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BAYESIAN CONFIDENCE INTERVALS FOR THE NUMBER AND THE SIZE OF LOSSES IN THE OPTIMAL BONUS–MALUS SYSTEM

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Abstract: Most of the so far proposed Bonus–Malus Systems (BMSs) establish a premium only according to the number of accidents, without paying attention to the vehicle damage severity. [Frangos and Vrontos 2001] proposed the optimal BMS design based not only on the number of accidents of a policyholder, but also on the size of loss of each accident. In our work, we apply the approach presented by Frangos and Vrontos to construct the Bayesian confidence intervals for both the number of accidents and the amount of damage caused by these accidents. We also conduct some simulations in order to create tables of estimates for both the numbers and the sizes of losses and to compute the realizations of the corresponding Bayesian confidence intervals. We compare the results obtained by using our simulation studies with the appropriate results derived through an application of an asymmetric loss function and its certain modification.

Keywords: optimal BMS, number of claims, severity of claims, Bayesian analysis, Bayesian confidence intervals, asymmetric loss functions

INTRODUCTION

The Bonus-Malus Systems are commonly used in the calculation of insurance premiums in the area of vehicle insurance. They penalize the car owners who caused accidents by premium surcharges and reward the drivers with accident-free year(s) by discounts. The term "Bonus" means a discount in the premium of the policyholder, which is given on the renewal of the policy if no at-fault accident occurred to that client in the previous year. In turn, the term "Malus" denotes an increase in the premium of the insured driver who caused an accident in the last year. The main drawback of the BMSs is that they calculate a premium only according to the number of accidents, disregarding the size (severity) of loss that each accident incurred; in this way, a policyholder who had an accident with a small size of loss is unfairly penalized like a policyholder who had an accident with a big size of loss (thus, the policyholders with the same number of accidents pay the same malus, irrespective of the size of damage). [Frangos and Vrontos 2001] proposed the optimal BMS design based not only on the number of accidents, but also on the size of losses. It was a certain development of the method introduced by [Dionne and Vanasse 1989]. The works of the cited authors attracted our attention towards the subject of the optimal BMSs and encouraged us to undertake some research in this field.

The objectives of our studies are:

(i) application of the Frangos and Vrontos approach in the construction of the Bayesian confidence intervals for both the number of accidents and the size (severity) of damage that these accidents incurred,

(ii) application of the loss functions in the estimation of the expected number of losses and the expected size of losses in the year $t+1$, given the numbers and the sizes of losses in the previous years (i.e., in the years $1,...,t$),

(iii) conducting some simulation studies in order to create tables of estimates for both the number and the size of losses and to compute the realizations of the Bayesian confidence intervals for the expected number and size of claims,

(iv) calculation of the change in the frequency of the number of claims and the net premium in the optimal BMS,

(v) making comparisons of the results obtained by using our simulation studies with the appropriate results derived through an application of an asymmetric loss function and its certain modification.

BAYESIAN CONFIDENCE INTERVALS FOR THE EXPECTED NUMBER AND THE EXPECTED SIZE OF LOSSES

We denote by X the number of accidents (claims) and by λ the underlying risk of a policyholder to have an at-fault accident. We assume that the conditional distribution of X given the parameter $\lambda > 0$ is the $Poisson(\lambda)$ distribution, i.e.,

$$P(X = k \mid \lambda) = \frac{e^{-\lambda}\lambda^k}{k!}, \ \lambda > 0, k = 0,1,2,... \qquad (1)$$

The Poisson distribution is often applied to describe random and independent events, such as vehicle accidents. It is a particularly useful distribution for modeling numbers of automobile accidents in the case when a random variable is introduced into the regression component in the formula for the risk λ. In this setting the regression component contains all significant information about individual characteristics of the insured driver, which may affect the policyholder's driving skills and habits. For more details on this issue see the paper of [Dionne and Vanasse 1989].

In addition, we assume that λ is a random variable with the prior distribution $Gamma\left(a,b\right)$, i.e., its density function is given by

$$g(\lambda)=\frac{\lambda^{a-1}b^{a}e^{-b\lambda}}{\Gamma(a)},\ \ a>0,\ b>0. \tag{2}$$

Such a determination of λ gives a certain a priori knowledge about the *proneness* of a policyholder to cause an accident (parameters a, b specify our a priori knowledge about how *accident-prone* a policyholder is).

Denote by $X_1,...,X_t$ the numbers of accidents that a policyholder caused in the i th year, $i=1,...,t$. We assume that $X_1,...,X_t$ are conditionally independent given unobserved variable λ. It may be verified that $X_1,...,X_t$ are unconditionally dependent and that, under the conditions in (1), (2), the unconditional distribution of the number of claims X is the *Negative Binomial* $\left(a,b\right)$ distribution, with a probability function of the form

$$P\left(X=k\right)=\binom{k+a-1}{k}\left(\frac{b}{1+b}\right)^{a}\left(\frac{1}{1+b}\right)^{k},\ \ k=0,1,2... \tag{3}$$

Then, we have that the posterior distribution $\lambda|X_1,...,X_t$ (i.e., the a posteriori structure function of λ for a policyholder with the historical claim numbers $X_1,...,X_t$) is the $Gamma(a+K,b+t)$ distribution, where $K=\sum_{i=1}^{t}X_i$. Consequently, the $\left(1-\alpha\right)\cdot 100\%$ Bayesian confidence interval for $\lambda_{t+1}=\lambda_{t+1}\left(X_1,...,X_t\right)$ – the expected number of losses in the year $t+1$ of a policyholder with the claim numbers history $X_1,...,X_t$ – may be derived from the relation $P\left(A_\alpha\le\lambda_{t+1}\le B_\alpha|X_1,...,X_t\right)=1-\alpha$ by putting:

$$A_\alpha=\Gamma^{-1}\left(\alpha/2,a+K,b+t\right), \tag{4}$$

$$B_\alpha=\Gamma^{-1}\left(1-\alpha/2,a+K,b+t\right), \tag{5}$$

where $\Gamma^{-1}\left(\cdot,a+K,b+t\right)$ stands for the corresponding quantile of the $Gamma(a+K,b+t)$ distribution.

Now, let us refer to the issue concerning the size of claims of the insured drivers.We denote by Y the size of claims (losses) and by β the mean size of claims of each insured. We assume that the conditional distribution of Y given the parameter $\beta > 0$ is the $Exponential(\beta)$ distribution, i.e.,

$$P(Y \le y \mid \beta) = 1 - e^{-y/\beta}, \quad \beta > 0. \tag{6}$$

Furthermore, we also assume that β is a random variable having the prior distribution $Inverse\ Gamma(s, m)$, i.e., its density function is given by

$$g(\beta) = \frac{\frac{1}{m} e^{-m/\beta}}{\left(\frac{\beta}{m}\right)^{s+1} \Gamma(s)}, \quad s > 0, m > 0. \tag{7}$$

Denote by $Y_1,...,Y_t$ the sizes of losses incurred as a result of accidents that a policyholder caused in the i th year, $i = 1,...,t$. We assume that $Y_1,...,Y_t$ are conditionally independent given unobserved variable β. It can be checked that $Y_1,...,Y_t$ are unconditionally dependent and that, under the conditions in (6), (7), the unconditional distribution of the size of losses Y is the $Pareto(s,m)$ distribution, i.e., it has a density function of the form

$$f(y) = \frac{sm^s}{y^{s+1}} I(y \ge m), \quad s > 0, \ m > 0, \tag{8}$$

where I stands for the indicator function.

Then, the posterior distribution $\beta | Y_1,...,Y_t$ (i.e., the a posteriori structure function of β for a policyholder with the historical claim sizes $Y_1,...,Y_t$) is the $Inverse\ Gamma(s+t, m+L)$ distribution, where $L = \sum_{i=1}^{t} Y_i$.

Therefore, the $(1-\alpha)\cdot 100\%$ Bayesian confidence interval for $\beta_{t+1} = \beta_{t+1}(Y_1,...,Y_t)$ – the expected size of losses in the year $t+1$ for a policyholder with the claim sizes history $Y_1,...,Y_t$ – may be easily derived from the relation $P(C_\alpha \le \beta_{t+1} \le D_\alpha | Y_1,...,Y_t) = 1-\alpha$ by making the following substitutions:

$$C_\alpha = I\Gamma^{-1}(\alpha/2, s+t, m+L), \tag{9}$$

$$D_\alpha = I\Gamma^{-1}(1-\alpha/2, s+t, m+L), \tag{10}$$

where $I\Gamma^{-1}(\cdot, s+t, m+L)$ denotes the corresponding quantile of the $Inverse\ Gamma(s+t, m+L)$ distribution.

APPLICATION OF THE LOSS FUNCTIONS IN THE ESTIMATION OF THE EXPECTED NUMBER AND THE EXPECTED SIZE OF LOSSES

A map of the form $L(x) = e^{-cx} + cx - 1$ is called an asymmetric Linex Loss function. It is now a widely used function in the actuarial statistics (for its applications in the area of BMSs, we refer to [Bermudez et al. 2001]). In contrast to the quadratic loss function, this type of loss function avoids high penalties by breaking the symmetry between the overcharges and underchages. If $c < 0$, it gives a greater penalty for overestimation than for underestimation of losses. If $c > 0$, it gives a greater penalty for underestimation than for overestimation of losses.

It can be shown that in our model, the optimal Bayesian estimator of the parameter λ_{t+1} (interpreted as the expected number of losses in the year $t+1$), obtained by minimizing the expectation $E_{post}L\left(\hat{\lambda}_{t+1} - \lambda_{t+1}\right)$, where E_{post} is calculated with respect to the posterior distribution $\lambda | X_1,..., X_t$, has the following form (see Appendix for the corresponding proof)

$$\hat{\lambda}_{t+1(Linex)} = \frac{1}{c}\ln\left(\frac{b+t}{b+t-c}\right)^{a+K}, \quad K = \sum_{i=1}^{t} X_i. \tag{11}$$

For comparison, the mean of the posterior distribution $\lambda | X_1,..., X_t$ (i.e., an average of the $Gamma(a+K, b+t)$ distribution) is equal to

$$\hat{\lambda}_{t+1} = (a+K)/(b+t). \tag{12}$$

Let us now move on to the issue concerning the estimation of the parameter relating to the size of losses. Since there is no optimal Bayesian estimate of β_{t+1} for any $c > 0$ in our model, the Linex1 Loss function, instead of the Linex Loss function, is used in the estimation of β_{t+1}. [Basu and Ebrahimi 1991] proved that the optimal Bayesian estimator of β_{t+1}, obtained through an application of the Linex1 Loss function, has the following form

$$\hat{\beta}_{t+1(Linex1)} = \left(-\frac{L+m}{c}\right)\left(1 - \exp\left(\frac{c}{t+s+1}\right)\right), \quad \text{where} \quad L = \sum_{i=1}^{t} Y_i. \tag{13}$$

In order to obtain (13), we minimize $E_{post}L\left(\hat{\beta}_{t+1}/\beta_{t+1} - 1\right)$, where L is the Linex function and E_{post} is calculated with respect to the posterior distribution $\beta | Y_1,...,Y_t$. For comparison, the mean of the posterior distribution $\beta | Y_1,...,Y_t$ (i.e., an average of the $Inverse\ Gamma(s+t, m+L)$ distribution) is equal to

$$\hat{\beta}_{t+1} = (m+L)/(s+t-1). \tag{14}$$

CALCULATION OF THE NET PREMIUM IN THE OPTIMAL BMS

In this part of our work, we give the formulas for the changes in the frequency of the number of claims and for the net premium in the optimal BMS. Namely, let us notice that:

(i) By using (12), we have the following formula for the change in the frequency of the number of losses

$$(\hat{\lambda}_{t+1} / EX)\cdot 100\%, \tag{15}$$

where $EX = a/b$ is an unconditional expected value of the number of claims (we recall that $X \sim Negative\ Binomial\,(a,b)$);

(ii) By using (11) (and the Linex Loss function in particular), we have the following formula for the change in the frequency of the number of claims

$$(\hat{\lambda}_{t+1(Linex)} / EX)\cdot 100\%; \tag{16}$$

(iii) By using (12), (14), we obtain the following formula for the optimal BMS net premium (interpreted as the total loss in the year $t+1$)

$$\hat{\lambda}_{t+1}\cdot\hat{\beta}_{t+1}; \tag{17}$$

(iv) By using (11), (13) (and the Linex Loss functions in particular), we get the following formula for the optimal BMS net premium (interpreted as the total loss in the year $t+1$)

$$\hat{\lambda}_{t+1(Linex)}\cdot\hat{\beta}_{t+1(Linex1)}. \tag{18}$$

By putting for the values of parameters a, b the values considered in [Frangos and Vrontos 2001, p. 16], we may compute the quantity in (15) for different t, K and give the interpretations of the obtained results as follows:

(1) Put: $a = 0.228, b = 2.825, t = 3, K = 0$.

Then: $X \sim Negative\ Binomial\,(0.228, 2.825)$, $EX = a/b = 0.08$, and $\left(\hat{\lambda}_4 / EX\right)\cdot 100\% = 48\%$. It means that provided a policyholder has not had any accident for the first 3 years of insurance duration, then the premium paid at the beginning of the 4th year of insurance duration should amount to 48% of the premium paid at the beginning of the 1st insurance year;

(2) Put: $a = 0.228, b = 2.825, t = 4, K = 1$.

Then: $X \sim Negative\ Binomial\,(0.228, 2.825)$, $EX = a/b = 0.08$, and $\left(\hat{\lambda}_5 / EX\right)\cdot 100\% = 223\%$. It means that provided a policyholder has had 1 accident for the first 4 years of insurance duration, then the premium paid at the beginning of the 5th year of insurance duration should amount to 223% of the premium paid at the beginning of the 1st insurance year.

SIMULATION STUDIES

I. Simulation of the numbers of losses (claims)

This subsection consists of two parts. In the first one, we construct (based on the previously introduced theoretical background) the procedure leading to the simulation of the numbers of losses; the simulated numbers of losses are collected in the appropriate table. In the second part, the realizations of the confidence intervals for the expected number of losses are computed by using the earlier derived formulas.

I.1. Table of the numbers of losses

We simulated the numbers of claims for a portfolio of 100 polyholders by applying the following procedure:

(i) We generated a sample of size 100 from the $Gamma(0.3, 1/3)$ distribution (i.e., we generated 100 values of the random variable $\lambda \sim Gamma(0.3, 1/3)$ (see (2))),

(ii) For the given $\lambda(s)$ ($s = 1,...,100$) from the generation in (i), we generated 100 independent 10–element samples $\mathbf{x}^{(s)} = \left(x_1^{(s)}, x_2^{(s)}, ..., x_{10}^{(s)}\right)$ from the $Poisson(\lambda(s))$ distribution, which represented the numbers of losses in a portfolio of 100 polycyholders in the period of 10 years.

Based on the generations in (i), (ii), we obtained the simulated numbers of losses in the course of 10 years. The corresponding results are collected in Table 1.

Table 1. The simulated numbers of losses (claims)

Years/Numbers of losses												
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*≥11*
1	67	16	7	6		1		1		2		
2	73	11	5	3	3	2	2			1		
3	78	9	4	3	2	1		1	2			
4	77	9	4	6	2	1	1					
5	71	14	6	3	2	2					1	1
6	69	12	8	5	2	1	2					1
7	70	15	8	2	1	2		1				1
8	76	9	4	5	2		1	2				1
9	74	13	6	2	1	1	2					1
10	78	8	5	4		1	1	1				2

Source: own calculations

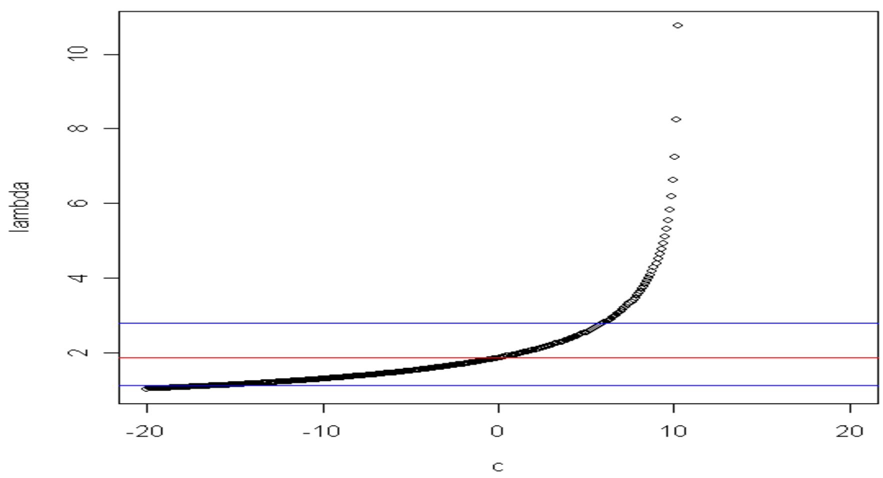
I.2. Realizations of the confidence intervals for the expected number of losses

Based on the first 10 observations $x_1^{(1)},...,x_{10}^{(1)}$ (relating to the simulated numbers of losses of the first polycyholder - see the generation $\mathbf{x}^{(1)}$ in (ii) above), we obtained the realization of the 95% confidence interval (CI) for $\lambda_{t+1} = \lambda_{11}$ by putting: $a = 0.3$, $b = 1/3$, $K = \sum_{i=1}^{10} x_i^{(1)} = 19$, $t = 10$ into the formulas in (4), (5).

The received realization of the 95% CI for λ_{11} - the expected number of losses in the 11th year - was $[1.13; 2.79]$.

Fig. 1 depicts the Bayesian estimates of λ_{11}, obtained through an application of the Linex Loss function $L(x) = e^{-cx} + cx - 1$ for c from -20 to 20 with the step 0.1. The outer horizontal lines correspond to the limits 1.13, 2.79, of the estimated 95% Bayesian CI for λ_{11}. In addition, by the inner horizontal line the expected value 1.87, of the posterior distribution $\lambda | X_1,..., X_{10}$, which is the $Gamma(0.3+19, 1/3+10)$ distribution, is presented (this expected value has been calculated by substituting the values: $a = 0.3$, $K = 19$, $b = 1/3$, $t = 10$ into (12)).

Figure 1. The Bayesian estimates of λ_{11} (based on the application of the Linex Loss function), the estimated 95% Bayesian CI for λ_{11} and the expectation of an appropriate posterior distribution



Source: own calculations

In practise, the values of a, b are unknown, but we may estimate them from a portfolio by the method of moments (MM). Since X - the number of losses - has the negative binomial distribution with the mean $EX = a/b$ and the variance $VarX = (a/b)(1+1/b)$, then: $\hat{a} = \overline{X}\hat{b}, \hat{b} = \overline{X}/(S^2 - \overline{X})$, where $\overline{X}$ and S^2 are the sample mean and the sample variance. The obtained values of the MM estimators (calculated for the previously generated samples relating to 100 policyholders): $\hat{a} = 0.25, \hat{b} = 0.35$ are very close to the true values of $a = 0.3$, $b = 1/3$. The realization of the approximated Bayesian CI for λ_{11}, computed by substituting the estimates $\hat{a}$, $\hat{b}$ for a, b into (4), (5), is $[1.13; 2.78]$.

It may be verified that the MM estimators $\hat{a}$, $\hat{b}$ are the consistent estimators of the parameters a, b, respectively.

II. Simulation of the sizes of losses (claims)

We simulated the sizes of claims by using the following scheme:

(i) We generated a 1-element sample from the *Inverse Gamma* $(2.5, 1/5000)$ distribution (i.e., we generated a value of the random variable $\beta \sim Inverse\ Gamma(2.5, 1/5000)$ (see (7))),

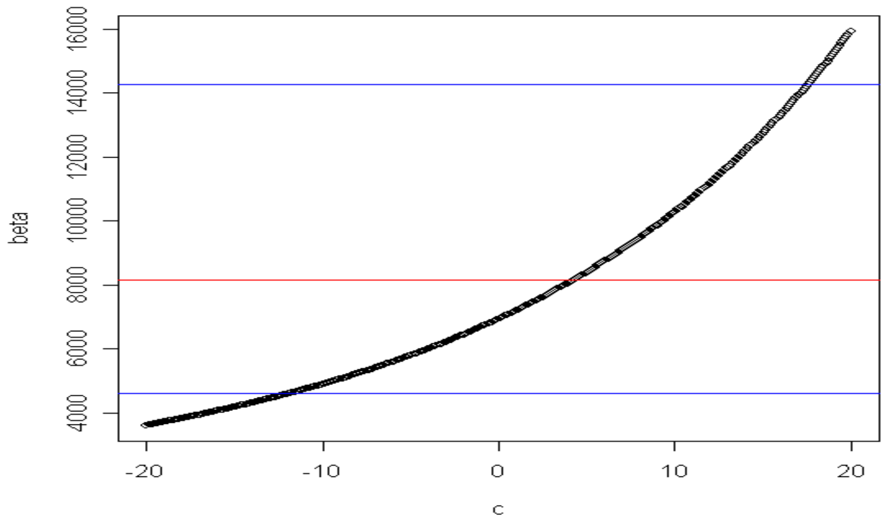
(ii) From the *Exponential(β)* distribution, where β was a value generated in (i), we generated a sample $y_1, y_2, ..., y_{10}$, of the sizes of losses in the period of 10 years.

Based on the first 10 observations $y_1, y_2, ..., y_{10}$, of the sample generated in (ii), we obtained the realization of the 95% CI for $\beta_{t+1} = \beta_{11}$ by substituting the values: $s = 2.5$, $m = 1/5000$, $t = 10$, $L = \sum_{i=1}^{10} y_i = 93608.96$ into (9), (10).

The received realization of the 95% CI for β_{11} – an average size of losses in the 11th year – was [4606.00; 14296.96]. Furthermore, we simulated the sizes of losses from the last 10 years and obtained the following values: 1351.418, 24872.665, 15063.568, 1083.870, 27508.688, 14729.373, 1839.331, 1332.476, 4024.451, 1803.115.

Figure 2 depicts the Bayesian estimates of β_{11}, obtained through an application of the Linex1 Loss function for c from -20 to 20, with the step 0.1. The outer horizontal lines correspond to the limits 4606.00, 14296.96, of the estimated 95% Bayesian CI for β_{11}. Furthermore, by the inner horizontal line the expected value 8139.91, of the posterior distribution $\beta | Y_1, ..., Y_{10}$, which is the $Inverse\ Gamma\,(2.5+10, 1/5000+93608.96)$ distribution, is presented (this expected value has been calculated by substituting the values: $m = 1/5000$, L = 93608.96, s = 2.5, t = 10 into (14)).

Figure 2. The Bayesian estimates of β_{11} (based on the application of the Linex1 Loss function), the estimated 95% Bayesian CI for β_{11} and the expectation of an appropriate posterior distribution



Source: own calculations

If the prior parameters s, m are unknown, we may estimate them by the method of moments (MM). We proceed as follows. By generating from the *Inverse Gamma*(2.5,1/5000) and the appropriate *Expotential(β)* distributions, we simulate 100 independent sizes of losses according to the steps (i), (ii) from the current subsection (with the difference that, we generate a sample of size 100 in (i) and 100 10–element samples in (ii)). Then, by using the formulas for the mean and the variance of the Pareto distribution and the mentioned method of moments, we have the following formulas for the MM estimators: $\hat{m} = (\hat{s}-1)\overline{Y}/\hat{s},\ \hat{s} = 1+\sqrt{1+c}$, where $c = \overline{Y}^2/S^2$. Based on these formulas and the generated sample, we obtain the following values of the MM estimators: $\hat{m} = 0.8 \cdot 10^{-4}$, $\hat{s} = 2.13$.

The realization of the approximated Bayesian CI for β_{11}, computed by substituting the estimates $\hat{m}$, $\hat{s}$ for m, s into (9), (10), was $[4716.05; 14873.42]$.

It may be verified that the MM estimators $\hat{m}$, $\hat{s}$ are the consistent estimators of the parameters m, s, respectively.

III. Some simulations of the net BMS premium and the related quantities

In this subsection, the values of the derived estimators are presented. The calculations have been carried out for the following parameter values: $a = 0.3, b = 1/3, t = 10, K = 19, L = 93608.96, s = 2.5, m = 1/5000$. The remarks "*a, b, s, m* - estimated" mean that the values *a, b, s, m* are replaced by the MM estimates: $\hat{a} = 0.25,\ \hat{b} = 0.35,\ \hat{s} = 2.13,\ \hat{m} = 0.8 \cdot 10^{-4}$.

Table 2. The simulated net BMS premiums and the related estimates

Estimators	Computed values	Numbers of the applied formulas
$\frac{\hat{\lambda}_{11}}{EX} \cdot 100\%$	208	(12), (15)
$\frac{\hat{\lambda}_{11}}{EX} \cdot 100\%$ (*a, b* - estimated)	260	(12), (15)
$\frac{\hat{\lambda}_{11(Linex)}}{EX} \cdot 100\%$	169, c=-5 284, c=5	(11), (16)
$\frac{\hat{\lambda}_{11(Linex)}}{EX} \cdot 100\%$ (*a, b* - estimated)	212, c=-5 356, c=5	(11), (16)
95% Bayesian CI (in %)	$\lambda_{11} \in [126;$ $310]$	(4), (5)
95% Bayesian CI (in %) (*a, b*- estimated)	$\lambda_{11} \in [157;$ $389]$	(4), (5)
$\hat{\lambda}_{11} \cdot \hat{\beta}_{11}$	15203	(12), (14), (17)
$\hat{\lambda}_{11} \cdot \hat{\beta}_{11}$ (*a, b, s ,m* - estimated)	15643	(12), (14), (17)
$\hat{\lambda}_{11(Linex)} \cdot \hat{\beta}_{11(Linex1)}$	8828, c=-5 21426, c=5	(11), (13), (18)
$\hat{\lambda}_{11(Linex)} \cdot \hat{\beta}_{11(Linex1)}$ (*a, b, s, m* - estimated)	8997, c=-5 22044, c=5	(11), (13), (18)

Source: own calculations

COROLLARY

In our paper, the Bayesian confidence intervals for the expected number of losses and the expected size of losses in the optimal Bonus-Malus Systems have been established. Although both of the parameters of the prior distribution, relating to the number and the size of loss, respectively, are unknown, they may be estimated by the method of moments. The realizations of the obtained confidence intervals have been compared with the Bayesian estimates of the corresponding parameters, obtained with the help of the Linex Loss function and its modification, called the Linex1 Loss function. The proposed constructions of the Bayesian confidence intervals can be easily generalized to the models with additional explanatory, deterministic variables. Apart from the constructions of the confidence intervals, the procedures leading to the simulations of the numbers and the sizes of losses are also presented. Furthermore, the formulas for the net premiums and the related quantities have been established and applied.

APPENDIX

Our objective here is to prove the formula in (11).

Let $\Delta = \hat{\theta} - \theta$. For the Linex Loss function $L(x) = e^{-cx} + cx - 1$, we search $\hat{\theta}$ minimizing the posterior risk $E_{post}L(\hat{\theta} - \theta)$, where the expected value is computed with respect to the posterior distribution $\theta|X_1, \dots, X_t$. Since the function L is convex, the given expected value attains its minimum if it is finite.

Thus, it is sufficient to find the solution of the equation $\frac{\partial E_{post} L(\Delta)}{\partial \hat{\theta}} = 0$. By taking the derivative of the integral, we obtain that $-cE_{post}e^{c\hat{\theta} - c\theta} + c = 0$. Therefore, $\hat{\theta} = \frac{1}{c} ln\left(E_{post}e^{c\theta}\right)$.

In the case when $X_1, \dots, X_t$ have the marginal Poisson distribution with the parameter λ, the prior distribution of λ is the $Gamma(a, b)$ distribution and the corresponding posterior distribution is the $Gamma(a + K, b + t)$ one, where $K = \sum_{i=1}^{t} X_i$. Hence, $E_{post}e^{c\lambda} = \int_0^\infty e^{c\lambda} \frac{\lambda^{a+K-1}(b+t)^{a+K}}{\Gamma(a+K)} e^{-(b+t)\lambda} d\lambda = \left(\frac{b+t}{b+t-c}\right)^{a+K}$.

Consequently, we obtain $\hat{\lambda}_{(Linex)} = \frac{1}{c} \ln\left(\frac{b+t}{b+t-c}\right)^{a+K}$, which is a desired result (11).

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