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PORTFOLIO PERFORMANCE MEASUREMENT BASED ON THE MULTIHORIZON SHARPE RATIO - WAVELET ANALYSIS APPROACH

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Abstract: The aim of the study is to evaluate and compare the performance of mutual funds. The proposed approach evaluates the effectiveness of the fund's investment strategy in terms of the investment horizon. As a tool wavelet analysis that has been applied to the decomposition of the excess returns of funds for the six scales was used. The Sharpe ratios calculated on the basis of the so-transformed series formed the basis for the arrangement of funds. The results indicate that the variance of rate of return in the analyzed funds decline as the wavelet scale increases.

Keywords: wavelet analysis, multihorizon Sharpe ratio, investment horizon

INTRODUCTION

The Sharpe ratio is one of the oldest and the most popular measures of the performance of assets allocation portfolios. This ratio is sensitive to the selected sample. A common approach in portfolio evaluation is the calculation of the one-period investment Sharpe ratio [Levy 1972]. It is suggested in the literature that for *n*-period investment the scaling factor should be taken into account. In empirical analysis researchers and experts usually calculate only the one-period Sharpe ratio [Kim, In 2005]. The portfolio managers make decisions over different time periods, and they especially concentrate on the performance at the end of the clearing period. So the problem is how to create the performance measure resistant to above restrictions. The main purpose of this paper is to investigate the usefulness of wavelet analysis for evaluating the performance of mutual funds.

The starting point in applied methodology is the Sharpe ratio, defined as the level of the expected excess of return of portfolio per unit of risk associated with the portfolio. Then wavelet analysis is suggested, in terms of wavelet and scaling filters. Such an approach allows for decomposing the unconditional variance into different time scales, which means different sample windows. The advantage of the approach is that it enables the analysis of the non-normally distributed portfolio returns [Bruzda 2003]. The results of the empirical analysis of mutual fund portfolios, operating in the financial market in Poland, support the fact that the application of the wavelet analysis in multihorizon evaluation of performance enables providing more useful information about behaviour of portfolios.

PORTFOLIO MANAGEMENT AND INVESTMENT HORIZON

Asset allocation is a process that identifies the optimal portfolio for a particular investor over a given investment horizon. An investment horizon depends on when and how much profit the investor expects in the future. This assumption implies that the duration of the investment horizon influences the optimal investment strategy. Investment horizon identifies the total duration that an investor/portfolio manager expects to hold the portfolio. The investment horizon is used to determine the investor's income needs and desired risk exposure. In general, the shorter the investor’s horizon, the lower the acceptance for a given profit. According to the above, in portfolio performance evaluation process the investment horizon should be taken into account.

Table 1. The ranks of mutual funds of Legg Mason and WIG according to the Sharpe ratio

Duration (year)	Fund type				WIG
	MIXED	STOCK	BOND	MONEY	
1	3	2	4	5	1
2	4	2	3	5	1
3	4	2	3	5	1
4	4	1	3	5	2
5	4	1	3	5	2
6	3	1	4	5	2
7	4	1	3	5	2
8	3	1	4	5	2
9	4	1	3	5	2
10	4	1	3	5	2

Source: Author’s calculation

There are many portfolio performance ratios in use [Cogneau, Hubner 2009a, 2009b]. Their disadvantage is that they refer to only one period of time. It is obvious that they change and consequently the rank of portfolio changes as well. Table 1 presents the ranks of market index and four mutual funds with different investment style according to the Sharpe ratio. It is interesting to point out that the market index WIG was in the lead if the portfolio was held up to 3 years. After that the stock fund was the leader. Also, mixed fund and the bond fund switched in position a few times.

The Sharpe ratio is the oldest and the most popular measure of the performance of a portfolio in assets allocation. It was proposed by Sharpe as a reward to variability ratio in 1966 [Sharpe 1966]:

$$S_p = \frac{E\left[r_p - r_f\right]}{\sqrt{E\left[r_p\right]^2}} \tag{1}$$

where: r_p is the rate of return of the portfolio, r_f is the return of the risk-free rate, $E[r_p-r_f]$ is the expected excess returns and $E[r_p]^2$ is the risk of portfolio measured by variance of the portfolio return. After Sharpe's 1994 revision the measure took the following formula [Sharpe 1994]:

$$S_p = \frac{E\left[r_p - r_f\right]}{\sqrt{E\left[r_p - r_f\right]^2}} \tag{2}$$

The Sharpe ratio for portfolio (S_p) describes the share of units of the expected excess return per unit of risk. The ratio is in common use mainly because of simplicity of calculation, interpretation and possibility of ranking of portfolios. Additionally, the Sharpe ratio should be applied only if the rates of return are normally, identically and independently distributed and only when the average excess return is positive. Also, the value of the ratio is sensitive to the range of the sample selected for an analysis. The Sharpe ratio is sensitive to the window of the selected sample, but a common approach in portfolio evaluation is calculation of the one-period investment Sharpe ratio. From the time horizon point of view it is necessary to point out that the fact that the relation of risk premium to the level of risk changes in time that comes from structural changes in the capital market should be taken into account. As Levy [1972] notes, the Sharpe ratio is strictly dependent on the investment horizon, so it is very important to include it in the evaluation of portfolio performance.

In the Capital Asset Pricing Theory interpretation of the utility is as follows: investors look for such investment that maximized their utility wealth defined as $U=E(R_p)-A\sigma_p^2$, where $E(R_p)$ is the expected rate of return of portfolio, σ_p^2 is the variance of rate of returns of portfolio, A is the risk-aversion coefficient [Berk 1997]. Maximization of such defined utility is equivalent to maximization the Sharpe ratio given by $S_p=E(R_p-R_f)/\sigma_p$. In the formula an ex ante approach appears. An ex post version is possible as the historical average rate of return and it's standard deviation is applied.

If rates of return of portfolios have finite expected rate of return and variance and the assumption of normality of distribution of returns is fulfilled, then the Sharpe ratio estimator is a function of expected return and the variance of returns of portfolios [Lo 2002]. The asymptotic distribution of the Sharpe ratio is as follows:

$$\sqrt{T}\left(\hat{S}_p - \bar{S}_p\right) \overset{a}{\sim} N\left(0, V_{iid}\right) \tag{3}$$

The asymptotic variance V is weighted average of the asymptotic variances of expected values and variances:

$$V_{iid} = \left(\frac{\partial g}{\partial \mu}\right)^2 \sigma^2 + \left(\frac{\partial g}{\partial \sigma^2}\right)^2 2\sigma^4 \tag{4}$$

Where g is the function of the Sharpe ratio defined as $g(\mu,\sigma^2)$. The asymptotic variance of the Sharpe ratio can be calculated as follows [Lo, 2002]:

$$V_{iid} = 1 + \frac{1}{2} S_p^2 \tag{5}$$

And the standard error of the Sharpe ratio estimator [Lo, 2002]:

$$SE\left(S_p\right) = \sqrt{\left(1 + \frac{1}{2} S_p^2\right) / T} \tag{6}$$

In the formula (6) the number of periods is included.

It is suggested in the literature that for T-period investment scaling factor should be taken into account, but in empirical analysis researchers and experts usually calculate only one-period Sharpe ratio [Kim, In 2005]. The portfolio managers make decisions over different time periods and they especially focus on the performance at the end of the clearing period. So the problem is how to create the performance measure resistant to the above restrictions. Kim and In proposed the wavelet analysis as a tool for measurement of portfolio performance. That approach allows for decomposition of the unconditional variance into a different time scale, which means different sample windows. The advantage of the approach is the possibility of analysis of the non-normally distributed portfolio returns and non-stationary series.

WAVELET ANALYSIS

Wavelet analysis is a natural tool available to investigate the disaggregation of performance into various time scales, as it enables decomposing of the data on a

scale-by-scale basis. Wavelet analysis is an approach used to evaluating the portfolio performance which enables analysis the multihorizon Sharpe ratio. The great advantage of this approach is a possibility to decompose data into several time scales so we can observe which investment horizons are important contributors to time series variance. We can also construct the local average of the portfolio returns over each scale. These properties provide an effective way of constructing the multihorizon Sharpe ratio.
Wavelet analysis is a kind of frequency analysis, in which the process is represented in both: the time and the frequency domain. This analysis is a technique of windows, the size of which can increase or decrease depending on the fluctuation (long-or short-term analysis) [Bruzda 2003]. Due to its local nature the wavelet analysis is appropriate for testing non-stationary processes. Wavelet analysis, also known as a filtering technique, provides a tool that allows taking the dynamic of economic/financial time series into account [Gencay et al. 2002].

The purpose of the wavelet analysis is to decompose process into components, which are shifted and scaled versions of the basic function (called the mother wavelet). Wavelet analysis has a time-scale nature, which means that there is correspondence between the small scale and high frequency as well as large scale and low frequency [Bruzda 2003]. The Haar wavelet is the simplest dyadic wavelet. The Haar wavelet can be used to decompose a time series into two components: a high-frequency oscillations represent deviations from the time trend and the coefficients of the smoothed time trend. The decomposition is performed by using two filters called (quadrature) mirror filters [Bruzda 2003]: a low-pass filter (coarse representation), high-pass filter (detailed representation).

The wavelet transformation is able to capture all the information in a time series and associate it with a specific time horizon and locations in time [Gencay et al. 2002]. Kim and In [2005] proposed to use the wavelet analysis as a portfolio performance measure. This approach allows for decomposition of the unconditional variance of excess returns on various scales. The Sharpe ratio for different scales reflects the assessment of the effectiveness of portfolio management for different horizons. An additional advantage of using wavelet analysis is that is not necessary to fulfil the assumptions of normality and stationarity of returns.

Multihorizon Sharpe ratio is created in the same way as its standard version (1). The main difference is the application of the wavelet variance ($\sigma_p^2(k)$) and the local average of the excess rate of return ($\hat{R}_p(k)$) for the scale k:

$$WS_p = \frac{\overline{R}_p(k)}{\sqrt{\sigma_p^2(k)}} \tag{7}$$

The application of the wavelet analysis is the evaluation of the results of portfolio management which assumes treating returns as signals. So it is possible to analyse results for different frequencies and investment horizons if the time series is decomposed according to the wavelet transformation.

DATA AND EMPIRICAL ANALYSIS

The sample covers 256 weeks, which comes from $2^8=256$ (the dyadic wavelet). According to the different investment strategies four types of funds of TFI Legg Mason were selected: mixed, stock, bond, money. The sample included weekly data from January 2008 to December 2013. Also the WIG index and the weight average return from treasury bills were applied. The table below includes the average yearly returns from funds and their risk level measured by annualized standard deviation. Also statistics of the normality and stationarity tests are given. The highest average yearly return for the stock fund, 5.06%, with a high level of risk, 20.07%, can be noticed. The mixed fund reached lower return than the stock one but the risk level was almost twice lower. Yearly average return for the money fund was almost 3%, with the risk level of 0.54%. The normality (JB) test shows that the returns were not normally distributed. The results of ADF test confirm stationarity of returns (nonexistence of the unit root).

Table 2. Descriptive statistics of Legg Mason mutual fund return and diagnostic tests.

Fund type	Annual rate of return	Annualized risk	Normality test JB	Unit root test ADF
MIXED	3.65%	10.62%	51.15*	-4.393*
STOCK	5.06%	20.07%	54.31*	-4.088*
BOND	2.90%	3.75%	56.66*	-4.099*
MONEY	2.79%	0.54%	190.37*	-4.194*

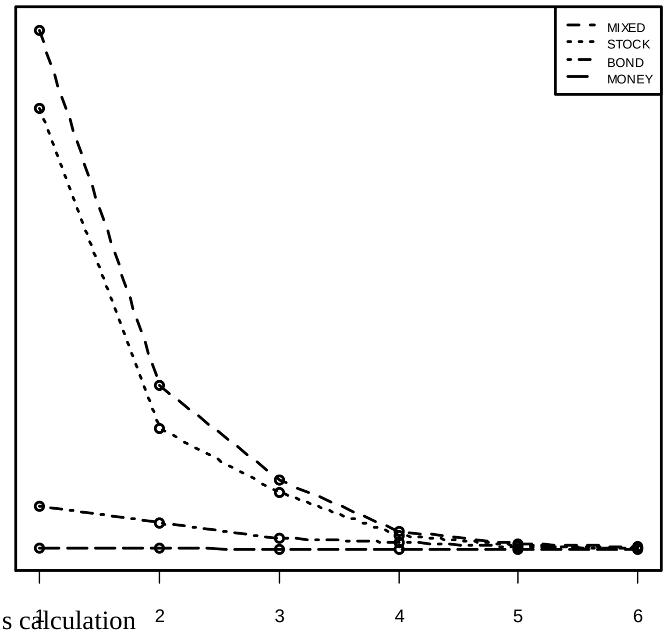
Note: JB indicates the Jarque-Bera statistic for normality of rate of return distributed as χ^2 with 2 degree of freedom; ADF indicates the Augmented Dickey-Fuller statistics for unit root; * - significant at 5% of significance level

Source: Author's calculation.

The characteristics of distributions of returns from funds point out that the application of the standard Sharpe ratio can lead to misleading conclusions of portfolio management results. As a consequence, in the next step of the analysis the multihorizon Sharpe ratio is applied. Time series of prices of units of investments funds were decomposed using the Haar wavelet transformat for six scales, representing filters with different frequencies. Then, for each transformed series the local means and wavelet variance were calculated. The last measure was applied for finding the multihorizon Sharpe ratios. The Sharpe ratios at different scales represent the performance measures of portfolio at various frequencies (various time scales). Scale 1 represents a period from 2 to 4 weeks, scale 2 represents 4 to 8, scale 3 represents 8 to 16, scale 4 represents 16 to 32, scale 5 represents 32 to 64, scale 6 represents 64 to 128. We use a simple Haar wavelet, which is the first wavelet filter. Such simple wavelet allows for receiving two types of elements of the time series. The first one (d_k, $k=1,..,6$), which captures the higher-frequency oscillations, represents increasingly fine scale deviations' from the smooth trend, and the second (a_k, $k=1,..,6$), represents the smooth coefficients that capture the trend.

We analyzed the variance of the mutual fund rates of return against various time scales. Figure 1 presents that the variance of all funds decline as the wavelet scale increases and all curves peak at scale 5. We can notice a big difference between curves at scale 1 and almost no difference at scales 5 and 6. It implies that investor with a short investment horizon reacts to every fluctuation in analysed rates of return, while the investor with a medium and long horizon would react much slower. In the long-run the investment risk is significantly lower. Figure 1 shows a pattern similar to that presented by Siegiel [2008] and Kim, In [2005]. They suggest that the standard deviation of average return falls with the square root of the length of the holding period, due to the random walk of the asset returns [Kim, In 2005].

Figure 1. Estimated Wavelet Variance



Source: Author's calculation

Multihorizon Sharpe ratios, using treasury bills as a benchmark, are presented in table 3. The value of Sharpe ratio for each fund rises as the holding period lengthens. The Sharpe ratio for a mixed fund is -0.08 for the first wavelet scale which is a 2- to 4-week period and increases to 231.88 for the longest wavelet scale 64- to 128-week period. We can conclude that the Sharpe ratio is not time-consistent. So when the Sharpe ratio is applied as a performance measure, then the investment horizon should be take into account.

Table 3. Multihorizon Sharpe ratio in the period 2003-2013

Scale	MIXED	STOCK	BOND	MONEY
Scale 1	-0.80	-0.50	0.45	0.56
Scale 2	1.67	-0.29	1.02	1.40
Scale 3	2.75	1.48	3.65	4.64
Scale 4	7.27	16.33	16.23	14.62
Scale 5	21.57	35.74	21.14	38.86
Scale 6	231.88	241.63	145.25	113.42

Source: Author’s calculation

SUMMARY AND REMARKS

Measuring the quality of portfolio management is still at the stage of improvement and the search for objective measures of evaluation of the results. The importance of the idea of the quality management as an important element in the future prosperity of society in the context of an aging population and the associated challenges facing the pension system should be emphasized. It is therefore important to pay attention to the quality and relevance of the indicator which is a commonly used measure of assessing the quality of portfolio management. Among these measures the most popular one is Sharpe Ratio, which is commonly used selectively for a single period of time only. The study of assessing the quality of the portfolio management used multihorizon Sharpe Ratio. The proposed measure of the effectiveness uses wavelet analysis to determine the average excess returns and portfolio risk for a variety of scales. The obtained results show that it is important to take the investment horizon into account, due to the fact that for a variety of scales which were obtained for the studied arrangement of funds.

The results indicate that the variance of rate of return in analyzed funds decline as the wavelet scale increases and this implies that an investor with a short horizon reacts to every fluctuation in an observed rate of return, while an investor with a medium and long horizon, doesn’t react to fluctuation in the long-run and risk is significantly lower. Furthermore, the value of Sharpe ratio for each fund rises as the period holding lengthens, which implies that the Sharpe ratio is not time-consistent.

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