QUANTITATIVE METHODS IN ECONOMICS
Vol. XVI, No. 1, 2015, pp. 137 – 146

TRADE DURATION AND MARKET IMPACT

Marek Andrzej Kociński
Department of Applied Mathematics
Warsaw University of Life Sciences – SGGW
e-mail: marek_kocinski@sggw.pl

Abstract: In this article the problem of the algorithm of the transaction execution as the factor in market impact modelling is studied. The current state of research in this area is presented and discussed. The paper adds new arguments to the discussion on this topic. Moreover, the solution to the problem of the trade execution's duration in practical application of [Almgren et al. 2005] market impact model is proposed.

Keywords: market impact, square root impact law, trade duration, transaction cost

INTRODUCTION

Market impact (also called price impact) can be defined as some sort of a change in the asset price with respect to adequate reference price, caused by trading. This change, if occurs, is against the trade initiator, that is the price grows when buying and drops when selling, and thus the price impact is a source of transaction costs. It in intuitive and a standard in economic theory that a demand increase should result in growth of the price and a supply increase should result in the price drop. The concept of market impact is closely related to the notion of bid-ask spread, which is the difference between the best available in the market bid and ask prices (called just bid and ask prices, respectively) and often is expressed as a fraction of a so called mid-price which is defined as the average between best bid and ask prices and represents the market value of an asset. If, as some authors do, the mid-price is the reference price, then bid-ask spread is a part of market impact. However, it is often assumed that the reference price is a bid price in case of a seller-initiated trade and an ask price in case of a buyer-initiated trade. In such approach the spread and price impact are treated separately.

Market impact is the main source of liquidity risk. It is the reason of not being able to execute a transaction at the current quoted price because execution moves the price in an unfavourable manner. Spectacular examples showing how important is market impact are: the fiasco of Metallgesellschaft in 1993, the LTCM crisis in 1998 and the cancelling of the portfolio of Jérôme Kerviel by Societé Générale in 2008 [Schied and Slynko 2011]. Since market impact moves adversely the prices at which transactions are made, it can, sometimes significantly, reduce profits and turn theoretically profitable strategy into a financial failure.

Therefore, it is no surprise that modelling, estimation and analysis of market impact interests many asset managers and scholars. In fact, research on price impact has become one of the most popular activities in quantitative finance since the mid-1990s [Tóth et al. 2011].

The aim of this article is to analyse the problem of importance of the algorithm of the trade execution as the variable in modelling of market impact. The paper contributes to the literature on market impact by adding new arguments confirming that the execution speed is of minor significance in price impact modelling. Next contribution is a new view on the model of [Almgren et al. 2005] which allows for better practical use of this model. This article contains also the example of the calculation of market impact in Warsaw Stock Exchange, with use of order book.

MARKET IMPACT MODELS AND TRADE DURATION

Market impact modelling and estimation has been very important to scholars interested in market microstructure and practitioners. A well-calibrated price impact model is an important part of quantitative investment management. It is a useful tool in predicting transaction costs and price changes due to trading activity. Such expectations allow to forecast the consequences of implementing portfolio strategies. Today, any decent pre-trade analytic software takes into account the price impact of a proposed transactions as a function of trade-based parameters and characteristics of the traded security [Gatheral 2010].

The simple and popular approach to modelling price impact suggested in the literature is to consider it as one of the components of transaction costs. Then the formula for market impact as a relative fraction of the price of the traded security at the beginning of the trade, is given as follows:

$$MI = c\sigma\left(\frac{V_{trade}}{\overline{V}}\right)^{\delta} \tag{1}$$

where σ is the daily volatility, V_{trade} is the volume of the executed trade, $\overline{V}$ is the average daily volume, c is the numerical constant of order unity that can be estimated from the representative sample of transactions and the exponent δ does not exceed 1 and its estimation has often the range between 0.4 and 0.7, however an important practically and theoretically case is linear function of market impact with $\delta = 1$.

A particular variant of the formula (1) is the so-called square root impact law which is widely used in academia and financial service industry:

$$MI = c\sigma\sqrt{\frac{V_{trade}}{\bar{V}}} \tag{2}$$

Equation (2) is strongly supported by the empirical data, reasonable arguments given in [Grinold and Kahn 2000] and it is consistent with a trading rule of thumb according to which the transaction cost of the volume equal to the average one day's volume, costs roughly one day's volatility of the price. [Grinold and Kahn 2000], [Gatheral 2010].

Formula (1) suggests that the only trade-based variable which is necessary to calculate the market impact is the transaction size V_{trade}, it does not take into account the execution algorithm used by the trader. In this context it should be noticed that there is a great variety of execution strategies – apart from static (determined in advance of trading) there are also dynamic which are conditioned on movement of the security price during execution of transaction. The trade execution is roughly characterized by duration which describes how long the executing lasts. The duration is determined by the trading rate (the speed of execution) and the transaction volume. Low importance of the execution characteristics is more emphasised by some authors [Tóth et al. 2011], [Zarinelli et al. 2014] by using, for the volume of the executed trade, in the market impact equation the name "metaorder", which denotes the sequence of trading decisions. A metaorder is usually fragmented and traded incrementally by single orders which are, in this context, called child orders.

Such approach is however contrary to the widespread opinion that market impact can be reduced by dividing intended transaction into smaller orders and placing them in separate time intervals. In short, the popular view is that slower trade execution lowers price impact. It is also empirically confirmed that the trading rate can, in some circumstances, significantly affect the market impact. There is an extensive theoretical research and practical solutions on the problem of optimal counteracting market impact while executing transaction. The question arises, therefore, about the explanation of this conceptual contradiction. In order to answer this question, it is worth pointing out that the observations where the duration was important pertain to the cases of very large trading rates were trading sizes were large relative to the volume of trade offers in the order book. For reasonable trading rates (about 1% to 25% of average daily volume per day), it seems that the market impact is roughly independent of trade duration [Gatheral 2010]. It is even presented in [Gatheral and Schied 2013] as the empirical rule of thumb that market impact is roughly proportional to the size of the transaction and not very dependent on the trading rate. There is also a heuristic argument that duration can or maybe even should be omitted as variable in formula (1). Namely, [Grinold and Kahn 2000] claimed that in a framework of inventory risk model, for a proposed trade of size V_{trade}, the estimated time before a sufficient number of opposing trades appears in the market to execute the transaction (time to clear the transaction) is given by the formula

$$\tau_{clear} \propto \frac{V_{trade}}{\overline{V}} \tag{3}$$

Formula (3) establishes a strictly linear relationship between the size of the transaction and the time of execution. Thus, since it is natural that the durations can be measured by time τ_{clear}, it is, according to (3), fully characterized by the trading volume. Consequently, since the size of the transaction is a variable in (1), duration does not have to appear there.

Weak dependence of market impact on trade duration is also confirmed by the empirical data in [Engle et al. 2008].

In order to provide new arguments for discussion on the meaning of execution algorithm in market impact modelling I would like to notice that it is not uncommon to consider the problem of the optimal portfolio selection in multiperiod setting where neither the total transaction volume nor the investment horizon has to bounded in advance. Then, it is appropriate to ask how long lasts the market impact effect of trading in one period, on the asset's price dynamics. It is clear that the value of market impact in next time period strongly depends on the answer to this question. Most practitioners in execution models use the decomposition of the market impact into permanent and temporary market impact [Guéant 2014]. Temporary price impact affects a single transaction and may be considered as the cost of providing enough liquidity to absorb the trade. The permanent price impact component is an information-based effect and measures the change of the market value before and after trade. This is due to the fact that there is no easy, method to distinguish not informed traders from informed traders and therefore each transaction is considered as a source of information on the market value of the traded asset. Thus, a buyer-initiated transaction tells the market participants that an asset may be underpiced and a seller-initiated transaction is a signal that an asset is overvalued. As a result, the transaction causes the change in the theoretical value of the asset which is unfavourable to the initiator of the trade.

It seems that the speed execution has different effects on the levels of the considered components of market impact. The higher trading rate results in larger temporary impact and lower permanent impact, in case of lower trading rate it is the other way round. Therefore the coexistence of the two components of market impact which differently react on the speed of trading I find as one of the arguments for low significance of the execution style in modelling market impact.

In case of informed traders there is also another factor that counteracts the effect of reducing market impact of the strategy of slower execution. It is opportunity cost. This notion assumes that, if the motivation of the trade is information on the future value of the traded assets, then quick execution is necessary because such information can be used only for a limited time. Rapid execution enables to benefit from the underpricing in case of buying and from overpricing in case of selling.

Physical time is not the only method of measuring the duration of the execution. The duration is sometimes quantified in so called volume time [Almgren et al. 2005], [Zarinelli et al. 2014] which is calculated for time periods shorter than trading day, as the fraction of an average daily volume that has been executed up to physical time t. Speaking formally let $V(t)$ be the total volume traded in the market from the trading day's open up to physical time t. Volume time (also called volume duration) is defined as $v = \frac{V(t)}{V(t_c)}$ or $v = \frac{V(t)}{\bar{V}}$, where t_c is the market close. It is easily seen that independently of the total daily volume, the volume time defined that way equals 0 at the market opening and 1 at market closing time [Zarinelli et al. 2014].

The duration measured in volume time is an input variable in the elaborately worked out and seemingly ready for use model used by [Almgren et al. 2005]. In its estimation [Almgren et al. 2005] used the data set of almost 700,000 trade orders from the US market, executed by Citigroup equity trading desk from December 2001 to June 2003, in which a direction of the trade (buyer or seller initiated) is known. The market impact defined as the execution cost in the model of [Almgren et al. 2005], assuming that price impact is positive for buy as well as for sell orders (in the original version of this model the execution cost can be negative), is given by the formula:

$$MI = \frac{1}{2}\gamma\sigma\frac{V_{trade}}{\bar{V}}\left(\frac{\Theta}{\bar{V}}\right)^{1/4} + \eta\sigma\sqrt{\frac{V_{trade}}{\bar{V}T}}^{3/5} \tag{4}$$

where Θ is the total number of shares outstanding, T is volume duration of active trading, γ, η are the constants.

The estimated values of γ and η were calculated by linear regression [Almgren et al. 2005] and they calculated that $\gamma = 0.314 \pm 0.041$ and $\eta = 0.142 \pm 0.0062$.

An example of the application of this model is presented in [Kociński 2014] where the duration was assumed to be an arbitrary value. The variable T I find the most problematic in the model given in [Almgren et al. 2005]. It seems that the trader is rather not able to control the duration of execution to the extent which is necessary to produce reliable estimator of the volume duration. However, by reasonable assumptions, applying of low-frequency estimator which uses only the daily volumes in estimating the time to clear the trade and application of linear approximation, it is possible to eliminate T from the formula (3).

Namely, to justify the use of low frequency estimation, it seems sound to assume that average time to fulfil an order does not depend on whether it is a buy or sell order. By this assumption, the formula (3) and the fact that the coefficient of proportionality in (3) which is estimated by using daily volumes, equals 1, it is possible to write the following formula for physical time of transaction execution:

$$\tau_{clear} = \frac{V_{trade}}{\overline{\overline{V}}} \tag{5}$$

The volume time given in [Almgren et al. 2005] is defined as $v = \frac{V(t)}{\overline{\overline{V}}}$. From the assumption that $V(t)$ is a linear function of t, it follows that physical time is identical with volume time and consequently from (6) it follows that the value of T is given by the formula:

$$T = \frac{V_{trade}}{\overline{\overline{V}}} \tag{6}$$

The equations (5) and (6) imply the following, simplified expression for the market impact:

$$MI = \frac{1}{2}\gamma\sigma\frac{V_{trade}}{\overline{\overline{V}}}\left(\frac{\Theta}{\overline{\overline{V}}}\right)^{1/4} + \eta\sigma \tag{7}$$

According to (7) the price impact is a linear function of the traded volume, what is an interesting in view of the popularity of the square root model. However the assumption that market impact is linear in the traded volume one can meet in the literature (see for example [DeMiguel et al. 2014]). Such approach can be partly justified in market microstructure theory by the Kyle model [Kyle 1985].

The model given in [Almgren et al. 2005] treats the bid-ask spread as a part of the market impact and this allows to interpret the second component of the sum in (7) as the bid-ask spread, and in this approach it is a linear function of volatility. Moreover, an interesting observation is that in view of (7) the average volume is not a significant determinant of the spread. The negligibility of the market volume in case of the bid-ask spread in the option market was found in [Cho and Engle 1999].

EMPIRICAL RESEARCH

To verify the conclusions on the role volatility and volume in determining bid-ask spread following from formula (7) the empirical research was carried out on a random sample of 300 stocks quoted on the Warsaw Stock Exchange (WSE) in 2014. The annual volatility was computed from the formula:

$$volatility = \ln\left(\frac{P_{\max}}{P_{\min}}\right) \tag{8}$$

where $P_{\max}$ and $P_{\min}$ denote the maximal and minimal price of the stock in year 2014, respectively.

Using data 2014 WSE Statistic Bulletin I calculated the Pearson correlation coefficients between the average spread and volatility ($r_{volatility}$) and the average spread and the average daily volume (r_{volume}). Then, I verified their significance by the standard significance test with test statistics $t = r\sqrt{\frac{n-2}{1-r^2}}$ where r is the correlation coefficient and n is a number of observations. The following results were obtained (p is the p-value):

$$r_{volatility} = 0{,}256; p = 0{,}0000068$$

$$r_{volume} = 0{,}037, p = 0{,}5256843.$$

The correlation between volatility and spread is highly statistically significant, in contrary to the correlation between the spread and volume. I also estimated the regression coefficients of the spread on volatility:

$$spread = \underset{(2{,}95*10^{-25})}{142{,}48} + \underset{(6{,}94*10^{-6})}{63{,}18} * volatility \qquad (9)$$

The results concerning correlation coefficients presented above are consistent with equation (7). However, the non-zero constant in regression equation suggests that formula (7) should be supplemented by an additive constant.

PRICE IMPACT IN MARKET MICROSTRUCTURE: AN EXAMPLE FROM WARSAW STOCK EXCHANGE

One of the most important characteristics of a market is the type of its execution system. In this respect there are three major types of markets: quote driven markets, order driven markets and brokered markets. Warsaw Stock Exchange (WSE) is classified as order driven market [Doman 2011]. Table 1 shows the first five rows of an order book for the stock of the company Stalexport Autostrady S.A. (denoted as STALEXPORT), from the WSE at some point in time during the trading session on 09 September 2015.

Table 1. The first five rows of the order book for the shares of STALEXPORT at some moment during the trading day in WSE on 09 Sep 2015

Bid size	Bid price (PLN)	Ask price (PLN)	Ask size
2000	3.21	3.24	2700
1600	3.18	3.25	3582
2375	3.17	3.27	1500
1900	3.16	3.28	5821
4433	3.15	3.29	2550

Source: http://biznes.onet.pl/gielda/notowania/gpw-rynek-glowny/akcje-wszystkie,101,notowania-gpw-ciagle-szczegolowe.html

The second row of Table 1 represents the highest available bid price (3.21), the number of stocks to buy at this price (2000), the lowest available ask price (3.24) and the number of stocks to sell at this price (2700). The theoretical price of the stock STALEXPORT is the average of the best bid and ask prices and equals 3.225. The spread is computed as $\frac{3.24-3.21}{3.225}$ and equals approximately 0.009.

Consider an investor who wants to buy some shares and places market buy order. For the sake of transparency of this example I assume that commission here is negligible, since this sort of cost is just an additive constant. On the frictionless market the cost of his or her transaction would be 3.225 PLN per share. The transaction cost here is calculated as the relative increase in average price per share with respect to the theoretical price. On real market if the number of shares does not exceed 2700 then the required by the market price per share is 3.24 PLN and transaction cost is just half of the spread. However, if the trading volume increases then the average price per share grows. It is clear that the average price per share can be computed from the formula:

$$\overline{S} = \frac{\sum_{i=1}^{n} x_i S_i}{\sum_{i=1}^{n} x_i} \tag{10}$$

where n is the number of price levels in the order book, S_i is the i-th price level and x_i is the number of shares for which the ask price is S_i, respectively. Then the average cost per share equals $\frac{\overline{S}-S_0}{S_0}$ where S_0 is the theoretical price. Market impact is here the difference between transaction cost and the half of the spread. The calculation of the average costs and market impact for trading volumes corresponding to the cumulated values of ask sizes from the order book are shown in Table 2.

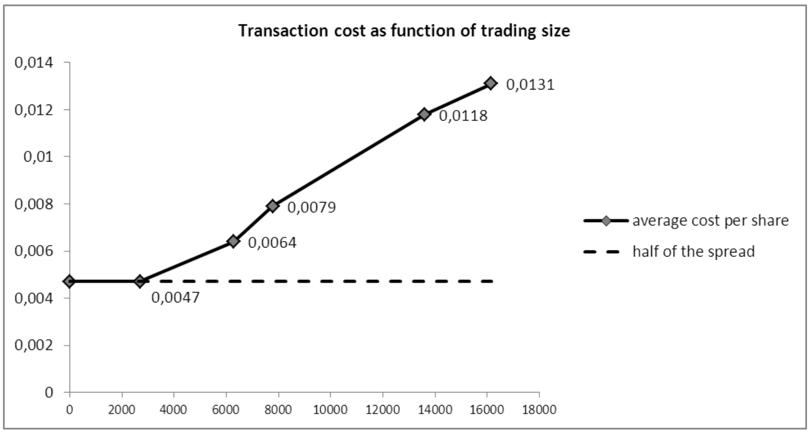
Table 2. The average cost per share for purchase transactions of the stock

Trading size	Average cost	Market impact
2700	0.0047	0.0000
3582	0.0064	0.0018
1500	0.0079	0.0032
5821	0.0118	0.0071
2550	0.0131	0.0085

Source: data from Table 1 and own elaboration

The graph of the transaction costs function is presented in Figure 1.

Figure 1. The function of transaction cost



Source: data from Table 2 and own elaboration

The dashed line corresponds to the case of proportional transactions costs, where the only source of payments for exchange of shares is the bid-ask spread. The market model with proportional cost is much more realistic then the assumption of frictionless market where there are no costs associated with trading. Figure 1, however, shows, that in order to precisely estimate the risk in asset management, one should take the price impact into account.

CONCLUSIONS

The paper presents the problem of the influence of the method of transaction execution on the magnitude of market impact. Neglecting the style of trading as factor affecting price impact is popular among both theoreticians and practitioners. Taking into account the literature on this subject and the arguments presented in this article, It appears reasonable to assume that when the transaction volume is not extremely large and the investment horizon is not very long the execution style is not very important. The effect of lowering market impact with execution method is usually of minor order than the value of price impact, and therefore the execution algorithm need not be taken into account when estimating market impact.

REFERENCES

Almgren R., Thum C., Hauptmann E., Li H. (2005) Direct Estimation of Equity Market Impact Risk 18, pp. 58–62.

Cho Y.-H., Engle R. F. (1999) Modeling the Impacts of Market Activity on Bid-Ask Spreads in the Option Market, http://www.nber.org/papers/w7331.pdf

DeMiguel V., Martín-Utrera A., Nogales F. J. (2014) Parameter uncertainty in multiperiod portfolio optimization with transaction costs, http://faculty.london.edu/avmiguel/DMN-2014-01-09.pdf

Doman M. (2011) Mikrostruktura giełd papierów wartościowych”, Wydawnictwo Uniwersytetu Ekonomicznego w Poznaniu, Poznań.

Elton R. J., Gruber M. J., Brown S. J., Goetzmann W. N. (2010) Modern Portfolio Theory and Investment Analysis, John Wiley & Sons, Hoboken.

Engle R., Ferstenberg R., Russel J. (2008) Measuring and modeling execution cost and risk, http://faculty.chicagobooth.edu/jeffrey.russell/research/transcost.pdf

Gatheral J., No-Dynamic-Arbitrage and Market Impact (2010) Quantitative Finance 10, pp. 749–759.

Gatheral J., Schied A. (2013) Dynamic models of market impact and algorithm for order execution, [in:] Fouque J.-P., Langsam J. A. (eds.) Handbook on Systemic Risk, Cambridge University Press, Cambridge.

Grinold R. C., Kahn R. N. (2000) Active Portfolio Management, McGraw Hill, New York.

Guéant O. (2014) Permanent market impact can be nonlinear, http://arxiv.org/pdf/1305.0413v4.pdf

Kociński M. (2014) Transaction Costs and Market Impact in Investment Management, e-Finanse, pp. 28–35.

Kyle A. S., (1985) Continuous auctions and insider trading, Econometrica 53, pp. 1315-1335.

Scherer B. (2010) Portfolio Construction and Risk Budgeting, Risk Books, London.

Schied A., Slynko A. (2011) [in:] Blath J., Imkeller P., Roelly S. (eds.) Surveys in Stochastic Processes, European Mathematical Society Publishing House, Zürich.

Tóth B., Lempérière Y., Deremble C., de Lataillade J., Kockelkoren J., Bouchaud J.-P. (2011) Anomalous Price Impact and the Critical Nature of Liquidity in Financial Markets, http://journals.aps.org/prx/pdf/10.1103/PhysRevX.1.021006

Zarinelli W., Treccani M., Doyne Farmer J., Lilo F. (2014) Beyond the square root: evidence for logarithmic dependence of market impact on size and participation rate, http://arxiv.org/pdf/1412.2152v1.pdf

WSE Statistics Bulletin (2014) http://www.gpw.pl/statystyki_roczne.