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ROBUSTNESS OF TWEEDIE MODEL OF RESERVES WITH RESPECT TO DISTRIBUTION OF SEVERITY OF CLAIMS

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Abstract: The aim of the work is to discuss the robustness of estimation procedures and robustness of prediction in Tweedie's compound Poisson model. This model is applied to the claim reserving problem. The quality of parameter estimators and predictors is studied when the distribution of severity of claims is disturbed. The ε-contamination class of distributions is considered. The example, where errors of estimators are large is presented. The simulation methods, using the R programming environment, are applied.

Keywords: loss reserves, Tweedie model, Poisson and gamma distribution, ε-contamination, generalized linear model, mean square error, bias, prediction

INTRODUCTION

The claim reserving problem is one of the most important in the insurance mathematics and is the main task of insurance actuaries. There are a lot of models, methods and algorithms setting claim reserves. We can mention chain ladder method [Mack 1991, 1993, 1999], [Wüthrich et al. 2009], Bornhuetter-Ferguson method [Bornhuetter, Ferguson 1972], [Taylor 2000], [Mack 2008], bootstrap methods [England, Verrall 1999], [Liu, Verrall 2009], lognormal model [Han, Gau 2008], Bayesian and credibility methods [Ntzoufras, Dellaportas 2002], [Gisler, Wüthrich 2008], [Sánchez, Vilar 2011]. For excellent overview we refer to [England, Verrall 2002] and [Wüthrich, Merz 2008]. The main aim is to predict an appropriate random variable which describes future payments or estimate parameters of that random variable distribution like mean, variance. It is also interesting to estimate prediction and estimation errors.

To do this we select a statistical model, methods of estimation and prediction. But the observed random variables sometimes do not satisfy the assumptions of a chosen model. It is interesting to verify how our estimators or predictors, their errors and parameters of their distributions can change when we have some deviation from the model. In mathematical statistics such a problem is called the problem of examine the robustness of a statistical procedure. The concept of robustness is presented in [Huber 1981], [Hampel 1986] and [Zieliński 1983]. We will use the concept of Zieliński and will measure the difference in bias and mean square error (*MSE*) of estimators and predictors when model assumptions are not satisfied and the distribution of number of claims and severity of claims are different from the distributions in the choosen model. We assume the Tweedie model as the main (basic) model. The numbers of payments are Poisson random variables and severity of payments is Gamma distributed. These assumptions lead to Tweedie's compound Poisson model (see [Jørgensen, de Souza 1994], [Jørgensen 1997], [Smyth, Jørgensen 2002] and [Ohlsson, Johansson 2006]). The model belongs to the exponential dispersion family and we use GLM for the parameter estimation. The model considered in this paper was described in [Wüthrich 2003], [Peterset al. 2009] and [Boucher, Davidov 2011]. In the first paper the method of parameter estimation is presented and the model is applied to the motor insurance data. It is compared with the chain ladder and lognormal models. The dispersion parameter is assumed to be constant. There is not presented a method of verifying the assumptions of the model. In the second paper the model is compared with a Bayes model and the last paper presents the extension and the dispersion parameter is not assumed to be constant. We use the model and methods of estimation described in [Wüthrich 2003] and measure the variation of *MSE* of parameter estimation and prediction when the distribution of severity of claims is not gamma distribution.

So far there have not been papers which present results about robustness of the Tweedie model for prediction reserves and the verification of model assumptions may be difficult or even impossible. We consider the ε-contamination distribution of the value of claim, where $\varepsilon \in [0,1]$ is the power of disturbance. If $\varepsilon = 0$ we have the model distribution, if $\varepsilon = 1$ we have total departure from this distribution and we replace it with other, differing in shape, variance, coefficient of asymmetry. By introducing this type of disturbances we move away from the exponential family of distributions. This may affect the quality of the GLM estimators. As a basic model we consider the model with parameters estimated in [Wüthrich 2003]. We will consider a model with constant Tweedie's dispersion coefficient, the work associated with its variation we plan to in the future.

Applying simulation methods we generate run-off triangles of numbers and severities of claims using disturbed distributions, and next compute *MSE* and bias of estimation and prediction. For all simulations and GLM procedures the R language environment was used.

DEFINITION OF THE BASIC MODEL

Let C_{ij} be incremental claim payments, where i denotes the accident year and j development year and $i \in \{0,1,\ldots,I\}$, $j \in \{0,1,\ldots,J\}$. Let R_{ij} be numbers of claims corresponding to these payments. At time J we observe the following sets

$$D_J = \{C_{ij} : i = 0,1,\ldots,I; j = 0,1,\ldots,J-i\}$$

and

$$DR_J = \{R_{ij} : i = 0,1,\ldots,I; j = 0,1,\ldots,J-i\}.$$

If $I = J$ the sets are upper triangles. In this paper $J > I$, but despite that, these sets are called upper triangles. The main aim is prediction of the lower triangle i.e. variables C_{ij}, where $i + j > J$ and $i \le I$, $j \le J$, or estimation of parameters of their distributions. It is also interesting to estimate the expected value of a random variable $S_i = \sum_{j=J-i+1}^{J} C_{ij}$, $i = 1,2,\ldots,I$, called the reserves for the year i, and a random variable $S = \sum_{i=1}^{I} S_i$, called the total reserve, as well as the prediction of these variables.

In the basic model we assume that R_{ij} are independent and have Poisson distributions $Poiss(\lambda_{ij} w_i)$, where $\lambda_{ij} w_i > 0$ are expected values and w_i numbers of policies. The individual payments $X_{ij}^{(k)}$ for $k = 1,2,\ldots$ are independent and $Gamma(\gamma, s_{ij})$ distributed with shape parameter $\gamma > 0$, expected value $EX_{ij}^{k} = \tau_{ij} = \gamma s_{ij}$ and variance $VarX_{ij}^{(k)} = \gamma s_{ij}^2 = \tau_{ij}^2 / \gamma$. Variables R_{ij} i $X_{lm}^{(k)}$ are independent for all indices. Then

$$C_{ij} = \sum_{k=1}^{R_{ij}} X_{ij}^{(k)},$$

if $R_{ij} > 0$, and $C_{ij} = 0$ otherwise, hence $P(C_{ij} = 0) = \exp(-\lambda_{ij} w_i)$. Conditionally, given $R_{ij} \neq 0$, the variable C_{ij} has $Gamma(R_{ij}\gamma, s_{ij})$ distribution.

Let $Y_{ij} = C_{ij} / w_i$. Random variables Y_{ij} have Tweedie distribution with the density function (for $y > 0$)

$$f_{Y_{ij}}(y; \mu_{ij}, \phi, w_i, p) = \sum_{r=1}^{\infty} \left(\frac{(w_i/\phi)^{\gamma+1} y^{\gamma}}{(p-1)^{\gamma}(2-p)} \right)^{r} \frac{1}{r!\Gamma(r\gamma) y} \exp\left(\frac{w_i}{\phi} \left(y \frac{\mu_{ij}^{1-p}}{1-p} - \frac{\mu_{ij}^{2-p}}{2-p} \right) \right),$$

where

$$p = \frac{\gamma+2}{\gamma+1} \in (1,2), \mu_{ij} = \lambda_{ij}\tau_{ij}, \phi = \lambda_{ij}^{1-p}\tau_{ij}^{2-p}(2-p)^{-1}$$

(for details see [Wüthrich 2003]).
We have

$$EY_{ij} = \mu_{ij}, \qquad VarY_{ij} = \frac{\phi}{w_i}V(p) = \frac{\phi}{w_i}\mu_{ij}^p, \qquad \frac{E(Y_{ij}-EY_{ij})^3}{Var^{1,5}Y_{ij}} = \frac{p}{\sqrt{\lambda w_i(2-p)}}.$$

In this paper, as in [Wüthrich 2003], the parameter p is estimated. Additionally we assume a multiplicative model, i.e. there exist parameters $\alpha(i)$, $\beta(j)$, $i = 0,1,\ldots,I$, $j = 0,1,\ldots,J$ such that

$$\mu_{ij} = \alpha(i)\beta(j).$$

Hence choosing the logarithmic link function we have

$$\eta_{ij} = \ln(\mu_{ij}) = \mathbf{x}_{ij}b$$

where $b^T = [b_{0,0}, b_{1,0},\ldots,b_{I,0}, b_{0,1},\ldots,b_{0,J}]$ and a matrix **x** of rows $\mathbf{x}_{ij}$ is defined so that $\ln(\mu_{ij}) = \ln\alpha(i) + \ln\beta(j) = b_{0,0} + b_{i,0} + b_{0,j}$. The vector of unknown parameters is equal to $B = [b_{0,0}, b_{1,0},\ldots,b_{I,0}, b_{0,1},\ldots,b_{0,J}, \phi, p]$, and the logarithm of the likelihood function (for $R_{ij} = r_{ij}$ and $Y_{ij} = y_{ij}$, $i = 0,1,\ldots,I$, $j = 0,1,\ldots,J-i$) is equal to

$$L(\mu, p, \phi) = \sum_{\{i,j:r_{ij}\neq 0\}}\left[r_{ij}\ln\left(\frac{(w_i/\phi)^{\gamma+1}y_{ij}^{\gamma}}{(p-1)^{\gamma}(2-p)}\right) - \ln(r_{ij}!\Gamma(r_{ij}\gamma)y_{ij})\right] + \sum_{i,j}\frac{w_i}{\phi}\left(y_{ij}\frac{\mu_{ij}^{1-p}}{1-p} - \frac{\mu_{ij}^{2-p}}{2-p}\right),$$

where $\gamma = (p-2)(1-p)^{-1}$ and $\mu = [\mu_{ij}]_{i=0,1,\ldots,I,\ j=0,1,\ldots,J}$ is a matrix.

The estimation in this model is described in [Wüthrich 2003]. We remind the most important steps.

1. Choose an initial $\hat{p}$.
2. For $\hat{p}$ known, use GLM and find estimates for $b_{0,0}, b_{1,0},\ldots,b_{I,0}, b_{0,1},\ldots,b_{0,J}$ and estimates for μ_{ij} equal to $\hat{\mu}_{ij} = \exp(\hat{b}_{0,0} + \hat{b}_{i,0} + \hat{b}_{0,j})$.
3. For given $p = \hat{p}$ and $\mu = \hat{\mu}$ find MLE for ϕ equal to
$$\hat{\phi} = -\left((1+\gamma)\sum_{i,j}r_{ij}\right)^{-1}\sum_{i,j}w_i\left(y_{ij}\frac{\mu_{ij}^{1-p}}{1-p} - \frac{\mu_{ij}^{2-p}}{2-p}\right).$$
4. Given $\hat{\phi}$ and $\hat{\mu}$, find maximum of the likelihood function and choose the next $\hat{p}$.
5. Repeat steps 2, 3, 4, 5 until convergence of $\hat{p}, \hat{\mu}$.

The vector

$$\hat{B} = [\hat{b}_{0,0}, \hat{b}_{1,0}, \ldots, \hat{b}_{I,0}, \hat{b}_{0,1}, \ldots, \hat{b}_{0,J}, \hat{\phi}, \hat{p}]$$

is the estimator for *B*, and applying equalities

$$\gamma = \frac{p-2}{1-p}, \qquad \tau_{ij} = (2-p)\phi\mu_{ij}^{p-1}, \qquad \lambda_{ij} = \frac{\mu_{ij}}{\tau_{ij}}$$

we obtain the estimators for γ, τ_{ij} and λ_{ij}. The predictor for C_{ij}, $i + j > J$, is defined as $\hat{C}_{ij} = w_i \hat{\mu}_{ij}$ and it is an estimator for EC_{ij}. Hence, the predictor for S_i (S) and the estimator for ES_i (ES) is defined as $\hat{S}_i = \sum_{j=J-i+1}^{J} \hat{C}_{ij}$ ($\hat{S} = \sum_{i=1}^{I} \hat{S}_i$).

DISTURBED MODELS

We use simulation methods to obtain data from assumed distributions. Values of the parameters of the basic model are equal to estimated values presented in [Wüthrich 2003] and [Boucher, Davidov 2011], (see Appendix 1). Assume that $I = 8$, $J = 10$. Table 1.3 (Appendix 1) provides values of w_i and p, γ, ϕ. Tables 1.1 and 1.2 (Appendix 1) provide values of the parameters λ_{ij} and τ_{ij}. The parameters μ_{ij} satisfy $\mu_{ij} = \lambda_{ij}\tau_{ij}$. Given these values the basic model and parameters of distributions of numbers and severity of claims for all periods are uniquely determined. Entries in the upper and lower triangle in Tables 1.1 and 1.2 are parameters of observed and predicted distribution respectively.

Disturbances in the model concern distributions of severity of claims. Distributions of severity of claims incurred in accident year i and reported in development year j are in the form of ε-contamination distributions

$$F(x) = (1-\varepsilon)F_0(x) + \varepsilon G(x),$$

where F_0 is a distribution function of severity of claims in the basic model and *G* is a distribution function of contamination. The parameter $\varepsilon \in [0,1]$ determines the power of disturbance. In particular, $\varepsilon = 0$ means that there is no disturbance, for $\varepsilon = 1$ distribution F_0 is replaced by *G*.

Table 1 provides the basic and contaminating distributions and their parameters. A disturbance by means of a given distribution is called a type. Thus there are three types of disturbances according to type of distribution (I – lognormal, II –Weibull, III – Pareto). In all cases the contaminating distribution has the same mean as the basic distribution, but different variance and coefficient of asymmetry. Chosen distributions have variance the same as the basic distribution (type Ia and IIa) or $z\gamma$ times greater than the one in the basic model (then its value is such that square of the coefficient of variation is equal to 1.5 or 2, while in the basic distribution it equals 1/γ = 0.211). Thus we assume that the parameter z is equal to 1.5 and 2. An asymmetry of contaminating distribution differs from that in the basic model (see Table 1). Only in type IIa it is less than asymmetry in the basic model.

Table 1. Contaminating distributions (a_z denotes coefficient of asymmetry of distribution with parameter z)

Type	Distribution	Parameters	Coefficient of asymmetry a	value of a
Ia	Lognormal $L(m_{ij},\sigma^2)$	$\sigma^2=\ln(1+\gamma^{-1})$ $m_{ij}=\ln(\tau_{ij})-0{,}5\sigma^2$	$3(\sqrt{\gamma})^{-1}+(\sqrt{\gamma})^{-3}$	1.47
Ib	Lognormal $L_z(m_{z,ij},\sigma_z^2)$	$\sigma_z^2=\ln(1+z)$ $m_{z,ij}=\ln(\tau_{ij})-0{,}5\sigma_z^2$	$3\sqrt{z}+(\sqrt{z})^3$	$a_{1,5}$ = 5.51 a_2 = 7.07
IIa	Weibull $W(c_{ij},t)$	t satisfies $\frac{\Gamma^2(1+t^{-1})}{\Gamma(1+2t^{-1})-\Gamma^2(1+t^{-1})}=\gamma$ $c_{ij}=(\Gamma(1+t^{-1})\tau_{ij}^{-1})^t$	$\frac{\Gamma(\frac{t+3}{t})-3\Gamma(\frac{t+2}{t})\Gamma(\frac{t+1}{t})+2\Gamma^3(\frac{t+1}{t})}{[\Gamma(\frac{t+2}{t})-\Gamma^2(\frac{t+1}{t})]^{1,5}}$	0,45
IIb	Weibull $W_z(c_{z,ij},t_z)$	t_z satisfies $\frac{\Gamma^2(1+t_z^{-1})}{\Gamma(1+2t_z^{-1})-\Gamma^2(1+t_z^{-1})}=\frac{1}{z}$ $c_{z,ij}=(\Gamma(1+t_z^{-1})\tau_{ij}^{-1})^{t_z}$	$\frac{\Gamma(\frac{t_z+3}{t_z})-3\Gamma(\frac{t_z+2}{t_z})\Gamma(\frac{t_z+1}{t_z})+2\Gamma^3(\frac{t_z+1}{t_z})}{[\Gamma(\frac{t_z+2}{t_z})-\Gamma^2(\frac{t_z+1}{t_z})]^{1,5}}$	$a_{1,5}$ = 2.70 a_2 = 3.33
IIIb	Pareto $Pa_z(\theta_z,\varphi_{z,ij})$	$\theta_z=2z(z-1)^{-1}$ $\varphi_{z,ij}=\tau_{ij}(\theta_z-1)$	$\frac{6z-2}{(3-z)\sqrt{z}}$	$a_{1,5}$ = 3.81 a_2 = 7.07
IV	gamma $G(\gamma,s_{ij})$	$s_{ij}=\tau_{ij}\gamma^{-1}$ or $s_{ij}=\tilde{\tau}_{ij}\gamma^{-1}$	$2\gamma^{-0,5}$	0.92

Type IV consists in using gamma distribution as a contamination. Its mean is equal to average value of payments for data given in [Wüthrich 2003] (see Table 2.2, Appendix 2). The average payments computed on the basis of that paper differ much from τ_{ij} (cf. Tables 1.2 and 2.2) and are almost the same in successive development years, while parameters τ_{ij} decrease with increasing development period. It might indicate the difference between distributions. So it is the type worth carrying out. In this case two types of distributions of number of claims are considered. In the first case (type IVa) it is Poisson distribution with parameters given in Table 1.1 (Appendix 1). Then (similar to types I, II, III) distribution of number of claims is the same as the one in the basic model and distribution of severity of claims is of the form of ε-contamination. In the second case (type IVb) numbers of claims are generated from ε-contamination distributions where Poisson distributions with parameters given in Table 2.1 (Appendix 2) are contaminating distributions. The parameters are equal to average numbers of claims per policy obtained on the basis of paper [Wüthrich 2003]. It means that for every i, j we generate numbers of claims according to distributions $(1-\varepsilon)Poiss(\lambda_{ij}) + \varepsilon Poiss(\tilde{\lambda}_{ij})$ where parameters λ_{ij} and $\tilde{\lambda}_{ij}$ are given in Table 1.1 and 2.1 respectively. If the number of claims is generated from $Poiss(\lambda_{ij})$ distribution then severity of claims is generated from $Gamma(\gamma, \tau_{ij}/\gamma)$ distribution, otherwise from $Gamma(\gamma, \tilde{\tau}_{ij}/\gamma)$ distribution. Parameters τ_{ij} and $\tilde{\tau}_{ij}$ are given in Table 1.2 and 2.2 respectively. The data presented in the paper [Wüthrich 2003] allows us to determine parameters (in particular expected values) of distributions of number and severity of claims for the upper triangle. Parameters for the lower triangle of contaminating distributions are calculated by applying chain ladder method to the run-off triangles of the paper.

The goal is to check how the choice of different distributions influences the bias of a parameter estimator, its mean square error and the mean square error of prediction.

A study for a chosen contaminating distribution and the value of ε is called a scenario. A case of generating random variables from the basic model is called the zero scenario. For every scenario we generate independently 10000 tables of numbers of claims and the corresponding ones of sums of payments, that is the variables R_{ij} and C_{ij}. Based on data from the upper triangle we estimate elements of the vector B according to the steps described in the previous section and then we estimate the other ones. We also predict the lower triangle, i.e., quantities C_{ij} for $i + j > J$.

Let $\hat{\theta}_l$ be the value of an estimator $\hat{\theta}$ of a parameter θ based on the *l*-th simulation, $l = 1,2,\ldots,10000$. As the parameter estimates of distribution of variable $\hat{\theta}$ we take:

- expectation: $E\hat{\theta} = \frac{1}{10000}\sum_{l=1}^{10000}\hat{\theta}_l$ and variance: $Var\hat{\theta} = \frac{1}{10000}\sum_{l=1}^{10000}(\hat{\theta}_l - E\hat{\theta})^2$;
- mean square error: $MSE(\hat{\theta}) = Var\hat{\theta} + bias^2(\hat{\theta})$, where $bias(\hat{\theta}) = E\hat{\theta} - \theta$;
- standard error: $SE(\hat{\theta}) = \sqrt{MSE(\hat{\theta})}$;
- relative bias: $Rbias(\hat{\theta}) = \frac{bias(\hat{\theta})}{\theta} \cdot 100\%$;

- variation coefficient: $\upsilon(\hat{\theta}) = \frac{(Var\hat{\theta})^{1/2}}{E\hat{\theta}} \cdot 100\%$;
- standard percentage error: $SPE(\hat{\theta}) = \frac{SE(\hat{\theta})}{\theta} \cdot 100\%$ which measures percentage of the standard error of the estimate in actual value of the parameter.

Analogously, let $\hat{Z}_l$ be the value of a predictor $\hat{Z}$ of some random variable Z based on the *l*-th simulation, $l = 1,2,\ldots,10000$, and let Z_l be an observed value of the variable. We take the following measures:

- mean square error of prediction: $MSEP(\hat{Z}) = \frac{1}{10000}\sum_{l=1}^{10000}(\hat{Z}_l - Z_l)^2$;
- standard error of prediction: $SEP(\hat{Z}) = \sqrt{MSEP(\hat{Z})}$;
- bias of predictor: $bias(\hat{Z}) = \frac{1}{10000}\sum_{l=1}^{10000}(\hat{Z}_l - Z_l)$;
- relative bias of predictor: $RbiasP(\hat{Z}) = \frac{bias(\hat{Z})}{\bar{Z}} \cdot 100\%$, where $\bar{Z} = \frac{1}{10000}\sum_{l=1}^{10000} Z_l$;
- standard percentage error of prediction: $SPEP(\hat{Z}) = \frac{SEP(\hat{Z})}{\bar{Z}} \cdot 100\%$.

PROPERTIES OF ESTIMATORS AND PREDICTORS IN THE BASIC MODEL

As a result of simulations for the zero scenario the bias and mean square error of estimators of elements of vector *B*, matrices λ, τ, μ and vector of expected reserves for successive accident years and expected value of the total reserve are obtained. Relative bias of estimators of parameters p, γ, ϕ is of the order of $10^{-3}\%$ and standard error is less than 0.9% of actual value of the parameter (see Table 5, the third row).

Table 2. Relative bias and *SPE* of estimators $\hat{\lambda}_{ij}$, $\hat{\tau}_{ij}$, $\hat{\mu}_{ij}$

	j										
	0	1	2	3	4	5	6	7	8	9	10
Rbias for $\hat{\lambda}_{ij}$											
min	-0.003	-0.010	-0.020	-0.004	-0.025	-0.052	0.003	-0.008	-0.038	-0.12	-14.27
max	0.021	0.015	0.005	0.020	-0.001	-0.027	0.027	0.019	-0.013	-0.10	-14.23
Rbias for $\hat{\tau}_{ij}$											
min	-0.003	-0.006	-0.013	-0.013	-0.020	-0.037	-0.037	-0.046	-0.123	-0.19	-42.37
max	0.002	-0.001	-0.007	-0.008	-0.015	-0.032	-0.032	-0.041	-0.117	-0.18	-42.36
Rbias for $\hat{\mu}_{ij}$											
min	-0.004	-0.014	-0.028	-0.005	-0.028	-0.048	0.035	0.033	0.100	0.07	-1.50
max	0.025	0.016	0.003	0.023	0.000	-0.018	0.063	0.065	0.131	0.10	-1.45

SPE for $\hat{\lambda}_{ij}$											
min	0.94	1.05	1.73	2.51	3.02	4.63	5.87	6.58	11.19	13.51	104.28
max	1.04	1.16	1.79	2.60	3.10	4.70	5.94	6.64	11.24	13.55	104.36
SPE for $\hat{\tau}_{ij}$											
min	0.25	0.27	0.50	0.70	0.81	1.14	1.39	1.54	2.51	2.98	68.220
max	0.27	0.29	0.51	0.72	0.82	1.15	1.41	1.55	2.52	2.99	68.221
SPE for $\hat{\mu}_{ij}$											
min	1.12	1.26	2.06	2.99	3.61	5.54	7.06	7.92	13.54	16.32	128.32
max	1.25	1.39	2.12	3.10	3.71	5.63	7.13	8.00	13.59	16.38	128.47

Source: Authors' calculations

Table 2 provides the minimum and maximum value (over i) of the relative bias and *SPE* of estimators of parameters λ_{ij}, τ_{ij}, μ_{ij} for all development years. Variation of *Rbias* and *SPE* with respect to *i*, for fixed *j*, is small. *SPE* is increasing function of development year *j*. Absolute value of relative bias of estimators of parameters λ_{ij}, τ_{ij}, μ_{ij} for $j < 10$ is not greater than 0.2%, for $j = 10$ it grows tens times. In the last development year bias of all these estimates is negative. Thus even in the basic model it can be observed large average underestimate of both expected values (severity and number of claims) in the last development year, compared to the others. There is an even bigger jump for *SPE* in the last development year.

Table 3. Estimation of expected value and prediction of reserves

			Estimation of ES_i				Prediction of S_i		
i	ES_i	$\hat{ES}_i$	*Rbias*	υ	*SE*	*SPE*	*RbiasP*	*SEP*	*SPEP*
1	326	321	-1.45	130.33	418	128.44	3.58	600	180.41
2	21575	21588	0.06	16.24	3506	16.25	0.29	6042	27.91
3	40746	40779	0.08	10.61	4325	10.62	-0.31	7966	19.60
4	89306	89371	0.07	6.55	5855	6.56	-0.18	11935	13.38
5	138537	138630	0.07	4.96	6872	4.96	-0.01	14888	10.74
6	204637	204723	0.04	3.88	7939	3.88	-0.06	18138	8.86
7	361456	361481	0.01	2.83	10245	2.83	-0.06	25456	7.05
8	598005	597967	-0.01	2.29	13678	2.29	-0.03	33644	5.63
total	1454587	1454860	0.02	2.79	40528	2.79	-0.05	60912	4.19

Source: Authors' calculations

The results for estimation of expected values of reserves and prediction of reserves are shown in Table 3. The last row shows the errors for the total reserve.

Table 4. Errors: *SPE* and *SPEP* (in parentheses) for reserves C_{ij}

	j							
i	3	4	5	6	7	8	9	10
1								128.4(180.4)
2							16.4(28.2)	128.5(182.3)
3						13.6(27.1)	16.4(28.7)	128.4(187.8)
4					8.0(18.1)	13.6(27.6)	16.4(29.5)	128.4(185.8)
5				7.1(17.5)	8.0(18.0)	13.6(27.7)	16.4(28.8)	128.5(185.3)
6			5.6(15.1)	7.1(18.0)	8.0(18.7)	13.6(28.2)	16.4(29.5)	128.4(185.5)
7		3.7(10.6)	5.6(15.5)	7.1(18.4)	8.0(19.1)	13.6(28.6)	16.4(30.0)	128.3(194.2)
8	3.1(9.2)	3.7(10.5)	5.6(15.5)	7.1(18.2)	8.0(19.2)	13.6(28.9)	16.4(30.1)	128.4(186.7)

Source: Authors' calculations

Absolute relative bias of estimates, υ , *SPE* and *SPEP* are decreasing functions of accident year *i*. Large values of υ , *SPE* and *SPEP* for $i = 1$ (compared to the other *i*) are related to the consideration of the reserve only for development year $j = 10$. Table 4 presents *SPE* for estimates of EC_{ij}, $i + j > J$, and *SPEP* for prediction of C_{ij}, $i + j > J$. For all *i* both the errors are increasing functions of *j*, however for $j = 10$ are much greater than the others.

PROPERTIES OF ESTIMATORS AND PREDICTORS IN THE DISTURBED MODEL

We analyze behavior of estimators and predictors in contaminated models with respect to the type and the power of disturbance.

First consider disturbances of types I, II, III. It turns out that the type of disturbance does not influence on results. The power of disturbance and the variance of contaminating distribution are much more important. In all tables below we denote:

m_a, M_a – the minimum and maximum value of a concerned error in all disturbances of the type "a",

$m_{b;z}$, $M_{b;z}$ – the minimum and maximum value of a concerned error in all disturbances of the type "b" given *z*,

$r_a = M_a - m_a$ and $r_{b,z} = M_{b,z} - m_{b,z}$.

The minima and maxima are taken also over $\varepsilon \in \{0; 0.01; 0.1; 0.2; 0.5; 1\}$.

Let us remind that the type "a" of disturbance denotes contamination by lognormal or Weibull distribution with the variance as the one in the basic model and type "b" denotes contamination by lognormal, Weibull or Pareto distribution with the variance $z\gamma$ times greater than the one in the basic model, $z \in \{1.5; 2\}$ (cf. Table 1).

Table 5. Behavior of errors of estimators $\hat{p}$, $\hat{\gamma}$, $\hat{\phi}$ for disturbances of types I, II, III

	Rbias			*SPE*		
	$\hat{p}$	$\hat{\gamma}$	$\hat{\phi}$	$\hat{p}$	$\hat{\gamma}$	$\hat{\phi}$
$\varepsilon = 0$	0.0010	-0.0024	-0.0050	0.1054	0.8608	0.4404
m_a	-0.0002	-0.0144	-0.0117	0.1046	0.8546	0.4356
M_a	0.0025	0.0080	0.0018	0.1071	0.8745	0.4462
$m_{b;1,5}$	0.0010	-1.3545	-0.6749	0.1054	0.8608	0.4404
$M_{b;1,5}$	0.1722	-0.0024	-0.0050	0.3105	2.4678	1.2611
$m_{b;2}$	0.0010	-1.8251	-0.9076	0.1054	0.8608	0.4404
$M_{b;2}$	0.2328	-0.0024	-0.0050	0.3757	2.9667	1.5124

Source: Authors' calculations

Table 5 presents behavior of errors of estimators of parameters p , γ , ϕ . For each type of contamination $|Rbias|$ and *SPE* are increasing functions of variance of contaminating distribution. For disturbances of the type "b" these errors are also increasing functions of the power of disturbance. For $\varepsilon = 0{,}01$ values of the errors are close to those in the basic model regardless of variance of contaminating distribution. But for $\varepsilon = 1$ and for contaminating distribution of the type "b" with parameter $z = 2$, $|Rbias|$ increases more than one hundred times and *SPE* more than three times, compared to the results for the basic model. However, in all the types of disturbances the errors are less than 3%.

Tables 6, 7, 8 provide these errors for estimators $\hat{\lambda}_{ij}$, $\hat{\tau}_{ij}$, $\hat{\mu}_{ij}$. Here the minimum and maximum for a given type is taken also over all accident years *i*. The reason is that for every estimator, for every $\varepsilon \in \{0;\ 0.01;\ 0.1;\ 0.2;\ 0.5;\ 1\}$ and for every type I, II, III the oscillation of the errors with respect to *i* is similar to the one in the basic model (see Table 2) and it does not exceed 1%.

Table 6. Behavior of errors of estimators $\hat{\lambda}_{ij}$ for disturbances of types I, II, III

	j										
	0	1	2	3	4	5	6	7	8	9	10
	Rbias										
M_a	0.03	0.04	0.05	0.07	0.04	0.07	0.07	0.16	0.08	0.18	-11.09
r_a	0.05	0.07	0.07	0.12	0.10	0.16	0.21	0.19	0.35	0.63	3.22
$M_{b,1,5}$	0.02	0.13	0.57	0.76	0.80	1.01	1.13	1.05	1.07	0.75	-11.76
$r_{b,1,5}$	0.19	0.15	0.60	0.78	0.81	1.06	1.31	1.21	1.18	0.97	4.63
$M_{b,2}$	0.02	0.17	0.69	1.02	1.12	1.29	1.35	1.38	1.62	1.54	-12.20
$r_{b,2}$	0.24	0.18	0.71	1.07	1.15	1.29	1.52	1.52	1.84	1.85	5.05

SPE											
M_a	1.07	1.18	1.81	2.64	3.12	4.73	5.97	6.77	11.14	13.71	107.04
r_a	0.14	0.13	0.10	0.14	0.11	0.18	0.14	0.22	0.19	0.27	3.42
$M_{b,1.5}$	1.48	1.64	2.52	3.68	4.36	6.69	8.56	9.67	15.89	19.64	139.05
$r_{b,1.5}$	0.54	0.59	0.80	1.17	1.38	2.04	2.76	3.02	4.85	6.13	34.15
$M_{b,2}$	1.59	1.78	2.76	4.07	4.81	7.34	9.32	10.53	17.67	22.22	145.14
$r_{b,2}$	0.65	0.73	1.03	1.55	1.78	2.71	3.43	3.88	6.55	8.72	40.27

Source: Authors' calculations

The oscillation of *Rbias* of every estimator is very small for all the types of disturbance (not greater than 6.4%). For estimators $\hat{\lambda}_{ij}$ and $\hat{\mu}_{ij}$ the standard percentage error and its oscillation are increasing functions of variance of contaminating distribution. The power of disturbance also affects these values (the greatest ones are achieved for $\varepsilon = 0.5$ or $\varepsilon = 1$). For estimators $\hat{\lambda}_{i,10}$ and for disturbances of the type "a" the oscillation of *SPE* does not exceed 4% and for disturbances of the type "b" with parameter $z = 2$ exceeds 40%. For estimators $\hat{\mu}_{i,10}$ the oscillation is even greater. However, *SPE* is stable for estimators $\hat{\tau}_{ij}$ and for every *i* it is increasing function of development year *j*.

Table 7. Behavior of errors of estimators $\hat{\tau}_{ij}$ for disturbances of types I, II, III

	j										
	0	1	2	3	4	5	6	7	8	9	10
					Rbias						
M_a	0.005	0.004	0.003	0.002	0.002	-0.016	-0.026	-0.014	-0.11	-0.13	-41.04
r_a	0.015	0.014	0.018	0.030	0.035	0.031	0.049	0.050	0.08	0.15	1.27
$M_{b,1.5}$	0.137	0.001	-0.010	-0.011	-0.015	-0.022	-0.040	-0.045	-0.10	-0.19	-41.33
$r_{b,1.5}$	0.142	0.123	0.530	0.741	0.808	1.005	1.086	1.135	1.40	1.50	5.47
$M_{b,2}$	0.186	0.000	-0.007	-0.013	-0.020	-0.023	-0.037	-0.059	-0.14	-0.19	-41.40
$r_{b,2}$	0.192	0.154	0.729	0.983	1.075	1.356	1.511	1.490	1.86	1.98	6.32
					SPE						
M_a	0.28	0.29	0.52	0.72	0.83	1.17	1.42	1.57	2.49	3.03	68.13
r_a	0.03	0.03	0.02	0.03	0.03	0.04	0.03	0.04	0.02	0.07	0.80
$M_{b,1.5}$	0.59	0.58	1.26	1.71	1.89	2.47	2.82	3.01	4.27	5.01	68.92
$r_{b,1.5}$	0.33	0.31	0.75	1.00	1.08	1.32	1.42	1.45	1.80	2.01	1.40
$M_{b,2}$	0.67	0.66	1.49	2.00	2.22	2.87	3.29	3.44	4.88	5.65	68.99
$r_{b,2}$	0.41	0.39	0.98	1.29	1.41	1.71	1.88	1.87	2.39	2.64	1.76

Source: Authors' calculations

Table 8. Behavior of errors of estimators $\hat{\mu}_{ij}$ for disturbances of types I, II, III

	j										
	0	1	2	3	4	5	6	7	8	9	10
					Rbias						
M_a	0.04	0.05	0.05	0.08	0.05	0.09	0.10	0.23	0.23	0.43	2.70
r_a	0.06	0.08	0.08	0.15	0.13	0.18	0.24	0.24	0.43	0.77	4.30
$M_{b,1.5}$	0.04	0.03	0.06	0.10	0.11	0.14	0.22	0.15	0.25	0.45	2.38
$r_{b,1.5}$	0.08	0.08	0.11	0.16	0.19	0.22	0.41	0.45	0.57	0.73	4.00
$M_{b,2}$	0.03	0.06	0.04	0.10	0.09	0.18	0.15	0.17	0.31	0.49	2.00
$r_{b,2}$	0.08	0.10	0.13	0.18	0.17	0.27	0.36	0.33	0.56	0.79	3.90
					SPE						
M_a	1.28	1.41	2.15	3.15	3.72	5.67	7.17	8.16	13.45	16.56	133.06
r_a	0.17	0.16	0.12	0.17	0.13	0.21	0.15	0.27	0.24	0.31	5.45
$M_{b,1.5}$	1.83	2.03	3.10	4.50	5.31	8.11	10.32	11.68	19.15	23.70	189.12
$r_{b,1.5}$	0.71	0.77	1.05	1.50	1.74	2.53	3.33	3.67	5.81	7.37	59.45
$M_{b,2}$	1.98	2.20	3.39	4.96	5.84	8.89	11.25	12.71	21.36	27.27	207.28
$r_{b,2}$	0.86	0.94	1.33	1.95	2.24	3.33	4.16	4.71	7.93	10.94	77.56

Source: Authors' calculations

Table 9. Behavior of errors of estimation of expected value of reserve ES_i and the total reserve *ES* and behavior of errors of prediction of reserve S_i and the total reserve *S* for disturbances of types I, II, III

	Estimation						Prediction					
i	M_a	r_a	$M_{b,1.5}$	$r_{b,1.5}$	$M_{b,2}$	$r_{b,2}$	M_a	r_a	$M_{b,1.5}$	$r_{b,1.5}$	$M_{b,2}$	$r_{b,2}$
	Rbias						*RbiasP*					
1	2,67	4,24	2,35	3,90	1,93	3,80	3,58	5,14	4,29	7,00	3,58	6,84
2	0,43	0,75	0,42	0,71	0,43	0,69	0,29	0,84	0,72	1,46	0,56	0,86
3	0,15	0,27	0,19	0,35	0,29	0,55	0,40	0,71	0,30	0,83	0,38	1,01
4	0,15	0,18	0,11	0,32	0,20	0,30	0,13	0,44	0,26	0,57	0,28	0,59
5	0,07	0,12	0,07	0,17	0,18	0,26	0,05	0,23	0,17	0,41	0,27	0,58
6	0,05	0,08	0,09	0,15	0,09	0,14	0,10	0,25	0,13	0,37	0,19	0,35
7	0,05	0,09	0,05	0,11	0,08	0,11	0,18	0,33	0,11	0,23	0,11	0,27
8	0,03	0,08	0,08	0,11	0,08	0,15	0,06	0,12	0,12	0,17	0,13	0,27
total	0,06	0,09	0,06	0,09	0,10	0,16	0,07	0,18	0,09	0,14	0,05	0,12
	SPE						*SPEP*					
1	132,98	5,30	189,12	60,68	206,54	78,10	183,91	6,12	264,49	84,25	284,70	104,29
2	16,39	0,21	23,52	7,36	26,96	10,71	28,21	0,40	40,20	12,30	44,20	16,29
3	10,73	0,33	15,19	4,66	17,25	6,63	19,92	0,57	28,57	8,99	31,84	12,31
4	6,61	0,15	9,40	2,89	10,50	3,98	13,56	0,33	19,31	6,21	21,30	7,92
5	5,04	0,14	7,13	2,18	7,97	3,01	10,78	0,29	15,51	4,77	17,12	6,39
6	3,91	0,09	5,59	1,71	6,15	2,27	8,97	0,20	12,98	4,11	14,08	5,21
7	2,88	0,06	4,11	1,28	4,49	1,65	7,05	0,16	10,05	3,09	10,99	4,06
8	2,33	0,06	3,30	1,02	3,63	1,34	5,69	0,15	8,07	2,44	8,91	3,28
total	2,83	0,08	4,00	1,22	4,47	1,68	4,27	0,10	6,10	1,91	6,66	2,47

Source: Authors' calculations

Consider behavior of errors of estimators of expected value of reserves and predictors of reserves. In all types of disturbances the errors behave similarly (see Table 9). The oscillation of relative bias is less than the one of standard percentage error. *SPE* is increasing function of variance of contaminating distribution and increasing function of power of disturbance. Owing to oscillation of errors and their maxima it may be concluded that estimators of expected reserves are robust with respect to the relative bias. However, on account to the other measures of quality, the variance of contaminating distribution and the power of disturbance influence behavior of estimators. For disturbances of the type "b" even for $\varepsilon = 0{,}1$ *SPE* is larger than the one in disturbances of the type "a", moreover they are increasing functions of ε .The errors of prediction are larger than errors of estimation of ES_i, but exhibit analogous trend.

Let us consider disturbances of type IV. Here all distributions that disturb distributions of severity of claims have γ and *p* equal to ones in zero scenario but, compared to this, *Rbias* for $\hat{\gamma}$ is more than 100 times larger, even for ε = 0.01 (cf. Table 10). If power of disturbance increases then *Rbias* also increases significantly, for ε = 0.2 is greater than 23, while for $\varepsilon = 0$ equals -0.0024. For $\varepsilon = 1$, for all i, j, there is no mixture of gamma distributions but, similar to the case of $\varepsilon = 0$, we have a gamma distribution. It may explain lower values of errors than that for $\varepsilon \in \{0.1; 0.2; 0.5\}$. In value of *SPE* the bias plays major role, variance of estimator does not change so much (cf. Table 10). Errors of estimation of *p* do not rise as much as errors of $\hat{\gamma}$, they are about 10 times smaller. Comparing Tables 5 and 10 it may be concluded that type IV has larger influence on behavior of estimators than changing of the shape of distribution in the types I, II, III. Moreover, regardless of the power of disturbance, errors for type IVa are greater than that for type IVb.

Table 10. Behavior of errors of estimators $\hat{p}$, $\hat{\gamma}$, $\hat{\phi}$ for disturbance of type IV

	ε									
	0.01		0.1		0.20		0.50		1.00	
	IVa	IVb	IVa	IVb	IVa	IVb	IVa	IVb	Iva	IVb
	Rbias									
$\hat{p}$	-0.2014	-0.0645	-1.5552	-0.6086	-2.3717	-1.1207	-2.3714	-1.8847	0.8831	0.0396
$\hat{\gamma}$	1.6748	0.5356	14.1979	5.1915	23.0648	9.9105	23.0759	17.6414	-6.7869	-0.3153
$\hat{\phi}$	0.8613	0.2543	6.9897	2.4319	11.1164	4.5339	12.7848	7.7715	2.0335	-0.1608
	SPE									
$\hat{p}$	0.2340	0.1264	1.5609	0.6218	2.3754	1.1291	2.3784	1.8893	0.9046	0.1214
$\hat{\gamma}$	1.9512	1.0437	14.2619	5.3132	23.1158	9.9976	23.1726	17.6968	6.9348	0.9839
$\hat{\phi}$	0.9988	0.5202	7.0168	2.4945	11.1336	4.5765	12.8064	7.7962	2.1280	0.4980

Source: Authors' calculations

Analogously to the basic distribution and disturbances of types I, II, III, errors *Rbias* and *SPE* of estimators $\hat{\lambda}_{ij}$, $\hat{\tau}_{ij}$, $\hat{\mu}_{ij}$ oscillate little with accident year *i* and the largest are for development year $j = 10$ (for disturbance of type IVb). Therefore tables presenting these errors for chosen ε include minima (min) and maxima (max) taken over $i \in \{0,1,\ldots,8\}$ or maxima and the upper bound for the oscillation *r*=max-min (see Tables 11, 12, 13).

For disturbances of type IVa distributions of number of claims are the same as that in the zero scenario. Thus it is suprising that values of the errors of $\hat{\lambda}_{ij}$ are very large in comparison to the ones for disturbances of the other types (cf. Tables 6 and 11) and are increasing functions of ε for $j < 10$. Even for $\varepsilon = 0.1$ the minimum of absolute values (over $i \in \{0,1,\ldots,8\}$) of errors is larger than the maximum of the corresponding ones for disturbances of types I, II, III. For disturbances of type IVb distributions of number of claims are mixture of Poisson distributions with different expectations. Thus expected values of generated variables differ from parameters λ_{ij} of the basic model. Here values of errors are larger than for disturbances of the type I, II, III but not as large as in type IVa (see Table 11). The maximum of *Rbias* is reached for $\varepsilon = 0.5$, while the maximum of *SPE* for $\varepsilon = 0.5$ if $j \leq 5$, for $\varepsilon = 1$ if $5 < j < 10$ and for $\varepsilon = 0.01$ if $j = 10$.

Table 11. Behavior of errors of estimators $\hat{\lambda}_{ij}$ for type IV

j	0	1	2	3	4	5	6	7	8	9	10
type IVa, *Rbias*											
	$\varepsilon = 0.1$, $r < 2.5$										
max	1.32	-2.12	-2.21	1.07	12.40	8.46	31.90	41.50	18.90	18.50	-28.24
	$\varepsilon = 1$										
min	-11.79	-22.87	11.49	56.76	153.5	140.1	330.3	409.3	255.4	260.7	-31.60
max	-0.38	-12.89	25.92	77.04	186.3	171.2	386.2	475.3	301.5	307.4	-22.76t
type IVa, *SPE*											
	$\varepsilon = 0.01$, $r \leq 0.15$										
max	1.07	1.21	1.85	2.65	3.42	4.84	7.96	10.27	12.51	14.89	103.1
	$\varepsilon = 1$, *r* similar to the corresponding *r* for *Rbias*										
max	11.83	22.89	26.03	77.18	186.6	171.6	388.1	478.3	306.2	312.3	95.98
type IVb, *Rbias*											
	$\varepsilon = 0.01$, $r \leq 0.04$										
max	0.07	-0.03	-0.18	-0.29	-0.33	-0.46	-0.39	-0.39	-0.56	-0.72	-14.19
	$\varepsilon = 0.5$, $r \leq 0.3$										
max	1.49	-0.97	-5.48	-7.61	-8.29	-10.07	-10.84	-11.56	-13.19	-12.85	-28.34
	$\varepsilon = 1$, $r \leq 0.1$										
max	-0.01	0.03	0.13	0.21	0.12	0.22	0.07	-0.09	-0.45	-0.69	-8.05
type IVb, *SPE*											
	$\varepsilon = 0.01$, $r \leq 0.2$										
max	1.04	1.15	1.80	2.64	3.17	4.77	6.12	7.01	11.55	13.92	103.8
	$\varepsilon = 0.5$, $r \leq 0.3$										
max	1.81	1.65	6.00	8.36	9.62	11.98	16.51	19.61	23.96	24.65	84.80
	$\varepsilon = 1$, $r \leq 0.3$										
max	1.07	1.18	1.99	3.45	6.42	8.43	18.36	24.21	29.76	30.98	91.21

Source: Authors' calculations

Table 12 presents errors of estimator $\hat{\tau}_{ij}$. For disturbance of type IVb, minima (*m*) and maxima (*M*) are taken over $i \in \{0,1,\ldots,8\}$ and $\varepsilon \in \{0;\ 0.01;\ 0.1;\ 0.2;\ 0.5;\ 1\}$. For type IVa, if $j < 10$, the errors reach the largest values for $\varepsilon = 0{,}5$, but they are less than maximum values for errors of $\hat{\lambda}_{ij}$. In this type only for $\varepsilon = 0{,}01$ or $j = 10$ the errors are close to the ones in the other types. For $\varepsilon > 0{,}01$ (except $j = 10$) the errors in type IVb are smaller than in type IVa. It is worth noting that for $j=10$ the errors *SPE* are close to each other and are in the interval $(56;68)$ for the type IVb, and interval $(65;69{,}5)$ for the other types.

Table 12. Behavior of errors of estimators $\hat{\tau}_{ij}$ for type IV

j	0	1	2	3	4	5	6	7	8	9	10
type IVa, *Rbias*											
	$\varepsilon = 1.01$, $r \le 0.03$										
max	-0.08	0.16	0.74	1.07	1.42	1.59	2.19	2.41	2.05	2.01	-41.11
	$\varepsilon = 0.1$, $r \le 0.2$										
max	-0.48	1.31	6.01	8.99	11.94	13.47	18.55	20.15	18.59	18.50	-27.88
	$\varepsilon = 0.5$, $r \le 1.5$										
max	1.31	3.20	12.97	20.44	29.31	31.51	44.72	48.55	43.82	43.92	-17.12t
type IVa, *SPE*											
	$\varepsilon = 0.01$, $r \le 0.03$										
max	0.29	0.33	0.92	1.32	1.69	2.01	2.84	3.26	3.47	3.83	68.40
	$\varepsilon = 0.1$, $r \le 0.2$										
max	0.66	1.34	6.04	9.03	12.00	13.56	18.76	20.44	19.06	19.00	67.85
	$\varepsilon = 0.5$, $r \le 1.5$										
max	1.37	3.21	12.99	20.46	29.33	31.55	44.78	48.62	44.03	44.16	67.18
type IVb, *Rbias*											
m	-1.47	-0.04	-0.14	-0.18	-0.24	-0.30	-0.57	-0.82	-1.26	-1.40	-41.33
M	0.03	1.22	6.05	8.40	9.19	11.48	12.32	12.20	14.34	14.38	-18.31
type IVb, *SPE*											
m	0.26	0.27	0.54	0.77	0.88	1.23	1.52	1.70	2.60	3.09	56.53
M	1.49	1.25	6.08	8.46	9.28	11.62	12.71	12.79	15.23	15.34	67.81

Source: Authors' calculations

Consider the behavior of errors of estimators $\hat{\mu}_{ij}$ (cf. Table 13). For type IVb and *Rbias*, because of very small oscillation, we give minima (*m*) and maxima (*M*) over *i* and ε. Both *Rbias* and *SPE* are increasing functions of power of disturbance for $j<10$ (excluding *Rbias* for type IVb). The errors are very large for type IVa even for $\varepsilon = 0{,}1$, compared to that in types I, II, III and IVb. Notice that parameters μ_{ij} in type IVb differ less from parameters μ_{ij} of the basic model than in type IVa. It may explain such large difference in errors between these types of disturbance.

Table 14 presents behavior of estimators of expected reserves and predictors of reserves. It gives maxima (*M*) and the oscillation taken over ε. For type IVb values of *Rbias* and *RbiasP* are close to that for types I, II, III. *SPE* and *SPEP* are decreasing functions of ε for $i=1$ and increasing ones for $i>1$. The oscillation of these errors is smaller for $i=1$ and larger for $i>1$ than the oscillation for the types I, II, III. The errors of estimation (for $\varepsilon \ge 0{,}1$) are lower (even more than 10 times) than for type IVa. For the case IVa only for $i=1$ or

$\varepsilon = 0{,}01$ the errors have values comparable to those in the basic model. For type IVa the errors of prediction are smaller than the ones of estimation of expected value of reserve and they are more stable when ε is changing. It may be caused by moving away from the values of μ_{ij} in the basic model. As it was mentioned, the errors of estimators of these parameters are large, they estimate completely different values, better corresponding to the future reserves.

Table 13. Behavior of errors of estimators $\hat{\mu}_{ij}$ for type IV

j	0	1	2	3	4	5	6	7	8	9	10
type IVa, *Rbias*											
	$\varepsilon = 0.01$, $r \le 0.25$										
max	0.07	-0.09	0.39	0.97	2.57	2.27	5.52	6.94	3.91	4.06	-1.27
	$\varepsilon = 0.1$, $r < 3$										
max	0.83	-0.83	3.67	10.17	25.86	23.13	56.73	70.67	41.81	41.32	-1.41
	$\varepsilon = 1$										
min	-7.62	-22.75	18.18	77.22	218.2	194.2	498.9	636.1	368.8	377.4	-29.27
max	7.37	-10.22	37.35	106.0	269.8	241.9	596.4	755.8	445.0	454.9	-17.81
type IVa, *SPE*											
	$\varepsilon = 0.01$, $r \le 0.4$										
max	1.27	1.41	2.17	3.30	4.70	6.26	10.69	13.68	15.88	18.75	129.6
	$\varepsilon = 0.1$, $r < 3$										
max	1.47	2.83	4.27	10.75	26.47	24.47	60.23	76.42	50.92	51.43	127.6
	$\varepsilon = 1$										
min	1.26	10.30	18.36	77.42	218.6	194.9	501.8	641.0	376.4	385.42	97.24
max	7.70	22.77	37.47	106.2	270.3	242.7	600.1	761.6	453.7	464.0	109.2
type IVb, *Rbias*											
m	-0.09	-0.07	-0.07	-0.14	-0.12	-0.11	-0.01	-0.49	-0.18	-0.28	-1.49
M	0.06	0.08	0.08	0.07	0.06	0.19	0.34	0.31	0.29	0.60	1.81
type IVb, *SPE*											
	$\varepsilon = 0.01$, $r \le 0.15$										
max	1.25	1.37	2.12	3.15	3.78	5.73	7.41	8.52	14.01	16.88	128.8
	$\varepsilon = 1$, $r \le 0.5$										
max	1.29	1.40	2.37	4.21	7.66	10.27	22.17	29.31	36.03	37.50	110.5

Source: Authors' calculations

CONCLUSION

Except the disturbance of type IVa, in behavior of the errors the power of disturbance and the variance of contaminating distribution play a key role, i.e., the errors are robust to considered disturbances of shape of distribution. The relative bias oscillates less with increasing variance of contaminating distribution than standard percentage error. If variance of contaminating distribution equals the variance of the basic model then oscillation of *SPE* is several times smaller than for contaminating distribution with larger variance. Moreover, *SPE* is increasing function of the power of disturbance. However, for type IVa both these errors are larger. Here products $\lambda_{ij}\tau_{ij}$ of contaminating distributions differ more from μ_{ij} in the basic model than for the other types. This departure from the assumptions causes the largest differences in the errors of estimation. The difference of expected value of claims does not have significant influence if this is accompanied by the appropriate difference of the expectation of number of claims (see Type IVb). Notice that in the other types of disturbances values μ_{ij} are preserved. The conclusion from analysis is that however the use of the Tweedie model and presented method of estimation allows us to predict reserves, the inference about expected value of payments and of number of payments and of reserves may lead to significant errors. For type IVb and $\varepsilon = 1$ estimates of expected values of number and severity of claims are closer, in the sense of *Rbias* and *SPE*, to parameters in the basic model than to values of actual distributions. The contamination by distributions with parameters μ_{ij} different from that in the basic model (even for small ε) leads also to larger errors of estimation of expected values of reserves than contamination of shape of distributions of claim severities without changing parameters μ_{ij}.

Table 14. Behavior of errors of estimation of ES_i and *ES* and bahavior of errors of prediction of reserve S_i and total reserve *S* for type IV

	type IVa				type IVb			
	Estimation		Prediction		Estimation		Prediction	
i	*M*	*r*	*M*	*r*	*M*	*r*	*M*	*r*
	Rbias		*RbiasP*		*Rbias*		*RbiasP*	
1	-1.31	22.20	3.58	7.90	1.80	3.25	3.58	4.02
2	423.33	423.27	2.66	2.37	0.54	0.82	0.54	1.04
3	409.08	409.00	-0.01	2.06	0.29	0.44	0.36	0.67
4	617.81	617.74	-0.18	21.54	0.22	0.41	0.02	0.97
5	575.38	575.31	-0.01	17.96	0.20	0.26	0.68	0.86
6	472.18	472.14	-0.06	19.30	0.14	0.22	0.13	0.61
7	348.97	348.96	-0.06	7.33	0.06	0.10	0.57	0.63
8	260.79	260.79	-0.03	4.79	0.07	0.09	0.82	0.86
Total	370.82	370.80	-0.05	10.64	0.10	0.14	0.42	0.48
	SPE		*SPEP*		*SPE*		*SPEP*	
1	129.5	26.5	190.30	9.89	128.7	18.23	184.19	29.38
2	432.0	415.7	37.86	9.96	37.0	20.72	64.15	36.25
3	413.3	402.6	26.86	7.27	26.0	15.34	45.38	25.79
4	620.7	614.1	26.83	13.45	19.8	13.26	35.25	21.87
5	576.9	572.0	21.62	10.88	15.0	10.01	27.15	16.41
6	473.1	469.3	21.97	13.11	10.5	6.57	20.46	11.59
7	349.4	346.5	10.61	3.56	6.7	3.82	14.58	7.54
8	261.0	258.7	7.93	2.30	4.5	2.18	10.32	4.69
Total	371.4	368.6	12.14	7.95	7.5	4.73	9.94	5.75

Source: Authors' calculations

REFERENCES

Antonio K., Beirlant J. (2007) Actuarial statistics with generalized linear mixed models, Insurance: Mathematics and Economics 40(1), 58–76.

Bornhuetter R. L., Ferguson R. E. (1972) The actuary and IBNR. Proceedings of the Casualty Actuarial Soc. 59, 181–195.

Boucher J. P., Davidov D. (2011) On the Importance of Dispersion Modeling for Claims Reserving: An Application with the Tweedie Distribution, Variance 5(2), 158–172.

England P. D., Verrall R. J. (1999) Analytic and bootstrap estimates of prediction errors in claims reserving, Insurance: Mathematics and Economics 25(3), 281–293.

England P. D., Verrall R. J. (2002) Stochastic claims reserving in general insurance, British Actuarial Journal 8(3), 443–518.

Gisler A., Wüthrich M. V. (2008) Credibility for the Chain Ladder Reserving Method, ASTIN Bulletin 38(2), 565–600.

Hampel F. R., Ronchetti E. M., Rousseeuw P. J., Stahel W. A. (1986) Robust statistics: the approach based on influence functions. Wiley, New York.

Han Z., Gau W. (2008) Estimation of loss reserves with lognormal development factors, Insurance: Mathematics and Economics 42(1), 389–395.

Huber P. J. (1981) Robust statistics. Wiley, New York.

Jasiulewicz H. (2013) Przestrzeń stanów i filtr Kalmana w teorii ubezpieczeń, Roczniki Kolegium Analiz Ekonomicznych z. 34, Oficyna Wydawnicza SGH, Warszawa, 101–116.

Jørgensen B. (1997) The Theory of Dispersion Model, Chapman and Hall, London.

Jørgensen B., de Souza M. C. P. (1994) Fitting Tweedie's Compound Poisson Model to Insurance Claims Data, Scandinavian Actuarial Journal 1, 69–93.

Liu H., Verrall R. J. (2009) A Bootstrap Estimate of the Predictive Distribution of Outstanding Claims for the Schnieper Model, ASTIN Bulletin 39(2), 677–689.

Mack T. (1991) A simple parametric model for rating automobile insurance or estimating IBNR claims reserves, ASTIN Bulletin 21(1), 93–109.

Mack T. (1993) Distribution-free calculation of the standard error of chain ladder reserve estimates, ASTIN Bulletin 23(2), 213–225.

Mack T. (1999) The standard error of chain ladder reserve estimates: recursive calculation and inclusion of a tail factor, ASTIN Bulletin 29, 361–366.

Ntzoufras I., Dellaportas P. (2002) Bayesian Modelling of Outstanding Liabilities Incorporating Claim Count Uncertainty, North American Actuarial Journal 6(1), 113–128.

Ohlsson E., Johansson B. (2006) Exact credibility and Tweedie models, ASTIN Bulletin 36(1), 121–133.

Peters G. W., Shevchenko P. V., Wüthrich M. V. (2009) Model uncertainty in claims reserving within Tweedie's compound Poisson models, ASTIN Bulletin 39(1), 1–33.

Sánchez J. R., Vilar J. L. (2011) Bayesian and credibility estimation for the chain ladder reserving method, Anales 2011, Instituto de Actuarios Españoles, 51–74.

Smyth G. K., Jørgensen B. (2002) Fitting Tweedie's compound Poisson model to insurance claims data: dispersion modelling, ASTIN Bulletin 32(1), 143–157.

Taylor G. C. (2000) Claim Reserving. An Actuarial Perspective, Elsevier Science Publishers, New York.

Wüthrich M. V. (2003) Claims reserving using Tweedie's compound Poisson model, ASTIN Bulletin 33(2), 331–346.

Wüthrich M. V., Merz M. (2008) Stochastic claims reserving methods in insurance, Wiley.

Wüthrich M. V., Merz M., Lysenko N. (2009) Uncertainty in the claims development result in the chain ladder method, Scand. Actuar. J. 109(1), 63–84.

Zieliński R. (1983) Robust statistical procedures: a general approach, Lecture Notes in Mathematics 982, Springer-Verlag, 283–295.

APPENDIX 1

Table 1.1. Values of parameters $10000\lambda_{ij}$ in the basic model

	j										
i	0	1	2	3	4	5	6	7	8	9	10
0	571.74	227.45	39.944	17.485	13.291	6.1327	4.4623	4.2884	2.0140	2.0033	0.0645
1	589.64	234.57	41.194	18.032	13.707	6.3246	4.6019	4.4226	2.0770	2.0660	0.0665
2	621.31	247.17	43.407	19.001	14.443	6.6644	4.8492	4.6602	2.1886	2.1770	0.0701
3	610.80	242.99	42.673	18.679	14.199	6.5516	4.7671	4.5813	2.1515	2.1401	0.0689
4	613.92	244.23	42.891	18.775	14.271	6.5850	4.7915	4.6047	2.1625	2.1510	0.0692
5	592.70	235.79	41.408	18.126	13.778	6.3574	4.6258	4.4455	2.0878	2.0767	0.0668
6	605.79	241.00	42.323	18.526	14.083	6.4979	4.7281	4.5438	2.1339	2.1226	0.0683
7	584.01	232.33	40.801	17.860	13.576	6.2642	4.5580	4.3804	2.0572	2.0463	0.0659
8	597.80	237.81	41.764	18.282	13.897	6.4121	4.6656	4.4838	2.1057	2.0946	0.0674

Table 1.2. Values of parameters τ_{ij} in the basic model

	j										
i	0	1	2	3	4	5	6	7	8	9	10
0	2997.0	2467.6	1710.0	1436.6	1355.9	1151.8	1077.1	1068.1	910.77	909.75	440.79
1	3016.6	2483.7	1721.1	1445.9	1364.7	1159.3	1084.1	1075.1	916.71	915.68	443.67
2	3050.0	2511.3	1740.2	1462.0	1379.8	1172.2	1096.2	1087.0	926.88	925.84	448.59
3	3039.1	2502.2	1733.9	1456.7	1374.9	1168.0	1092.2	1083.1	923.55	922.51	446.98
4	3042.3	2504.9	1735.8	1458.3	1376.4	1169.2	1093.4	1084.3	924.54	923.51	447.46
5	3019.8	2486.4	1723.0	1447.5	1366.2	1160.6	1085.3	1076.3	917.71	916.68	444.15
6	3033.8	2497.9	1730.9	1454.2	1372.5	1165.9	1090.3	1081.2	921.95	920.92	446.20
7	3010.5	2478.7	1717.6	1443.0	1361.9	1157.0	1082.0	1072.9	914.86	913.83	442.77
8	3025.3	2490.9	1726.1	1450.1	1368.7	1162.7	1087.3	1078.2	919.37	918.34	444.96

Table 1.3. Number of policies and values of parameters p, γ, ϕ

I	0	1	2	3	4	5	6	7	8
w_i	112953	110364	105400	102067	99124	101460	94753	92326	89545
p = 1.1741431	γ = 4.7424055		ϕ = 1481.7243						

Source: Authors' calculations based on data from Wüthrich [2003]

APPENDIX 2

Table 2.1. Values of parameters $10000\tilde{\lambda}_{ij}$

	j										
i	0	1	2	3	4	5	6	7	8	9	10
0	551.47	309.86	37.626	11.863	4.5152	2.1248	1.1509	1.0624	0.5312	0.3541	0.0885
1	579.45	302.82	36.425	9.786	2.8089	1.2685	1.0873	0.4530	0.5437	0.4530	0.0899
2	607.78	278.94	38.046	9.298	3.9848	1.7078	0.4744	0.2846	0.2846	0.4089	0.0905
3	602.35	283.93	29.490	9.014	4.0170	2.2534	1.1757	0.9797	0.4573	0.4058	0.0898
4	600.46	272.29	30.669	9.483	4.9433	2.2194	0.7062	0.6888	0.4515	0.4007	0.0887
5	583.88	265.33	29.568	8.969	3.1540	2.2669	0.8898	0.6688	0.4384	0.3890	0.0861
6	585.21	290.65	30.817	8.126	3.6938	1.9596	0.9170	0.6892	0.4518	0.4009	0.0888
7	597.88	266.34	28.919	8.773	3.8086	1.9323	0.9043	0.6796	0.4456	0.3953	0.0875
8	601.93	248.37	24.904	9.154	3.7345	1.8947	0.8867	0.6664	0.4369	0.3876	0.0858

Table 2.2. Values of parameters $\tilde{\tau}_{ij}$

	j										
i	0	1	2	3	4	5	6	7	8	9	10
0	2864.2	2126.4	2106.9	3042.9	4061.4	2565.4	1229.1	2077.0	206.0	3910.8	321.0
1	3052.2	1991.8	2341.9	1438.8	2240.6	2697.8	4486.0	22278.2	7043.8	5166.6	331.8
2	3120.7	2152.2	2743.6	2853.6	3872.7	3888.9	11375.6	3293.7	6552.0	4949.2	350.5
3	3140.2	2032.4	2634.6	3359.2	3559.0	4237.6	2293.6	6192.0	4370.8	4898.6	346.9
4	3073.2	2146.5	2267.9	3070.5	7051.5	5026.6	16549.0	7178.1	4395.9	4926.8	348.9
5	3179.0	2132.7	1939.3	2731.5	3339.8	4096.2	5706.1	7228.6	4426.9	4961.4	351.4
6	3093.4	2145.3	1797.3	2992.9	9911.5	3851.2	5611.5	7108.7	4353.5	4879.2	345.6
7	3010.0	2078.6	2072.2	3121.9	4870.7	3766.0	5487.4	6951.6	4257.2	4771.3	337.9
8	3123.5	2249.1	2194.4	2921.9	5129.4	3966.0	5778.8	7320.8	4483.3	5024.7	355.9

Source: Authors' calculations based on data from Wüthrich [2003]