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ENDOGENOUS TECHNOLOGICAL PROGRESS AND ECONOMIC GROWTH IN A MODEL WITH NATURAL RESOURCES

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Abstract: Idea of technological progress based on two different types of research that generate radical and incremental innovations, became new approach in endogenous growth modeling. This approach seems to be useful in modeling relationships between technological progress, natural resources, environmental quality and economic growth.

The purpose of this paper is to answer questions about relationships between long-term economic growth, technological progress and use of natural resources. The main object is an impact of natural resources use on growth rate and a role of endogenous technological progress.

Keywords: economic growth, technological progress, natural resources, technological opportunities

INTRODUCTION

Formal description of innovation process is not easy, it is even much more complicated to include it in a standard economic growth model, especially when natural resources use also has to be included. There are at least three dominating approaches to technological progress modeling: human capital approach (considered extensively, for example, in [Lucas 1988]), research and development sector with rising number of patents (like in [Romer 1990]), and, recently, technological opportunities approach, introduced by [Olsson 2000]. The latter concept is mostly based on [Kuhn's 2012] theory of scientific revolutions.

The purpose of this paper is an attempt to answer question about theoretical dependences among long-run economic growth, technological progress and natural resources use, with implication of technological opportunities approach. Main subject of this research is influence of natural resources on rate of economic growth and a role of endogenous technological progress in that dependence.

The structure of this paper is as follows. In part one we describe Olsson's idea of endogenous technical change, based on technological opportunities. Construction of an economic growth model based on this idea is described in section two. After that, in section three, we present solution of the model and perform its analysis. Whole paper is ended with a short summary.

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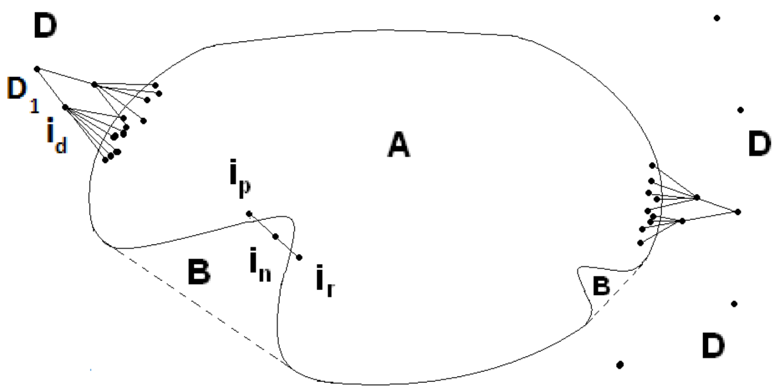
IDEA OF TECHNOLOGICAL OPPORTUNITIES

Concept of technological opportunities was introduced by [Olsson 2000, 2005], but even earlier some authors mentioned that new knowledge is a result of combining few old existing theories. Figure 1 shows basics of this idea.

Figure 1. Idea of technological opportunities



Source: [Olsson 2000]

Description of Olsson's idea is as follows. Let $A \subset R^k$ be a set of all existing, known and widespread ideas. In this set can be found all basic ideas (like adding and subtracting natural numbers), but also more complex ones (like e.g. quantum mechanics). New ideas appear in three different ways: as a scientific discovery, as a radical innovation or as an incremental innovation.

Incremental innovation is a result of regular research work, which is based on use of existing ideas and combine them, which leads to form new idea. Simple example of incremental innovation might be a new application for smartphones which is created with a use of existing hardware (phone), its software and some algorithmic schemes. These are not revolutionary ideas, because they need existing ones, they do not lead to significant expansion of knowledge.

New idea i_n arises as a convex combination of early existing ideas i_p and i_r, so:

$$i_n = \alpha \cdot i_p + (1 - \alpha) \cdot i_r, \tag{1}$$

where $\alpha \in (0; 1)$. It is possible to match in convex combination more than two existing ideas. Convexity of this linear combination comes as a result of mixing ideas, if one of ideas i_p, i_r is used more (with greater share) that result would be different[1]. Convexity of linear combinations implies that new ideas, emerge as a result of incremental innovations, are in a convex hull of A (which we denote as a $conv(A)$). Set $B = conv(A) - A$ is a set of technological opportunities – it contains all of ideas, that are reachable with existing state of knowledge, but still not invented. Whenever new idea is invented as an incremental innovation set of technological opportunities become smaller and set of all ideas become greater. It is easy to see, that if set B would not grow eventually all of ideas possible to invent will be invented. So there must be some way to increase size of set of technological possibilities. This is based on another two types of scientific innovations.

The most important are scientific discoveries, which are more accidental, non intentional side-effects of research than effects of directed research. Appearance of scientific discoveries always creates new paradigm, they are anomalies with respect to current set of knowledge. These discoveries (D) are outside of set A, in some distance from it. With existing level of knowledge it is not clear what are the causes of these discoveries, but they become inspiration to directed and intentional research.

Existence of ideas outside of set A implies possibilities of combine ideas from set A and scientific discovery (for example D_1). That combination, which is an effect of intentional research, creates new knowledge (i_d) outside of set A, leads to its enlargement and enlargement of convex hull, and leads to enlargement of set B. New research might be continued until scientific discovery would be included into set A and would become a part of general knowledge. Scientific discovery leads to new paradigm, which opens new technological opportunities. That leads to creation of new ideas.

In the next section we describe a model of economic growth based on idea of technological opportunities.

[1] It is hard to imagine what would arise as a result of mixing idea of car tires (with a share, let's say, 0.98) and idea of Phillips Curve (with a share 0.02). Therefore concept of Olsson seems to be more adequate inside of a single scientific discipline or on a cross-section of similar disciplines.

MODEL

Concept of technological opportunities, described in details in section one, seems quite interesting but not easy to apply in a growth theory. There are not many papers which include that idea in standard economic growth model. One, that need to be mentioned, is a paper by Growiec and Schumacher [2013]. In that paper standard economic growth model with endogenous technological progress is presented. This progress may only take place if there are technological opportunities, and those opportunities may be created only with a use of existing knowledge. Appearance of new knowledge raises TFP, which leads to increase in production. Paper considers both centralized economy and social planner case, results are similar in both cases. It seems to be much harder to apply technological opportunities approach to modeling of natural resources use. Probably the only paper considering this problem was [Lundström 2003].

Our model is based on [Olsson 2000, 2005], [Lundström 2003] and [Growiec, Schumacher 2013] models. We use standard optimal control approach to long-run economic growth modeling. We consider closed economy. Households contains L citizens, their number increase with a rate of growth equal to n:

$$\dot{L} = nL \tag{2}$$

We assume, for simplicity, that at the beginning the number of citizens is equal to 1. Labor supply is shared between three different activities: production, basic research and applied research.

Applied research creates new ideas. Evolution of level of knowledge A is of following form:

$$\dot{A} = \delta(u_A L)^\gamma B^\mu \tag{3}$$

where u_A is a share of time devoted to applied research, B is a level of technological opportunity, $\delta, \gamma, \mu > 0$ are constant parameters.

Basic research creates new technological opportunities:

$$\dot{B} = \zeta(u_B L)^\gamma A^\mu - \delta(u_A L)^\gamma B^\mu \tag{4}$$

where u_B is a share of time devoted by households to basic research, $\zeta > 0$ is a parameter. Creation of new technological opportunities depends on existing level of knowledge, but amount of technological opportunities decreases by an amount of new, created knowledge.

Level of production is given by production function of standard, Cobb-Douglas form:

$$Y = A^\sigma M^\alpha (u_Y L)^{1-\alpha} \tag{5}$$

where $\sigma > 0, \alpha \in (0,1)$, M is effective physical capital. By effective physical capital we understand physical capital that may be powered with produced energy:

$$M = \min\{aK, bE\} \tag{6}$$

where K is physical capital stock and E is a flow of produced energy. Energy production function uses two factors – existing stock of capital and flow of natural resources:

$$E = A^{\kappa} K^{\beta} R^{1-\beta} \tag{7}$$

where $\kappa > 0, \beta \in (0,1)$, R is natural resources flow used to energy production. We assume that there is always enough of physical capital, the problem lies only in energy production, so

$$M = bE. \tag{8}$$

Evolution of physical capital is in a standard form:

$$\dot{K} = Y - C - dK \tag{9}$$

where C is a level of consumption and d is depreciation rate.

Economy is endowed with supply S of natural resources, which is extracted successively:

$$\dot{S} = -R \tag{10}$$

Households maximize their lifetime utility from present moment to infinity given by following utility function:

$$L_0 \int_0^{+\infty} e^{-(\rho-n)t} \frac{1}{1-\theta} \left(c^{1-\theta} - 1\right) dt \to max \tag{11}$$

where $\rho > 0$ is a discount rate, $L_0 = 1$ and $-\theta$ is equal to elasticity of marginal utility. Small letter c denotes consumption per capita.

We express the model in variables in per capita terms and denote them with small letters, for example, $k = \frac{K}{L}$. Our model is now of the following form:

$$\int_0^{+\infty} e^{-(\rho-n)t} \frac{1}{1-\theta} \left(c^{1-\theta} - 1\right) dt \to max \tag{12}$$

$$\dot{k} = A^{\sigma+\kappa\alpha} b^{\kappa\alpha} k^{\beta\alpha} r^{(1-\beta)\alpha} (1 - u_A - u_B)^{1-\alpha} - c - (d+n)k \tag{13}$$

$$\dot{A} = \delta u_A^{\gamma} e^{\gamma n t} B^{\mu} \tag{14}$$

$$\dot{B} = \zeta u_B^{\gamma} e^{\gamma n t} A^{\mu} - \delta u_A^{\gamma} e^{\gamma n t} B^{\mu} \tag{15}$$

$$\dot{s} = -r - ns \tag{16}$$

Households choose their level of per capita consumption c, use of natural resource per capita r and share of time devoted between work, applied research and basic research u_Y, u_A, u_B. Obviously

$$u_Y = 1 - u_A - u_B. \tag{17}$$

In the next section we derive solution of this model and draw some conclusions.

SOLUTION AND DISCUSSION

We maximize present-value Hamiltonian, given by:

$$H(u_A, u_B, r, c, A, B, s, k, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = e^{-(\rho-n)t}\frac{1}{1-\theta}\left(c^{1-\theta} - 1\right) + \lambda_1\left(\delta u_A^{\gamma} e^{\gamma nt} B^{\mu}\right) + \lambda_2\left(\zeta u_B^{\gamma} e^{\gamma nt} A^{\mu} - \delta u_A^{\gamma} e^{\gamma nt} B^{\mu}\right) + \lambda_3(-r - ns) + \lambda_4\left(A^{\sigma+\kappa\alpha} b^{\kappa\alpha} k^{\alpha\beta} r^{\alpha(1-\beta)}(1-u_A-u_B)^{1-\alpha} - c - (d+n)k\right). \quad (18)$$

Transversality conditions are as follows:

$$\lim_{t\to+\infty} \lambda_1 A = 0, \quad (19)$$

$$\lim_{t\to+\infty} \lambda_2 B = 0, \quad (20)$$

$$\lim_{t\to+\infty} \lambda_3 s = 0, \quad (21)$$

$$\lim_{t\to+\infty} \lambda_4 k = 0. \quad (22)$$

First order conditions are of following form:

$$e^{-(\rho-n)\,t} c^{-\theta} = \lambda_4, \quad (23)$$

$$\lambda_4 \alpha(1-\beta) A^{\sigma+\kappa\alpha} b^{\kappa\alpha} k^{\alpha\beta} r^{\alpha(1-\beta)-1}(1-u_A-u_B)^{1-\alpha} = \lambda_3, \quad (24)$$

$$(\lambda_1 - \lambda_2)\delta\gamma u_A^{\gamma-1} e^{\gamma nt} B^{\mu} = \lambda_4(1-\alpha) A^{\sigma+\kappa\alpha} b^{\kappa\alpha} k^{\alpha\beta} r^{\alpha(1-\beta)}(1-u_A-u_B)^{(1-\alpha)-1} \quad (25)$$

$$\lambda_2 \zeta\eta u_B^{\gamma-1} e^{\gamma nt} A^{\mu} = \lambda_4(1-\alpha) A^{\sigma+\kappa\alpha} b^{\kappa\alpha} k^{\alpha\beta} r^{\alpha(1-\beta)}(1-u_A-u_B)^{-\alpha}, \quad (26)$$

$$-\dot{\lambda}_1 = \lambda_2 \zeta\mu u_B^{\gamma} e^{\gamma nt} A^{\mu-1} + \lambda_4(\sigma+\kappa\alpha) A^{\sigma+\kappa\alpha-1} b^{\kappa\alpha} k^{\alpha\beta} r^{\alpha(1-\beta)}(1-u_A-u_B)^{1-\alpha} \quad (27)$$

$$-\dot{\lambda}_2 = \lambda_1 \delta\mu u_A^{\gamma} e^{\gamma nt} B^{\mu-1} - \lambda_2 \delta\mu u_A^{\gamma} e^{\gamma nt} B^{\mu-1} \quad (28)$$

$$-\dot{\lambda}_3 = -\lambda_3 n \quad (29)$$

$$-\dot{\lambda}_4 = \lambda_4 \left(\alpha\beta A^{\sigma+\kappa\alpha} b^{\kappa\alpha} k^{\alpha\beta-1} r^{\alpha(1-\beta)}(1-u_A-u_B)^{1-\alpha} - (d+n)\right). \quad (30)$$

We define steady-state as a state when all variables grow at constant rates. With this assumption, we use standard procedure to obtain solution. Growth rates of variables are as follows:

$$g_y = g_c = g_k = \frac{(\sigma+\kappa\alpha)\frac{\gamma n}{1-\mu} - \alpha(1-\beta)\rho}{1-\alpha+\alpha\theta(1-\beta)} > 0 \quad (31)$$

$$g_s = g_r = (1-\theta)g_y - \rho < 0 \quad (32)$$

$$g_A = g_B = \frac{\gamma n}{1-\mu} \quad (33)$$

All four transversality conditions come down to a single inequality:

$$-\rho + n + (1-\theta)g_y < 0 \quad (34)$$

which is fulfilled[2].

Formulas for u_Y, u_A and u_B are also possible to derive, but they are too complex to present them here[3]. All rates of growth depend only on parameters. Table 1 presented below contains signs of first partial derivatives of rates of growth with respect to chosen parameters. During process of obtaining those signs we assumed for simplicity that $\theta > 1$.

Table 1. Signs of first partial derivatives (with assumption $\theta > 1$)

	$x=\rho$	$x=\sigma$	$x=\kappa$	$x=\gamma$	$x=n$	$x=\mu$	$x=\theta$
$\frac{\partial g_y}{\partial x}=\frac{\partial g_k}{\partial x}=\frac{\partial g_c}{\partial x}$	<0	>0	>0	>0	>0	>0	<0
$\frac{\partial g_r}{\partial x}=\frac{\partial g_s}{\partial x}$	<0	<0	<0	<0	<0	<0	<0
$\frac{\partial g_A}{\partial x}=\frac{\partial g_B}{\partial x}$	$=0$	$=0$	$=0$	>0	>0	>0	$=0$

Source: own calculations

The most important conclusions drawn from Table 1 are as follows.

- Higher discount rate leads to lower rate of economic growth. This result might be understood with a following logic. In two identical economies, which are different only in a size of ρ, the one with greater discount rate has lower rate of growth of production per capita and natural resources are used in a more intensive way. To maximize utility this economy switch production from future to present moments, which decreases rate of capital accumulation. Size of ρ has no effect on rate of growth of technological progress.
- Increase in σ or κ leads to increase in g_y and more intense use of natural resources (lower g_r). It also has no effect on rate of technological progress.
- Whenever γ, n or μ is higher it is connected to higher rate of growth of A and production per capita, higher intensity of use of natural resources and higher rate of growth of technological opportunities.
- θ represents tendency of consumers to smooth (less volatile) path of consumption in time. Higher θ implies lower g_y, in the limit, when $\theta \to +\infty$, g_y reaches zero.

Performed analysis leads to conclusion, that technological progress in general leads to more intensive extraction of natural resources – higher rate of growth of A is related to higher (in absolute value) g_r. This interesting impact of technological

[2] $-\rho + n + (1-\theta)g_y$ is equal to rate of use of natural resources (g_R), so it has to be negative.

[3] Available upon request from the author.

progress on natural resource use is due to substitutability of natural resources and physical capital – it is optimal to extract natural resources as soon as possible to produce enough energy to power more physical capital and use it in production. This obviously increases level of production and level of investments, which leads to higher stock of K. Higher level of physical capital substitute natural resources in production and allow to entirely exploit them sooner.

SUMMARY

Concept of technological opportunities, introduced by [Olsson 2000], opens many interesting directions of research in theory of economic growth. Obviously, this idea leads to mathematically more complex endogenous growth models, much harder in analysis, but with interesting consequences. The first attempt of formulating Olsson's theory in economic growth model, [Lundström 2003], cannot be treated as a good example, because in modeled economy natural resources were not a production factor, but source of all income. On the other hand, [Growiec, Schumacher 2013] model includes concept of technological opportunities, but without natural resources.

An attempt to modeling an economy with natural resources in an economic growth model taken in this paper should not be treated as final one. Substitutability between physical capital and natural resources is a flaw of proposed model. In further research this substitutability should be replaced by complementarity between physical capital and natural resources or (as in some papers) by treating natural resource as a factor of production of physical capital.

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