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A COMPARISON OF THE USEFULNESS OF WINTERS' AND SARIMA MODELS IN FORECASTING OF PROCUREMENT PRICES OF MILK IN POLAND

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Abstract: In the paper the Winters' model has been studied as one from adaptive models based on exponential smoothing methods as well as seasonal autoregressive integrated moving average model SARIMA. The aim of the paper is the assessment of accuracy of short-term forecasts of procurement prices of milk in Poland. Empirical verification of ex post forecasts of monthly procurement prices of milk on the basis of 109 time series with 12-month forecast horizon was conducted. Forecasts constructed with the use of SARIMA model are more often exact than when additive and multiplicative Winters' model are used.

Keywords: forecast, forecast accuracy, forecast error measures (*ex post*)

INTRODUCTION

In Poland after 1989 annual milk production was decreasing greatly and fell from about 16 billion litres to its lowest value of 11.3 billion litres in 1995. In the years 2000-2004 the production remained constant at about 11.5 billion litres and it was increasing slightly from 2005. The growth of production was limited by the quota system to about 12.1 billion in 2011. The largest purchaser of milk has been the milk industry that bought 71.4 % of production in 1989 and this percentage decreased over the following years down to 51.8% in 1994. The reconstitution of milk procurement to the level from the end of 1980s took 15 years. The procurement exceeded 60% only in 2002, and 70% in 2005. It reached 74.8% of milk production in 2011 [Urban 2011].

Procurement and prices of milk industry vary between seasons in a given year; it is connected with different levels of milk production. The level of milk production, in turn, first of all depends on the cattle feeding system and gestation chart [Iwan 2005; Majewski 2006].

The aim of the paper is to assess the accuracy of short term forecasts of procurement prices of milk after Poland's accession to the EU, obtained with regard to seasonal differences and trend on the basis of additive and multiplicative Winter's model and SARIMA model. Monthly expired forecasts of procurement prices obtained on the basis of 109 time series of 12-month forecast horizon were subjected to empirical verifiability. The analysis of forecasts will make it possible to search for a better forecast model [Makridakis and Hibon 2000]. The research material constituted mean monthly procurement prices of milk in Poland from January 1990 to December 2012 on the basis of the Statistical Bulletin of the Central Statistical Office (1990-2012).

RESEARCH METHODOLOGY

In time series made up of mean milk procurement prices the following elements may be distinguished: trend, seasonal and irregular component. In practice two decomposition formulas of observed time series values are used with the condition of elements being independent. The first formula is based on the application of additive model of the elements of the series, and the second one on the application of the multiplicative one. Therefore, to forecast the procurement prices on the basis of time series both additive and multiplicative models of Winter's exponential smoothing methods can be used [Winters 1960; Stańko 1999, Tłuczak 2009, Stańko 2013]. Moreover, factors influencing the price with certain delay, through the use of seasonal autoregressive integrated moving average model SARIMA, may be taken into account [Zeliaś et al. 2003; Cieślak 2008, Hamulczuk 2011].

To assess the accuracy of forecasts of procurement prices of milk the basic forecast error assessment methods were used [Zeliaś et al. 2003]: percentage error (PE), mean squared error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). For errors $|\mathrm{PE}|$ (absolute value) and MAPE it was assumed that the forecasts made with the adopted model are very accurate if they do not exceed 3%, are good in the range of $(3\%, 5\% >$, are acceptable (the recipient may decide they are adequately accurate or reliable) in the range of $(5\%, 10\% >$ and are unacceptable for error above 10%. In the case of forecast error: MSE and MAE forecasts are the more accurate the more their values approach 0.

Average monthly procurement prices of milk in Poland from January 1990 to December 2012 were grouped in 109 time series. The first series included 168 time units reflecting monthly milk procurement prices from the years 1990-2003 and the consecutive time series were one element longer from the previous one, moreover, the last time series included 276 units, e.g. consecutive months from the period of 1990-2012.

For monthly procurement prices both additive and multiplicative models of Winter's exponential smoothing methods with linear form of trend, with MAPE as adopted minimizing criterium. Additionally, seasonal model of autoregression and moving average SARIMA $(0,1,3)\times(0,1,1)_{12}$ was used. Its construction process was based on single differentiation of adjacent elements in a series. For a stationary series, Box-Jenkins procedure was used to determine the order of autoregression and wandering mean on the basis of autocorrelation (ACF) and partial autocorrelation (PACF). During statistical verification of the model, the significance of its parameters was confirmed.

To find the best forecasting model prediction criteria were used. Every time series had smoothed procurement prices of milk attributed with the use of studied models and 12-month forecasts were constructed. Next, procurement prices and respective smoothed values were used to determine percentage errors and summary accuracy measures: MSE, MAE and MAPE. In turn, actual procurement prices from the forecast horizon and *ex post* forecasts were used to calculate percentage errors. Moreover, analysis of *ex post* forecasts of procurement prices were based on blocked one-way ANOVA, where the factor was the type of model and consecutive time series were blocks. After rejecting the null hypothesis of equal mean forecasts values of procurement prices obtained on the basis of studied models, multiple comparisons procedure based on Tukey method for all pairwise comparisons was used. Necessary numerical calculations were made on the basis of *STATISTICA 10* package and *gretl* programme.

THE RESULTS OF EMPIRICAL RESEARCH

During the first stage of research percentage errors were analysed, then measures of ex post forecast accuracy. For the first time series made of consecutive months from the period of 1990-2003 the Winter's models yielded 168 percentage errors. In the case of SARIMA$(0,1,3)\times(0,1,1)_{12}$ model, 155 percentage errors were obtained as the stationarity of time series required single differentiation both of seasonal fluctuatons and trend. As a result, the model was 13 elements shorter.

In every series one more error was obtained, compared with the precedent time series. In turn, for the last series containing monthly procurement prices from the period of 1990-2012 respectively 276 and 263 percentage errors were found. Analysing the median of ex post percentage errors shown on Figure 1 for Winter's model especially their large values, e.g. above 10% or below -10% may be noted for the years 1990-2012. In the case of Winter's additive model those errors were observed in January (-12,2%), May (12,6%), September (-18,2%), October (-16,7%) and November (-11,2%) of 1990 as well as January (13,3%) 1991. For the multiplicative model, however, errors were observed only in January (25,5%) and May (-12,7%) 1990. High value of the first percentage error (January 1990) may be connected with the starting point, and the other ones with considerable fluctuations of procurement prices in the first years after the transformation in Poland. Only for the SARIMA(0,1,3)×$(0,1,1)_{12}$ no particularly high values of percentage errors were found, as they ranged from -4,8% in April 2012 to 5,3% in October 1991.

Table 1 shows the basic characteristics for ex post forecast errors calculated on the basis of milk procurement prices and their smoothed values for 109 time series. On the basis of mean values of these errors it can be stated that for the SARIMA(0,1,3)×$(0,1,1)_{12}$ model the obtained values were the lowest, the next was Winter's multiplicative model, and the highest error values were observed for additive version of Winter's model. Therefore, for SARIMA(0,1,3)×$(0,1,1)_{12}$ model the mean variation from forecast error amounted to about 1.05 PLN for 1 hl of milk, forecast bias (as for the absolute value) on the level of about 0.68 PLN per hl and relative total adjustment of the model to procurement prices – about 1.17%. Respective values of forecast errors for Winter's multiplicative model amounted to 1.11 PLN/hl, 0.77 PLN/hl and 1.89%, and for Winter's additive model – 1.22 PLN/hl, 0.80 PLN/hl and 2,02%.

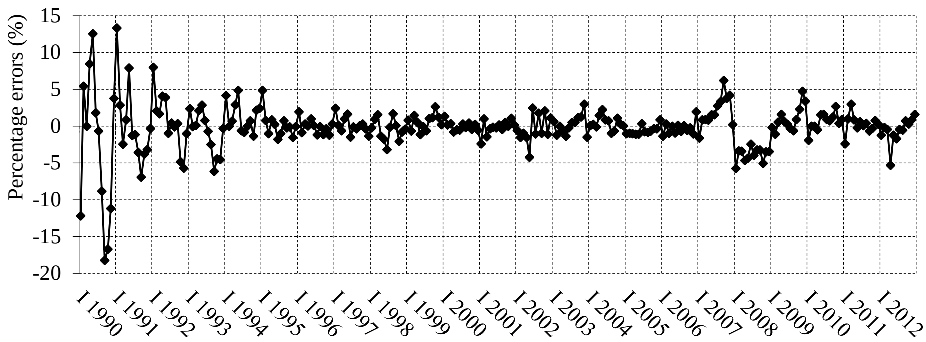
Table 1. Descriptive statistics of ex post forecasting errors

Ex post forecast errors	Minimum	Lower quartile	Median	Upper quartile	Maximum
Additive Winters' model					
MSE [zł/hl]^2	0,5378	0,6182	1,4904	2,4693	9,9006
$\sqrt{\text{MSE}}$ (zł/hl)	0,7334	0,7863	1,2208	1,5714	3,1465
MAE (zł/hl)	0,5482	0,5773	0,8026	1,0582	2,1685
MAPE (%)	1,60	1,69	2,02	2,44	4,69
Multiplicative Winters' model					
MSE [zł/hl]^2	0,4493	0,9391	1,2217	2,2206	2,2870
$\sqrt{\text{MSE}}$ (zł/hl)	0,6703	0,9691	1,1053	1,4902	1,5123
MAE (zł/hl)	0,4719	0,7062	0,7680	0,9799	1,0331
MAPE (%)	1,34	1,85	1,89	1,94	1,99
SARIMA(0,1,3)×$(0,1,1)_{12}$					
MSE [zł/hl]^2	0,5389	0,5908	1,1059	1,3170	1,3981
$\sqrt{\text{MSE}}$ (zł/hl)	0,7341	0,7686	1,0516	1,1476	1,1824
MAE (zł/hl)	0,5135	0,5424	0,6806	0,7646	0,7945
MAPE (%)	1,09	1,12	1,17	1,19	1,20

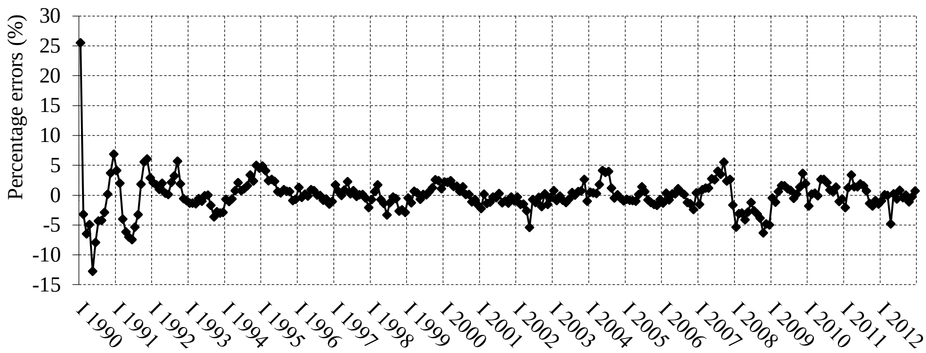
Adopted acronyms: mean squared error (MSE), mean absolute error (MAE), Mean absolute percentage error (MAPE).
Source: own study based on Statistical Bulletin from the period of 1990-2012.

Figure 1. Median of ex post percentage errors for individual months from 1990 to 2012[1)]

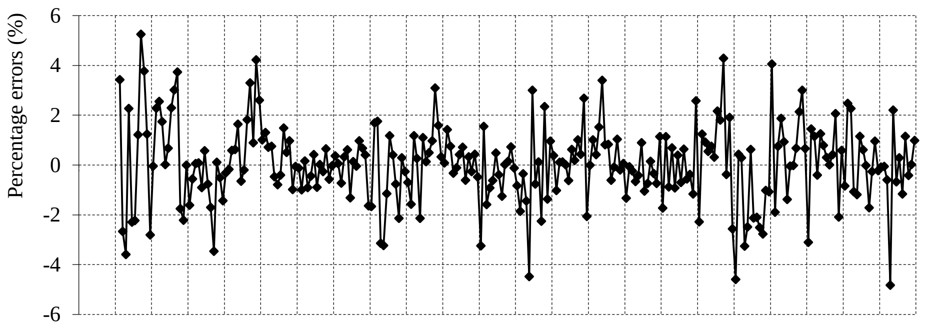
a) additive Winters' model



b) multiplicative Winters' model



c) SARIMA$(0,1,3)\times(0,1,1)_{12}$



[1)] From February 1991 for model SARIMA$(0,1,3)\times(0,1,1)_{12}$.

Source: see Table 1.

In the second stage of the study ex post forecasts were subjected to empirical verifiability. For every time series obtained due to grouping milk procurement prices, 12-month forecast horizon was adopted. In the case of the first series, constructed forecasts included consecutive months from January to December 2004. For every next time series the forecast horizon was moved by one month, e.g. from February 2004 to January 2005 (the second series), from March 2004 to February 2005 (the third series), ..., from January to December 2013 (the last series, no 109). In total 1308 forecasts were constructed while only ex post forecasts from January 2004 to December 2012 were subjected to empirical verifiability i.e. 1230 ex post forecasts, including 108 forecasts with one-month advance, 107 with two-month advance and 97 with 12 months of advance. In Table 2 basic characteristics concerning percentage errors are presented for SARIMA$(0,1,3)\times(0,1,1)_{12}$ model and both additive and multiplicative Winter's model. The structure of these errors is, in turn, presented in Table 3.

Table 2. Characteristics of percentage errors for ex post forecasts and milk procurement prices in Poland (%)

Step-ahead of forecast	Lower quartile			Median			Upper quartile			Percentage of positive errors PE (%)		
	AMW	MMW	SAR	AMW	MMW	SAR	AMW	MMW	SAR	AMW	MMW	SAR
1	-0,83	-1,11	-0,72	0,01	0,09	0,18	0,86	1,27	1,07	51,9	50,9	54,6
2	-1,86	-1,93	-1,22	0,19	-0,14	0,34	1,56	2,19	2,00	50,5	49,5	54,2
3	-2,35	-3,45	-1,93	0,21	0,08	0,08	2,62	3,58	3,33	51,9	50,9	50,9
4	-3,21	-4,34	-2,60	0,62	0,20	0,37	3,64	4,77	4,98	56,2	53,3	52,4
5	-3,57	-4,99	-2,98	0,70	1,10	0,54	4,82	5,33	5,68	57,7	53,8	52,9
6	-4,66	-5,56	-3,52	0,95	0,38	0,54	5,35	6,17	6,42	57,3	52,4	55,3
7	-6,04	-5,66	-4,38	1,08	0,37	0,73	6,80	6,77	7,58	55,9	52,0	51,0
8	-7,35	-5,78	-5,63	1,35	0,38	1,10	6,69	7,56	8,65	55,4	52,5	54,5
9	-9,09	-5,80	-6,59	0,91	-0,55	0,94	7,76	8,08	9,15	56,0	47,0	51,0
10	-10,16	-6,47	-6,85	1,45	-0,05	0,21	8,69	8,49	9,16	55,6	49,5	51,5
11	-14,35	-6,75	-7,26	3,06	0,45	0,77	10,17	9,42	10,88	55,1	50,0	52,0
12	-14,53	-6,31	-7,00	3,33	1,76	-0,23	10,63	9,75	11,21	54,6	54,6	49,5

Explanation: AMW – additive Winters' model; MMW – multiplicative Winters' model, SAR – SARIMA$(0,1,3)\times(0,1,1)_{12}$.
Source: see Table 1

Forecasts constructed on the basis of Winter's additive model were more often undervalued for every forecast advance from 0.5 pp (two months) to 7.7 pp (five months). The mean value of percentage error ex post increased from 0.01% for the advance of one month to 3.33% for the advance of 12 months. The highest percentage of very good forecasts was obtained for the advance of one to three months, respectively: 84.3%, 67.3% and 54.7%. Moreover, for this model it was discovered that for the advance of 11 and 12 months the percentage of inadmissible forecasts exceeded 50%. More than a half of both good and very good forecasts were noted for the advance of one (95.4%) to five months (51.9%).

Table 3. Structure of percentage errors for ex post forecasts and milk procurement prices in Poland (%)

Percentage errors *ex post*	Step-ahead of forecast (months)												
	1	2	3	4	5	6	7	8	9	10	11	12	total
Additive Winters' model													
(-3%; 3%)	84,3	67,3	54,7	44,8	39,4	32,0	22,5	18,8	17,0	15,2	11,2	7,2	35,3
(-5%; -3%> and <3%; 5%)	11,1	16,8	16,0	16,2	12,5	16,5	17,6	17,8	12,0	8,1	8,2	8,3	13,5
(-10%; -5%> and <5%; 10%)	3,7	12,2	18,9	25,7	28,9	26,2	27,5	25,8	32,0	34,3	28,6	27,8	24,1
≤-10% and ≥10%	0,9	3,7	10,4	13,3	19,2	25,3	32,4	37,6	39,0	42,4	52,0	56,7	27,1
Multiplicative Winters' model													
(-3%; 3%)	85,2	62,6	45,3	37,2	28,8	22,3	23,5	17,8	15,0	17,2	14,3	11,3	32,4
(-5%; -3%> and <3%; 5%)	11,1	17,8	20,8	17,1	20,2	22,3	15,7	16,8	16,0	9,1	8,2	11,3	15,6
(-10%; -5%> and <5%; 10%)	3,7	15,9	19,8	28,6	27,9	28,2	31,4	32,7	36,0	37,4	37,7	35,1	27,5
≤-10% and ≥10%	0,0	3,7	14,1	17,1	23,1	27,2	29,4	32,7	33,0	36,3	39,8	42,3	24,5
SARIMA$(0,1,3)\times(0,1,1)_{12}$													
(-3%; 3%)	92,6	73,8	53,8	43,8	40,4	29,1	26,5	19,8	17,0	15,2	15,3	10,3	37,2
(-5%; -3%> and <3%; 5%)	7,4	9,4	19,8	15,2	11,5	21,4	14,7	16,8	14,0	14,1	9,2	13,4	13,9
(-10%; -5%> and <5%; 10%)	0,0	16,8	17,0	24,8	26,9	18,4	26,5	30,7	32,0	31,3	34,7	27,8	23,7
≤-10% and ≥10%	0,0	0,0	9,4	16,2	21,2	31,1	32,3	32,7	37,0	39,4	40,8	48,5	25,2

Source: see Table 1

Analysing ex post forecasts obtained with the use of Winter's multiplicative model it may be observed that the forecasts were more often overvalued for advance of 2, 9 and 10 months and they were overvalued by, respectively, 0.5 pp, 3.0 pp and 0.5 pp. For the advance of 1, 3-8 and 12 months, however, undervalued forecasts were more frequent. The mean value of percentage error for advance of one to 12 months ranged from 0.55% (for 9 months) to 1.76% (for 12 months). More than half of very good forecasts for this model were observed only for the advance of one (85.2%) and two months (62.6%), and analysing both good and very good forecasts, for the advance of one (96.3%) to four months (54.3%).

Forecasts obtained on the basis of SARIMA$(0,1,3)\times(0,1,1)_{12}$ model were more often undervalued for the advance of one to 11 months and overvalued only by 0.5 pp for 12 months. In the case of one-month advance 100% of both good and very good forecasts were obtained, and for the advance of 6 months there were more than 50% of good and very good forecasts. In total 25% of inadmissible forecasts were found.

CONCLUSION

Winters' model based on exponential smoothing methods both in additive and multiplicative versions as well as seasonal integrated autoregression model SARIMA allow to construct short-term forecasts of agricultural produce prices subject to seasonal variations. Those forecasts inform agricultural producers of possible level of prices in consecutive months in a year and may influence their actions on the agricultural market.

The lowest mean overall values of ex post forecasts errors were obtained for SARIMA$(0,1,3)\times(0,1,1)_{12}$ model, that showed the best fit of this model for the empirical data. The next place was taken by Winter's multiplicative model and Winter's additive model followed. For SARIMA$(0,1,3)\times(0,1,1)_{12}$ model, compared with Winter's additive model, the mean forecast error for milk procurement prices for 1 hl was 0.17 PLN less, forecast bias (as to absolute value) was 0.12 PLN less and relative total adjustment of the model to procurement prices – 0.85 pp.

The highest accuracy of forecasts for advance of one to three months was obtained for Winter's additive model and SARIMA$(0,1,3)\times(0,1,1)_{12}$ model, and in the case of Winter's multiplicative model for one (85.2%) to two (62.6%) months. In turn, more than half of both good and very good forecasts were obtained for the advance of one (100%) to six (50.5%) months for the SARIMA$(0,1,3)\times(0,1,1)_{12}$ model; from one (95.4%) to five (51.9%) months for Winter's additive model, and one (96.3%) to four (54.3%) months for Winter's multiplicative model.

Forecasts constructed with the use of SARIMA$(0,1,3)\times(0,1,1)_{12}$ model are more often exact than when additive (for the advance of 1, 3, 6-7 and 9-11 months) and multiplicative (for the advance of 1-8 and 10-12 months) Winters' model based on exponential smoothing methods are used. In turn, the additive Winters' model based on exponential smoothing methods gives more frequently more exact forecasts than its multiplicative version, as for the advance of two to six months there were more very good forecasts, respectively 4.7 pp, 9.4 pp, 7.6 pp, 10.6 pp and 9.7 pp. However, the percentage of both good and very good forecasts for SARIMA$(0,1,3)\times(0,1,1)_{12}$ model was 51.1% and exceeded by 2.3 pp the percentage of those forecasts for Winter's additive model and by 3.1 pp their percentage for Winter's multiplicative model. For every studied model it can be stated that the smaller the variability of milk procurement prices in forecast horizon, the higher the accuracy of the forecasts.

After rejecting the null hypothesis of the same mean values of milk procurement prices constructed on the basis of Winter's additive and multiplicative models as well as SARIMA$(0,1,3)\times(0,1,1)_{12}$ model (statistics F=7.3 and probability p = 0.001) it was stated on the basis of Tukey method for all pairwise comparisons only significant differences in mean values of forecasts for both Winter's models.

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