QUANTITATIVE METHODS IN ECONOMICS
Vol. XV, No. 2, 2014, pp. 84 –93

MODIFICATIONS OF THE MAXIMIN JOY CRITERION FOR DECISION MAKING UNDER UNCERTAINTY

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Abstract: Decision making under uncertainty (DMU) occurs when the decision maker (DM) has to choose an appropriate alternative on the basis of a set of decisions and a set of scenarios (with an unknown probability distribution). The author suggests two modifications of the maximin joy criterion (MJC) - one of the classical decision rules used in DMU by pessimists searching for an optimal pure strategy. The goal of the alterations for MJC is to accentuate more effectively the position of particular outcomes in comparison with other outcomes connected with a given scenario.

Keywords: decision making under uncertainty, optimal pure strategy, maximin joy criterion, payoff matrix

INTRODUCTION

According to the Knightian definition [Knight 1921], decision making under uncertainty (DMU), in contrast to decision making under certainty (DMC) or risk (DMR), is characterized by a situation where future factors are neither deterministic nor probabilistic at the time of the decision. Actually the decision maker (DM) has to choose the appropriate alternative (decision, strategy) on the basis of some scenarios (events, states of nature) whose probabilities are not known [Groenewald and Pretorius 2011, Neumman and Morgenstern 1944, Sikora 2008][1]. Usually the DM can describe the problem on the basis of a payoff matrix representing possible states of nature, decisions and outcomes. There are many decision rules for pure and mixed strategy searching. They differ one from another

[1] The forth category, decision making under partial information, is characterized by probabilities not known completely [Cannon and Kmietowicz 1974, Weber 1987].

with respect to DM's attitude towards risk. Additionally, some classical decision rules and a considerable number of extended decision rules take into account how particular outcomes assigned to alternatives are ordered in the payoff matrix and what the position of a given result is in comparison with other payoffs of the same state of nature. This feature is rather an advantage since it enables to obtain different rankings depending on the status of particular scenarios (dominated or not) and depending on the superiority of a given outcome to other values of the same scenario. The maximin joy criterion (MJC) is one of the classical procedures whose rankings vary after transposing the profits related to particular alternatives. In this paper we present the benefits of this rule and suggest two possible modifications whose aim is to accentuate more effectively the rank of a given outcome in comparison with other results connected with the same scenario. The remainder of this paper is organized as follows. Section DECISION MAKING UNDER UNCERTAINTY deals with the main features of DMU. Section THE MAXIMIN JOY CRITERION briefly describes the maximin joy criterion. Section POSSIBLE MODIFICATIONS OF THE MAXIMIN JOY CRITERION is devoted to the presentation and illustration of possible modifications of MJC. Conclusions are gathered in the last Section.

DECISION MAKING UNDER UNCERTAINTY

As it was mentioned in the introduction, DMU may be presented by means of a profit matrix where m is the number of mutually exclusive scenarios (S_1, …, S_m), n denotes the number of decisions (D_1, …, D_n) and a_{ij} stands for the profit connected with scenario S_i and alternative D_j. In this paper we assume that the distribution of payoffs related to a given decision is discrete. We only discuss one-criterion decision problems and focus on optimal pure strategy searching. A pure strategy is a solution assuming that the DM chooses only one decision. Meanwhile the mixed strategy allows him or her to select a weighted combination of several alternatives [Gaspars-Wieloch 2014b, Puppe and Schlag 2009, Sikora 2008].

Among classical decision (CD) rules designed for DMU one can enumerate the Wald's criterion, maximax criterion, Hurwicz's criterion, Savage's criterion, maximin joy criterion and Bayes' (Laplace's) criterion [Wald 1950, Hurwicz 1952, Savage 1961, Hayashi 2008]. The literature also offers many extensions of these methods, e.g. [Basili et al. 2008, Basili and Chateauneuf 2011, de Finetti 1974, Ellsberg 2001, Etner et al. 2012, Gaspars 2007, Gaspars-Wieloch 2012, 2013, 2014a, 2014c, 2014d, 2015, Ghirardato et al. 2004, Gilboa 2009, Gilboa and Schmeidler 1989, Marinacci 2002, Piasecki 1990, Schmeidler 1986, Tversky and Kahneman 1992], which can be named extended decision (ED) rules. The majority of them refers to the probability calculus (e.g. expected utility maximization, α-maximin expected utility, restricted Bayes/Hurwicz, cumulative prospect theory, Choquet expected utility), which is rather characteristic of DMR where the likelihood is given – let us remind that according to the Knight's definition the uncertainty occurs when we do not know the probabilities of particular scenarios [Knight 1921, Chateauneuf et al. 2008, Domurat and Tyszka 2004].

THE MAXIMIN JOY CRITERION

The concept of the maximin joy criterion [Hayashi 2008] is extremely similar to the reasoning characteristic of the Savage's procedure which uses a regret matrix and minimizes the maximal opportunity loss, but here, instead of a regret table, a matrix of relative profits is applied.

The first step consists in computing an index for each alternative according to Equation (1) which represents the worst relative profit connected with D_j:

$$J_j = \min_i \{t_{ij}\} \qquad j = 1,\ldots,n \tag{1}$$

$$t_{ij} = a_{ij} - \min_j \{a_{ij}\} \qquad i = 1,\ldots,m;\ j = 1,\ldots,n \tag{2}$$

where a_{ij} is the payoff related to decision D_j and scenario S_i. Symbol t_{ij} denotes the relative profit which is always a non-negative number.

The second step is to choose the alternative that has the highest index:

$$j^* = \arg\max_j J_j \tag{3}$$

Table 1. Payoff matrices for cases I, II and III

Case	Case I			Case II			Case III		
Scen./Dec.	D1	D2	D3	D1	D2	D3	D1	D2	D3
S1	5	1	3	-6	1	3	2	0	3
S2	2	-4	8	2	-4	2	5	1	8
S3	-6	4	-10	5	4	8	4	4	-10
S4	4	3	2	1	3	-10	-6	-4	2
S5	1	0	5	4	0	5	1	3	5

Source: created by the Author

Table 2. Relative payoff matrices for cases I, II and III

Case	Case I			Case II			Case III		
Scen./Dec.	D1	D2	D3	D1	D2	D3	D1	D2	D3
S1	4	0	2	0	7	9	2	0	3
S2	6	0	12	6	0	6	4	0	7
S3	4	14	0	1	0	4	14	14	0
S4	2	1	0	11	13	0	0	2	8
S5	1	0	5	4	0	5	0	2	4
J_j	1	0	0	0	0	0	0	0	0
Ranking	I	II	II	I	I	I	I	I	I

Source: own calculations

The MJC is a decision rule for pessimists, i.e. for people representing a risk-averse behavior. Notice that in this context we do no treat risk as a situation where the probability distribution of each parameter of the decision problem is known, but we mean the possibility that some bad circumstances will happen (losses or low outcomes). The alternative recommended by MJC may be executed only once (see one-shot decisions [Guo 2011]). Despite the fact that the steps of the Savage's rule are very similar to the steps of MJC, there are cases where rankings generated by both methods are different [Gaspars-Wieloch 2015]. The goal of MJC is to show the superiority of particular outcomes connected with a given scenario to its worst result, while within the framework of the Savage's rule the target is to demonstrate the inferiority of particular payoffs related to a state of nature to its best result.

The serious weak point of MJC concerns final rankings. That is, imagine that each decision is the worst for at least one state of nature. In such a case one can find at least one zero in each column (for each decision) of the relative profit table, which means that the MJC's index is equal to zero for each alternative. And then, all the strategies are treaded as optimal solutions (they all obtain the first rank in the ranking), which certainly does not facilitate the decision making process. The problem aforementioned is illustrated in Tables 1-2. Table 1 presents three payoff matrices for three decision problems. Note that these cases are very similar. In each example the sets of payoffs for particular alternatives are identical. Outcomes have only different positions. Table 2 demonstrates relative payoff matrices and final indices for each decision. MJC indices are almost always equal to zero, which does not allow the DM to obtain a reliable ranking. When applying MJC, the size of particular relative profits may be totally ignored if each decision is the worst for at least one event. That feature of MJC is quite alarming. Notice that this phenomenon does not occur when the Savage's rule is used. For our three cases the rankings would be as follows: D_1(10), D_2(12), D_3(14); D_2(6), D_1(9), D_3(13) and D_2(7), D_1(8), D_3(14) respectively (numbers in brackets indicate the Savage's indices). Therefore, in the next section, two possible amendments are proposed.

POSSIBLE MODIFICATIONS OF THE MAXIMIN JOY CRITERION

The alterations of MJC may result from the observation that the relative profit should consider the superiority of a given outcome to all remaining outcomes of the same scenario, not merely to the worst one. If so, the modified relative profit will contain a more precise information about the position of particular results in comparison with other payoffs. The following subsections describe two approaches enabling one to include this information in the final index.

Dominance joy criterion (DJC)

In the first procedure (the dominance joy criterion), instead of computing the relative profit, the number of "dominance cases" is calculated, i.e. the number of payoffs lower than a given outcome within the framework of a scenario:

1. Calculate an index for each alternative according to Equation (4) which represents the sum of all "dominance cases" connected with D_j:

$$J_j^D = \sum_{i=1}^{m} t_{ij}^D \qquad j = 1,...,n \tag{4}$$

$$t_{ij}^D = n - \max\{p^i(a_{ij})\} \qquad i = 1,...,m;\ j = 1,...,n \tag{5}$$

where J^D_j denotes the dominance joy criterion, t^D_{ij} stands for the sum of "dominance cases" related to a_{ij} and $p^i(a_{ij})$ is the position of payoff a_{ij} in the non-increasing sequence of all results connected with scenario S_i (when a_{ij} is equal to at least one other payoff concerning a given event, then it is suggested to choose the farthest position of this payoff in the sequence, Equation 5).

2. Choose the alternative that has the highest index:

$$j^* = \arg\max_j J_j^D \tag{6}$$

Cumulative maximin joy criterion (CMJC)

Now let us have a look at the second possible procedure (the cumulative maximin joy criterion), where instead of computing the number of "dominance cases", the sum of all relative profits concerning a given payoff is calculated and further steps do not differ from those used in the original MJC:

1. Calculate an index for each alternative according to Equation (7) which represents the worst cumulative relative profit (CRP) connected with D_j:

$$J_j^C = \min_i\{t_{ij}^C\} \qquad j = 1,...,n \tag{7}$$

$$t_{ij}^C = n \cdot a_{ij} - \sum_{j=1}^{n} a_{ij} \qquad i = 1,...,m;\ j = 1,...,n \tag{8}$$

where J^C_j denotes the cumulative maximin joy criterion and t^C_{ij} stands for the cumulative relative profit.

2. Choose the alternative that has the highest index:

$$j^* = \arg\max_j J_j^C \tag{9}$$

Results and discussion

As it can be observed, in the case of the dominance joy criterion we do not analyze differences between the lowest outcome and the remaining outcomes belonging to the same state of nature, but we check the rank of each payoff in the non-increasing sequence of all results related to this scenario.

The modified relative payoff matrix for the cumulative maximin joy criterion also allows the DM to take into account the position of a given outcome in comparison with other outcomes of the same scenario, but in this case, just like in the original MJC, the superiority is measured by means of relative profits rather than by dominance. Nevertheless, this time the way the relative profit is calculated differs from that used in MJC, because within the framework of CMJC a given outcome is compared with all outcomes of the same scenario (not only with the lowest one) and those differences are summed up. If the CRPs are computed in such a way, condition (10) is always fulfilled. Additionally, irrespective of how payoffs related to a given decision are distributed in the matrix (see the three cases presented in Table 1), the sum of all cumulative relative payoffs concerning this decision does never change (analyze Equation 11 and Table 4). Thus, for CMJC we can not use an index being the sum of all cumulative relative profits (instead of the minimal cumulative relative profit) as a decision criterion, because such a measure will not vary depending on the position of particular outcomes in the payoff table.

$$\sum_{j=1}^{n} t_{ij}^{C} = (a_{i1} - a_{i1} + a_{i1} - a_{i2} + \ldots + a_{i1} - a_{in}) + (a_{i2} - a_{i1} + a_{i2} - a_{i2} + \ldots + a_{i2} - a_{in}) + \\ + \ldots + (a_{in} - a_{i1} + a_{in} - a_{i2} + \ldots + a_{in} - a_{in}) = 0 \qquad i = 1,\ldots,m \tag{10}$$

$$\sum_{i=1}^{m} t_{ij}^{C} = \left(n \cdot a_{1j} - \sum_{j=1}^{n} a_{1j} \right) + \left(n \cdot a_{2j} - \sum_{j=1}^{n} a_{2j} \right) + \ldots + \left(n \cdot a_{mj} - \sum_{j=1}^{n} a_{mj} \right) \\ = n \sum_{i=1}^{m} a_{ij} - \left(\sum_{j=1}^{n} a_{1j} + \sum_{j=1}^{n} a_{2j} + \ldots + \sum_{j=1}^{n} a_{mj} \right) \qquad j = 1,\ldots,n \tag{11}$$

Table 3. Dominance case matrices for cases I, II and III

Case	Case I			Case II			Case III		
Scen./Dec.	D1	D2	D3	D1	D2	D3	D1	D2	D3
S1	2	0	1	0	1	2	1	0	2
S2	1	0	2	1	0	1	1	0	2
S3	1	2	0	1	0	2	1	1	0
S4	2	1	0	1	2	0	0	1	2
S5	1	0	2	1	0	2	0	1	2
J^{D}_{j}	7	3	5	4	3	7	3	3	8
Ranking	I	III	II	II	III	I	II	II	I

Source: own calculations

Table 4. Cumulative relative payoff matrices for cases I, II and III

Case	Case I			Case II			Case III		
Scen./Dec.	D1	D2	D3	D1	D2	D3	D1	D2	D3
S1	6	-6	0	-16	5	11	1	-5	4
S2	0	-18	18	6	-12	6	1	-11	10
S3	-6	24	-18	-2	-5	7	14	14	-28
S4	3	0	-3	9	15	-24	-10	-4	14
S5	-3	-6	9	3	-9	6	-6	0	6
Sum of t^C_{ij}	0	-6	6	0	-6	6	0	-6	6
J^C_j	-6	-18	-18	-16	-12	-24	-10	-11	-28
Ranking	I	II	II	II	I	III	I	II	III

Source: own calculations

Tables 3 and 4 contain intermediate calculations and final indices for the three cases described in Table 1 according to both recommended procedures. Thanks to the suggested modifications it is possible to obtain varied indices for particular alternatives (compare the measure values in Tables 2, 3 and 4). Rankings generated by these decision rules depend on the structure of the payoff matrix and they do not have the flaw characteristic of the original MJC, i.e. the indices of particular actions are equal to different values (compare with Table 2).

At first glance DJC seems to be logic and easy in use, but it is worth emphasizing that the analysis of "dominance cases" does not reveal the size of this dominance. For instance, we see that outcome "2" dominates "-4" and "-4" dominates "-6" (case III, scenario S4), but these two examples of dominance are totally different. We would say that the first dominance is more significant than the second one. Meanwhile, when the DJC is applied, both dominances receive the same weight equal to 1, which may be a little questionable and controversial.

Therefore, if the size of dominance is a crucial factor for the DM, it is recommended to use CMJC, where "dominance cases" are computed more conscientiously. In the example aforementioned (case III, scenario S4) the superiority (measured by cumulative relative profits) for the outcome "2", "-4" and "-6" equals to (2-(-6))+(2-(-4))=14, (-4-(-6))+(-4-2)=-4 and (-6-(-4))+(-6-2)=-10 respectively. Note that the change of the decision rule (from DJC to CMJC) has an enormous impact on final rankings. For instance, in case II decision D3 has the first rank according to DJC and the last rank when CMJC is applied. In our example CMJC rankings resemble Savage's orders. The choice of the procedure (DJC or CMJC) should depend on the DM's preferences, i.e. whether he/she only analyzes the order of payoffs related to a given state of nature or whether the distance between particular results is of great importance for him/her. Note that, as a matter of fact, the first modification of MJC (DJC) can be reduced to the simplest case of stochastic dominance, i.e. the state-by-state dominance (or statewise dominance).

Table 5. Payoff matrices for cases IV and V.

Case	Case IV				Case V			
Scen./Dec.	D1	D2	D3	D4	D1	D2	D3	D4
S1	11	9	9.5	9	5	9	9.5	8
S2	8	9	7.5	8	8	6	7.5	9
S3	11	10	7.5	8	11	10	7.5	8
S4	5	6	8	8	11	9	8	8

Source: created by the Author

Let us analyze one more example (cases IV and V) where payoff matrices differ one from another only in terms of the structure of profits (Tables 5-6). This time, we present rankings for all classical rules and two new modifications of MJC. The general conclusion that can be made when comparing all results (Tables 1-6) is as follows: DJC is rather devoted to optimists (its rankings are similar to maximax rankings) since it considers only the superiority of particular payoffs to other outcomes, whereas CMJC is rather designed for pessimists (its rankings resemble Wald's rankings) because in this case that superiority is conscientiously measured.

Table 6. Rankings for cases IV and V.

Case	Case IV				Case V			
Crit. / Dec.	D1	D2	D3	D4	D1	D2	D3	D4
Wald criterion	IV	III	II	I	IV	III	II	I
Maximax criterion	I	II	III	IV	I	II	III	IV
Bayes criterion	I	II	IV	III	I	II	IV	III
Hurwicz crit. (α=0.4)	II	III	I	II	II	III	I	II
Savage criterion	II	I	III	II	III	I	II	I
Maximin joy criterion	I	I	I	I	I	I	I	I
Dominance joy criterion	I	II	III	III	I	II	IV	III
Cumulative maximin joy criterion	IV	I	III	II	III	II	II	I

Source: own calculations

CONCLUSIONS

The factor demonstrating the significance of the rank of a payoff in the set of all outcomes in a given matrix is the fact that people consider not only the value they receive, but also the value received by others [Frank 1997]. Some classical rules and many extended procedures designed for DMU take into account this aspect and for that reason they are quite often applied by DMs. In this paper we suggest two possible alterations for the maximin joy criterion. They may interest people who want to know how their outcome compares with other results, not only with the worst one (maximin joy criterion), the best one (Savage's rule) or the reference point (prospect theory). The proposed methods focus on payoffs, but the concept of dominance or cumulative relative profits can also be used for utilities.

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