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PARAMETRIC AND NON-PARAMETRIC EFFICIENCY MEASUREMENT – THE COMPARISON OF RESULTS[1]

Sebastian Jarzębowski
Department of Economics and Organisation of Enterprises
Warsaw University of Life Sciences – SGGW
e-mail: sebastian_jarzebowski@sggw.pl

Abstract: In the paper the author considered estimation of efficiency, which measures the ability of the company to obtain the maximum output from given inputs. The comparison of results obtained by using two approaches: parametric (on the example of the SFA method, Stochastic Frontier Analysis) and non-parametric (on the example of the DEA method, Data Envelopment Analysis) has been carried out. In the paper the data from the companies of a key food processing sector in Poland, namely the meat processing sector, was used. The analysis covered the period 2006–2011, the sample covered from 195 up to 210 enterprises (depending on the analyzed year).

Keywords: efficiency, the SFA method, the DEA method, food processing sector

INTRODUCTION

The aim of the article was the comparison of the parametric (using the SFA method) and non-parametric (using the DEA method) approach to measurement of the basic economic category, which is efficiency. The discussed methods have been applied to the meat processing industry in Poland. This sector was chosen due to the large size of the sample, as well as the strategic importance and significant contribution to the production of the entire agri-food sector. Furthermore, with respect to the meat processing sector there are no comparative analyzes carried out, which justifies the need for their conduction.

[1] Research granted by National Science Centre from the funds for science in years 2011-2013 as a scientific project no 2011/01/B/HS4/02612 „Stopień integracji w łańcuchu dostaw a efektywność przedsiębiorstw przetwórstwa rolno-spożywczego"(„Degree of integration in supply chain and efficiency of food processing enterprises").

The discussion on assessing efficiency of economic entities should start with the precise definition of efficiency, which is not an unambiguous term. M. Bielski claims that there are several different concepts of efficiency, its measurement and expressions. He states that within the framework of the concept of efficiency, many terms of similar meaning may be applied: effectiveness, productivity, profitability [Bielski 2002, p. 54]. However, these concepts are not identical, and the actual concept of efficiency is derived from the structure of the production function, therefore, is conditioned by changes in the productivity of production factors and their remuneration and refers to the allocation of production factors in the most technically efficient way. W. Rembisz presented argument that the growth in efficiency is a function of changes in the productivity of capital and labour productivity and changes in the structure of expenditures (in production technology) [see Rembisz 2011]. Improvement in the efficiency level can lead to the increase in profitability. According to the author of the paper, these three concepts discussed, such as productivity, efficiency and profitability can be a reference point for assessing the degree of achievement of the objectives (*effectiveness*). A broader concept is effectiveness that focuses on the results and the degree of the objectives' achievement.

In the literature, there is likewise a concept of economic efficiency that determines the ratio of outputs achieved and inputs used. Z. Adamowski assumes that economic efficiency may be understood as the ratio of output to input, or cost, or vice versa – input to output. The first case concerns input-oriented efficiency, second one – output-oriented efficiency (capital intensity of output) [Adamowski 1983, p. 70]. The dual approach to efficiency is a result of the existence of two variants of the economic rationality principle. Adherence to this principle is understood as achieving given outputs by using minimal inputs or achieving maximal outputs by using a given level of inputs [Telep 2004, p. 9]. The aspect of efficiency is perceived similarly by C. Skowronek, who claims that maximization of the output/input ratio (or the difference between output and input), as a measure of economic efficiency, can be achieved by maximizing outputs with given inputs, or by minimizing inputs with given outputs [Skowronek 1987, p. 241].

The dual approach to efficiency is also presented by S.C. Krumbhakar and C.A.K. Lovell, according to whom an elementary objective of producers can be avoiding waste, by obtaining maximum outputs from given inputs or by minimizing inputs used in the production of given outputs [Krumbhakar and Lovell 2004, pp. 15], which is defined by the authors as technical efficiency. At a higher level, the objective of producers might entail the production of given outputs at minimum cost or the utilization of given inputs to maximize revenue, or the allocation of inputs and outputs to maximize profit. In these cases, productive efficiency corresponds to what the authors call economic (cost, revenue or profit) efficiency [Krumbhakar and Lovell 2004, p. 16]. They indicate that technical efficiency can be graphically defined in terms of distance to a production frontier, and economic efficiency is defined in terms of distance to a cost, revenue or profit frontier [Krumbhakar and Lovell 2004, p. 17]. Whereas technical efficiency is a purely physical notion that can be measured without recourse to price information, cost, revenue, and profit efficiency are economic concepts whose measurement requires price information.

T.J. Coelli, D.S.P. Rao, Ch.J. O`Donnell and G.E. Battese, that refer to the dual approach in their researches on the efficiency, argue that the efficiency ratio increases by maximizing outputs with given inputs (an output-oriented approach), or by minimizing inputs with given outputs (an input-oriented approach). A company uses materials, labour and capital (inputs) in order to manufacture the final product (output), on the basis of which the authors define efficiency of companies as their ability to transform inputs into outputs [Coelli et al. 2005].

COMPARISON OF SFA AND DEA METHODS

According to the mathematical models of general equilibrium of L. Walras, A. Wald, K.J. Arrow or G. Debreu, a company can be described as a mathematical function depending on the technology applied (without innovation) for transformation of inputs into outputs [Noga 2009, p. 134]. In the literature it is assumed that the production function illustrates available and effectively used manufacturing techniques. It determines the maximum quantity of product (production) (y) that is possible to be obtained by a given of production factor(s): (x). In this sense, the production function is a reflection of the production technique used, the technical relationship of a given state of technology. Related to this are: organization, knowledge and experience (which is accepted on the basis of the *implicite* principle) [Rembisz 2011, p. 10]. In the literature, it is emphasized that the production function is a defined mental structure expressed in an algebraic notation, in which the above-mentioned relationships are included, defining the nature of production in economic and technical terms[2]. The analytic form of the function reflects above all changes in production efficiency (in terms of the relationship between input and output as the factors' involvement and production increase). The production function enables to explain the reasons for changes in technical relationships and the consequent changes in the production efficiency and productivity of individual factors. These are important relationships, as somehow they exemplify the source of changes in production efficiency (related to the changes in manufacturing techniques). They reflect the structural changes.

[2] S.C. Krumbhakar and C.A.K. Lovell assume that producers use a nonnegative vector of inputs to produce a nonnegative vector of outputs. Although the analytical foundations developed by S.C. Krumbhakar and C.A.K. Lovell readily accommodate zero values for some inputs and some outputs; see: Krumbhakar S.C., Lovell C.A.K. (2004) Stochastic Frontier Analysis. Cambridge University Press, United Kingdom, Cambridge.

The method of efficiency measurement, basing on the production function, is the SFA method (Stochastic Frontier Approach). However, in the literature it is also common to use deterministic tools, where the analytical basis is an optimization problem (e.g. DEA method – Data Envelopment Analysis). Here benchmarks (the best objects in an analyzed group) are determined which are de facto the solution/solutions for the optimization problem. Apart from the undeniable advantage of the SFA method, which is using the analytical tool well-established in economic theory (i.e. production function), a number of other advantages may be presented[3].

In the literature, one can find an approach that in a case if there is a random component in the analyzed sample, the application of the SFA method outweighs the DEA method [Krumbhakar and Lovell 2004, p. 1][4]. Considering the random component as inefficiency, as in the DEA method, affects the location of efficiency frontier, and thus the final value of the efficiency ratio. Due to a number of specifics, the analyzed agri-food processing industry is characterized by a certain degree of randomness, which confirms the rightness of the SFA method's use. The application of this method allows conducting the statistical analysis of the significance of the obtained results [Krumbhakar and Lovell 2004, pp. 69].

Due to the infirmities of the deterministic methods (in the context of the validation of the obtained results), the efficiency measurement basing on integrated use of the SFA and DEA method was applied. According to this approach, the specification of the models in the DEA method was made based on the results of parameters' estimation of stochastic frontiers in the SFA method[5]. Hence, the problems associated with the verification of the correctness of variables' selection,

[3] More about the weaknesses of the SFA and DEA methods in the publication: Bezat A. (2009) Comparison of the deterministic and stochastic approaches for estimating technical efficiency on the example of non-parametric DEA and parametric SFA methods, [w:] Witkowska D. (ed.): Metody ilościowe w badaniach ekonomicznych, Vol. 10, Wyd. SGGW, Warszawa, s. 20-29.

[4] S.C. Krumbhakar and C.A.K. Lovell indicate that efficiency may be determined by using a deterministic production function or its stochastic counterpart. The authors state further that because the first model ignores the effect of random shocks, and the second one takes it into account, the preferred approach to the efficiency evaluation is the stochastic frontier; see: Krumbhakar S.C., Lovell C.A.K. (2004) Stochastic…, op. cit., pp. 65-66. This means that the stochastic model is less – in comparison to the deterministic model – vulnerable to the influence of outliers; see: Sellers-Rubio R., Más-Ruiz F.J. (2009) Technical efficiency in the retail food industry: the influence of inventory investment, wage levels and age of the firm, European Journal of Marketing, Vol., 43, No. 5/6, pp. 663.

[5] These conceptions were presented in detail in the monograph: Bezat A. (2012) Efficiency of Polish grain trade companies: an integrated application of SFA and DEA methods, Universität Bonn-ILB Press, Bonn.

orientation of models and economies of scale. The aim of the DEA method's use was to obtain the detailed results for individual companies [Jarzębowski 2011].

EVALUATION OF ENTERPRISES' EFFICIENY USING SFA AND DEA METHODS

The efficiency assessment was carried out on the basis of data collected from meat processing enterprises across Poland (a panel data for the period 2006–2011). The sample covers from 195 up to 210 companies, depending on the analyzed year. The production data is reported as revenue/expenditure denominated in PLN in constant prices. The production frontiers are fitted for a single output and two inputs. The inputs are: value of fixed assets (x_1), operating costs (x_2), and the output is net revenues from sales of goods and materials (y).

Selection of a functional form and specification of the SFA and DEA models

As a parametric approach, the SFA requires assuming a specific functional form that determines the input(s)-output relation a priori [Coelli et al. 2005]. The Cobb-Douglas function is the most commonly used. The adequacy of this function should be tested against a less restricted functional form, which is the trans-logarithmic function [Piesse and Thirtle 2000, p. 474]. Thus, the study involves two functional forms describing the input(s)-output relations, namely the Cobb-Douglas (equation 1) and trans-logarithmic model (equation 2). The tested frontier models take following form:

$$\ln y_i = \beta_0 + \sum_{j=1}^{k} \beta_j \ln x_{ij} + v_i - u_i \qquad (1)$$

and

$$\ln y_i = \beta_0 + \sum_{j=1}^{k} \beta_j \ln x_{ij} + \frac{1}{2} \sum_{j=1}^{k} \sum_{l=1}^{k} \beta_{jl} \ln x_j \ln x_l + v_i - u_i \qquad (2)$$

where:

i – index indicating objects $i=1,...,I$, where I is a number of objects in a sample,

j – index indicating inputs $j=1,...,l$,

y_i – output of an object i,

x_{ij} – input j of an object i,

β – vector of parameters to be estimated,

v_i – random variable representing the random error, so called statistical noise,

u_i – a positive random variable associated with technical efficiency (TE).

Comparison of the selected functional forms is carried out basing on the likelihood ratio test statistics (LR, Table 1). The LR statistics has the following form:

$$LR = -2[\ln L(R) - \ln L(N)] \quad (3)$$

where:

lnL(R) – logarithm of the maximum likelihood value in the restricted model,

lnL(N) – logarithm of the maximum likelihood value in the non-restricted model.

Table 1. Likelihood ratio statistics and model's selection verification

years	$\ln L(\hat{\theta}_R)$	$\ln L(\hat{\theta}_N)$	LR	result [1]	model
2006	-324,69	-322,25	4,88**	fail to reject of H_0	Cobb-Douglas
2007	-346,47	-344,33	4,28**	fail to reject of H_0	Cobb-Douglas
2008	-329,28	-326,27	6,00**	fail to reject of H_0	Cobb-Douglas
2009	-346,17	-341,15	10,04*	fail to reject of H_0	Cobb-Douglas
2010	-348,03	-342,38	11,30*	fail to reject of H_0	Cobb-Douglas
2011	-327,77	-322,37	10,80*	fail to reject of H_0	Cobb-Douglas

[1] The value of test statistic for $\chi^2(3)$ distribution amounts to 7,82 at the significance level of 0,05 (**) and 11,34 at the significance level of 0,1 (*)

Source: own calculations

The likelihood ratio tests lead to acceptance of the null hypothesis, saying that the Cobb-Douglas function (a model with restrictions on parameters) better describes the inputs-output relations (equation 1). Therefore, the empirical results obtained from estimating only the Cobb-Douglas function are reported in Table 2.

The output-oriented efficiency ratio – in the case of the stochastic frontier function – is measured as a relation between an observed output (value y, equation 1) and maximum output possible to be achieved in environment characterized by *exp(v_i)* (value *y**). Hence, the ratio may be written as:

$$TE_i = \frac{y_i}{y_i^*} = \frac{\exp(\beta_0 + \sum \beta_1 \ln x_{1i} + \sum \beta_2 \ln x_{2i} + v_i - u_i)}{\exp(\beta_0 + \sum \beta_1 \ln x_{1i} + \sum \beta_2 \ln x_{2i} + v_i)} = \exp(-u_i) \quad (4)$$

On the basis of equation (4) it can be stated that the value of the TE ratio varies from 0 to 1, where the unity indicates that this company is technically efficient. Otherwise $TE_i < 1$ provides a measure of the shortfall of observed output from maximum feasible output in an environment characterized by *exp(v_i)*, and indicates the inefficiency of this company.

The same sample (data set) and variables, as in case of the SFA method's application, was applied while estimating performance indicators using the DEA method. In this way, the problem of accidental or build upon the insights of experts in the selection of variables in the model has been eliminated. On the basis of the sum of parameters $\sum_{j=1}^{k}\beta_j$ it was stated that the sample of companies operating in the meat processing sector in years 2006-2011 was characterized by decreasing returns to scale, hence the NIRS[6] model was applied with use of the DEA method. The output-oriented models were used in this method. It was the result of the reference to the results obtained using the SFA method, in which the production function may – depending on the progress within the framework of production factors involved – move upwards, which means that by a given input level one can achieve increasing level of output (i.e. orientation on output).

Comparison of the results of efficiency evaluation in terms of the stochastic and deterministic approach

The obtained, by using the SFA method, efficiency ratios are compiled in distinction to size classes (Table 2).

Table 2. The average efficiency ratio calculated by using the SFA method by size of companies in years 2006-2011

Year/size of companies	2006	2007	2008	2009	2010	2011
micro	0,239	0,326	0,266	0,271	0,300	0,307
small	0,378	0,423	0,344	0,362	0,378	0,397
medium	0,493	0,483	0,404	0,494	0,499	0,488
large	0,507	0,483	0,480	0,559	0,564	0,592

Source: own calculations

On the basis of the results presented in Table 3, it can be noticed that the average efficiency ratio increases with an increase in the size of the analyzed companies. Due to the fact that within the framework of the SFA method the relative efficiency is determined there is no possibility of comparison of the results obtained in different models[7]. The micro companies achieved the efficiency ratio

[6] NIRS, Non-increasing Returns to Scale, involves decreasing or constant returns to scale (CRS, *Constant Returns to Scale*). Although there was no constant returns to scale noted in the analyzed sector, in order to keep the complexity of the conducted analyses the possibility of their presence was assumed. For more about the DEA model see [Jarzębowski S. 2011]

[7] Within the framework of the SFA method the creation of a dynamic model for the balanced panel data is one of the approach of possibility to evaluate the changes in

ranging from 0,24 to 0,33; the average ratio for the small companies ranged from 0,34 to 0,42; the ratio for the medium companies ranged from 0,4 to 0,5; the lowest ratio for large companies was equal to 0,48 and the highest to 0,59.
The efficiency ratios obtained by using the DEA method are compiled in Table 3.

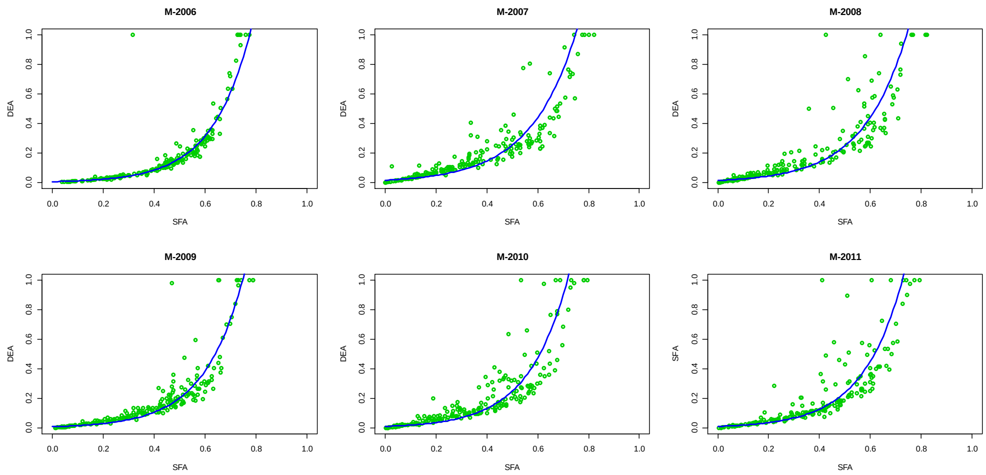
Table 3. The average efficiency ratio calculated by using the DEA method by size of companies in years 2006-2011

Year/size of companies	2006	2007	2008	2009	2010	2011
micro	0,164	0,207	0,185	0,174	0,181	0,185
small	0,182	0,175	0,187	0,171	0,174	0,168
medium	0,226	0,283	0,254	0,223	0,285	0,257
large	0,316	0,226	0,502	0,459	0,428	0,548

Source: own calculations

The efficiency ratios obtained for the stochastic (using the SFA method) and deterministic model (using the DEA method) are compiled according to each year in a form of correlation charts (Figure 1)

Figure 1. The relation between efficiency ratios determined using the SFA and DEA method for companies of the meat processing sector in years 2006-2011



Source: own calculations

efficiency over years; see Bezat A. (2011) Estimation of technical efficiency by application of the SFA method for panel data, Scientific Journal Warsaw University of Life Sciences – SGGW, Problems of World Agriculture 2011, Vol. 11, No. 3, p. 5-13.

The results obtained by using the SFA and DEA methods have been evaluated. On the basis of the correlation graphs it can be stated that the relation between the analyzed variables is best described by the exponential function. The matching of the functional form was based on the value of the coefficient of determination. The determination coefficients for the meat processing sector took values ranging from 0,77 to 0,94.

SUMMARY

A company uses inputs in order to manufacture the output, thus the author defines efficiency of companies as their ability to transform inputs into outputs. In the literature, it is assumed that the production frontier illustrates available and effectively used manufacturing techniques, since the function determines the maximum size of production (*Y*) to be achieved by a given level of production factor(s) (*X*). Thus, the production function is a reflection of the state of technology, including applied technique, organization, knowledge and experience. The production function is defined as the base function for analysing production process, and it was always considered as a kind of the foundation of theoretical analyses in the neoclassical economics.

The SFA method (*Stochastic Frontier Analysis*) is a method of efficiency evaluation. Nevertheless, the deterministic tools are used in the literature as well. Their analytical background is not the production function but the optimization problem (e.g. the DEA method, *Data Envelopment Analysis*). The both methods require all decision making units to have comparable inputs and outputs and both can handle multiple input and multiple output models.

The SFA and DEA methods were applied to evaluate the efficiency of companies of the meat processing sector. The similar results were obtained in case of both methods. On the basis of the conducted analysis it was stated that the results obtained using the DEA method (after the models' specification basing on the SFA method's results) and the results obtained using the SFA method indicate the exponential dependence for the analyzed period.

Basing on the conducted analysis it was claimed that the use of the SFA and the DEA methods integrally - combining advantages of both methods – allows preserving the analogy when comparing the results and formulating reliable conclusions.

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