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SUBSAMPLING APPROACH FOR STATISTICAL INFERENCE WITHIN STOCHASTIC DEA MODELS[1]

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Abstract: In the current literature, many different stochastic extensions of the DEA framework can be found. Such generalized approaches enable one to model the uncertainty inherent to the form of the production possibility set and the value of the technical efficiency measure at any given point of the former. In the paper we provide a detailed discussion of some statistical model and the subsampling algorithm which are used in statistical inference. The methodology is then illustrated with an empirical example using the real-world data from the Polish energy sector.

Keywords: DEA method, technical efficiency, statistical inference, subsampling

INTRODUCTION

Essentially, data envelopment analysis (or DEA, in short) is applied to measure technical efficiency within a group of n production (or, more generally, decision-making) units producing s sorts of outputs (arrayed in vector $\mathbf{y}_j = [y_{1j}, \ldots, y_{sj}]$ for $j = 1, \ldots, n$) out of m sorts of inputs ($\mathbf{x}_j = [x_{1j}, \ldots, x_{mj}]$).

In what follows we assume that the producers are technologically homogenous and the technology itself is represented by the production set T, the specifics of which are discussed in the following section. Additionally, in a deterministic DEA framework, T is specified as a minimal set (in terms of inclusion), containing all the data points $(\mathbf{x}_j, \mathbf{y}_j)$, $j = 1, \ldots, n$. Such an approach

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enables one to derive its form explicitly. On the other hand, such an assumption is invalid within stochastic variations of DEA, which is attributable to uncertainty inherent to the data collected[2]. Consequently, the production set is of an unknown form.

Based on T, the so-called Farrell technical efficiency measure (for some feasible production plan $(\mathbf{x}_o, \mathbf{y}_o)$) can be defined. Such a measure underlies studies employing DEA methods and is usually formulated in the input- and output-oriented settings, respectively:

$$\theta\,(\mathbf{x}_o, \mathbf{y}_o) = \min\{\theta \in R: (\theta\mathbf{x}_o, \mathbf{y}_o) \in T\},$$
$$\lambda\,(\mathbf{x}_o, \mathbf{y}_o) = \max\{\lambda \in R: (\mathbf{x}_o, \lambda\mathbf{y}_o) \in T\}.$$

In the research we restrict ourselves to the input orientation solely. In a deterministic DEA framework, owing to a known form of the production set, a unique value of the measure can be calculated by solving appropriate linear programs (see, e.g., [Guzik 2009]). However, it is no longer the case in stochastic DEA methods, where the actual form of T remains unknown.

STATISTICAL MODEL

In the paper we focus on one of the stochastic DEA methods, developed by a team of researchers gathered around Professor Léopold Simar[3]. According to the preceding remarks, the setting requires the form of T to be approximated, so that technical efficiency measure can be further estimated at any given point of T. For the approximation and estimation to be valid one needs to formulate a relevant model. In the present study we resort to the methodology presented by Kneip et al. [2008], recognized for its transparency as compared with earlier suggestions found in the literature (see [Kneip et al. 1998], [Gijbels et al. 1999] and [Park et al. 2000]).

Below we present three most fundamental assumptions that are material to further considerations, starting with the one specifying properties of the production set.

Assumption 1. The units produce *s* sorts of goods out of *m* sorts of inputs and use the same technology, represented by T – a closed and convex production set, satisfying the 'free disposal' (or: inefficiency) condition and the 'no free lunch' condition (i.e. no output can be produced out of zero inputs).

For more details and a review of other possible properties of the production set we refer to, e.g., [Prędki 2012].

The second assumption pertains to the data generating process.

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[2] The uncertainty may arise as a result of measurement errors or data incompleteness, for instance.

[3] Léopold Simar (born 1946) is Full Professor of Statistics at the Department of Statistics, Université Catholique de Louvain (Belgium).

Assumption 2. The sample $\{(\mathbf{x}_j, \mathbf{y}_j) \in R_{0+}^{m+s}: j = 1, \ldots, n\}$ is drawn from a sequence of independent random vectors (X_j, Y_j) following the same distribution with a continuous (over support T) density function $f_T(\mathbf{x}, \mathbf{y})$: $R_{0+}^{m+s} \to R$.
The presumption above underpins the proof of consistency of the technical efficiency measure estimator (at a given point of T) and implies obvious fact of all the data points $(\mathbf{x}_j, \mathbf{y}_j)$ $(j = 1, \ldots, n)$ are feasible production plans (i.e. fall into T) with probability 1.

The final assumption is meant to lend due 'smoothness' to the efficiency measure as a function of inputs and outputs.

Assumption 3. The function $\theta(\mathbf{x}, \mathbf{y})$ is of class $C^2(T)$ (i.e. the second-order derivatives exist and are continuous).
The above requirement aims at obtaining additional information on asymptotic sample distribution of the estimator of technical efficiency measure (at a given point of T).

EFFICIENCY MEASURE ESTIMATOR

As an approximation of the production set we shall use one of its well-known forms obtained in the deterministic setting. Specifically, T is approximated with a minimal set (in terms of inclusion) $\hat{T}$ that satisfies Assumption 1 and includes all the data points[4]:

$$\hat{T} = \{(\mathbf{x}, \mathbf{y}) \in R_{0+}^{m+s}: \exists \lambda_j \geq 0: \sum_{j=1}^{n} \lambda_j = 1, \mathbf{x} \geq \sum_{j=1}^{n} \lambda_j \mathbf{x}_j, \mathbf{y} \leq \sum_{j=1}^{n} \lambda_j \mathbf{y}_j\}.$$

Replacing T with $\hat{T}$ in the definition of $\theta(\mathbf{x}_o, \mathbf{y}_o)$ one obtains the estimator of input-oriented technical efficiency estimator at $(\mathbf{x}_o, \mathbf{y}_o)$:

$$\hat{\theta}(\mathbf{x}_o, \mathbf{y}_o) = \min\{\theta: (\theta \mathbf{x}_o, \mathbf{y}_o) \in \hat{T}\}.$$

In [Kneip et al. 1998] it has been proved that for any $(\mathbf{x}_o, \mathbf{y}_o)$ lying in the interior of $\hat{T}$ the above formula yields a consistent estimator of the true efficiency measure (with the convergence rate of $n^{-2/(m+s+1)}$), whereas in [Kneip et al. 2008] the asymptotic sample distribution of $\hat{\theta}(\mathbf{x}_o, \mathbf{y}_o) - \theta(\mathbf{x}_o, \mathbf{y}_o)$ has been derived. Regrettably, its form is dependent on $(m + s)(m + s + 1)/2 + 2$ unknown parameters related to the density f_T and the multivariate production frontier. To this day, no consistent method of estimation for these parameters has been designed. This, in turn, prohibits construction of asymptotic confidence intervals and other quantities which could possibly shed some light on the relation between the data uncertainty and the one of efficiency measure estimation.

[4] Inequalities are understood "by coordinates".

BOOTSTRAP

In the situation outlined above, one is prompted to resort to the bootstrap methods, which have gained much favor since the late 1990s. Such an approach, based on simulating additional samples out the original one, allow for approximation of the aforementioned confidence intervals and some measures of statistical dispersion. A necessary condition for any bootstrap procedure to be valid is its consistency, which requires (in the limiting case of $n\to\infty$) the distribution of the bootstrap estimator of an efficiency measure to be corresponding (in a certain manner) with the actual asymptotic distribution of the estimator considered in the previous section. A formal definition of the bootstrap consistency can be found in, e.g., [Simar and Wilson 2000, p. 61]. It seems that early attempts of designing relevant bootstrap procedures (such as the ones presented in [Löthgren and Tambour 1999], [Simar and Wilson 1998, 2000]) have not been quite successful as they have resulted in either methods consistency of which could not be formally verified, or in ones proved positively inconsistent[5]. Moreover, some of these have been founded upon a quite restrictive and non-verifiable assumption of homogeneity of technical efficiency measure. Only recently, in [Kneip et al. 2008], have two consistent bootstrap methods been devised. However, one of them requires solving a huge number of linear programs at each bootstrap iteration, which renders the method (already complex due to its multilevelness and intricate formulae) computationally formidable. Therefore, the other of the two – resting upon the idea of the so-called subsampling – seems a more attractive alternative.

SUBSAMPLING

Since the subsampling approach is the one employed in our empirical study to follow, we present in some more detail the way the method proceeds.

Step 1. Based on the original sample, the value of $\hat{\theta}(\mathbf{x}_o, \mathbf{y}_o)$ is calculated.

Step 2. Generate a bootstrap sample $\{(\mathbf{x}_j^*, \mathbf{y}_j^*)\text{: } j = 1, \ldots, k\}$ by randomly drawing (independently, uniformly, and with replacement) $k < n$ observations from the original sample[6].

Step 3. The bootstrap estimate of $\theta(\mathbf{x}_o, \mathbf{y}_o)$ is computed according to the formula:

$$\hat{\theta}^*(\mathbf{x}_o, \mathbf{y}_o) = \min\{\theta\text{: } (\theta\,\mathbf{x}_o, \mathbf{y}_o) \in \hat{T}^*\},$$

where

[5] Only some simulation studies have been carried out, the results of which seem to corroborate consistency of some procedures (under a given Data Generating Process).

[6] While specifying value of $k = k(n)$ in simulation studies one needs to guarantee that $k(n)\to\infty$ and $k(n)/n\to 0$ in the limiting case of $n\to\infty$. Assuming $k = n$ (which corresponds with the so-called naive bootstrap) results in inconsistency of the procedure.

$$\hat{T}^* = \{(\mathbf{x}, \mathbf{y}) \in R_{0+}^{m+s}: \exists \lambda_j \geq 0: \sum_{j=1}^{k} \lambda_j = 1, \mathbf{x} \geq \sum_{j=1}^{k} \lambda_j \mathbf{x}_j^*, \mathbf{y} \leq \sum_{j=1}^{k} \lambda_j \mathbf{y}_j^* \}.$$

Repeat steps 2-3 *B* times, obtaining a sequence of bootstrap estimates:

$$\{\hat{\theta}_b^*(\mathbf{x}_o, \mathbf{y}_o): b = 1, ..., B\}.$$

The way the algorithm proceeds next depends on the particular aim of the inference. In the current study, for instance, we are to analyze uncertainty related to the efficiency measure calculated in Step 1, which stems from an unknown form of the production set. We start with approximating asymptotic confidence interval for $\theta(\mathbf{x}_o, \mathbf{y}_o)$.

Step 4. For each b = 1, …, *B*, compute the expression:

$$k^{2/(m+s+1)}\left(\frac{\hat{\theta}_b^*(\mathbf{x}_o, \mathbf{y}_o)}{\hat{\theta}(\mathbf{x}_o, \mathbf{y}_o)} - 1\right),$$

and put the values obtained in a non-decreasing order.

Step 5. For a given $\alpha \in (0, 1)$, discard $\alpha/2 \cdot 100\%$ elements in both tails of the sequence. The minimum and maximum of the remaining values are the so-called bootstrap quantiles, denoted by $\delta_{\alpha/2,k}$ and $\delta_{1-\alpha/2,k}$, respectively.

Step 6. Compute the interval:

$$[\hat{\theta}(\mathbf{x}_o, \mathbf{y}_o)/(1 + n^{-2/(m+s+1)}\delta_{1-\alpha/2,k}); \hat{\theta}(\mathbf{x}_o, \mathbf{y}_o)/(1 + n^{-2/(m+s+1)}\delta_{\alpha/2,k})],$$

henceforth referred to as the asymptotic bootstrap confidence interval for $\theta(\mathbf{x}_o, \mathbf{y}_o)$.

Some dispersion measures providing one with information about uncertainty related to $\hat{\theta}(\mathbf{x}_o, \mathbf{y}_o)$ can be calculated as well, including:

- sample variance of the bootstrap estimates:

$$\hat{\sigma}_o^2 = \left(\frac{k}{n}\right)^{\frac{4}{m+s+1}} B^{-1}\left[\sum_{b=1}^{B} \hat{\theta}_b^*(\mathbf{x}_o, \mathbf{y}_o) - B^{-1}\left(\sum_{b=1}^{B} \hat{\theta}_b^*(\mathbf{x}_o, \mathbf{y}_o)\right)\right]^2,$$

- bootstrap bias of $\hat{\theta}(\mathbf{x}_o, \mathbf{y}_o)$:

$$\text{bias}_o = \left(\frac{k}{n}\right)^{\frac{2}{m+s+1}}\left[B^{-1}\left(\sum_{b=1}^{B} \hat{\theta}_b^*(\mathbf{x}_o, \mathbf{y}_o)\right) - \hat{\theta}(\mathbf{x}_o, \mathbf{y}_o)\right].$$

Then, taking the bootstrap bias into account, a revised estimate of technical efficiency measure is readily available:

$$\hat{\hat{\theta}}(\mathbf{x}_o, \mathbf{y}_o) = \hat{\theta}(\mathbf{x}_o, \mathbf{y}_o) - \text{bias}_o.$$

Note, however, that such a correction may result in an increase of the mean square error of the revised estimator (with respect to $\hat{\theta}(\mathbf{x}_o, \mathbf{y}_o)$). In the literature (see, e.g., [Simar and Wilson 2000, p. 63]) it is postulated that the revision should be applied once the following inequality holds:

$$\text{bias}_o^2 > 3\hat{\sigma}_o^2.$$

SPECIFIC PROBLEMS

While using the DEA methods, values of the efficiency measure are calculated for each of the units in the original sample, i.e. for o = 1, …, *n*, as it is the main objective of the approach to compare and rank the units with respect to their efficiency. Analogously, one may wish to obtain (approximated) confidence intervals and the dispersion measures for each unit. However, theoretical results and simulation studies presented in the relevant literature (see [Kneip et al. 2008] and [Simar and Wilson 2011]) consider only a single object $(\mathbf{x}_o, \mathbf{y}_o)$ that is assumed to be independent of the ones in the original sample $\{(\mathbf{x}_j, \mathbf{y}_j) \in R_{0+}^{m+s}: j = 1, \ldots, n\}$[7]. Aiming at employing the subsampling approach in order to compute the confidence intervals and dispersion measures we slightly modify the procedure outlined above. We proceed along some hints found in [Simar and Wilson 2011, p. 40]. Firstly, the subsampling procedure is repeated *n* times for each o = 1, …, *n*. For a given unit, the algorithm proceeds similarly as before with the only modification in Step 2, now consisting in drawing *k* out of $n - 1$ elements (excluding object o). Naturally, in such a case $k < n - 1$, so some information is lost, yet the required independence is guaranteed.

Choosing an appropriate value for *k* is another key issue. Some simulation studies carried out in [Kneip et al. 2008] indicate a critical role of the parameter to final results of the subsampling procedure, thereby proving their pronounced sensitivity. Nevertheless, no universal approach (featuring some desired statistical properties) to selecting the 'right' number for *k* can be pointed. Therefore, we resort to (quite an arbitrary) empirical rule that has been formulated in [Simar and Wilson 2011, pp. 42-43]. The method – presented below – is a particular version of a general procedure designed in [Politis et al. 2001], and has specifically been developed to deal with approximation of confidence intervals.

Step 1. For a given point $(\mathbf{x}_o, \mathbf{y}_o)$, conduct the subsampling scheme for arbitrarily specified values of *k*: $k_1 < k_2 < \ldots < k_W$. As a result one obtains a sequence of corresponding confidence intervals:$\{[L(k_w); R(k_w)]: w = 1, \ldots, W\}$.

Step 2. Choose some arbitrary small natural number *v* (usually, in practice, *v* assumes the value of one, two or three).

Step 3. For any $w \in [v + 1, W - v]$,calculate a sum of standard deviations of the left-hand side endpoints: $L(k_{w-v}),\ldots, L(k_{w+v})$, and of the right-hand side ones: $R(k_{w-v}), \ldots, R(k_{w+v})$.

Step 4. For a given $w \in [v + 1, W - v]$, select the value of $k_w \in \{k_1, k_2, \ldots, k_W\}$ that minimizes the sum computed in Step 3.

[7] In the sense of the random vector (X_o, Y_o) being stochastically independent of each (X_j, Y_j), j = 1, …, *n*.

EMPIRICAL STUDY

We finally proceed to an empirical illustration of the presented methodology, analyzing a real-world data set comprising of 32 Polish power stations and thermal-electric power stations in 1995. Previously, and for the first time, the data have been analyzed in [Osiewalski and Wróbel–Rotter 2002]. With a view to make inference about technical efficiency of these production units, we employ the following variables as the inputs:

x_1 – real capital (gross value of capital assets in thousands of Polish zloty);
x_2 – labor (number of workers);
x_3 – feed energy(in TJ).[8]

The only output of the units is the energy produced, y, measured in TJ.

Utilizing the author's own numerical procedure programmed in GAUSS 12.0, subsampling routines presented in the previous sections (with suitable modifications) have been run.[9] The results are presented in Table 1.

Table 1. Subsampling results

j	$\hat{\theta}_j$	$\hat{\sigma}_j^2$	bias_j	$\hat{\hat{\theta}}_j$	L_j	R_j	k_j
1	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
2	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
3	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
4	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
5	0.67971	0.00810	0.07037	0.60935	0.47925	0.67954	23
6	0.77265	0.00557	0.08513	0.68752	0.60434	0.77246	27
7	0.74739	0.00577	0.07962	0.66777	0.56619	0.74720	27
8	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
9	0.96423	0.00025	0.02402	0.94021	0.93145	0.96399	27
10	0.69205	0.00614	0.07542	0.61663	0.49809	0.69188	24
11	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
12	0.63646	0.00983	0.10151	0.53496	0.43344	0.63630	20
13	0.58258	0.01551	0.14586	0.43671	0.38674	0.58243	16
14	0.97465	0.00011	0.01522	0.95943	0.95105	0.97440	27
15	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
16	0.82456	0.00307	0.07183	0.75273	0.68625	0.82435	27

[8] TJ – terajoule, 1 GWh = 3,6 TJ.

[9] The author is deeply indebted to Professor Anna Pajor for her advice and unwavering support as well as for providing him with additional numerical procedures of her own.

j	$\hat{\theta}_j$	$\hat{\sigma}_j^2$	bias_j	$\hat{\hat{\theta}}_j$	L_j	R_j	k_j
17	0.82574	0.00283	0.05880	0.76694	0.68825	0.82553	27
18	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
19	0.96811	0.00016	0.01122	0.95689	0.93864	0.96786	27
20	0.95963	0.00021	0.02988	0.92974	0.92272	0.95939	27
21	0.65916	0.00217	0.06664	0.59252	0.54118	0.65899	19
22	0.62917	0.00411	0.06698	0.56219	0.47174	0.62901	25
23	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
24	0.86040	0.00338	0.06460	0.79581	0.74578	0.86019	27
25	0.81771	0.00221	0.04918	0.76853	0.67515	0.81751	27
26	0.75999	0.00506	0.05942	0.70057	0.58505	0.75980	27
27	0.75424	0.00629	0.06550	0.68875	0.57647	0.75405	27
28	0.89069	0.00190	0.04488	0.84582	0.79796	0.89047	27
29	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
30	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
31	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5
32	1.00000	0.00000	0.00000	1.00000	0.99975	0.99975	5

Source: own elaboration

In columns 2-4 of Table 1 we report estimates obtained for the measure of efficiency and measures of its dispersion, whereas in columns 5-7 – the revised estimates of the efficiency measure along with approximated asymptotic confidence intervals (with L_j and R_j standing for the left- and the right-hand side endpoints, respectively). The last column displays values of k_j's, i.e. the number of generated bootstrap samples for each data point $(\mathbf{x}_j, \mathbf{y}_j)$ (j = 1, …, n), which have been specified according to the empirical rule discussed in the previous section.[10]

As regards selecting particular values for B and v, we follow the mainstream literature and set $B = 2000$ and $v = 2$, which obviously does not shed the arbitrariness of such a choice. Reflecting on specific choices for k, again, we resorted to general suggestions found in the literature and – avoiding extreme possible values of the parameter – considered $k \in \{3, 4, \ldots, 29\}$. Since $n = 32$ in our study, setting k close to 32 would imply slower convergence due to the algorithm "nearing" the (inconsistent) naive bootstrap (see Footnote 6). On the other hand, low values of the parameter result in a sizeable loss of information and enforce low number of bootstrap subsamples available to be generated.

[10] Let us recall that the bootstrap samples generated for any j do not include the unit itself.

According to values reported in the last column of Table 1, it is clear that following the empirical rule discussed above the most frequent choice for the number of elements in a subsample is 27. With regard to revision of the point estimates of the efficiency measure, only in the case of unit j = 20 the relevant inequality holds so that the correction is then justified.[11] Values of the bias-revised estimator for the other units are presented only for the sake of completeness.

Estimates of the efficiency measures hover between 0.586 and 1.000. The lowest efficiency is indicated for unit $j = 13$, whereas power stations no.: 1-4, 8, 11, 15, 18, 23, 29-32, appear to be the most efficient. Note, however, that the subsampling procedure is not valid for the latter, as it was not possible to obtain either non-zero values of the dispersion measures or reasonable approximations of the confidence intervals. On a technical note, it is caused by a lack of objects with a higher value of the efficiency measure in the original sample. Partly, it is also in accord with the theoretical results discussed above, which hold only for the data points lying in the interior of the production set, thereby lacking in efficiency. Admittedly, one cannot dismiss a possibility that our estimates of technical efficiency are incorrect and that the units may in fact be inefficient. Nevertheless, with the data set in hand it is not possible to diagnose conclusively whether it is actually the case.

For the units that emerged to be inefficient we managed to compute both the bootstrap measures of dispersion as well as approximated confidence intervals. One needs to perceive the results with some caution, however, as the low number of units in the sample undermines the asymptotics.

Table 2 presents relative and absolute measures of dispersion (only for the inefficient stations), sorted increasingly with respect to technical efficiency estimates.[12]

[11] See the end part of the "Subsampling" section.

[12] As an absolute measure of dispersion we consider the diameter of a confidence interval, i.e. $R_j - L_j$, the values of which are reported in the last column of Table 2.

Table 2. Relative and absolute measures of dispersion

j	$\hat{\theta}_j$	$\hat{\sigma}_j / \hat{\theta}_j$	$bias_j / \hat{\theta}_j$	$R_j - L_j$
13	0.58258	0.21380	0.25038	0.195692
22	0.62917	0.101879	0.106458	0.157266
12	0.63646	0.15575	0.15949	0.202866
21	0.65916	0.070723	0.101093	0.117812
5	0.67971	0.13243	0.10353	0.200289
10	0.69205	0.11327	0.10898	0.193788
7	0.74739	0.10166	0.10654	0.181013
27	0.75424	0.10517	0.08684	0.177587
26	0.75999	0.09364	0.07818	0.17475
6	0.77265	0.09656	0.11018	0.168121
25	0.81771	0.05749	0.06015	0.14236
16	0.82456	0.06721	0.08711	0.138101
17	0.82574	0.06441	0.07121	0.137282
24	0.86040	0.06754	0.07508	0.114401
28	0.89069	0.04894	0.05039	0.09251
20	0.95963	0.015051	0.031142	0.03667
9	0.96423	0.01654	0.02491	0.032538
19	0.96811	0.01289	0.01159	0.029225
14	0.97465	0.01083	0.01561	0.023352

Source: own elaboration

Note a clear negative correlation between estimates of efficiency measure and its dispersion. It follows that higher values of $\hat{\theta}_j$ are usually accompanied with tighter confidence intervals. Conversely, for units of lesser efficiency higher uncertainty is typically indicated. A technical reason behind that observation resides in the fact that the generated subsamples contain a large fraction of highly efficient units, with respect to which the highly inefficient ones are compared. It should be pointed, however, that values obtained for the relative dispersion measures do not exceed 25% of the point estimate. Therefore, even for the most inefficient power stations the relative uncertainty is not considerable.

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