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DEMOGRAPHIC DEVELOPMENT OF THE POWIATS OF THE MAŁOPOLSKIE VOIVODESHIP

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Abstract: The evolution of the demographic phenomena both in spatial and time terms allows to assess the development of the region. The purpose of the article was an attempt to identify similarities in the selected demographic processes in the powiats of the Małopolskie voivodeship in the years 2002-2011. In the first part of the analysis of the phenomenon the powiats are organized using indicator of demographic development estimated in the first and in the last year of the analysis. For separated four groups of powiats, similar in terms of the analysed indicator, representatives were selected using the method of the centre of gravity, for which shows the process of changes of demographic characteristics such as: birth rate per 1000 population; gross reproduction rate; non-productive age population, per 100 persons of working age; the number of infant deaths per 1000 live births in terms of time.

Keywords: indicator of demographic development, classification, the method of the centre of gravity

INTRODUCTION

Forming of the demographic phenomena both in terms of space and time allows to assess the development of the region. The purpose of this article is an attempt to identify similarities in the selected demographic processes in different powiats of the Małopolskie voivodeship in the years 2002-2011. In the first part of the consideration of the phenomenon the powiats are organized using indicator of demographic development estimated in the first and last year of the analysis. For separated groups of powiats similar in terms of selected indicator, representatives were chosen, for which it was shown the process for selected demographics phenomenon in terms of time.

METHOD OF ANALYSIS

Population of *n* objects O_i $(i = 1,\ 2,\ ...,\ n)$ in defined unit of time is characterized by *m* characteristics. Values of characteristics X_j $(j = 1,\ 2,\ ...,\ m)$ corresponding to objects are described by matrix:

$$[x_{ij}] = \begin{bmatrix} x_{11} & x_{12} & ... & x_{1m} \\ x_{21} & x_{22} & ... & x_{2m} \\ ... & ... & ... & ... \\ x_{n1} & x_{n2} & ... & x_{nm} \end{bmatrix} \qquad (i = 1,\ 2,\ ...,\ n;\ j = 1,\ 2,\ ...,\ m),$$

where x_{ij} - value of the *j*-characteristic for the *i*-object in defined unit of time.

Every object we can equate with a point in the *m*-dimensional Euclidean space, which the individual axes correspond to the variable X_j.

To determine the indicator of the relative demographic development for each object, you should:

- standardize values of characteristics X_j according to the formula:

$$z_{ij} = \begin{cases} \dfrac{x_{ij} - \bar{x}_j}{S_j}, & X_j \in S \\ \dfrac{\bar{x}_j - x_{ij}}{S_j}, & X_j \in D \end{cases} \tag{1}$$

where: x_{ij} – value of the *j*-characteristic for the *i*-object,

$\bar{x}_j$ – the arithmetic mean of the *j*-characteristic,

S_j – the standard deviation of the *j*-characteristic,

S, D – accordingly, a set of stimulant and destimulant[1];

- transform standardized characteristics' values in accordance with the formula:

$$z_{ij}^{*} = z_{ij} - \min_i \{z_{ij}\} \tag{2}$$

- get the value of the indicator of demographic development for each object O_i [Pociecha, 1988]:

[1] The term of stimulant and destimulant was introduced by Z. Hellwig [1968].

$$W_i = \frac{\sum_{j=1}^{m} z_{ij}^*}{\sum_{j=1}^{m} \max_i \left\{ z_{ij}^* \right\}} \tag{3}$$

Indicator W_i takes the values from the range $\langle 0,\ 1 \rangle$, higher values mean a higher level of development. This measure is relative, based on it you can evaluate the level of development of the object in relation to the level of the rest of the analyzed objects and group tested objects.

CLASSIFICATION OF POWIATS IN RELATION TO THE INDICATOR OF DEMOGRAPHIC DEVELOPMENT

Quantitative and qualitative changes in the population in the area, describe number of statistical characteristics. Using the criterion of low correlation between variables forming set of variables to determine the indicator of demographic development in one year, which is the basis for the classification of powiats of the Małopolskie voivodeship, the following variables were selected:

X_1 - natural growth per 1000 population,

X_2 - gross reproduction rate,

X_3 - non-productive age population, per 100 persons of working age,

X_4 - the number of infant deaths per 1000 live births.

Thanks to this method, powiats were organized based on value of the indicator of demographic development. Values of indicator and positions occupied by individual powiats due to the value of this measure in two years is shown in the table 1.

The highest value of the indicator of demographic development in both presented years reached bocheński powiat. In turn, the lowest value W_i in year 2002 reached olkuski powiat, and in the year 2011 chrzanowski powiat. Diversity of values of estimated indicator was higher in 2002. The coefficient of variation of the estimated indicators of development in 2002 was 48,9%, and in 2011 - 36,8%. In 2011 average value of W_i was 0,432 and it was higher than average value of his measure estimated in 2002. Powiats with indicators of demographic development higher than average value W_i form a coherent whole territory (south part of voivodeship) both in 2002 (bocheński, nowosądecki, nowotarski, suski, tatrzański, limanowski, gorlicki, tarnowski, brzeski, wadowicki powiats) and in 2011 (bocheński, nowosądecki, nowotarski, limanowski, gorlicki, suski, brzeski, tatrzański powiats).

Table 1. Values of the indicator of demographic development for powiats

Powiat	Year 2002		Year 2011	
	W_i	position (d_{li})	W_i	position (d_{ki})
bocheński	0,860	1	0,807	1
brzeski	0,363	9	0,471	7
chrzanowski	0,140	18	0,158	19
dąbrowski	0,261	12	0,244	18
gorlicki	0,378	7	0,559	5
krakowski	0,256	13	0,311	14
limanowski	0,406	6	0,581	4
miechowski	0,255	14	0,295	16
myślenicki	0,306	11	0,426	9
nowosądecki	0,478	2	0,679	2
nowotarski	0,442	3	0,608	3
olkuski	0,128	19	0,354	13
oświęcimski	0,211	15	0,252	17
proszowicki	0,167	16	0,415	10
suski	0,424	4	0,478	6
tarnowski	0,376	8	0,393	12
tatrzański	0,414	5	0,470	8
wadowicki	0,343	10	0,304	15
wielicki	0,148	17	0,397	11

Source: own elaboration

Conformity assessment of hierarchy of sorting out of powiats in two classifications we can make estimate of the Spearman's ranks correlation coefficient, using the formula:

$$r_S = 1 - \frac{6\sum_{i=1}^{n}(d_{li} - d_{ki})^2}{n(n^2 - 1)} \tag{4}$$

where: d_{li}, d_{ki} - position of the *i*-object respectively in sort outs of *l* and *k*; *n* – number of objects.

To test the compatibility of the sort out measure, we use statistics:

$$u = r_S\sqrt{n-1} \tag{5}$$

that is, assuming, that the sort outs are only coincide at random, has a normal distribution.

The value of the Spearman's ranks correlation coefficient for presented arrangement of powiats is 0,804, which is statistically significant ($u = 3{,}409 > u_{\alpha=0,05} = 1{,}960$), so there is no major changes in hierarchy of powiats.

Within the arranged set of powiats, respectively in 2002 and in 2011, four disjoint subsets of similar objects were separated in the following ways:

– I a group of objects, for which: $W_i > \overline{W} + S_W$,

– II a group of objects, for which: $\overline{W} < W_i \leq \overline{W} + S_W$,

– III a group of objects, for which: $\overline{W} - S_W < W_i \leq \overline{W}$,

– IV a group composed of objects, for which: $W_i \leq \overline{W} - S_W$,

where: $\overline{W}$ - the arithmetic mean of W_i, S_W - the standard deviation of W_i.

Summary of the results of grouping of powiats against the designated demographic development indicator measure shows graphically on the figure 1.

In the first year of the analysis only one powiat (bocheński) was assigned to the group I, to group II nine powiats, mostly southern and central Małopolska. The third and the fourth in 2002 formed the powiats adjacent to the city of Kraków, and powiats put forth the most of the northwest and dąbrowski powiat.

Table 2. The minimum and maximum characteristics in groups of powiats

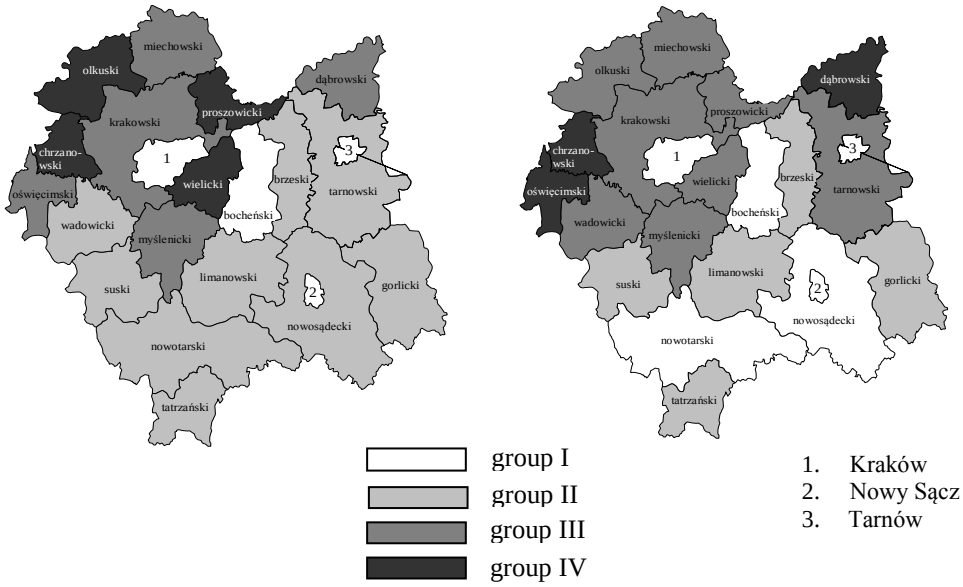
Group	Description	Year 2002				Year 2011			
		X_1	X_2	X_3	X_4	X_1	X_2	X_3	X_4
I	$\min_i x_{ij}$	1,9	0,764	70,2	6,4	3,5	0,722	58,1	2,6
	$\max_i x_{ij}$	1,9	0,764	70,2	6,4	5,6	0,844	62,0	6,4
II	$\min_i x_{ij}$	1,8	0,625	66,0	1,6	2,3	0,639	59,0	3,0
	$\max_i x_{ij}$	6,4	0,920	77,5	9,3	5,3	0,833	64,2	4,2
III	$\min_i x_{ij}$	-3,3	0,585	61,5	2,1	-4,3	0,554	54,9	2,7
	$\max_i x_{ij}$	3,9	0,725	71,6	7,5	4,7	0,746	63,0	6,2
IV	$\min_i x_{ij}$	-1,7	0,564	59,1	6,0	-1,1	0,570	54,4	5,3
	$\max_i x_{ij}$	0,8	0,723	68,2	12,1	0,3	0,717	56,9	7,5

Source: own elaboration

In 2011, there was a slight realignment, the largest group was the third group. In the group of top classified it was next to bocheński powiat, nowotarski and nowosądecki powiats. In group IV remained chrzanowski powiat and joined oświęcimski and dąbrowski powiats.

The minimum and maximum values of characteristics, on the basis of which it was estimated the economic development level indicators in separated groups of powiats is shown in the Table 2.

Figure 1. Diversity of powiats according to the indicator of level demographic development in 2002 and 2011



Source: own elaboration based on the Table 1

In 2011, comparing to the year 2002 decreased within each group, both the maximum and minimum values for the number of people in the non-productive age, per 100 persons of working age. The minimum natural growth in groups generally increased (the exception is in a group three), the maximum values of that characteristic decreased in the second and third group. Changes in the value of the minimum and maximum number of live-born girls per one woman of child-bearing age have the same direction as the natural growth change (exception - first group). The maximum values of the number of deaths of infants per 1000 live births in all separated groups of powiats in 2011 comparing to 2002 did not increase.

To assess the compliance of designated classification of powiats we apply measures [Podolec, 1978]:

$$S_{lk} = 1 - \frac{z_1}{n(n-1)} \tag{6}$$

$$S_{lk}^{*} = 1 - \frac{2(z_2 - n)}{\sum_{i=1}^{4}(n_{li}^2 - n_{li}) + \sum_{i=1}^{4}(n_{ki}^2 - n_{ki})} \tag{7}$$

where:

n – the number of tested objects,

$\left[z_{ij}\right]=\left[p_{ij}^{l}\right]+\left[p_{ij}^{k}\right]$ – compatibility assignment matrix, where $\left[p_{ij}^{l}\right]$ - ordering classification matrix l, for which $p_{ij}^{l}=1$, where objects O_i and O_j were assigned to the same subset, and $p_{ij}^{l}=0$, where objects O_i and O_j were in different subsets; $\left[p_{ij}^{k}\right]$ - ordering classification matrix *k*, for which $p_{ij}^{k}=1$, where objects O_i and O_j were assigned to the same subset, and $p_{ij}^{l}=0$, where objects O_i and O_j were in different subsets,

z_1 – number of ones in compatibility assignment matrix,

z_2 – number of twos in compatibility assignment matrix,

i – subset's number,

n_{li} – number of objects in i-subset created in classification 1,

n_{ki} – number of objects in i-subset created in classification k.

Measures S_{lk}, S_{lk}^{*} take a value in range $\langle 0,\ 1\rangle$. Value S_{lk} tells you, what is the probability that a randomly chosen pair of objects were compatibly allocated under the classification 1 and k. If $S_{lk}>0{,}8$ it can be concluded that the divisions are compatible. Measure S_{lk}^{*} specifies, what part of the "connections" between objects created by one of the divisions is covered in the second division.

For two classifications of powiats of the Małopolskie voivodeship, respectively in 2002 and 2011, $S_{lk}=0{,}661$ and $S_{lk}^{*}=0{,}376$, this shows very weak compatibility of divisions.

Selection of representatives of groups of powiats in 2002 was conducted by the method of the centre of gravity, as a measure of distance, Euclidean distance was selected [Pluta 1977]. How to select the representatives of the groups of this method depends on the size of groups of objects. Objects forming one-piece groups become automatically representatives. We choose representatives of the multiple-element groups (the number of elements greater than two) after the calculation of the sum of distances of each object from the other group's objects and indicate the representative object, for which the sum of the distances from other objects in the group is the smallest.

Group I represents bocheński powiat, group II – brzeski powiat, group III – krakowski powiat, and group IV – olkuski powiat.

CHARACTERISTIC OF DEMOGRAPHIC INDICATORS FOR SELECTED POWIATS

General trends, that characterize the development of the population in the years 2002-2011 in selected powiats of the Malopolskie voivodeship is shown in table 3. Linear trends presented in selected population of powiats are of good compatibility.

Table. 3. Population trend models in selected powiats in the years 2002-2011

Group	Powiat	The estimated trend model
I	bocheński	$\hat{y}_t = \underset{(415,55)}{97644,93} + \underset{(66,972)}{567,376}t, \quad t = 1, \ 2, \ ..., \ 10$ $V = 0,604\%, \qquad R^2 = 0,9$
II	brzeski	$\hat{y}_t = \underset{(263,675)}{88961,67} + \underset{(42,495)}{287,297}t, \quad t = 1, \ 2, \ ..., \ 10$ $V = 0,426\%, \qquad R^2 = 0,851$
III	krakowski	$\hat{y}_t = \underset{(2007,386)}{235572,9} + \underset{(323,520)}{2205,461}t, \quad t = 1, \ 2, \ ..., \ 10$ $V = 1,186\%, \qquad R^2 = 0,853$
IV	olkuski	$\hat{y}_t = \underset{(228,142)}{114776,1} - \underset{(36,768)}{75,721}t, \quad t = 1, \ 2, \ ..., \ 10$ $V = 0,292\%, \qquad R^2 = 0,346$

Source: own elaboration

The exception is olkuski powiat, in which the number of people in the last analyzed year increased significantly compared to previous years, which largely contributed to the mismatch of trend. If we assess the trend of olkuski powiat without taking into account the year 2011 we would get:

$$\hat{y}_t = \underset{(125,289)}{114980,1} - \underset{(22,264)}{131,483}t, \quad t = 1, \ 2,..., \ 9 \qquad (V = 1,509\%; \ R^2 = 0,832)$$

.

Models for the powiats of the first three groups provide for further increases in population, if you continue the trend so far. This can not be said about the representative of Group IV, olkuski powiat, in which the model predicts a further decrease in the number of population.

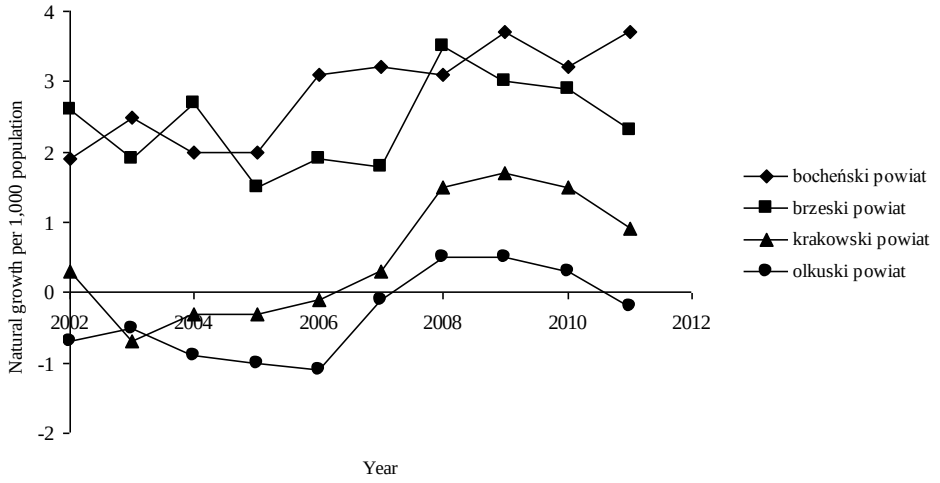
The observed changes in the characteristics, on the basis of which we defined the relative indicators of demographic development for selected powiats in the years 2002-2011 are presented at figures 2-5. On this basis, we conclude that:

– bocheński and brzeski powiats had throughout analyzed period of time a positive natural growth;

– only in bocheński powiat in the last analyzed year, natural growth was higher than in 2010;

– the lowest natural growth indicator in almost all years (an exception is the year 2003) had olkuski powiat;

Figure 2. Natural growth per 1,000 population in the years 2002-2011



Source: own elaboration

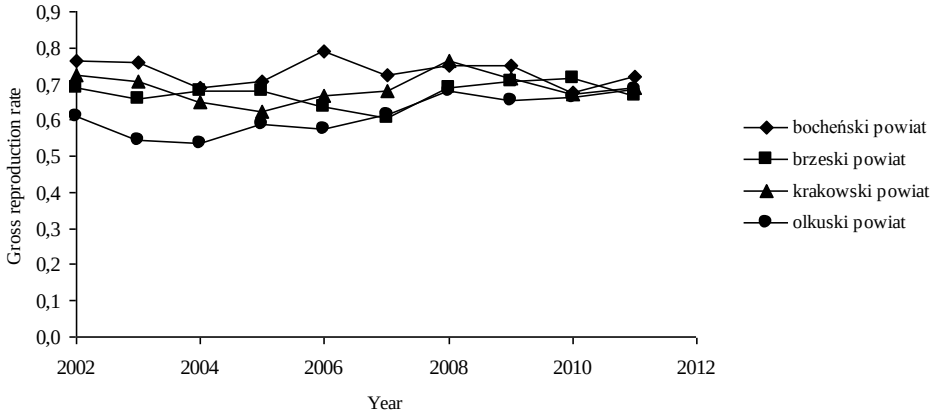
– the number of live-born girls per one woman is currently of childbearing age, showed throughout the analyzed period of time, slight fluctuation;

– differences (in absolute value) in the value of the gross reproduction rate in representative powiats decreased from year to year so that in 2011, were no more than 0,055;

– in 2002, the non-productive age population per 100 persons of working age was the biggest in brzeski powiat (70,2), the lowest in olkuski powiat (60,1);

– average rates of changes in non-productive age population per 100 persons of working age were less than 1, indicating that from year to year in these powiats this indicator decreased by 1% (olkuski powiat), and 2% (bocheński, brzeski, krakowski powiats);

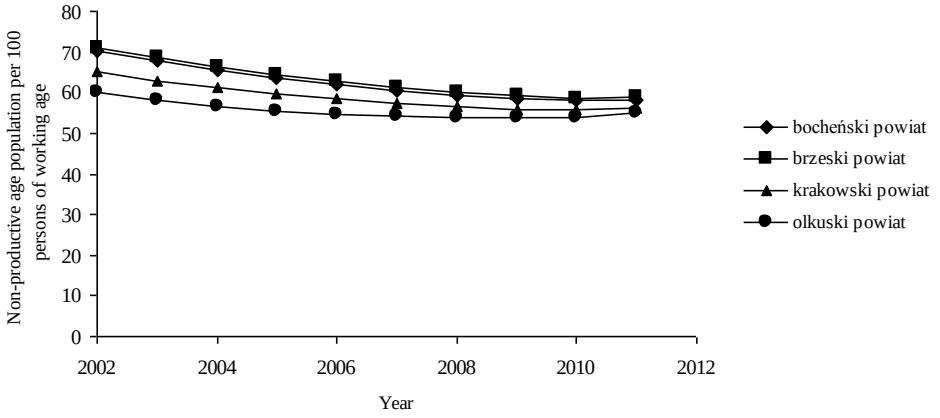
Figure 3. Gross reproduction rate in the years 2002-2011



Source: own elaboration

– in 2011, non-productive age population per 100 persons of working age in all powiats did not exceed 59,0 (brzeski powiat) and was not lower than 54,9 (olkuski powiat);

Figure 4. Non-productive age population, per 100 persons of working age in the years 2002-2011



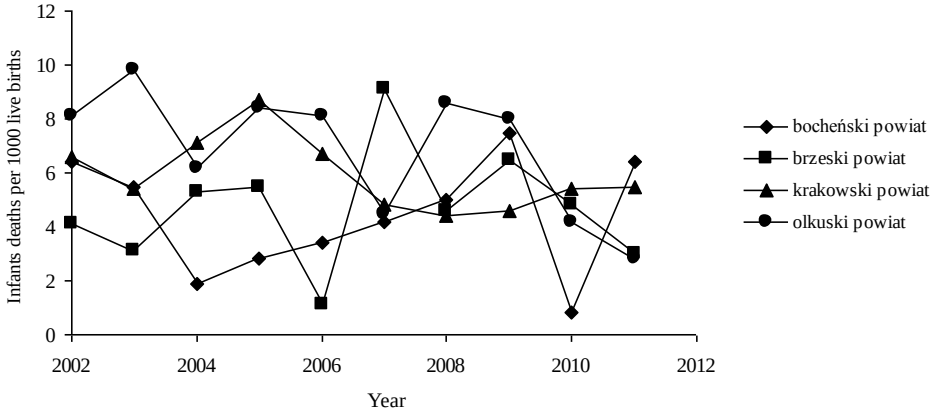
Source: own elaboration

– the number of infant deaths per 1000 live births in powiats in analyzed period of time did not show constant trends;

– in 2002, the number of infant deaths per 1,000 live births was the highest in olkuski powiat (8,1), but from year to year in this powiat decreased on average by 11% and in 2011, has reached a value of 2,8;

– in krakowski and brzeski powiats number of infant deaths per 1000 live births in the last analyzed year was lower than in the first year of analysis; in turn, within bocheński powiat, in those years was the same (6,4).

Figure 5. The number of infant deaths per 1000 live births during the years 2002-2011



Source: own elaboration

SUMMARY

1. The analysis was based on selected demographic indicators, which may decide about the development of the population in the area.
2. Powiats of the Małopolskie voivodeship are diverse in terms of the level of demographic development. We can distinguish four groups of powiats with similar characteristics describing analyzed phenomenon.
3. Assessment of the demographic development is definitely higher in powiats of South and central part of the Małopolskie voivodeship.
4. Linear trends of population in selected powiats generally provide further increase in population (the exception is olkuski powiat).
5. Natural growth in bocheński and brzeski powiats in all analysed years was much larger than in krakowski and olkuski powiats.

6. The differences in the number of live-born girls per one woman who is currently of childbearing and non-productive age population, per 100 persons of working age, in selected powiats were decreasing from year to year.
7. The number of infant deaths per 1000 live births in both powiats, as well as in years was varied.

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