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IMPRECISE RETURN RATES ON THE WARSAW STOCK EXCHANGE

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Abstract: The return rate in imprecision risk may be described as a fuzzy probabilistic set [Piasecki, 2011a]. On the other side, in [Piasecki, Tomasik 2013] is shown that the Normal Inverse Gaussiandistribution is the best matching probability distribution of logarithmic returns on Warsaw Stock Exchange. There will be presented the basic properties if imprecise return with the Normal Inverse Gaussian distribution of future value logarithm. The existence of distribution of expected return rate is discussed. All obtained results may be immediately applied for effectiveness analysis at risk of uncertainty and imprecision [Piasecki, 2011c]

Keywords: Normal Inverse Gaussian distribution, uncertainty risk, imprecision risk, fuzzy present value

INTRODUCTION

Typically, the analysis of properties of any security is kept, as analysis of return rate properties. The future value of a security is presented as a random variable. Distribution of this random variable is formal image of uncertainty risk. In [Piasecki, Tomasik2013] is shown that the Normal Inverse Gaussiandistribution is the best matching probability distribution of logarithmic one-day return rates on Warsaw Stock Exchange.

On the other side, any present value is approximately equal to market price. For this reason a present value may be given as a fuzzy number. Then the return rate of is given as a fuzzy probabilistic set. Properties of this return are considered in [Piasecki 2011b] for the case of any probability distribution of future value. In [Piasecki 2014] the fuzzy probabilistic return is applied for financial decision making.

Taking into account all above results, we see that imprecise return rates on Warsaw Stock Exchange may be determined by one-day logarithmic returns as fuzzy probabilistic set under the Normal Inverse Gaussian distribution. Basic properties of these returns will be investigated in this paper. The main goal of our considerations will be to define a three-dimensional risk image for such logarithmic return rate.

RETURN RATES

Let us assume that the time horizon $t > 0$ of an investment is fixed. Then any security is determined by two values:

– anticipated future value (FV) $V_t \in \mathbb{R}^+$,

– assessed present value (PV) $V_0 \in \mathbb{R}^+$.

The basic characteristic of benefits from owning this instrument is a return rate $r \in \mathbb{R}$ given by the identity

$$r = r(V_0, V_t). \tag{1}$$

In the general case, the function $r: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$is a decreasing function of PV and an increasing function of FV.

Let the set of all securities be denoted by the symbol$\mathbb{Y}$. Each security $\breve{Y} \epsilon \mathbb{Y}$ is represented by its return rate r_Y. According to the principle of maximizing benefits, the set of all securities may be ordered by the relation $R[r] \subset \mathbb{Y} \times \mathbb{Y}$ defined as follows

$$\breve{Y} R[r] \breve{Z} \Leftrightarrow r_Y \geq r_Z. \tag{2}$$

In the special case we have here logarithmic return rate

$$R = \ln \frac{V_t}{V_0}. \tag{3}$$

For any returns r and logarithmic returns R we have

$$r = r(1, e^R). \tag{4}$$

It means that any return rate is an increasing function of logarithmic returns. Therefore any return ϱ defines order $R[r]$ equivalent to the order $R[R]$ defined by logarithmic return$R$. This observation prompts us to replace the study of any returns by the study of logarithmic returns.

The FV is at risk of uncertainty. A formal model of this uncertainty is the presentation of FV V_t as a random variable $\tilde{V}_t: \Omega = \{\omega\} \longrightarrow \mathbb{R}^+$. The set Ω is a set of financial marketelementary states. In the classical approach to the problem of return rate determination, PV of a security is identified with the observed market price $\check{C}$. Then the return rate is a random variable, which is at uncertainty risk. This random variable is determined by the identity

$$\tilde{R}(\omega) = \ln \frac{\tilde{V}_t(\omega)}{\check{C}}. \tag{5}$$

In practice of financial markets analysis, the uncertainty risk is usually described by probability distribution of return rates. At the moment, we have an extensive knowledge on this subject. Empirical studies have shown that capital markets can differ from each other type of best suited distribution returns. That conclusion is the result of a comparison research results relating to the German capital market [Eberlein, Keller 1994] with the results from research dedicated to the American capital market.[Weron, Weron1999]. This points out validity of a search for type of return rates distribution applicable to the Polish capital market. From the literature it is known to a lot of sub-studies of empirical distributions of returns on the Polish capital market. These results have been discussed in [Piasecki, Tomasik 2013]. In the same book have been studied returns on shares making up the portfolios defining stock indexes WIG20,mWIG40 and sWIG80. The survey covered all quotations for the period from 09.21.1998 till 03.03.2010. This period is divided into a bull market periods and bear market periods. Distinct studies covered duration of each bull or bear market. In total, there were tested 694 time series of quotations. The study subject was one-day logarithmic return. Piasecki and Tomasik (2013)have shown that the Normal Inverse Gaussian distribution is the best matching probability distribution of logarithmic returns on Warsaw Stock Exchange.

TheNormal Inverse Gaussian distribution was introduced in [Barndorff-Nielsen 1977]. This distribution is characterized by $\varpi = (\alpha, \beta, \delta, \mu)$ of four parameters fulfilling $\alpha \in \mathbb{R}^+,\ \beta \in (-\alpha, \alpha),\ \delta \in \mathbb{R}_0^+,\ \mu \in \mathbb{R}$. The density function $f_{NIG}(\cdot\,|\varpi): \mathbb{R} \longrightarrow \mathbb{R}^+$of the Normal Inverse Gaussian distribution is given by the identity

$$f_{NIG}(x|\varpi) = f_{NIG}(x|\alpha, \beta, \delta, \mu) = \\ = \frac{\alpha\delta K_1\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)}{\pi\sqrt{\delta^2 + (x-\mu)^2}} \cdot \exp\left\{\delta\sqrt{\alpha^2 - \beta^2} + \beta(x-\mu)\right\} \tag{6}$$

where$K_1: \mathbb{R}^+ \longrightarrow \mathbb{R}^+$is the modified Bessel function of the third kind determined by the identity

$$K_1(x) = \frac{1}{2}\int_0^\infty exp\left\{-\frac{x}{2}\left(y + \frac{1}{y}\right)\right\} dy. \tag{7}$$

Let us take into account fixed security. If the distribution of its logarithmic return rate is given by the identity (6), then the density function $f_V\left(\cdot\,\middle|\check{C}, \varpi\right): \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ of FV distribution is defined as follows

$$f_V\left(x\middle|\check{C}, \varpi\right) = f_{NIG}\left(\ln\frac{x}{\check{C}}\middle|\varpi\right). \tag{8}$$

Assessment of security FV is based on objective measurement only. It means that the density function of FV distributionis independent of how the PV is determined. Expected FV and its variance exist always. This is due to the fact that expected value and variance exist for each Normal Inverse Gaussian distribution [Bølviken, Benth2000].

The security PV security is approximately equal to security market price $\check{C}$. Thus it may be at imprecision risk. Then PV is described by fuzzy number in the sense given by Dubois and Prade(1979). This approach is studied by Ward (1985), Buckley (1987, 1992), Gutierrez (1989), Greenhutet al. (1995), Kuchta(2000), Lesage (2001), Sheen (2005) and Piasecki(2011a, 2011b). The security PV is a fuzzy number dependent on market price $\check{C}$. Each PV membership function $\mu(\cdot\,|\check{C}): \mathbb{R}^+ \longrightarrow [0;1]$ fulfils following properties

$$\mu(\check{C}|\check{C}) = 1\ , \tag{9}$$

$$\forall x, y, z \epsilon \mathbb{R}^+:\ x \le y \le z \Rightarrow \mu(y|\check{C}) \ge min\{\mu(x|\check{C}), \mu(z|\check{C})\}. \tag{10}$$

The above-mentioned imprecision risk is caused by behavioural reasons. Each investor takes into account the lowest possible market price and the biggest one. The security PV should be greater than the lowest possible price. Also, the security PV should be less than the biggest possible price. Therefore, we additionally assume about any PV membership function $\mu(\cdot\,|\check{C}): \mathbb{R}^+ \longrightarrow [0;1]$that it fulfills following condition

$$\forall \check{C} \in \mathbb{R}^+: \exists \check{C}_{min}, \check{C}_{max} \in \mathbb{R}^+: \check{C}_{min} < \check{C} < \check{C}_{max}\ \wedge\ \mu(\check{C}_{min}|\check{C}) = \mu(\check{C}_{max}|\check{C}) = 0\ . \tag{11}$$

Immediately from (10) and (11) we obtain

$$\forall \check{C} \in \mathbb{R}^+: x \notin (\check{C}_{min},\ \check{C}_{max}) \Longrightarrow \wedge\ \mu(x|\check{C}) = 0 \tag{12}$$

Some example of defined above PV is described in [Piasecki2011a].

Then the return rate is at risk of coincidence uncertainty and imprecision. According to the Zadeh extension principle, for each fixed elementary state $\omega \epsilon \Omega$ of financial market, membership function $\rho(\cdot, \omega|\check{C}): \mathbb{R} \to [0;1]$ of logarithmic return rate is determined by the identity

$$\rho(\cdot, \omega|\check{C}) = max\left\{\mu(y|\check{C}): y \epsilon \mathbb{R}^+, r = \ln\frac{\tilde{V}_t(\omega)}{y}\right\} = \mu\left(e^{-R} \cdot \tilde{V}_t(\omega)\middle|\check{C}\right)\ . \tag{13}$$

It means that the logarithmic return rate considered here is represented by fuzzy probabilistic set defined by Hiroto (1981). For this reason, this logarithmic return rate is called fuzzy probabilistic logarithmic return.

IMPRECISE ASSESSMENT OF RETURN RATE

For any fuzzy probabilistic logarithmic return we determine the parameters of its distribution. We have here distribution of expected logarithmic return rate

$$\varrho(R|\check{C}) = \int_0^{+\infty} \mu(e^{-R} \cdot x|\check{C}) \cdot f_V(x|\check{C}, \varpi)dx. \tag{14}$$

Integrating by substitution we obtain

$$\varrho(R|\check{C}) = e^{-R} \cdot \int_0^{+\infty} \mu(t|\check{C}) \cdot f_V(e^R \cdot t|\check{C}, \varpi)dt = e^{-R} \cdot \int_{\check{C}_{min}}^{\check{C}_{max}} \mu(t|\check{C}) \cdot f_V(e^R \cdot t|\check{C}, \varpi)dt\ .$$

It proves that distribution of expected logarithmic return rate always exists. Distribution of expected return rate $\varrho(\cdot\,|\check{C}): \mathbb{R} \longrightarrow [0;1]$ is a membership function of fuzzy subset $\tilde{R}$ in the real line. This subset $\tilde{R}$ represents both rational and behavioural aspects in the approach to estimate the expected benefits. Then the expected logarithmic return rate is defined as follows

$$\bar{R} = \bar{R}(\check{C}) = \frac{\int_{-\infty}^{+\infty} R \cdot \varrho(R|\check{C})dR}{\int_{-\infty}^{+\infty} \varrho(R|\check{C})dR}\ . \tag{15}$$

Similarly as in the case of precisely defined return rate, there are such distributions of expected logarithmic return rate for which the expected logarithmic return rate does not exist. We then replace this distribution with a distribution truncated on both sides, for which the expected logarithmic return rate always exists. This procedure finds its justification in the theory of perspective [Kahneman, Tversky 1979]. Among other things, this theory describes the behavioural phenomenon of the extremes' rejection.

The expected logarithmic return rate is at risk of uncertainty and imprecision. The image of this risk is described in [Piasecki 2011c].

We use the following variance of return rate as the assessment of the risk uncertainty

$$\sigma^2 = \sigma^2(\check{C}) = \frac{\int_{-\infty}^{+\infty}\int_0^{+\infty} x\cdot \nu(x,y|\check{C})\cdot f_V(x|\check{C},\varpi)dydx}{\int_{-\infty}^{+\infty}\int_0^{+\infty} \nu(x,y|\check{C})\cdot f_V(x|\check{C},\varpi)dydx} \tag{16}$$

where

$$\nu(x,\tilde{V}_t(\omega)|\check{C}) = \begin{cases} max\{\rho(\bar{R}+\sqrt{x},\omega|\check{C}),\rho(\bar{R}+\sqrt{x},\omega|\check{C})\} & x \geq 0 \\ 0 & x < 0 \end{cases}. \tag{17}$$

Imprecision is composed of ambiguity and indistinctness. Ambiguity is the lack of clear recommendation of one alternative from among various alternatives. In accordance with the suggestion given in [Czogałaet al.1982], we evaluate the ambiguity risk by energy measure$d(\tilde{R})$of expected logarithmic return rate distribution $\tilde{R}$. This measure is determined by the identity

$$\delta = \delta(\check{C}) = \frac{\int_{-\infty}^{+\infty} \varrho(x|\check{C})dx}{1+\int_{-\infty}^{+\infty} \varrho(x|\check{C})dx}\cdot . \tag{18}$$

Indistinctness is the lack of explicit distinction between the information provided and its negation. According to the suggestion given in [Gottwald et al., 1982], we evaluate the indistinctness risk by entropy measure$e(\tilde{R})$ of distribution of expected logarithmic return rate $\tilde{R}$. This measure is described as follows

$$\varepsilon = \varepsilon(\check{C}) = \frac{\int_{-\infty}^{+\infty} min\{\varrho(x|\check{C}),1-\varrho(x|\check{C})\}dx}{1+\int_{-\infty}^{+\infty} min\{\varrho(x|\check{C}),1-\varrho(x|\check{C})\}dx}. \tag{19}$$

In this way we describe security with imprecision return as the pair $(\bar{R},(\sigma^2,\delta,\varepsilon))$ where $\bar{R}$ is expected logarithmic return rate and $(\sigma^2,\delta,\varepsilon)$ is the three-dimensional image of risk of uncertainty, ambiguity and indistinctness. In [Piasecki2011c] this pair is applied for analysis of security effectiveness.

CONCLUSIONS

In this paper is shown that for the Normal Inverse Gaussian distribution the expected logarithmic return rate distribution and three-dimensional risk image always exist. Due to results obtained in [Piasecki 2014] these tools may be applied for decision- making on Warsaw Stock Exchange. Let us note that for any security all above models are depend on its current market price.

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