QUANTITATIVE METHODS IN ECONOMICS
Vol. XIV, No. 1, 2013, pp. 127 – 136

APPLICATION OF MIGRATION MATRICES TO RISK EVALUATION AND THEIR IMPACT ON PORTFOLIO VALUE

Urszula Grzybowska, Marek Karwański
Department of Informatics
Warsaw University of Life Sciences – SGGW
e-mail: urszula_grzybowska@sggw.pl, marek_karwanski@sggw.pl

Abstract: Migration matrices are widely used in risk management. In particular, quality of financial products due to credit risk is described by assigning them to one of several rating categories. The probability of future rating is determined by a migration matrix. Portfolio's value depends on the rating and on market states. To find an optimal portfolio one should consider migration matrices and the dynamics of market changes. The main goal of our research was to investigate the impact of both risks, market risk and credit risk on portfolio value. On a real portfolio we show that differences in migration matrices that result from the state of economy influence considerably credit risk and portfolio value.

Keywords: migration matrices, portfolio value, market risk, credit risk

INTRODUCTION

The New Basel Accords (Basel II/III) have tightened the rules of managing financial risk and introduced the possibility of using the IRB systems, i.e., internal rating systems to support estimation of economic capital and internal risk. The IRB approach requires estimation of individual PD (probability of default) for each financial position. Within the IRB approach migration matrices are used. They are estimated based on historic data. In credit risk estimation an obligor is assigned to one of several rating classes under the assumption that his future rating is determined by a transition matrix of a Markov chain. The probability that the obligor will migrate to the default state can be read off a migration matrix. The obligors with the same credit quality are assigned to the same risk group. Following S&P the highest and the best rating category is AAA. An obligation rated AAA is judged to be the best quality, with the smallest degree of investment risk. On the other edge of the scale is category D, which is assigned to an obligation where a default has already occurred [Rachev et al. 2009, Crouhy et al. 2001].

Rating related issues were the key factor of the financial crises of 2007. When in 2007 most of investigated bank assets were downgraded by rating agencies an avalanche of bankruptcies was triggered off because the change in credit rating causes a change in the price of rated assets or liabilities. The mechanisms of the economic breakdown were described in [Crouhy et al. 2008].

In the paper we show how the change in rating due to the change of economic state influences credit risk and the portfolio value. By risk we mean here the probability to default. In our research we consider migration matrices estimated for two states of economy and their impact on portfolio value and credit risk.

APPLICATION OF MIGRATION MATRICES TO RISK EVALUATION

Data description

We have based our research on one year migration matrices which were calculated based on quarterly migration matrices for two states of economy sourced from [Bangia et al. 2000]. We denote by E the matrix for the state of economic expansion (Table 1) and by R the annual matrix for the state of recession (Table 2). Comparing the entries of both matrices we can notice that the probabilities of migration to default (PDs) are higher for the matrix R for all states but two highest. On the other hand the entries on the diagonal of the matrix R, i.e., the probabilities that the obligor will stay in the same class, are smaller than the corresponding entries of E. There were no migrations between distant states during recession as positive probabilities for the matrix R are concentrated along the diagonal.

Table 1. Annual migration matrix for expansion (matrix E)

Rating before	Rating after							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,9297	0,0628	0,0057	0,001	0,0008	0,0001	0	0
AA	0,0057	0,9257	0,0609	0,0056	0,0006	0,0012	0,0003	0,0001
A	0,0008	0,0201	0,9265	0,0451	0,0049	0,0025	0,0001	0,0001
BBB	0,0004	0,0031	0,0548	0,8855	0,0448	0,0096	0,0009	0,0011
BB	0,0004	0,0013	0,0086	0,0689	0,8287	0,0793	0,0064	0,0064
B	0	0,0008	0,0030	0,0056	0,0599	0,8511	0,0406	0,0390
CCC	0,0016	0,0002	0,0065	0,0081	0,0177	0,1108	0,5837	0,2716
D	0	0	0	0	0	0	0	1

Source: own calculations based on ¼ year US migration matrix sourced from [Bangia et al. 2000]

Table 2. Annual migration matrix for contraction (matrix R)

Rating before	Rating after							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,9222	0,0653	0,0120	0,0005	0,0001	0	0	0
AA	0,0067	0,8828	0,1009	0,0059	0,0034	0,0001	0	0
A	0,0009	0,0319	0,8680	0,0930	0,0058	0,0002	0	0,0002
BBB	0,0015	0,0021	0,0400	0,8658	0,0818	0,0035	0,0006	0,0047
BB	0	0,0023	0,0031	0,0490	0,8170	0,0936	0,0155	0,0194
B	0	0,0022	0,0023	0,0045	0,0249	0,8173	0,0672	0,0816
CCC	0	0	0	0,0001	0,0004	0,0355	0,5382	0,4258
D	0	0	0	0	0	0	0	1

Source: own calculations based on ¼ year US migration matrix soured from [Bangia et al. 2000]

Application of migration matrices to PD estimation

Credit risk calculations are based on PDs. Risk measures are calculated for a given time horizon. In practice, various risk indicators which are based on migration matrices are calculated for one year ahead. On the other hand risk figures are applied to evaluation of different business measures, e.g., profitability, where one has to take into account migration forecasts for the whole duration of a loan or liability. For time homogeneous Markov chains, changes of transition matrix in given period can be easily calculated by multiplying transition matrices for intermediate periods. In particular, entries of the n-th power of an annual migration matrix denote probabilities of migration in n-th year.

For the purpose of forecasting we have calculated five successive powers of both annual matrices E and R. The values that are essential in risk analysis are shown in Tables 3 and 4. The matrices obtained have decreasing entries on the diagonal and the corresponding values are smaller for the recession than expansion. On the other hand the values in last columns of successive matrices increase. They represent probabilities of migration to default in successive years. The increase is especially significant for lower rating categories in time of recession. In particular, the probability of migration to default from the state CCC within five years is 90%. The high PD values for relatively short time are the reason why the lowest rating categories are described as speculative.

Table 3. Entries of successive powers of an annual migration matrix E

Rating class	Entries on the diagonal					PDs				
	n=1	n=2	n=3	n=4	n=5	n=1	n=2	n=3	n=4	n=5
AAA	0,9297	0,8647	0,8046	0,749	0,6975	0	0	0,0001	0,0002	0,0003
AA	0,9257	0,8585	0,7977	0,7426	0,6927	0,0001	0,0003	0,0007	0,0012	0,0018
A	0,9265	0,8622	0,8057	0,756	0,7122	0,0001	0,0004	0,0009	0,0018	0,0029
BBB	0,8855	0,7897	0,7093	0,6416	0,5842	0,0011	0,003	0,0057	0,0093	0,0136
BB	0,8287	0,6947	0,5894	0,5062	0,4399	0,0064	0,0166	0,0298	0,0451	0,0619
B	0,8511	0,7337	0,6391	0,5617	0,4974	0,039	0,0836	0,1288	0,1722	0,2126
CCC	0,5837	0,3453	0,2083	0,1291	0,0829	0,2716	0,4346	0,5348	0,5986	0,6410

Source: own calculations

Table 4. Entries of the successive powers of an annual migration matrix R

Rating class	Entries on the diagonal					PDs				
	n=1	n=2	n=3	n=4	n=5	n=1	n=2	n=3	n=4	n=5
AAA	0,9222	0,8509	0,7855	0,7255	0,6705	0	0	0	0,0001	0,0003
AA	0,8828	0,7830	0,6977	0,6246	0,5617	0	0,0001	0,0005	0,0011	0,0021
A	0,8680	0,7604	0,6722	0,5996	0,5395	0,0002	0,0009	0,0024	0,0046	0,0079
BBB	0,8658	0,7574	0,6691	0,5966	0,5366	0,0047	0,0109	0,0193	0,0300	0,0433
BB	0,8170	0,6739	0,5612	0,4721	0,401	0,0194	0,0497	0,0874	0,1294	0,1735
B	0,8173	0,6727	0,5569	0,4634	0,3873	0,0816	0,1774	0,2721	0,3591	0,4361
CCC	0,5382	0,2921	0,1604	0,0897	0,0515	0,4258	0,6579	0,7862	0,8586	0,9007

Source: own calculations

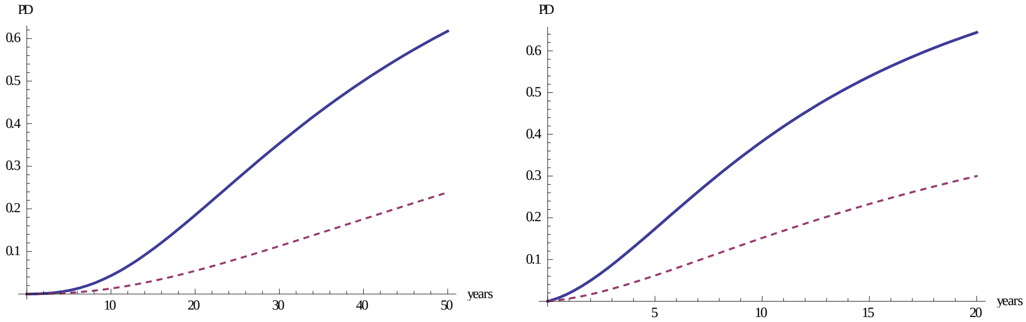
Using infinitesimal generator (or intensity matrix) Q and its eigenvalue decomposition we can make entries of the migration matrix time dependent. By D we denote a diagonal matrix of eigenvalues of Q and by V the corresponding matrix of its eigenvectors. Then for any $t > 0$

$$P(t) = \exp(tQ) = V \exp(tD) V^{-1},$$

where $\exp(tD) = diag(e^{\lambda_1 t}, .., e^{\lambda_n t})$ and λ_i are eigenvalues of matrix Q.

Functional representation of migration probabilities enables calculating PDs for an arbitrary $t > 0$. The graphs of functions representing probability of migration to default from the initial states A and BB are presented on Figure 1. For any $t > 0$ the probability of migration to default is much higher for the state of recession (thick line).

Figure 1. Probabilities of migration to default from BB (right) and A (left); (thick recession, dashed expansion)



Source: own calculations

For both matrices E and R we calculate the average number of years before entering the default state for each initial rating category. In order to obtain the desired result we use the notion of a fundamental matrix of an absorbing Markov chain. The values we have obtained are presented in Table 5. They indicate that in both cases the time before absorption for high initial rating categories is very long. On the other hand, for recession, the average time before absorption for initial class CCC is only 3 years and for initial class B only 12 years.

The results refer to relatively long time horizon that seem to be beyond the time used in practice. Banks however estimate risk for such a period that allows for possible changes in the portfolio structure. Products such as loans require several or a few dozen years.

Table 5. Average numbers of years before migration to the default state

States of economy	AAA	AA	A	BBB	BB	B	CCC
Recession	71	60	51	40	24	12	3
Expansion	162	150	138	120	91	59	27

Source: own calculations

APPLICATION OF MIGRATION MATRICES TO EVALUATION OF PORTFOLIO VALUE

The expected loss is usually measured based on the scoring models, i. e., econometric models of many dependent variables describing the client. There are a few models for calculating the percentile of loss. In the paper we focus on the models that involve migration matrices. One of the models is CreditMetrics™ [Bhatia et al. 1997]. The theoretical background of the model is built on the Merton's theory of firm evaluation. In that model credit quality of the firm can be measured comparing its liabilities and assets [Crouhy et al. 2001].

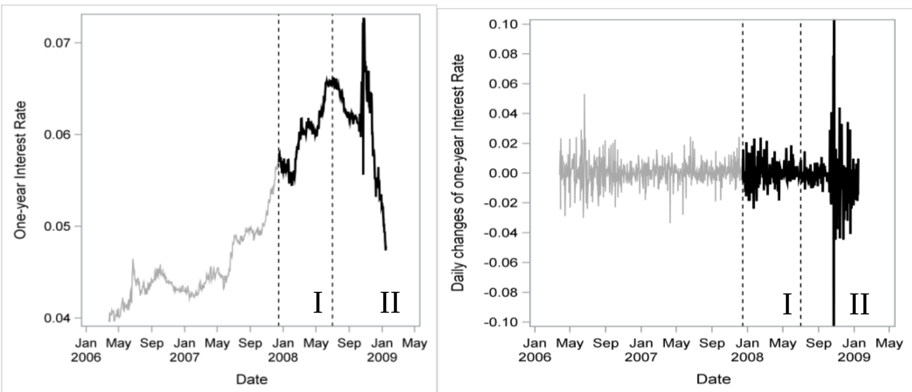
The CreditMetrics™ model goes beyond the approach suggested by Merton. In the model the assets volatility is consistent with the rating system. In other words the asset distribution parameters are modeled based on the migration matrix. In the CreditMetrics™ terminology the assets are called a portfolio. The pricing of bank product depends both on the rating category and the time structure of financial flows. The latter is expressed through the discount coefficients that make it possible to calculate financial flows in various moments. It is especially important when one wants to show differences in portfolio value and risk for different states of economy.

In our research we used two sub-portfolios of bonds sourced from one of Polish bank in year 2008. The first portfolio consisted of 138 corporate bonds issued by telecoms rated BB (according to the internal rating of the bank) and the second portfolio consisted of 164 corporate bonds issued by financial companies, all of them rated A. Default probabilities in both portfolios were higher than 0. In both portfolios there were coupon bonds indexed at fixed or variable interest rates - this has been included in the evaluation functions of the bonds. Duration of both portfolios was about 1.2 years.

Because the portfolios differ, for the purpose of comparison we used a scaling transformation so that the value of each portfolio was equal to 1 at the beginning of the observed period. The value of the portfolios was affected by various factors such as rating categories (A or BB), migration matrix (R or E) and forms of the yield curve.

In order to estimate portfolio value, computer simulations of influence of various factors on that portfolio were used. The results that we have obtained were then compared. The calculations were based on bond portfolio pricing with application of discount rate models estimated based on the market of the so called yield curves.

Figure 2. Annual interest rates and daily changes for expansion (I) and recession (II)



Source: own calculations

In our calculations we have used market data for two states of economy: expansion and contraction. The data used for calculations for the state of economic expansion were the migration matrix E (Table 1) for credit risk simulations and Polish market data covering the period 15.12.2007 – 1.06.2008 for modeling the yield curve. The data used for the state of recession were migration matrix R (Table 2) for credit risk simulations and Polish market data covering the period 15.10.2008 – 15.01.2009 for modeling the yield curve.

The models of interest rate scenarios

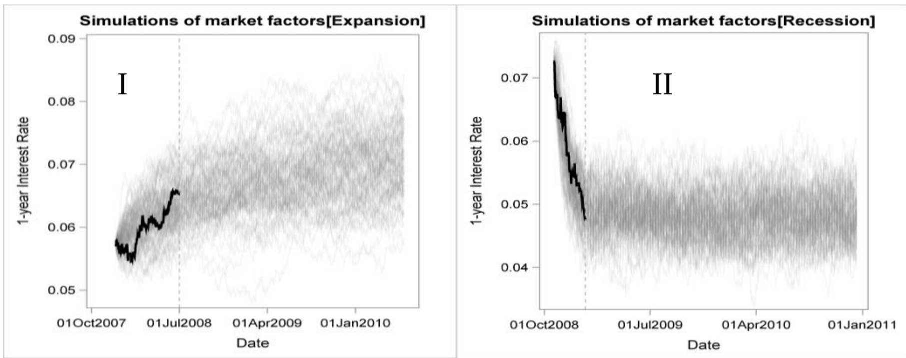
In order to investigate the behavior of portfolio we conducted prices scenarios of interest rates based on real market data. Models applied for generating interest rates were based on the theory of Brownian motion in continuous and discrete case. Brownian motion models satisfy the Stochastic Differential Equation (SDE):

$$dx = \mu dt + \sigma dz$$

where μ is the trend function and dz is Wiener process.

CIR (Cox-Ingersoll-Ross) is a popular model that belongs to the class of Brownian models [Bjoerk 2009]. We used CIR model for both building a generator and forecasts of market changes of rates of return which were then used in calculating the portfolio value.

Figure 3. Historical values and scenarios of interest rates



Source: own calculations

Bond pricing models

Market bond portfolio pricing P_T can be obtained using the equation:

$$P_T = E^M \left[exp \left(- \int_0^T R_t \, dt \right) X \right],$$

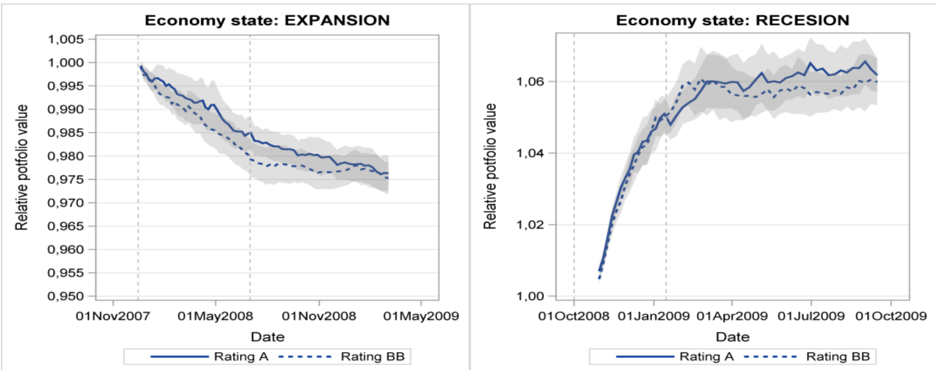
where X denotes the term structure of financial flows for bonds and other contingent debts, subject to default risk. E^M is the expected value for arbitrage free market, where all securities are priced based on a short term interest rate process r_t

on martingale measure M [Weron et al. 1998]. R_t is a yield curve for debt instruments obtained through replacing a process of short term interest rate r_t by a process corrected with respect to the risk of credit default: $R_t = r_t + Q_t L_{t.,}$ Q_t is an intensity function connected with the process of migration between rating categories and L_t is an expected rate of impairment. In the calculations Monte Carlo method was used based on generated scenarios of interest rates. The term structure of X was based on the portfolio description: dates of payments, coupon interest rates. The yield curve R_t was calculated based on a risk free yield curve obtained from treasury adjusted by spread. In order to calculate the spread the intensity matrices Q_E and Q_R obtained from migration matrices E and R were used. Because of the lack of data concerning the recovery rates after default, the rate was assumed to be 50%. It has to be mentioned, that the influence on credit risk of this factor has not been investigated in the paper.

Results

In the first step of our analysis we assumed that the credit spread for both rating classes is equal to 0. The portfolio pricing was performed for two periods reflecting two states of economy (expansion and recession). In figures below the risk parameters for both rating categories A and BB were changed in such a way that credit spread is 0.

Figure 4. Bond portfolio pricing for two states of economy and zero credit spread

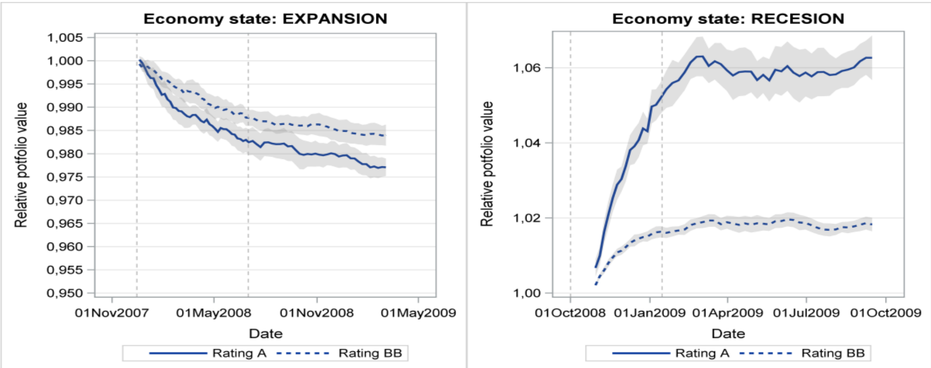


Source: own calculations

It can easily be seen that the portfolio value depends on the state of economy. During economic expansion the portfolio value decreases while during recession it increases. The behavior of the portfolio prices reflects the scenarios of interest rates (Figure 4). The portfolio values fall down during expansion because of the growth of interest rates. This decrease depends on the changes of interest rates (IR) with time. In our example the decline of IR by 1%-1.5% caused a 2% decline of the portfolio value. During recession IR declines about 2% and then it stabilizes. The portfolio value grows by 1% and then it remains constant. Due to the zero spread value the portfolio price does not depend on rating class.

In our further calculations we took into account credit risk captured by migration matrices and next we calculated the portfolio value.

Figure 5. Bond portfolio relative value for two states of economy and credit spread



Source: own calculations

In the case when credit risk is taken into account a visible change in dependence of portfolio value on the rating class can be noticed (Figure 5). One has to remember that economy state influences both IR and migration matrices. IR rates are the input for calculating the yield curve and spread in the pricing function.

During economic expansion the value of the portfolio with rating A rapidly decreases. The decrease is a response on the increase of IR during expansion. Credit risk causes an increase in price because of the so called risk premium, i.e., the price of the bond is modified by adding an additional component for financing a possible default. The difference between prices are due to differences in credit scoring for both portfolios. The difference of migration matrices shown in Table 5 and Figure 1 for ratings A and BB resulted in a 1.0% change of portfolio value within a 1 year horizon.

During recession the situation is reverse. The decrease of IR and the amendment with help of credit spread acts in the opposite direction. The difference between prices due to differences in credit scoring for both portfolios (A and BB) caused a 4.0% change of portfolio value within a 1 year horizon.

CONCLUSIONS

While examining changes in the portfolio value one must take into account not only market risk but also credit risk. In the paper we have shown the application of migration matrices to estimation of both risks: market and credit risk in a given time horizon for two states of economy. The probability of default in relatively short time (one year) is high for lower rating categories during recession.

We have shown the influence of market factors such as interest rates and credit rating on the value of real portfolio in a changing world of the whole economy. The results obtained indicate significant differences in the portfolio value with investing rating category A and speculative rating category BB. During recession the value of portfolio with rating BB is lower than that with rating A.

The value of the credit spread, which is a function of the credibility in credit risk has an important impact on the behaviour of the portfolio. Therefore, while comparing the quantitative models for financial instruments due to market risk it is very important to remember that the observed rates depend on the credit risk. Probability of default in case of rating A grows with the change of the economy state from 0.01% to 0.64%, while in case of rating BB from 0.02% to 1.94%. This entails a change in the portfolio value by 1% and 4%, respectively in one year. At the same time changes related to market risks are equal on average: 1.5% or 2.0% and 2% or 6%, respectively, depending on the rating class and the state of economy. It can be seen that influence of both types of risk on the portfolio value is comparable for rating A. The deterioration of portfolio rating however, brings about the domination of credit risk factors.

It might seem strange but during recession the changes of value of both portfolios is higher than during expansion. In the wake of this the business measure of risk, Value at Risk (VaR), is higher in time of recession. The higher risk is caused both by the form of a migration matrix and the change of interests rates – discount factors.

REFERENCES

Basel Committee on Banking Supervision (2001) The Internal Ratings-Based Approach. Consultative Document.

Basel II: Revised international capital framework. http://www.bis.org/publ/bcbsca.htm.

Besel III: http://www.bis.org/ bcbs/basel3.htm

Bangia A., Diebold F., Schuermann T. (2000) Rating Migration and the Business Cycle with Application to Credit Portfolio Stress Testing, The Wharton Financial Institution,http://www.ssc.upenn.edu/~diebold/index.html

Bhatia M., Gupton G., Finger C. (1997) CreditMetrics^{TM} -- Technical Document.

Bjoerk T. (2009) Arbitrage Theory in Continuous Time, Oxford University Press.

Crouhy M., Galai D., Mark R. (2001) Risk Management, McGraw-Hill, New York.

Crouhy M., Jarrow R., Turnbull S. (2008) The Subprime Credit Crisis of 07, http://www.fdic.gov/bank/analytical/cfr/bank_research_conference/annual_8th/Turnbull_Jarrow.pdf

Rachev S. T., Trueck S. (2009) Rating Based Modeling of Credit Risk Theory and Application of Migration Matrices, Academic Press.

Weron A., Weron R. (1998) Inżynieria finansowa, Wydawnictwo Naukowo Techniczne, Warszawa.