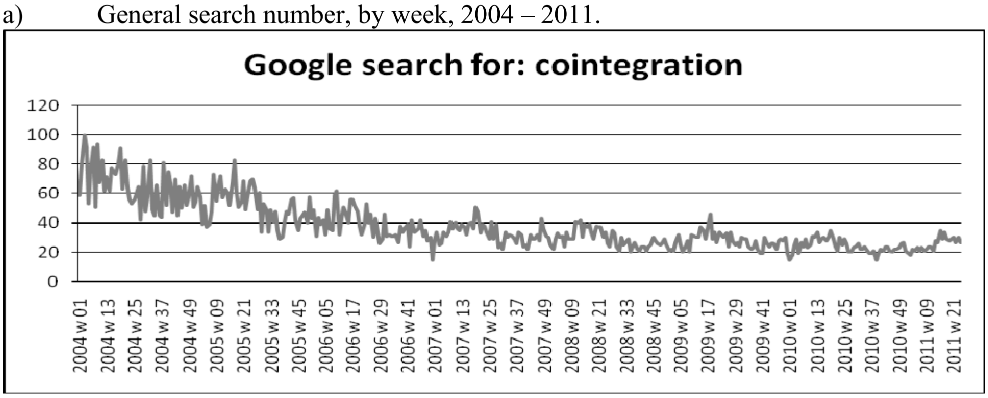


Source: http://ideas.repec.org

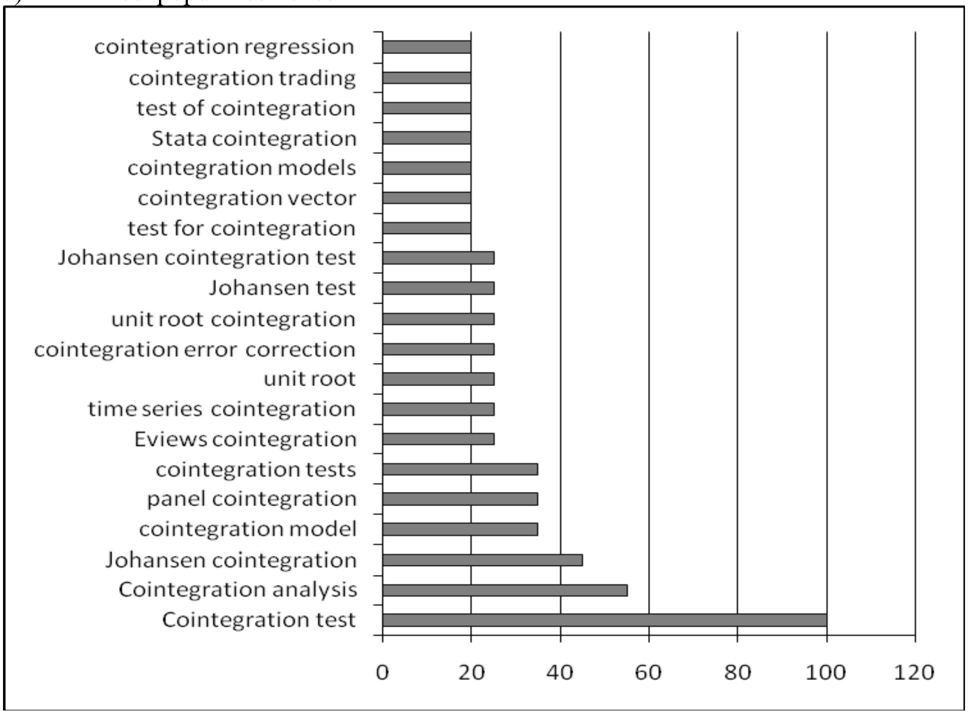
The first diagram in Fig. 2 reflects general search for cointegration in all meaning of the word. The second diagram is more informative about detailed formulations – users search both for methods and for software applications. The last diagram shows increase in number of searches – greatest for the Johansen method, cointegration tests, software applications and for general descriptions.

Figure 2. Google statistics on search for reference on cointegration in Internet

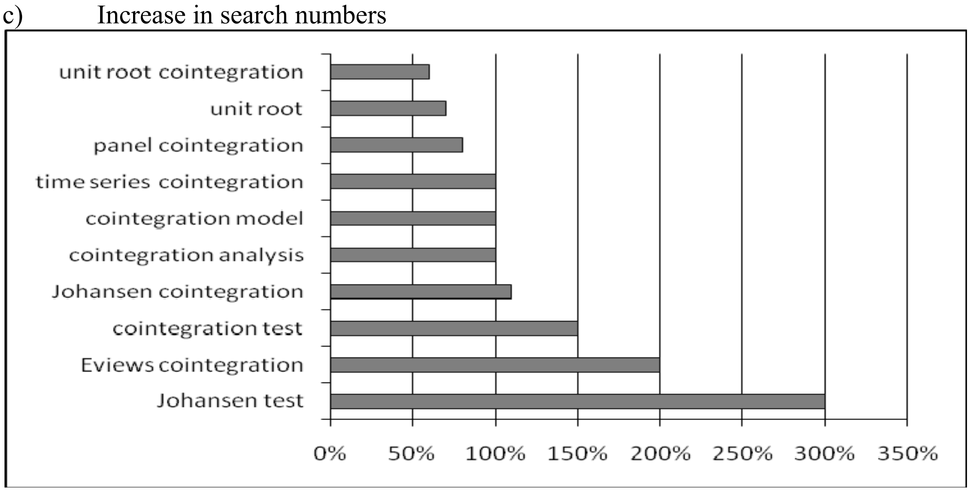
a) General search number, by week, 2004 – 2011.



b) Most popular searches



c) Increase in search numbers



Source: Google Statistics

Let us turn to Johansen and Juselius for comments on importance of cointegration – and of careful approach to its applications. In “Conversation with Søren Johansen and Katarina Juselius”, published in [Rosser et al. 2010] and available at Prof. Juselius web page[19], they talk on history of their cooperation, development of econometric methods, state of economic research and publishing, and give comments on the topic of cointegration:

Søren Johansen: "I think there was a time when all the theoretical econometricians were working on topics related to this methodology and many were applying it. Now many econometricians work on other topics – panel data, financial econometrics, factor models, analysis of large data sets, and so on. But the people who make a living on analyzing the usual monthly or quarterly data sets – they routinely apply cointegration methods, and that is possibly because the programs are there."

Katarina Juselius: "Unfortunately, this is not a method that can be applied routinely using standard cointegration software; it requires interaction between the analyst and the data; it is a powerful tool for an expert to use, not a tool for someone who doesn't understand the methodology. […] A good cointegration analysis is when you structure the information in the data, so that the complexity of the empirical reality can be grasped and better understood. […] Serious data analysis is a long process that requires a very systematic study; continually working with the data, trying different specifications until reaching the point where one can say: now I understand the basic features of the data (statistically as well as economically). This is a baseline model that it makes sense to continue working with. You have to carefully check for misspecifications […] These steps are enormously important."

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[19] The interview is available on Prof. Juselius web page: http://www.econ.ku.dk/okokj/#bio

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