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DEFAULT PREDICTION FOR SME USING DISCRIMINANT AND SURVIVAL MODELS, EVIDENCE FROM POLISH MARKET

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Abstract: The aim of this paper was to compare the new technique (survival analysis) used in the credit risk models with the traditional one (discriminant analysis), analyse the strengths and weaknesses of both methods and their usage in practice. This study attempts to use macroeconomic data to build models and examine its impact to the prediction. For this purpose, a number of models was built on the basis of the sample of 1547 enterprises including 494 defaults. The time range covered by sample was 2002-2012.

Keywords: survival analysis, discriminant analysis, macro variables, rating model

INTRODUCTION

Credit risk is the most important type of risk to which banks are exposed. This is due to their role as financial intermediaries. Also different factors have an impact on the credit risk level in banks, including the credit conditions and creditworthiness methods. The main factor limiting the risk of the credit portfolio is good economic and financial situation of the clients, in this case - company. Therefore, a special attention to the proper assessment of customers' creditworthiness should be paid and then the subsequent monitoring of their financial situation should be carried out.

The wide range and increasing availability of the credit in modern societies have led to the inordinate indebtedness of many borrowers (Allen and Rose, 2006). Since the problem of insolvency is getting bigger and bigger, the interest in the effective management of customer debt repayment is also growing.

The recent developments in the financial literature demonstrate the various questions posed in front of the financial analysts who build models based on survival analysis. Over the past 25 years, this method was implemented and successfully used in many fields of finance. In 1989, Altman suggested measuring the expected mortality of bonds and, consequently, loss rates in a similar way to that which actuaries use to evaluate human mortality. In 2000, Altman and Suggitt (2000) applied this analysis to assess the risk of corporate loans. In 1998, Lando estimated the bond default time, using a proportional hazard model for survival analysis, and applied macroeconomic variables as predictors (Lando 1998). The same approach was applied to modelling credit risk in the valuation of bonds and other financial instruments by Pierides (1997).

However it is recognized that the idea of the application of survival analysis in the credit risk models (credit scoring) was first used by Narain in 1992, and further developed by Carling et al. (1998), Stepanova and Thomas (2002), Allen and Rose (2006), Malik and Thomas (2006). In all these papers the parametric or semiparametric regression techniques for modelling the time to default (duration models) were used.

An interesting research was done by Nunes at al. (2014)[1] where authors using probit regressions and on the basis of two research samples: 1589 family-owned SMEs and 485 non family-owned SMEs checked whether there are significant differences between family-owned SMEs and non-family-owned SMEs for determinants of survival. The results obtained show the existence of significant differences between these two types of companies for the determinants of survival. In the context of family-owned SMEs, authors think that: size, age and R&D expenditure are neither positive nor restrictive determinants of survival; cash flow and labour productivity are positive determinants of survival; and, debt, interest paid and risk are restrictive determinants of survival. In case of the second group of SMEs, size, age, cash flow, debt and R&D expenditure are positive determinants of survival, with interest paid, risk and labour productivity being neither positive nor restrictive determinants of survival.

In the paper written by Glennon and Nigro (2005)[2], it was presented that the default behaviour of the analysed loans is time sensitive. The likelihood of default is pretty high at the beginning, then peaks in the second year, and declines thereafter. Authors used a discrete-time hazard model and from the received results showed that the likelihood of default is conditional on customer, lender, loan characteristics, and changes in economy.

[1] P. M. Nunes, Z. Serrasqueiro, J.V. da Silva: "Family-owned and non family-owned SMEs: empirical evidence of survival determinants ",Economics and Business Letters, 3(1), pp.68-76, 2014

[2] D.C.Glennon, P.Nigro: „Measuring the Default Risk of Small Business Loans: A Survival Analysis Approach", Journal of Money, Credit, and Banking, Volume 37, Number 5, October 2005, pp. 923-947.

Giovannetti et al. (2013) wanted to check the thesis that the firms' survival is often seen as crucial for economic growth and competitiveness[3]. In their research they considered business demography of Italian firms, using an original database. They considered consider the size effect, technology, trade, FDIs and innovation on companies' survival probability. The results obtained suggest that size and technological level positively affect the likelihood of firms' survival. It was also interesting that the internationalized firms showed higher failure risk: on average competition is stronger in international markets, forcing firms to be more efficient. However, large internationalized firms were more likely to 'survive'.

Different approach was proposed by Moon and Sohn (2011)[4]. According to authors, the scorecards are often filled-in based on the evaluator's total perception rather than the individuals' scores of which the scorecards are built. Authors proposed a survival model that considers the time to default as well as the total perception scoring phenomenon. Their approach can be used during the decision-making process in various areas of technology, (for example in R&D), alliances, transfers, and loans.

Papers regarding such types of models in Poland have much shorter history. The implementation of the western models to the market of enterprises which function in the transition economy such as Poland failed. It appeared that those models are not successful in conditions of the political and economic changes. Unsatisfactory effects of using foreign models in Polish conditions contributed to developing research into domestic models. The most popular became the models based on discriminatory analysis as it is the case abroad.

We propose a model for companies' prediction based on the survival analysis. This model will be compared with the model using the traditional method, i.e. discriminant analysis.

The paper is structured as follows. Section 2 describes the approaches used to estimate the probability of default. In Section 3 the data set was described. Results of the models are presented in Section 4. Section 5 contains conclusions.

DISCRIMINANT ANALYSIS

Discriminant analysis is used to determine which variables can be used to identify two or more groups from the analyzed data set. It allows identifying these

[3] G. Giovannetti, G. Ricchiuti, M. Velucchi: Size, innovation and internationalization: a survival analysis of Italian firms", Applied Economics,Volume 43, Issue 12, 2011, pp. 1511-1520.

[4] T.H.Moon, S.Y.Sohn: "Survival analysis for technology credit scoring adjusting total perception", Journal of the Operational Research Society (2011) 62, pp.1159–1168.

variables which allows classification of different groups with higher accuracy than the random ones.

The purpose of discriminant analysis is a correct classification of observations into two subspaces defined as groups. Discriminant function is defined as maximization of the distance between subpopulations (groups). In discriminant analysis the classification of units as defaults or non-defaults is based on minimum two explanatory variables. Simultaneously the analysis is carried out taking into account all selected ratios. It is crucial to find out dependencies between variables enabling a correct distinction of entities. In discriminant analysis the dependent variable is qualitative (binary). The classification of entities is based on linear discriminant function. Synthetic ratio arose as a result of applying the model (value of the function) makes it possible to classify the entity. However the discriminant analysis are limited to certain extent. It is possible to apply it when the analysed ratios are normally distributed. It is also necessary to meet the assumption of their independence and completeness. The lack of fulfillment of assumptions influence negatively the classification capacities of the model. Checking whether the assumptions about ratios were fulfilled can be verified by applying relevant tests and statistical procedures.

Discriminant functions, which are used to build multivariate warning bankruptcy models, take different forms – linear, square, etc.

Linear discriminant function takes the following form (Ptak-Chmielewska, Pęczkowski 2009):

$$Z = a_0 + a_1X_1 + a_2X_2 + \ldots + a_nX_n, \tag{1}$$

where:
Z – target (dependent variable),
a_0 – constant,
$a_{1} – a_n$ – parameter estimates (weights),
$X_1, X_2, \ldots, X_n$ – explanatory variables (financial ratios).

The presented discriminant function is also known as a Fisher discriminant function. Parameters a_i, are called discriminatory factors (weights). After determining the discriminant function the next step is to define the limit, allowing for classification of the individual being at a financial risk or not. Usually a mean value of the discriminant function is being determined for each group and then the cut-off value between the means. If the z value for the current company is lower than $Z_{cut\text{-}off}$ then the company is classified as being at bankruptcy risk otherwise is classified as a good company. The model's efficiency is assessed calculating the type I and type II error. The first one determines the percent of companies classified as being at bankruptcy risk, while the second category represents the percentage of companies classified as good ones.

Altman, who presented his model in 1968, is considered to be the precursor of multi-dimensional model. This model is a combination of ratio analysis and statistical method - multivariate discriminant analysis. The author analyzed 22 factors on a group of 66 companies (33 bankrupt and 33 non- bankrupt ones). In the subsequent stages of the analysis he discarded the weaker predictors and the final model included 5 indicators. In 1977, Altman and his team conducted further studies related to the companies' bankruptcy prediction. He analyzed 58 bankrupt and 58 healthy companies. The result is a model consisting of 7 variables without specified weights, and hence a discriminant function was not designated. Prediction of bankruptcy a year before the event reaches 90% and 5 years before the event amounts to 70%.

The next version of Altman's model was developed in 1983. E. I. Altman has made changes in the weights assigned to the variables of the first model. The value of the misclassification error was 6%. The next improvement of the Altman model concerned reducing the impact of economic and industry specificity to the Z index value. Altman models have been developed for companies operating in the U.S. market. Its application for companies operating in other conditions does not give very high discriminatory power.

The Z-Score Model selected for this paper application was based on the following revised model:

$$Z = 0.717 \cdot X_1 + 0.847 \cdot X_2 + 3.107 \cdot X_3 + 0.420 \cdot X_4 + 0.998 \cdot X_5$$

where:

X_1 = Working capital/Total assets

X_2 = Retained Earnings/Total assets

X_3 = Earnings before interest and taxes/Total assets

X_4 = Book value of equity/Book value of total debt

X_5 = Sales/Total assets

Z = Total Index

In the estimation data, the Z-Score Model classified correctly 90.9% of bankrupt firms and 97.0% of the non-bankrupt firms. In all, the data included 66 firms (33 + 33). Thus, the classification accuracy was only slightly less impressive than for the original model. Altman did not test the model on a secondary sample due to lack of a private firm data base. In concluding remarks, Altman (1983) regards the general applicability of his Z-Score Model as debatable. The model did not exclude very large and very small firms, the observation period was quite long, and the analysis included only manufacturing companies. Ideally, development of a bankruptcy predicting model should be done based on homogenous group of bankrupt companies and data as near to the present as possible. Altman advised the analysts interested in practical utilization of the Z-Score Model to be careful.

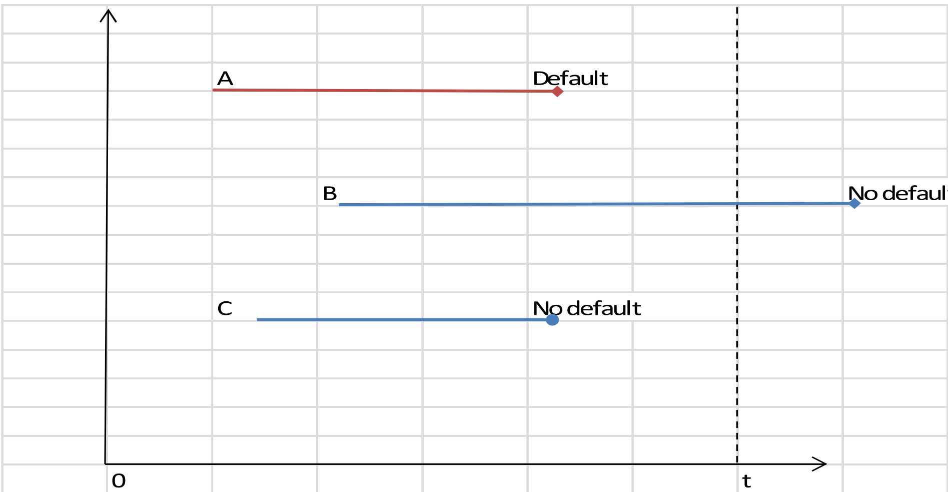
SURVIVAL ANALYSIS

Survival analysis is a collection of statistical procedures for data analysis, for which the analyzed variable is the time of the event. So it is a study in time, counted from the time when a case "came into" observation until the occurrence of the event.

The idea of using the survival analysis to assess credit risk, and more particularly to model PD (Probability of Default), is shown on Figure 1. It shows three cases that may occur in practice during the lifetime of a company.

The first customer (A) defaulted before the end of the credit. In this case, the lifetime of the customer (time to default), is observable during the analysed period. Customers: (B) and (C) present two different situations. In both of them it is not possible to observe the time of default, so the status of them is censored. In case of customer (B) it is only the time from the start of the loan to the end of the study, while in case of customer (C) presents a situation where the end of the loan occurred before default (i.e. early repayment).

Figure 1. Idea of the survival analysis



Source: own elaboration

Survival and hazard function

Two important functions in survival analysis are the survival function and the hazard function. The first function is a continuous function representing the probability that the 'failure time' T of an individual (company in this case) is greater than time t.

$$S(t|x) = P(T > t|X = x) = \int_t^{\infty} f(u|x)du \,, \tag{2}$$

The hazard function $h(t)$ represents the point in time default 'intensity' at time t conditional upon survival up to time t.

$$h(t|x) = \lim_{\Delta t \to 0} \frac{P(t \le T < t + \Delta t | T \ge t, X = x)}{\Delta t} = \frac{f(t|x)}{S(t|x)}, \tag{3}$$

There are many different models used in survival analysis. Models are differentiated according to assumptions about functional form of hazard rate and its variability in time. In practice the most frequently used model is proportional hazards Cox regression model. For this reason this model was presented in more details in this paper.

For Cox regression model the hazard function is given by formula:

$$h(t \mid x_1,...,x_k) = h_0(t)\exp(\alpha_1 x_1 + ... + \alpha_k x_k), \tag{4}$$

where: $h_0(t)$ - means base hazard, parametrically non-specified function of time and $x_1, x_2, \ldots x_k$ - means explanatory variables (including time dependent variables).

Cox proposed also the special type of estimation method called pseudo-likelihood (Cox, 1972). This method divides the likelihood function for proportional hazards model into two parts: first including only information about parameters and second, including information about parameters, and hazard function. Division into two components is justified because first depends only on sequence of events occurrence, does not depend on exact time of occurrence, and the second is 0 and is omitted.

Main advantage of Cox model (and other semi-parametric models) is assessment of many variables (including time dependent variables) influence on the process without necessity of base hazard $h_0(t)$ specification. The main disadvantage of Cox model is hazard proportionality assumption. This assumption impose that for each pair of individuals in any time the hazard rate is fixed. The relative hazard (ranking) for individuals is stable in time.

Despite this limitation of Cox model, it is especially attractive for researchers in case of (Blossfeld and Rohwer, 2002):

- unknown shape of hazard in time;
- no theoretical bases for parameterization;
- no possibility of functional shape of hazard specification;
- main interest is focused on explanatory variables influence on hazard.

Above mentioned advantages in using Cox regression model make this model useful in risk of enterprises' liquidation modeling. The only disadvantage of this model is proportionality assumption which implies fixed proportion of hazard for individuals during the observation time period. This problem may be solved by including additional time dependent variables in the model (like interaction between variable and the time). For checking the proportionality assumption the easy way is to include the interaction with time, the significance of this parameters confirm that the proportionality assumption is violated. In this case model is named non-proportional hazards Cox regression model. Results of Cox model estimation are parameters describing the influence of explanatory variables on the probability of event occurrence and on the base hazard (the same for all individuals, dependent only on time).

DATA DESCRIPTION

The available data comes from one source and covers a period from 2002 to 2010 (2004-2012 for defaults history), so the whole economic cycle was covered and therefore, the condition required for parameter estimation of PD is fulfilled. There are 1053 good and 494 bad companies in the sample. There are 2910 FS[5] in total. The sample was limited to companies with turnover between 2-35 million Euro.

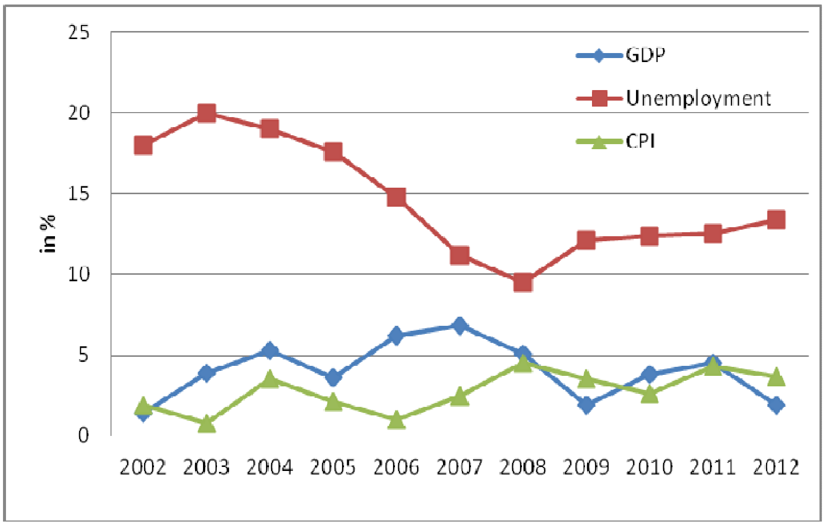
Additionally, the macroeconomic variables were included in the study. These variables are shown in Figure 2. It is worth mentioning that in the models using discriminant functions these variables were static (applied at a certain moment of time, e.g. at the date of bankruptcy). However, in the models using survival analysis, these variables were dynamic. All values were available at the time of FS date (end of calendar year).

For the purpose of this analysis only three variables were selected: GDP (Gross Domestic Product – dynamics), unemployment rate (in %) officially registered, CPI (Consumer Price Index). It is expected that high and increasing GDP should positively affect the probability of default. High unemployment rate is characteristic for downturn in economy and should increase the probability of default. The effect of CPI (inflation) is not obvious but higher inflation is rather positive in economy and should decrease the probability of default in enterprises segment indirectly.

An important issue, when estimating the PD parameter, is the fact that the length of the observation period of the data used for the parameter estimation must be at least five years, and come from at least one source, regardless of whether the source is internal, external, or a combination of both (Basel requirements).

[5] FS – financial statement

Figure 2. Macroeconomic variables in the analysed period



Source: GUS (Polish Central Statistical Office) database

THE RESULTS OF THE MODELS

In our empirical analysis we have applied Altman's Z-Score model to our sample. In the next step we estimated the survival Cox model with the same ratios as in Altman's model. In the final step we have included macroeconomic variables. In each step we checked the proportionality assumption in Cox model.

In a first step the Z-Score Altman's model was applied and Z-Score discriminatory power was not very high (AUC=0.699). It is assumed that AUC should be at least 0.75-0.80 to assume the discriminatory power as satisfactory. For the sample of Polish enterprises the effectiveness of this model was rather low-medium. It could be due to high heterogeneity of the sample. The sample represents rather homogenous group of enterprises as far as concerning the size but heterogeneous as far as concerning the type of activity (branch).

In the next step the Cox regression model with original Altman's variables (ratios) was estimated. All five ratios were significant (see Table 1).

The strongest influence was like in Altman's Z-Score for ratio X_3 (Earnings before interest and taxes/Total assets). Increase by 1 unit in this ratio leads to the decrease of default risk by about 90%.

Accuracy power of this model is much higher comparing to Altman's Z-Score and amounts to AUC=0.746. This level of predictive power may be assumed to be on the medium level.

Table 1. Results of the Cox regression model with original Z-Score ratios

Parameter	DF	Parameter Estimate	Standard Error	Chi-Square	Pr. > chi^2	Hazard Ratio
X1	1	-0.35351	0.08611	16.8557	.0001	0.702
X2	1	-0.92721	0.20987	19.5195	.0001	0.396
X3	1	-2.42348	0.23393	107.3235	.0001	0.089
X4	1	-0.41582	0.06930	36.0033	.0001	0.660
X5	1	-0.16972	0.03212	27.9141	.0001	0.844

Source: own elaboration

It was necessary to check the proportionality assumption in Cox regression model. One of the simplest ways of checking this assumption is to include the interactions with time for variables X_1-X_5 (see Table 2). Significant interaction effect means lack of proportionality assumption fulfillment. In our model the interaction with time was significant for X_3-X_5.

The interaction with time for variables X_1 and X_5 is significant. For those variables the assumption of proportionality is fulfilled. For variables X_3-X_5 the negative coefficient means that the negative influence on the risk is enhanced with time.

Cox survival model is named non-proportional model when the proportionality assumption is not fulfilled. The accuracy ratio for such a model is much higher AUC=0.827.

The next step was to include the macroeconomic variables in Cox semiparametric model. Results for such a combination are quite promising. The included variables were significant (except inflation). Results are presented in Table 3.

The higher the GDP and unemployment rate levels the lower the risk of enterprises' default. The direction of unemployment rate influence is not obvious. However it is not confirmed in other research results. The accuracy power of this model is even higher than the previous one. Macroeconomic variables increase the effectiveness of the model. The accuracy level may be assessed as satisfactory.

Table 2. Interaction with time in Cox regression model – proportionality assumption

Parameter	DF	Parameter Estimate	Standard Error	Chi-Square	Pr. > chi^2
duration*X1	1	0.00631	0.00599	1.1101	0.2921
duration*X2	1	-0.01065	0.01451	0.5381	0.4632
duration*X3	1	-0.06547	0.01536	18.1651	<.0001
duration*X4	1	-0.05530	0.00474	136.2655	<.0001
duration*X5	1	-0.02548	0.00216	138.7192	<.0001

Source: own elaboration

Table 3. Semiparametric Cox model with macroeconomic variables

Parameter	DF	Parameter Estimate	Standard Error	Chi-Square	Pr. > chi^2	Hazard Ratio
X1	1	-0.56683	0.08378	45.7800	<.0001	.567
X2	1	-1.39851	0.20301	47.4556	<.0001	.247
X3	1	-2.62719	0.22354	138.1217	<.0001	.072
X4	1	-0.45197	0.07127	40.2140	<.0001	.636
X5	1	-0.17275	0.03288	27.6053	<.0001	.841
GDP	1	-0.16940	0.02839	35.5974	<.0001	.844
Unemployment	1	-0.16784	0.02647	40.2175	<.0001	.845
CPI	1	0.08828	0.06511	1.8383	0.1752	.092

Source: own elaboration

Table 4. Cox regression model with macro variables and interactions with process duration - nonproportional hazards

Parameter	DF	Parameter Estimate	Standard Error	Chi-Square	Pr. > chi^2
duration*X	1	-0.00945	0.00906	1.0874	.2971
duration*X2	1	-0.02461	0.02165	1.2925	.2556
duration*X3	1	-0.05049	0.02224	5.1548	.0232
duration*X4	1	-0.01474	0.00489	9.0809	.0026
duration*X5	1	-0.00411	0.00220	3.4828	.0620
duration*GDP	1	-0.03799	0.00185	422.9914	.0001
duration*Unemployment	1	-0.07100	0.00300	559.2011	.0001
duration*CPI	1	-0.12050	0.00540	497.1255	.0001

Source: own elaboration

The interaction with time for CPI is significant; however the estimation for this variable was not significant.

CONCLUSIONS

In recent years there have been many changes in the credit environment. Banks offer a variety of financial products to a wide range of customers, including those who do not know the law. In view of the rapid increase in the volume of information on the applicants, the financial institutions have the ability to seek out and create newer and more sophisticated credit models.

In view of the recent financial crisis, banks have realized the need to take account of macroeconomic variables in these models, since the economy has a huge impact on the ability of customers to settle liabilities. Since the method used does not allow taking into account in the models time-dependent variables (time dependent), there is a need to find such methods. It seems that survival analysis is a technique that is facing these requirements, because it helps to determine when a specific event occurs in the future, and not just to predict whether it occurs at all. In the case of credit, this event is of course the insolvency of the borrower. Banks want to know when a customer ceases to repay the commitment to be able to prepare in advance for this event, and possibly take action to minimize losses. The possibility of macroeconomic variables, makes these models dynamic, and the banks can observe how these variables affect the level of bad debts.

Currently the most common method used in the default models for SMEs is the discriminant analysis or logistic regression. It seems, however, that more and more importance is put on the survival analysis, due to its properties. From the presented description of the survival analysis it can be noted that there are several reasons why it is worth using it as an alternative method to traditionally used static models (e.g. logistic regression models). First of all, it should be emphasized that the use of survival analysis in modeling the companies bankruptcy risk can extend the standard static approach into the dynamic one.

Other advantages of using the survival analysis method are as follows:

1) possibility to use censored data - event occurs when the company is eliminated from the observation data set before registering the default,
2) avoiding the instability which can appear due to rigidly fixed length of the observation time,
3) event time estimation allows following the risk default intensity,
4) obtaining the "dynamic" probability forecasts of event (forecast value is a function of time), which is very useful when determining the appropriate strategy and policy,
5) ability to implement changes in the economic environment in credit risk assessment using time dependent variables such as macro variables.

Generally, usage of the survival analysis as an alternative approach to model bankruptcy (default) risk, gives the wider chance to use the results of these models than when using the standard statistical methods, gaining also the improvement of the model bankruptcy prediction. In our further research we are going to include more macroeconomic variables with potential influence on the analysed process of bankruptcy. We will apply this model also on more homogeneous sample with the latest available data.

REFERENCES

Allen L.N., Rose L.C., Financial survival analysis of defaulted debtors, "Journal of Research Society", 57/2006.

Allison P., Logistic Regression Using SAS: Theory and Application, Second Edition, 2012,

Allison P., Survival Analysis Using SAS: A Practical Guide, Second Edition, 2010.

Allison P.D., Regression for Longitudinal Event Data, Sage University Paper Series on Quantitative Applications in the Social Sciences, Nr 46, Publications, Sage 1984.

Altman E., Suggitt H.J., Default rates in the syndicated bank loan market: A mortality analysis, "Journal of Banking and Finance", 2000, vol. 24, No 1.

Altman, E. I., Measuring corporate bond mortality and performance, “Journal of Finance” 44(4)/1989.

Blossfeld H.P., G. Rohwer, Techniques of Event History Modeling. New Approaches to Causal Analysis, Lawrence Elbaum Associates Publishers, London, 2002.

Cox D. R., and Oakes D., Analysis of Survival Data, Chapman and Hall, 1984.

Cox D.R., Regression models and life tables, Journal of the Royal Statistical Society (Series B), no 34, 1972, pp.187-202.

Giovannetti G., Ricchiuti G., Velucchi M., Size, innovation and internationalization: a survival analysis of Italian firms, Applied Economics,Volume 43, Issue 12, 2011, pp. 1511-1520

Glennon D.C., Nigro P., Measuring the Default Risk of Small Business Loans: A Survival Analysis Approach, Journal of Money, Credit, and Banking, Volume 37, Number 5, October 2005, pp. 923-947

Lando D., On Cox Processes and Credit Risky Securities, Review of Derivatives Research, 2, 1998.

Malik, M., Thomas, L. Modelling credit risk in portfolios of consumer loans: transition matrix model for consumer credit ratings. Southampton, UK, University of Southampton, 21pp. (Discussion Papers in Centre for Risk Research), 2009.

McDonald R.A., Matuszyk A., Thomas L. C., Application of survival analysis to cash flow modelling for mortgage products, OR Insight (2010) 23.

Moon T.H., Sohn S.Y., Survival analysis for technology credit scoring adjusting total perception, Journal of the Operational Research Society (2011) 62, pp.1159–1168

Narain, B., Survival analysis and the credit granting decision, in: Thomas L. C., Crook J. N., Edelman D. B., eds. Credit Scoring and Credit Control. OUP, 1992, Oxford, U.K.

Nunes P. M., Serrasqueiro Z., da Silva J.V., Family-owned and non-family-owned SMEs: empirical evidence of survival determinants , Economics and Business Letters, 3(1), pp.68-76, 2014

Pierides, Y. A. ,The pricing of credit risk derivatives, Journal of Economic Dynamics and Control 21(10), 1997.

Ptak-Chmielewska A., M. Pęczkowski Analiza dyskryminacji (discriminant analysis), [in:] Wielowymiarowa analiza statystyczna. Teoria - przykłady zastosowań z systemem SAS (Multivariate statistics. Theory and applications in SAS)., ed. E.Frątczak, Oficyna Wydawnicza SGH, Warsaw, 2009.

Stepanova, M., Thomas, L.C., Survival analysis methods for personal loan data, “Journal of Operations Research” 50(2), 2002.