

HISTORICAL AND IMPLIED VOLATILITIES: A REVIEW OF METHODOLOGY

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Abstract: Volatility is a subject of numerous studies. Many of them focus on predictive power of different sources of volatility. Most often, the Black-Scholes implied volatility is believed to outperform historical volatility, although some research demonstrates that implied volatility is a biased forecast of future volatility. Taken into account different opinions, the paper aims at presenting alternative methods for estimating volatility.

Keywords: historical volatility, Black-Scholes implied volatility, model-free implied volatility

INTRODUCTION

In 1973 Black and Scholes developed an option pricing model that depends upon five variables: stock price, strike price, time to maturity, risk-free rate, and the standard deviation of returns from the underlying stock – volatility. Of the five variables that are necessary to specify the model, all are directly observable except the last one (the risk-free rate of interest may be closely approximated by the rate of return on short term government securities, however Beckers [1981] states the model is not very sensitive to an exact specification of the risk-free rate). Thus, the most important is to estimate the standard deviation of the stock's rate of return over remaining life of the option.

There are two basic ways to assess the volatility: the first one uses historical data on underlying asset prices, and the second technique uses option prices to find the option market's estimate of the stock's standard deviation. This estimate of the stock's standard deviation drawn from the options market is called an implied volatility [Kolb, Overdahl 2007]. When evaluating volatility using historical data, there is no general rule how far back in the history the data should be used to

estimate the parameter. According to Hull [2012] more data generally lead to more accuracy. On the contrary, too old data may not be relevant for predicting the future volatility. He suggests to use closing prices from daily data over the most recent 90 to 180 days.

There are also more sophisticated approaches to estimating historical volatility involving exponential weighted average or GARCH model. In practice, traders usually work with implied volatilities. They are used to monitor the market's opinion about the volatility of a particular stock. Whereas historical volatilities are referred to as backward looking, implied volatilities are referred to as forward looking. The implied volatility can be interpreted as the average volatility that the underlying asset will have from now to the option's expiration time, or it can be used to forecast the change of underlying asset price in a short term [Zahng 2006].

The volatility implied from option prices is widely believed to be informationally superior to the historical volatility of the underlying asset. Musiela and Rutkowski [2007] present studies, both confirming and negating the superiority of implied volatility over the historical one. They quote Latané and Rendleman [1976], Schmalensee and Trippi [1978], Beckers [1981] who found that estimates of the actual volatility based on market implied volatilities outperform, at least in terms of their predictive power, more straightforward estimates based on historical data. Contrary to these findings, subsequent studies of stock index options, reported in Canina and Figlewski [1992], Day and Lewis [1992], and Lamoureux and Lastrapes [1993] suggest that the implied volatility has virtually no correlation with future volatility. Moreover, Jiang and Tian [2005] note in their paper that nearly all research on the information content of implied volatility are focused on implied volatility derived from at-the-money options. By concentrating on at-the-money options alone, these studies fail to incorporate the information contained in other options. Taking into account all those pros and cons, the paper aims at presenting alternative methods for estimating, both historical and implied volatilities.

HISTORICAL VOLATILITY

To estimate volatility using historical data, several techniques could be used. In his book Haug [2007] presents historical volatility from close prices, high-low volatility, high-low-close volatility, and exponential weighted historical volatility.

Historical volatility from close prices

The most widely used method for estimating historical volatility is calculation of the annualized standard deviation given by the formula (1):

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n \ln\left(\frac{Close_i}{Close_{i-1}}\right)^2 - \frac{1}{n(n-1)} \left[\sum_{i=1}^n \ln\left(\frac{Close_i}{Close_{i-1}}\right) \right]^2}, \quad (1)$$

where n is the number of observations. When assuming 252 trading days in a year, the annualized close volatility is obtained by multiplying σ from formula (1) with the square root of 252.

High-Low volatility

Parkinson [1980] suggests estimating the standard deviation by:

$$\sigma = \frac{1}{2n\sqrt{\ln(2)}} \sum_{i=1}^n \ln\left(\frac{High_i}{Low_i}\right). \quad (2)$$

The result should be also multiplied with the square root of 252. The high-low method is statistically much more efficient than the standard close method (in terms of number of observations needed to get the same interval compared with the standard close method). However, it assumes continuous trading and observations of high and low prices. Thus the method can underestimate the true volatility. The same is true for the high-low-close volatility [Haug 2007].

High-Low-Close volatility

Garman and Klass (1980) propose using a volatility estimator of the following form:

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n \frac{1}{2} \left[\ln\left(\frac{High_i}{Low_i}\right) \right]^2 - \frac{1}{n} \sum_{i=1}^n [2\ln(2) - 1] \left[\ln\left(\frac{Close_i}{Close_{i-1}}\right) \right]^2}. \quad (3)$$

The annualized high-low-close volatility is obviously got by multiplying the result with the square root of 252.

Exponential weighted historical volatility

Exponential weighted volatility often referred to as exponentially weighted moving average (EWMA) puts more weight on more recent observations. It can be calculated as:

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) \left[\ln(S_t / S_{t-1}) \right]^2, \quad (4)$$

where σ_t is the current volatility and σ_{t-1} is the volatility as calculated one observation ago (see also Hull [2012]). If one uses daily data, the volatility is annualized by multiplying the result with the square root of the number of trading days per year. In most markets λ should be between 0.75 and 0.98. The RiskMetrics developed by J.P. Morgan uses an EWMA with $\lambda=0.94$ [Haug 2007].

GARCH-based volatility

A more sophisticated approach to estimating volatility is based on the GARCH (1,1) model. Since 1982 when Robert Engle first introduced the ARCH (autoregressive conditional heteroscedasticity) model, whose generalized version - GARCH was proposed by Bollerslev in 1986, a number of their extensions have arisen. Nevertheless, the simplest and the most widely used is the GARCH (1,1) model. It may be useful in predicting future volatility [Rouah, Veinberg 2007]:

$$\sigma_{t+1}^2 = \omega + \alpha r_t^2 + \beta \sigma_t^2, \quad (5)$$

where $\alpha + \beta$ measures the volatility persistence, and $\alpha + \beta < 1$ is required for the variance to be mean-reverting. Moreover, the closer the value of $\alpha + \beta$ to one, the more volatility will persist, and the closer $\alpha + \beta$ is to zero, the faster volatility will revert to the long run variance. In the model (5) the forecast variance for time $t+1$, σ_{t+1}^2 , is a weighted average of the squared return at time t , r_t^2 , and the time- t estimate of the variance, σ_t^2 (the GARCH(1,1) model puts more weight on the most recent squared return). The long-run variance in the GARCH (1,1) model (5) is:

$$\sigma^2 = \frac{\omega}{1 - \alpha - \beta}. \quad (6)$$

To annualize the volatility, one must multiply it with the square root of 252.

Although the methods presented in this section are often easy to implement, Rouah and Veinberg [2007] note they constitute a retrospective estimate of asset volatility, since they are based on historical prices. Hence, many authors advocate the use of implied volatility as it reflects future expectations about volatility rather than reflecting past realization.

BLACK-SCHOLES IMPLIED VOLATILITY

The most popular type of implied volatilities are Black-Scholes implied volatilities. They are obtained by equating an observed market price with a given strike price and maturity to the Black-Scholes formula with the same strike price and maturity. The value of volatility in the Black-Scholes formula that yields the observed option price is the implied volatility. To find implied volatilities, one begins with established values for the stock price (S), the exercise price (X), the interest rate (r), the time until expiration (T), and the option price (C). Although it is impossible to solve the Black-Scholes equation directly for the standard deviation, one can use numerical search to closely approximate the standard deviation by any given option price. To do this, some iterative methods are applied. Chriss [1997] describes two of them: the method of bisections, and the Newton-Raphson method.

The method of bisections

The method of bisections works as follows:

1. Step 1. Choose a first guess for the implied volatility that must be greater than the actual implied volatility, and write σ_0 for this guess (if the Black-Scholes value computed using σ_0 is greater than the market price of option, σ_0 is greater than implied volatility).
2. Step 2. This step produces the next implied volatility guess. As we have already ensured that σ_0 is greater than the actual implied volatility, we need to make our next volatility guess lower. We set σ_1 to σ_0 reduced by 50% ($\sigma_1 = \sigma_0 - \sigma_0/2 = 0,50\sigma_0$). This is the first “real” guess – σ_1 .
3. Step 3. In this step, we produce the next guess σ_2 . First, we need to compute the Black-Scholes value with given input parameters $C(S, X, T, \sigma_1, r)$. If it is larger than the actual option price, then we set the next guess by reducing σ_1 by half as much as we did last time ($\sigma_2 = \sigma_1 - \sigma_0/4$). If $C(S, X, T, \sigma_1, r)$ is smaller than the actual option price, we increase σ_1 by $\sigma_0/4$ ($\sigma_2 = \sigma_1 + \sigma_0/4$).
4. Step 4. Iterate the process: compute σ_k from σ_{k-1} , compute the Black-Scholes value using σ_{k-1} , the volatility guess from the previous step. If it is larger than the market price, form the next guess by reducing σ_{k-1} by $\sigma_0/2^k$ ($\sigma_k = \sigma_{k-1} - \sigma_0/2^k$). Otherwise, rise σ_{k-1} by $\sigma_0/2^k$ ($\sigma_k = \sigma_{k-1} + \sigma_0/2^k$).

The most obvious place to stop is when a volatility guess produces a Black-Scholes price exactly equal to the market price. We can also have a preset “error tolerance”. The only drawback of the method is its speed. It converges rather slowly. A faster technique is the Newton-Raphson method.

The Newton-Raphson method

The Newton-Raphson method works in a following way:

1. Step 1. The first step is to guess what the correct implied volatility is and call this guess σ_1 . Haug [2007] presents an efficient seed value when the Newton-Raphson method is used to compute the implied volatility. The seed value developed by Manaster and Koehler in 1982 will guarantee convergence (if the implied volatility exists) for European Black-Scholes stock options. The seed value is: $\sigma_1 = \sqrt{|\ln(S/X) + rT| \cdot 2/t}$. Next we compute the Black-Scholes

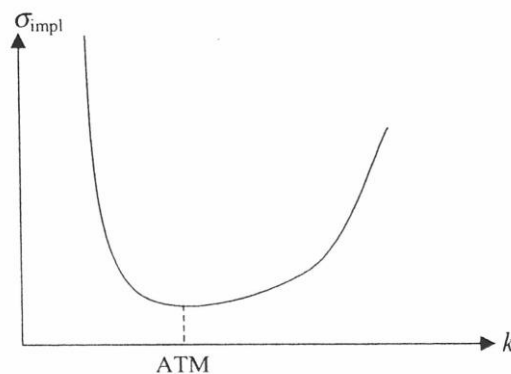
value of the option with σ_1 , denoted $C(\sigma_1)$, and the vega of this option, denoted $V(\sigma_1)$.

2. Step 2. We compute the value of our next volatility guess: $\sigma_2 = \sigma_1 - (C(\sigma_1) - C)/V(\sigma_1)$, where C is the market value of the option.
3. Step 3. The n^{th} volatility guess is given by: $\sigma_n = \sigma_{n-1} - (C(\sigma_{n-1}) - C)/V(\sigma_{n-1})$.

The Newton-Raphson method requires knowledge of the partial derivative of the option pricing formula with respect to volatility (vega) when searching for the implied volatility. For some options (exotic and American options in particular), vega is not known analytically and Haug [2007] suggests using the bisection method to estimate implied volatility when vega is unknown.

Normally, implied volatilities are larger for options with shorter time to maturity. This is the term structure of volatility that describes how implied volatility varies with time until expiration. Another important feature of Black-Scholes implied volatilities is that they resemble a smile or smirk when plotted against the strike or moneyness¹. Figure 1 presents an example of volatility smile. It shows that, all else equal, at-the-money options have a lower implied standard deviation than options that are in- or out-of-the money (the pattern can differ across option markets and across related options that differ only in their expiration dates).

Figure 1. Volatility smile



Source: Geman [2007], p. 101

As the implied volatility of a European call option is the same as that of European put option when they have the same strike price and time to maturity²,

¹ Plotting implied volatility as a function of moneyness and maturity produces a three-dimensional graph called an implied volatility surface.

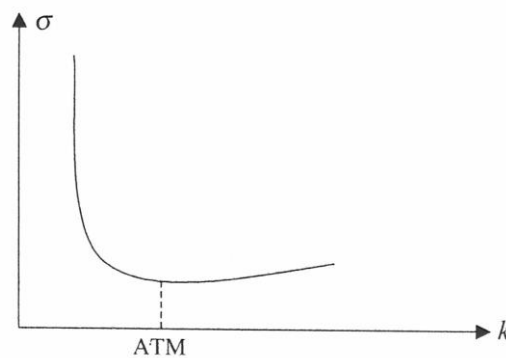
² Zahng [2006] presents examples showing that implied volatilities can be different for put and call options even with the same strike price and time to maturity. The difference may

this means that the volatility smile for European calls with a certain maturity is the same as that for European puts with the same maturity [Hull 2012]. In classical practice, call options are used to build the right part of the smile (i.e. out-of-the money calls), and out-of-the money put options are used for the left part. According to Rouah and Veinberg [2007] one explanation of a volatility smile is that the true distribution of asset prices has fatter tails than the distribution assumed in option pricing models. Smiles can occur, because returns show greater kurtosis than stipulated under normality, so that extreme returns are more likely. This implies that deep-in-the money options and deep-out-of-the money options are more expensive relative to the Black-Scholes price.

Kolb, Overdahl [2007] note that sometimes volatility smiles are an artifact of the exchange settlement procedure. For most listed options, at- or near-the-money options are the most liquid and quotations for this options are representative of market opinion. For deep in- or out-of-the money options, trades take place less frequently. For these illiquid options with stale prices, exchange settlement committees may set the price for clearing purposes only. This means that implied standard deviations obtained from these prices are not reflecting the market consensus.

In some markets, the volatility pattern resembles a smirk. Figure 2 presents an example of a volatility smirk with no symmetry between upward and downward movements.

Figure 2. Volatility smirk



Source: Geman [2007], p. 102

Rouah and Veinberg [2007] explain that smirks can occur because returns often show negative skewness, which of course the normal distribution does not

imply the imperfection of the actual market which violates the assumptions of the Black-Scholes model. The imperfect factors may include taxation, transaction costs, liquidity, and others.

allow. This implies that large negative returns are more likely, leading to implied volatilities for in-the-money calls that are higher than implied volatilities for out of the money calls. Similarly, implied volatilities for out-of-the money puts are higher than implied volatilities for in-the-money puts. They note that smiles and smirks are more pronounced for short-term options, and less pronounced for long-term options, which is synonymous with long-term returns being closer to normally distributed than short term returns.

At the market, there are several options on the same underlying with different strike prices and expirations traded at once. Each of them might have a different implied volatility. How to obtain a “collective assessment of volatility”, then? Trippi [1977] calculated an arithmetic average, although Latané and Rendleman had previously labeled the use of such an arithmetic average as “unreasonable”. In their paper [1976], they employed a weighted average of implied standard deviations as a measure of market forecasts of return variability. Their weighting system was:

$$WISD_{it} = \left[\sum_{j=1}^N ISD_{ijt}^2 \cdot d_{ijt}^2 \right]^{0.5} \cdot \left[\sum_{j=1}^N d_{ijt} \right]^{-1}, \quad (7)$$

where $WISD_{it}$ = weighted average implied standard deviation for company i in period t , ISD_{ijt} = implied standard deviation for option j of company i in period t , n denotes the number of options analyzed for company i and is always greater than or equal to 2, d_{ijt} = partial derivation of the price of option j of company i in period t with respect to its implied standard deviation using the Black-Scholes model.

Chiras and Manaster [1978] notice that Latané and Rendleman’s weighted average is not truly a weighted average since the sum of the weights is less than one. Therefore, the weighted average implied standard deviation ($WISD$) for Latané and Rendleman is biased towards zero. Furthermore, the bias increases with an increase in the sample size even if every option was observed to have the same ISD . They relate Latané and Rendleman intended to weight the $ISDs$ by the partial derivatives of the Black-Scholes model with respect to each implied standard deviation. That is equivalent to weighting $ISDs$ according to the sensitivity of the dollar price change for the options relative to the incremental change in the implied standard deviations. A rational investor measures returns as the ratio of the dollar price change to the size of the investment, but Chiras and Manaster point out that reasoning of Latané and Rendleman emphasizes the total dollar return without regard to the size of the investment (a one-dollar price change on a one-dollar stock is considered equivalent to the same price change on a fifty-dollar stock). They give an opinion that in order to be consistent with a rational measure of returns, the price elasticity of options with respect to their $ISDs$ must be considered. One must be concerned with the percentage change in the price of an option with respect to the percentage change in its ISD . To obtain the weighted implied standard

deviation of the options on one stock for each observation date, they use the following equation:

$$WISD = \frac{\sum_{j=1}^N ISD_j \frac{\partial W_j}{\partial v_j} \frac{v_j}{W_j}}{\sum_{j=1}^N \frac{\partial W_j}{\partial v_j} \frac{v_j}{W_j}}, \quad (8)$$

where N = the number of options recorded on a particular stock for the observation date, $WISD$ = the weighted implied standard deviation for a particular stock on the observation date, ISD_j = the implied standard deviation of option j for the stock,

$\frac{\partial W_j}{\partial v_j} \frac{v_j}{W_j}$ = the price elasticity of option j with respect to its implied standard deviation (v).

In an efficient market prices will fully reflect all available information. Therefore, estimated variances calculated from option prices should reflect not only the informational content of stock price history but also any other available information. Thus one may suspect that the $WISD$ values reflect future standard deviations more accurately than do the historic sample standard deviations [Chiras, Manaster 1978].

Beckers [1981] used an alternative weighting scheme that concentrates mainly on the implied standard deviations ($ISDs$) for at-the-money options. Specifically, on any single observation day the following loss function was minimized:

$$f(ISD) = \sum_{i=1}^I w_i [C_i - BS_i(ISD)]^2 / \sum_{i=1}^I w_i, \quad (9)$$

where C_i = market price of option i , BS_i = Black-Scholes option price as a function of the ISD , I = total number of options on a given stock with the same maturity, w_i = weight for the i -th option = $\delta BS_i(ISD) / \delta ISD$ (i.e., the first derivative of the Black-Scholes option pricing formula with respect to the standard deviation).

This procedure comes down to minimizing the weighted sum of the squared deviations between market value and the corresponding Black-Scholes price. The actual weights used in the procedure are proportional to the squared values of the Latané-Rendleman's weights. This method therefore tends to put more weight on the options that are highly sensitive to an exact specification of the standard deviation. Beckers believes his measure tends to outperform Latané-Rendleman's $WISD$. In contrary to the Latané-Rendleman's study suggesting that the best predictive performance could be obtained by using the information available in all options, he concludes that most of relevant information is reflected in the price of at-the-money options. However, he admits it is not clear whether this

result is due solely to the fact that the other options are not as sensitive to an exact specification of the underlying variance. Their prices could also have a higher tendency toward distortion. Systematic biases in the *ISD*'s due to the fact that the Black-Scholes model does not hold exactly for in-the-money or out-of-the money options could also influence this result [Beckers 1981].

The smile-shaped pattern, which constantly appears in volatilities extracted from a wide variety of options, has provided evidence against the constant volatility assumption inherent in the Black-Scholes model (if returns were normal, then implied volatility would be constant across moneyness and maturity). As a result, estimation procedures that use the Black-Scholes model to estimate implied volatilities may produce biased estimates. Moreover, using in forecasting experiments at-the-money implied volatility only, discards all potential information contained in the rest of option prices, especially that in practice options rarely trade exactly at-the-money. Britten-Jones and Neuberger [2000] derived a model-free implied volatility measure that incorporates the whole cross-section of option prices, not only at-the-money prices.

MODEL-FREE IMPLIED VOLATILITY

Under the assumptions that the underlying asset does not make dividend payments and the risk-free rate is zero, Britten-Jones and Neuberger [2000] derive the risk-neutral expected sum of squared returns between two dates (T_1, T_2) as:

$$E^Q \left[\int_{T_1}^{T_2} \left(\frac{dS_t}{S_t} \right)^2 \right] = 2 \int_0^\infty \frac{C(T_2, X) - C(T_1, X)}{X^2} dX, \quad (10)$$

where $E^Q[\]$ refers to expectation under the risk-neutral measure Q , $C(T, X)$ is an observed call price with maturity T and strike price X , and S_t is the asset price at time t . The asset return variance $(dS_t/S_t)^2$, which is also the squared volatility, is a function only of observed call prices at one point in time. No model for the underlying asset price is required in the derivation of (10), hence it is a "model-free" measure of variance. It only requires two cross sections of call prices with varying X , one with time to maturity T_1 and the other with time to maturity T_2 . Since (10) is the model-free implied variance, the model free implied volatility is obtained as its square root [Rouah, Veinberg 2007].

As Britten-Jones and Neuberger derived the model-free implied volatility under diffusion assumptions, it was unclear whether it was still valid when the underlying asset price process included jumps. This could be a serious limitation since random jumps are an important aspect of the price dynamics of many assets. Jiang and Tian [2005] extend the model and demonstrate that it is still valid even if the underlying asset price process has jumps. They also show how to relax original assumptions of no dividends and a zero risk-free rate. Results of their empirical

tests conducted using the S&P 500 index options traded on the Chicago Board of Trade (CBOE) ensure the generality of the model-free implied volatility.

The volatility index VIX published by the CBOE constitutes one important application of model-free volatility. The index was originally defined in terms of Black-Scholes implied volatilities calculated from at-the-money options on the S&P 100 index. The revision of the VIX uses options on the S&P 500 index, on a wide range of moneyness, not only at-the-money. It also uses model-free implied volatility rather than the Black-Scholes implied volatility [Rouah, Veinberg 2007].

CONCLUDING REMARKS

The volatility of an asset is a measure of our uncertainty about the returns provided by the asset. There has been extensive research regarding the prediction of future volatility. In particular, researchers have examined what sources of information are the best predictors of volatility. According to Chriss [1997], some obvious candidates are: historical volatility, implied volatility, some combination of the first two. The aim of the paper was presenting those alternative approaches to estimate volatility as in the literature there are different studies supporting different opinions on practical usefulness of separate methods.

Most often, the Black-Scholes implied volatility is believed to be superior to the historical volatility of the underlying asset, since it is derived from options prices that reflect market participants' expectations of future movements of the underlying asset. Even though early studies found that implied volatility was a biased forecast of future volatility and contained little incremental information beyond historical volatility, more recent studies present evidence that implied volatility is a more efficient forecast for future volatility than historical volatility. Research on the information content of implied volatility usually focuses on the Black-Scholes implied volatility from at-the-money options. Being more actively traded than other options, they can be a good starting point. However, by concentrating on at-the-money options alone, one omits the information embedded in other options.

An important departure from previous research is the model-free implied volatility derived in 2000 by Britten-Jones and Neuberger. Their model is not based on any specific option pricing model. It is derived entirely from no-arbitrage conditions and utilizes the whole cross-section of option prices. Although Britten-Jones and Neuberger had derived the model-free implied volatility under diffusion assumptions, Jiang and Tian [2005] extended their model to asset price processes with jumps and developed a simple method for implementing it using observed option prices. Their results obtained from the S&P 500 index options suggest that the model free implied volatility subsumes all information content in the Black-Scholes implied volatility and past realized volatility, and is a more efficient forecast for future realized volatility. As they write, their findings also provide theoretical and empirical support for the CBOE decision to modify its VIX index.

Now, it is based on the model-free implied volatility instead of the Black-Scholes volatility of at-the-money options.

In Poland, one of the first to study the subject was Piontek, who in 1999 published his paper presenting historical and the Black-Scholes implied volatilities obtained from currency options on PLN/USD exchange rate. He tried to assess the predictive power of historical and implied volatilities and concluded they both failed to forecast future volatility of the exchange rate as option market in Poland was small and illiquid at that time. Krawiec and Krawiec [2002] analyzed volatilities implied in commodity options traded at Poznań Exchange³ and at the Warsaw Commodity Exchange. In their opinion, implied volatilities derived from the options under consideration could not be a reliable source of information on realized volatilities. The Warsaw Stock Exchange, that introduced options on WIG20 index in September 2003, does not publish any own implied volatility index, although there have been already proposed some concepts [Ślepaczuk, Zakrzewski 2007, Rudzki 2008].

REFERENCES

- Beckers S. (1981) Standard deviations implied in option prices as predictors of future stock price variability, *Journal of Finance and Banking*, 5, 363 – 381.
- Black F., Scholes M. (1973) The pricing of options and corporate liabilities, *Journal of Political Economy*, May-June, 637 – 654.
- Bollerslev T. (1986) Generalized autoregressive conditional heteroscedasticity, *Journal of Econometrics*, 31, 307 – 327.
- Britten-Jones M., Neuberger A. (2000) Option prices, implied price processes, and stochastic volatility, *Journal of Finance*, 55, 839 – 866.
- Canina L., Figlewski S. (1992) The informational content of implied volatilities, *Review of Financial Studies*, 5, 659 – 682.
- Chiras D.P., Manaster S. (1978) The information content of option prices and a test of market efficiency, *Journal of Financial Economics*, 6, 213 – 234.
- Chriss N. A. (1997) *Black-Scholes and Beyond: Option Pricing Models*. McGraw-Hill, New York.
- Day T.E., Lewis C.M. (1992) Stock market volatility and the information content of stock index options, *Journal of Econometrics*, 52, 267 – 287.
- Engle R.F. (1982) Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, *Econometrica*, 50, 987 – 1007.
- Garman M.B., Klass M.J (1980) On the estimation of security price volatilities from historical data, *Journal of Business*, 53 (1), 67 – 78.
- Geman H. (2007) *Commodities and Commodity Derivatives*, John Wiley&Sons, Chichester, West Sussex.
- Haug E.G. (2007) *Option Pricing Formulas*, McGraw-Hill, New York.
- Hull J. (2012) *Options, Futures, and Other Derivatives*, Prentice Hall, Boston.

³ The Poznań Exchange was closed in 2001.

- Jiang G.J., Tian Y.S. (2005) The model-free implied volatility and its information content, *Review of Financial Studies*, Vol. 18, No. 14, 1305 – 1342.
- Kolb R.W., Overdahl J.A. (2007) *Futures, Options, and Swaps*, Blackwell Publishing, Malden.
- Krawiec B., Krawiec M. (2002) *Opcje na giełdach towarowych w Polsce*, PWN, Warszawa.
- Lamoureux C., Lastrapes W. (1993) Forecasting stock return variance: toward an understanding of stochastic implied volatilities, *Review of Financial Studies*, 6, 293 – 326.
- Latané H.A., Rendleman R.J. (1976) Standard deviations of stock price ratios implied in option prices, *Journal of Finance*, Vol. XXXI, No. 2, 369 – 381.
- Manaster S., Koehler G. (1982) The calculation of implied variances from the Black-Scholes model, *Journal of Finance*, 37 (1), 227 – 230.
- Musiela M., Rutkowski M. (2007) *Martingale Methods in Financial Modelling*, Springer-Verlag, Berlin Heidelberg.
- Parkinson M. (1980) The extreme value method for estimating the variance of the rate of return, *Journal of Business*, 53 (1), 61 – 65.
- Piontek K. (1999) Zmienność historyczna i implikowana jako prognozy zmienności instrumentów finansowych, część I materiałów konferencji „Modelowanie preferencji a ryzyko – Ustroń 1999”, 325 – 336.
- Rouah F.D., Veinberg G. (2007) *Option Pricing Models and Volatility Using EXCEL®-VBA*, John Wiley&Sons, Hoboken, New Jersey.
- Rudzki R. (2008) WIV20 – indeks zmienności implikowanej dla opcji na WIG 20, <http://wiv20.artim.waw.pl>.
- Schmalensee R., Trippi R. (1978) Common stock volatility expectations implied by option premia, *Journal of Finance*, 33, 129 – 147.
- Ślepaczuk R., Zakrzewski G. (2007) VIW20 – koncepcja indeksu zmienności dla polskiego rynku akcyjnego, finansowy kwartalnik internetowy e-Finanse, nr 4, www.e-finanse.com
- Trippi R. (1977) A test of option market efficiency using a random-walk valuation model, *Journal of Economics and Business*, 29, 93 – 98.
- Zahng P.G. (2006) *Exotic Options*, Word Scientific Publishing, Singapore.