METODY ILOŚCIOWE
W BADANIACH EKONOMICZNYCH

QUANTITATIVE METHODS IN ECONOMICS

Vol. XVIII, No. 1

Warsaw University of Life Sciences – SGGW
Faculty of Applied Informatics and Mathematics
Department of Econometrics and Statistics

QUANTITATIVE METHODS IN ECONOMICS

METODY ILOŚCIOWE W BADANIACH EKONOMICZNYCH

Volume XVIII, No. 1

Warsaw 2017

ISSN 2082-792X
e-ISSN 2543-8565

Warsaw 2017, Volume XVIII, No.1

The original version is the paper version
Journal homepage: qme.sggw.pl
Published by Warsaw University of Life Sciences Press

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII, No. 1, 2017

CONTENTS

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII, No. 1, 2017, pp. 7 – 17

Received: 28.01.2017
Accepted: 05.04.2017

ON THE CHOICE OF SYNTHETIC MEASURES FOR ASSESSING ECONOMIC EFFECTS

Zbigniew Binderman
Faculty of Economic Sciences
Jacob of Paradise University in Gorzów Wielkopolski, Poland
e-mail: zbigniew_binderman@sggw.pl
Bolesław Borkowski, Grzegorz Koszela (ORCID: 0000-0003-4175-4450)
Ryszard Kozera, Wiesław Szczesny
Faculty of Applied Informatics and Mathematics
Warsaw University of Life Sciences – SGGW, Poland
e-mail: boleslaw_borkowski@sggw.pl
grzegorz_koszela@sggw.pl, ryszard_kozera@sggw.pl
wieslaw_szczesny@sggw.pl

Abstract: Multidimensional analysis uses various measures for assessing economic effects. However, no single synthetic measure, regardless how popular, can give a satisfactory solution to the above problem. In general, various approaches of combining measures can lead to stable outcomes. Nevertheless, when combining "weak" classifiers one can obtain inevitably poorer classification. We propose here a new approach to construct doubly synthetic measures. The main goal of this work is to analyse the influence these new synthetic measures on the ranking of multidimensional objects.

Keywords: multidimensional analysis, synthetic measure, ranking methods

INTRODUCTION

Many scientific studies use multidimensional analysis to process their empirical data. It is also widely used in an enterprise environment. It is applied to compare objects defined as a set of *n*-indicator variables. Usually, the goal of such analysis is to reduce a large quantity of gathered data to a small number of simple categories (a few synthetic indicators) which is a subject to further analysis and allows the creation of uniform groups obtained and defined by the values of these categories. The bibliography in this area is extensive (i.a. Aczel 1989, Morrison

DOI: 10.22630/MIBE.2017.18.1.01

JEL: C15, C63

1990, Hair et al. 1995). Among the group of methods discussed in the related literature, the basic group contains the methods that utilize the so called model objects. Breiman proved in his works [Breiman, 1994, 1996, 1998] that using a single synthetic measure to either rank objects or to classify those objects can be far from optimal. Furthermore, a superposition of many measures gives a stable and close to optimal result. It should be noted, however when combining "weaker" classifiers one can obtain weaker classifier either.

According to Jackson [Jackson 1969, 1969a, 1970] our problem is correctly stated when:

- as a result of the applied algorithm we obtain a single result,
- the resulting classification is stable. The latter means that the resulting classification or order does not change "drastically" when the inputs are slightly varied,
- the applied algorithm is invariant with respect to the permutation of variables and names of objects that are to be ordered and classified,
- the applied algorithm scale insensitive in all cases when the values of variables belong to a scale with an absolute zero. The latter means that the algorithm is indifferent to multiplication of the matrix of distances.

In general, ranking methods can be split into model and non-model ones.

Non-model methods rely on constructing a synthetic aggregate measure based only on normalized values of features. Model methods rely on constructing taxonomic measures of growth (artificial reference points) and measuring distances from these models and on are based on creating a synthetic measure.

Naturally, models can also play an important part in normalization of variables (see [Kukula 2000]). Noticeably, most techniques that are commonly classified as non-model can be ultimately reduced to a form relying on the chosen explicit model.

The choice of a model in an automated reporting system is especially important in assessing, ordering and classifying objects according to the value of a synthetic measure within a specified period. Consequently, the goal of this work is to illustrate the influence of a choice of a proposed model on multidimensional objects' ranking.

MODEL MEASURES

Let, $X = \Re^n$, $\Re = (-\infty,\infty)$, $n \in N$, denote n-dimensional vector space. Consider now a problem of classifying $m \in N$ objects $Q_1, Q_2,...,Q_m$ of a studied phenomenon based on their variables (features). Without loss of generality, we assume that all features have the character of a stymulant.

Assume that vector $\mathbf{x_i} = (x_{i1}, x_{i2}, \ldots, x_{in}) \in X$, $i = 1,2,\ldots,m$, describe the i-th object. If $x_{ik} > x_{jk} (x_{ik} \geq x_{jk})$ for $k = 1,2,\ldots n$ then we will write:

$$\mathbf{x_i} > \mathbf{x_j} (\mathbf{x_i} \geq \mathbf{x_j}), \text{ where } i,j \in [1,m].$$

It is easy to see that if $\mathbf{x}_i \geq \mathbf{x}_j$ and $\mathbf{x}_i \neq \mathbf{x}_j$, then in some cases it is natural to say that object $\mathbf{x_i}$ is better (more highly rated) than object $\mathbf{x_j}$. This means that none of components of vector $\mathbf{x_i}$ is less than a corresponding component of vector $\mathbf{x_j}$, and at least one of them is greater, which implies the existence a *k* belonging to [1, n] such that $x_{ik} > x_{jk}$.

Let us use the following denotations:

$$x_{0,k} = \min_{1 \leq i \leq m} x_{ik}, \quad x_{m+1,k} = \max_{1 \leq i \leq m} x_{ik}, \quad k = 1,2 \ldots n,$$

and

$$\mathbf{x_0} := \left(x_{0,1}, x_{0,2}, \ldots, x_{0,n}\right),$$
$$\mathbf{x_{m+1}} := \left(x_{m+1,1}, x_{m+1,2}, \ldots, x_{m+1,n}\right).$$

It is obvious that objects $\mathbf{Q}_0$ - described by vector $\mathbf{x}_0$, $\mathbf{Q}_{m+1}$ described by vector $\mathbf{x}_{m+1}$ (perhaps fictitiously) are not worse nor better than the rest of objects $\mathbf{Q}_1$, $\mathbf{Q_2}$,...,$\mathbf{Q}_m$,. That is:

$$\mathbf{x}_{m+1} \geq \mathbf{x}_i \text{ and } \mathbf{x}_i \geq \mathbf{x}_0 \text{ for each } i : m \geq i \geq 1$$

In conjunction with the above let us denote by

$$\left\langle \mathbf{x_0}, \mathbf{x_{m+1}} \right\rangle := \left\{ \mathbf{x} \in \Re_+^n : \mathbf{x_0} \leq \mathbf{x} \leq \mathbf{x_{m+1}} \right\}$$

as an interval (hypercube) in an n-dimensional Euclidean space.

In the case when objects $\mathbf{Q}_0$ and $\mathbf{Q}_{m+1}$ are different from considered objects $\mathbf{Q}_1$, $\mathbf{Q}_2$,...,$\mathbf{Q}_m$, they fulfill the roles the worst and the best, respectively objects. Objects $\mathbf{Q}_0$ and $\mathbf{Q}_{m+1}$ can be treated as models.

Suppose *X* is a empty set. We say that a function d which projects a Cartesian product into a set of non-negative numbers $\Re_+^1 = \langle 0, +\infty)$ defines a distance between any elements $x, y \in X$ belonging to *X* if it fulfills the following criteria:

1. d(*x*, *y*) =d(*y*, *x*) (symmetric)
2. d(*x*, *x*) = 0.

A distance d(x, y) is a metric if it fulfills the triangle inequality

3. d(*x*, *y*) $\leq$ d(*x*, *z*) + d(*z*, *y*) for all $x, y, z \in X$

Let $\mathbf{x,y} \in \Re_+^n, \mathbf{x} = (x_1, x_2, \ldots, x_n)$, $\mathrm{y} = (y_1, y_2, \ldots, y_n)$ The following function is related to construction of radar measures [Binderman, Borkowski, Szczesny 2012]

$$d_{rad}(\mathbf{x,y}) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} |x_i - y_i| |x_{i+1} - y_{i+1}|} \tag{1}$$

where $x_{n+1} := x_1$, $y_{n+1} := y_1$ is a distance but not a metric. It can be easily verified that this function fulfills 1 and 2, but not 3.

Indeed, let $\mathbf{x} = (n,1,0,0,\ldots,0)$, $\mathbf{y} = 0 := (0,0,\ldots,0)$, $\mathbf{z} = (0,1,0,0,\ldots,0)$, then $d_{rad}(\mathbf{x}, \mathbf{y}) = 1$, $d_{rad}(\mathbf{x}, \mathbf{z}) = 0$, $d_{rad}(\mathbf{z}, \mathbf{y}) = 0$. Hence $1=d_{rad}(\mathbf{x}, \mathbf{y}) > d_{rad}(\mathbf{x}, \mathbf{z}) + d_{rad}(\mathbf{z}, \mathbf{y})=0$.
On the other hand, the function

$$d_p(\mathbf{x},\mathbf{y}) = \left[\sum_{j=1}^{n} \left|x_j - y_j\right|^p\right]^{\frac{1}{p}}, \quad \infty > p \geq 1, \tag{2}$$

is an example of a metric, and is also known as Minkowski's metric [Kukuła K, 2000].

Note 1. If $X = X_1 \times X_2 \times \ldots \times X_k \left(\sum_{j=1}^{k} dim\ X_j = n\right)$, ρ_i are distances in spaces $X_i\ (i = 1,2,\ldots,k)$ then a distance in space X can be defined by using distance $\rho_i\ (i = 1,2,\ldots,k)$. For example, a standard distance in space X is defined by:

$$\rho(\mathbf{x},\mathbf{y}) = \sqrt{\sum_{i=1}^{k} \rho_i^2(\mathbf{x},\mathbf{y})}\ ,$$

$\mathbf{x} = (\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_k), \mathbf{y} = (\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_k) \in X; \mathbf{x}_i,\mathbf{y}_i \in X_i; i = 1,2,\ldots,k$,

Especially, if $X = X_1 \times X_2, X_1 = X_2 = \Re^2; \rho_1 = d_{rad}, \rho_2 = d_p$ where functions d_{rad}, d_p are defined by equations (1) and (2), respectively, then a standard distance in space X is defined by

$$\rho(\mathbf{x},\mathbf{y}) = \sqrt{\sum_{i=1}^{2} (x_i - y_i)^2 + \frac{1}{2}\sum_{i=3}^{4} |x_i - y_i||x_{i+1} - y_{i+1}|}\ ,$$

$\mathbf{x} = (\mathbf{x}_1,\mathbf{x}_2), \mathbf{y} = (\mathbf{y}_1,\mathbf{y}_2) \in X, \mathbf{x}_1 = (x_1,x_2), \mathbf{y}_1 = (y_1,y_2) \in X_1$,
$\mathbf{x}_2 = (x_3,x_4), \mathbf{y}_2 = (y_3,y_4) \in X_2; x_5 := x_3, y_5 := y_3$.

Let $\rho^*(\mathbf{x},\mathbf{y})$ denote distance between vectors $\mathbf{x}, \mathbf{y} \in \Re_+^n$ and $\rho^*(\mathbf{x_0},\mathbf{x_{m+1}}) \neq 0$. In literature, classification of objects is performed by utilizing the following equations defining synthetic measures of the given vector $\mathbf{x} \in \langle \mathbf{x_0},\mathbf{x_{m+1}} \rangle$.

$$\mu_1(\mathbf{x}) = \frac{\rho^*(\mathbf{x_0},\mathbf{x})}{\rho^*(\mathbf{x_0},\mathbf{x_{m+1}})}, \tag{3}$$

$$\mu_2(\mathbf{x}) = 1 - \frac{\rho^*(\mathbf{x_{m+1}},\mathbf{x})}{\rho^*(\mathbf{x_0},\mathbf{x_{m+1}})}, \tag{4}$$

$$\mu_3(\mathbf{x}) = \frac{\mu_1(\mathbf{x}) + \mu_2(\mathbf{x})}{2} = \frac{1}{2} + \frac{\rho^*(\mathbf{x_0},\mathbf{x}) - \rho^*(\mathbf{x_{m+1}},\mathbf{x})}{2\rho^*(\mathbf{x_0},\mathbf{x_{m+1}})}, \tag{5}$$

$$\mu_4(\mathbf{x}) = \frac{\mu_1(\mathbf{x})}{1+\mu_1(\mathbf{x})+\mu_2(\mathbf{x})} = \frac{\rho^*(\mathbf{x_0},\mathbf{x})}{\rho^*(\mathbf{x_0},\mathbf{x})+\rho^*(\mathbf{x_{m+1}},\mathbf{x})}, \tag{6}$$

It can be easily shown that measures μ_1 and μ_2 use one model, while measures μ_3 and μ_4 resort to two models. These measures can be treated as tools for solving multi-criteria decision problems. Each of measures μ_1 and μ_2 uses only one criterion while measures μ_3 and μ_4 - two criteria.

In his work [Hellwig 1968] gave a measure that utilized only the best objects. The theory behind and applications of measure μ_3 were discussed in a series of works by Binderman [Binderman A. 2006] as well as in [Binderman Z. et al. 2012, 2013]. Measure μ_4 is linked with the TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution, see [Hwang, Yoon 1981]).

Note 2. If $X_1, \ldots, X_k$ are variables with values from an interval scale and variables $Z_1,\ldots,Z_k$ are derived from them by normalizing them with a zero unitarization method, then we receive

$$w_i = \frac{1}{n}\sum_{j=1}^{n} z_{ij} = \frac{\rho_1[(0,\ldots,0),(z_{i1},\ldots,z_{in})]}{\rho_1[(0,\ldots,0),(1,\ldots,1)]} = 1 - \frac{\rho_1[(1,\ldots,1),(z_{i1},\ldots,z_{in})]}{\rho_1[(0,\ldots,0),(1,\ldots,1)]}, \tag{7}$$

where $i = 1,\ldots,m$ and a ρ_1 denotes a Minkowski metric as defined by (2).

With this a typical synthetic measure which construction is based on variables normalized with a zero unitarization method is also an indicator which is received by using a standard technique of comparison with a negative model. It can be shown that if the zero unitarization method is replaced with standardization as the tool to normalize variables, then the values of the indicator can be expressed by using distances from a negative model, namely:

$$w_i = \frac{1}{n}\sum_{j=1}^{n} z_{ij} = \frac{1}{n}\left(\rho_1[(z_{01},\ldots,z_{0n}),(z_{i1},\ldots,z_{in})] - \rho_1[(0,\ldots,0),(z_{i1},\ldots,z_{0n})]\right),$$

where $(z_{01},\ldots,z_{0n})$ denotes a vector of values of the negative model. Which means that this indicator is also a synthetic indicator, which is constructed as a distance from the negative model.

In the next step we normalize the distance of vectors $\rho^*(\mathbf{x},\mathbf{y})$ to the established model vectors $\mathbf{x_0}$, $\mathbf{x_{m+1}}$, with (3):

$$\rho(\mathbf{x},\mathbf{y}) := \frac{\rho^*(\mathbf{x},\mathbf{y})}{\rho^*(\mathbf{x_0},\mathbf{x_{m+1}})},$$

Then $\rho^*(\mathbf{x_0},\mathbf{x_{m+1}}) = 1$ and equations (3)-(6) become:

$$\mu_1(\mathbf{x}) = \rho(\mathbf{x_0},\mathbf{x}), \tag{3'}$$

$$\mu_2(\mathbf{x}) = 1 - \rho(\mathbf{x_{m+1}},\mathbf{x}), \tag{4'}$$

$$\mu_3(\mathbf{x}) = \frac{1}{2}\left[1 + \rho(\mathbf{x_0}, \mathbf{x}) - \rho(\mathbf{x_{m+1}}, \mathbf{x})\right], \tag{5'}$$

$$\mu_4(\mathbf{x}) = \frac{\rho(\mathbf{x_0}, \mathbf{x})}{\rho(\mathbf{x_0}, \mathbf{x}) + \rho(\mathbf{x_{m+1}}, \mathbf{x})}, \tag{6'}$$

In the special case when vectors $\mathbf{x_0} = 0 = (0,0,\ldots,0)$, $\mathbf{x_{m+1}} = 1 := (1,1,...,1)$ then

$$\rho^*(0,1) = \begin{cases} 1 & dla \quad \rho^* = d_{rad} \\ n^{1/p} & dla \quad \rho^* = d_p \end{cases}, \tag{8}$$

Noticeably, the considered measures, as defined by (3')-(6') are normalized in terms of established models, that is:

$$\mu_i(\mathbf{x_0}) = 0, \ \ \mu_i(\mathbf{x_{m+1}}) = 1, \ dla \ \ i = 1,2,3,4\,, \tag{9}$$

The above measures are ones of the most commonly used measures to order objects. Nevertheless, one can give other measures based on averages, which utilize distances from models.

Let $\mathbf{x} \in \langle x_0, x_{m+1} \rangle$, numbers be defined by (3'), (4'). For a given vector **x** we can define the following measures:

$$\mu_5(\mathbf{x}) = \begin{cases} \dfrac{2\mu_1(\mathbf{x})\mu_2(\mathbf{x})}{\mu_1(\mathbf{x}) + \mu_2(\mathbf{x})} & \text{dla} \quad \mu_1(\mathbf{x}) + \mu_2(\mathbf{x}) \neq 0 \\ 0 & \text{dla} \quad \mu_1(\mathbf{x}) + \mu_2(\mathbf{x}) = 0 \end{cases} \text{- harmonic average,}$$

$$\mu_6(\mathrm{x}) = \sqrt{\mu_1(\mathrm{x})\mu_2(\mathrm{x})} \text{ - geometric average,}$$

$$\mu_7(\mathrm{x}) = \sqrt{\frac{\mu_1(\mathrm{x})\mu_2(\mathrm{x})}{2}} \text{ - root mean square.}$$

It can be shown [Mitrinovic 1993] that for a given vector the following inequalities hold:

$$\min(\mu_1, \mu_2) \leq \mu_5 \leq \mu_6 \leq \mu_3 \leq \mu_7 \leq \max(\mu_1, \mu_2).$$

NON-STATIONARY MODEL MEASURES

The described measures can be understood of as functions of any vector $\mathbf{x} \in \langle \mathbf{x}_0, \mathbf{x}_{m+1} \rangle$ (functions of n real variables) or as functions of vector $\mathbf{x} \in \langle \mathbf{x}_0, \mathbf{x}_{m+1} \rangle$ and vectors that define equivalent objects - (functions of $(n+1) \cdot m$ real variables) because model vectors are functions' values, depending on vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m$.

If μ denotes any set measure of vector $\mathbf{x} \in \langle \mathbf{x}_0, \mathbf{x}_{\mathrm{m+1}} \rangle$, defined by one of (3)-(6) then in the second case we should have: $\mu = \mu(\mathbf{x}, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)$.

As a result of the above, the considered measures can be used, as necessary, in one of two ways: static and dynamic. If objects Q_1, Q_2,...,Q_m of the studied phenomenon are considered in a time interval $\langle T_0, T_1 \rangle$ then their describing vectors should be treated as functional vectors $x_1, x_2, \ldots, x_m$ dependent on time.

Let $t_1, t_2, \ldots, t_q \in \langle T_0, T_1 \rangle, q \in N$. To order the objects Q_1, Q_2,...,Q_m with the considered measures at a given point in time $t_j \left(j \in \{1, 2, \ldots, q\} \right)$ or to order them based on their descriptions at points in time $t_1, t_2, \ldots, t_q$, we must compute the coordinates of model vectors $\mathbf{x}_0(t_j), \mathbf{x}_{\mathrm{m+1}}(t_j)$:

$$\begin{aligned} &x_{0,k}(t_j) = \min_{1 \le i \le m} x_{ik}(t_j), \quad x_{m+1,k}(t_j) = \max_{1 \le i \le m} x_{ik}(t_j), \\ &k = 1, 2, \ldots, n; \; j = 1, 2, \ldots, q \end{aligned} \tag{10}$$

or coordinates of model vectors $\mathbf{x}_0, \mathbf{x}_{\mathrm{m+1}}$:

$$\begin{aligned} &x_{0,k}(t_j) = \min_{1 \le i \le q} x_{0,k}(t_j), \quad x_{m+1,k}(t_j) = \max_{1 \le i \le q} x_{m+1,k}(t_j), \\ &k = 1, 2, \ldots, n \end{aligned}$$

As a direct result of the definition, the following inclusions are sound:

$$\underset{j \in \{1,2,\ldots,q\}}{\forall} \mathbf{x}_0(t_j), \mathbf{x}_{m+1}(t_j) \in \langle x_0, x_{m+1} \rangle$$

In the case when we want to order objects in the entire time interval (T_0, T_1), we must choose models $\mathbf{x}_0$, $\mathbf{x}_{\mathrm{m+1}}$, such that the following inequalities hold:

$$\begin{aligned} &x_{0,k} \le \inf_{1 \le i \le m} x_{ik}(t), \quad x_{m+1,k} \ge \sup_{1 \le i \le m} x_{ik}(t), \quad k = 1, 2, \ldots, n \\ &\text{for each } t \in \langle T_0, T_1 \rangle \end{aligned} \tag{11}$$

Especially, if the functional vectors $\mathbf{x}_i(t), i \in \{1, 2, \cdots, m\}, t \in \langle T_0, T_1 \rangle$ are continuous then:

$$x_{0,k} = \min_{1 \le i \le m} x_{ik}(t), \quad x_{m+1,k} = \max_{1 \le i \le m} x_{ik}(t), \quad k = 1, 2, \ldots, n,$$

Note 3. In a dynamic approach to the problem of ordering and classifying objects in the entire time interval $\langle T_0, T_1 \rangle$, we should assume that the obtained result, which uses "partial" results - got from pairs of model vectors $\mathbf{x}_0(t_j), \mathbf{x}_{m+1}(t_j); j = 1, 2, \ldots, q$ can be significantly different from the result received by means of "integral" models $\mathbf{x}_0, \mathbf{x}_{m+1}$.

Naturally, the choice of a model depends on the way of presenting/reporting the concrete phenomenon in a given period. However, the scale of differences can prove to be substantial, as the following analysis shows.

ILLUSTRATION OF CONSEQUENCES OF A CHOICE OF A MODEL

Ranking of objects is determined by, (in addition to the feature transformation method), the choice of model object. In this theoretical example we present different results of ordering objects in a dynamic approach depending on the method defining the model object.

In order to do that we created the following simulation which generates values for variables X_1-X_4, at given points in time T_1-T_5, with the distributions of their values:

$$X_1(T_1) \approx N(6{,}2),\ X_1(T_j) = X_1(T_{j-1}) + U_j,\ j = 2,\ldots,5,\ U_j \approx N(0{,}5;0{,}2),$$

$$X_2(T_1) \approx N(6{,}2),\ X_2(T_j) = X_2(T_{j-1}) + V_j,\ j = 2,\ldots,5,\ V_j \approx N(-0{,}5;0{,}2),$$

$$X_3(T_1) \approx N(6{,}2),\ X_3(T_j) = X_3(T_{j-1}) + \xi_j W_j,\ j = 2,\ldots,5,\ W_j \approx N(0{,}4;2),$$

$$X_4(T_1) \approx N(6{,}2),\ X_4(T_j) = X_4(T_{j-1}) + \eta_j Z_j,\ j = 2,\ldots,5,\ Z_j \approx N(0{,}4;2),$$

$$\xi_2 = \xi_3 = \eta_4 = \eta_5 = -1,\quad \eta_2 = \eta_3 = \xi_4 = \xi_5 = 1$$

$X_1(T_1),\ i = 1,\ldots,4,\ U_j, V_j, W_j, Z_j,\ j = 2,\ldots,5$, which are independent.

By using this model we generated data tables for 10 objects. One of these simulation is presented in Table 1. The last two rows of Table 1 contain the negative and positive model for each time point T_1-T_5, respectively.

Table 1. Sample data for simulations of 10 objects

	T_1				**T_2**				**T_3**				**T_4**				**T_5**			
	X_1	**X_2**	**X_3**	**X_4**	**X_1**	**X_2**	**X_3**	**X_4**	**X_1**	**X_2**	**X_3**	**X_4**	**X_1**	**X_2**	**X_3**	**X_4**	**X_1**	**X_2**	**X_3**	**X_4**
o01	3.17	5.76	6.64	3.79	3.79	5.38	6.2	5.41	3.96	4.99	10.33	2.16	4.56	4.27	12.99	-1.5	5.5	3.83	15.38	-0.11
o02	5.89	9.26	3.8	2.04	6.17	8.87	6.81	1.37	6.92	8.5	7.41	-2.93	7.32	7.94	6.82	-7.27	7.91	7.66	10.48	-9.63
o03	8.57	6.26	4.07	1.63	8.9	5.71	2.57	4.47	9.72	5.29	2.61	8.08	10.18	4.82	4.13	7.2	10.97	4.51	8.57	5.88
o04	7.04	2.2	9.56	3.19	7.68	2.45	6.69	6.48	8.01	1.89	6.06	5.09	8.29	1.17	9.06	6.01	8.61	0.65	5.01	7.81
o05	9.96	7.22	6.13	2.94	10.54	6.68	4.6	1.37	10.65	6.18	4.12	1.08	11.12	5.68	6.35	-0.19	11.5	4.87	7.99	-4.15
o06	8.37	4.83	5.91	1.97	8.82	4.51	8.42	3.29	9.27	3.99	6.77	5.23	9.87	3.37	8.5	6.55	10.55	2.85	8.78	8.62
o07	2.87	6.55	6.86	4.1	3.1	5.75	7.81	5.52	3.86	4.98	6.8	6.01	4.47	4.53	5	5.04	4.96	4.28	4.26	5.56
o08	3.24	7.22	6	4.59	3.89	6.84	9.26	7.11	4.65	6.25	8.72	7.06	5.03	5.91	11.49	7.6	5.75	5.64	12.09	11.95
o09	5.66	4.92	7.02	3.79	6.11	4.35	8.17	2.14	6.41	3.64	6.58	3.66	6.94	3.04	6.76	7.47	7.37	2.52	7.26	11.06
o10	3.08	6.1	9.75	4.78	3.75	5.44	12.88	7.73	3.99	4.83	10.2	10.08	4.61	4.53	11.49	10.55	5.25	3.86	12.6	12.7
min	**2.87**	**2.2**	**3.8**	**1.63**	**3.1**	**2.45**	**2.57**	**1.37**	**3.86**	**1.89**	**2.61**	**-2.93**	**4.47**	**1.17**	**4.13**	**-7.27**	**4.96**	**0.65**	**4.26**	**-9.63**
max	**9.96**	**9.26**	**9.75**	**4.78**	**10.54**	**8.87**	**12.88**	**7.73**	**10.65**	**8.5**	**10.33**	**10.08**	**11.12**	**7.94**	**12.99**	**10.55**	**11.5**	**7.66**	**15.38**	**12.7**

Source: own research

Naturally, the integral models are (2.87, 0.65, 2.57, -9.63) and (11.5, 9.26, 15.38, 12.7), respectively. We have used the most common synthetic indicator, defined by (7). For each of the objects, in each of the time periods, we have calculated the value of the indicator as well as the rank of the values in two approaches:

i. by normalizing the data by two integral models (W_1),
ii. by normalizing the data by five models from each period (W_2).

The obtained results are presented in Table 2. In the last 5 columns of Table 2 we have put the changes in positions in the order of objects according to the values of indicators W_1 and W_2 (a positive number means that object moves up the ranking by the number of spots, while a negative one means that object falls in the ranking). The differences are significant, but the simulation is specially chosen for this differentiating characteristic.

Table 3 presents the changes of order's positions in a ranking of objects for 8 simulations done, according to the values of indicators W_1 and W_2, same as in Table 2.

The changes to the order are not substantial in all cases, see, for example, simulation number 6. The choice of a model (an integral one for the entire period or a different one for each interval) has an impact regardless of the choice of a "distance from the model"-based on the indicator from the list $\mu_1 - \mu_7$.

For the given simulation from Table 1, we calculated the values of these 7 indicators by using an Euclidean metric (ρ_2). The latter is done after normalizing the data with a zero unitarization method. The changes in the order of the objects between these two applications of models are presented in Table 4. In each case we can see significant differences. These are similar to the differences observed in Table 2 when we have used distance ρ_1.

Table 2. Values of synthetic indicators defined by (7) using an integral model (W_1) and models for individual periods (W_2)

	W_1 (integral pattern)					W_2 (5 different patterns)					W_1 - ranks					W_2 - ranks					Change of position				
	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5
o01	0.387	0.403	0.441	0.448	0.525	0.427	0.374	0.469	0.449	0.491	10	10	7	9	6	7	9	7	7	7	3	1	0	2	-1
o02	0.492	0.540	0.515	0.450	0.504	0.389	0.433	0.518	0.433	0.503	2	4	6	8	8	9	6	6	9	6	-7	-2	0	-1	2
o03	0.483	0.479	0.532	0.552	0.637	0.356	0.432	0.556	0.552	0.638	3	6	2	5	2	10	7	4	5	1	-7	-1	-2	0	1
o04	0.446	0.452	0.418	0.474	0.409	0.513	0.455	0.419	0.469	0.352	6	7	8	6	9	4	4	9	6	9	2	3	-1	0	0
o05	0.606	0.560	0.536	0.565	0.540	0.630	0.449	0.538	0.579	0.546	1	2	1	3	5	2	5	5	4	5	-1	-3	-4	-1	0
o06	0.476	0.543	0.531	0.579	0.612	0.403	0.482	0.570	0.601	0.598	4	3	3	2	3	8	3	3	2	3	-4	0	0	0	0
o07	0.409	0.427	0.412	0.371	0.369	0.479	0.407	0.424	0.321	0.300	9	9	10	10	10	6	8	8	10	10	3	1	2	0	0
o08	0.428	0.527	0.521	0.582	0.656	0.518	0.570	0.584	0.612	0.626	8	5	5	1	1	3	2	2	1	2	5	3	3	0	-1
o09	0.442	0.442	0.416	0.461	0.508	0.501	0.334	0.415	0.443	0.458	7	8	9	7	7	5	10	10	8	8	2	-2	-1	-1	-1
o10	0.466	0.560	0.523	0.563	0.608	0.646	0.628	0.612	0.587	0.563	5	1	4	4	4	1	1	1	3	4	4	0	3	1	0

Source: own research

Table 3. Changes in the order of objects for a few sample data simulations

	Simulation 2					Simulation 3					Simulation 4					Simulation 5					Simulation 6					Simulation 7					Simulation 8				
	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5
o01	1	1	0	0	0	1	0	-1	0	1	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	0	2	0	-1	0	1	1	-1	0	-1	0
o02	4	0	1	0	0	0	0	0	0	-4	1	0	0	0	1	-2	0	0	0	0	1	0	0	-1	0	1	1	2	1	0	3	2	0	1	0
o03	-1	0	0	0	0	0	0	0	0	-1	-2	0	0	0	0	-1	0	0	0	0	0	0	0	0	-1	-1	-1	-2	2	1	0	0	1	0	0
o04	-5	-1	0	0	0	-1	0	0	0	0	3	0	0	0	0	2	-1	1	0	-1	1	-1	-1	0	0	2	0	-2	-4	-1	0	-1	0	-1	0
o05	-1	0	-2	0	0	-1	0	0	1	3	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	-5	-1	0	1	1	-1	-1	0	0	0
o06	0	0	-1	0	0	1	0	0	0	-1	0	0	-1	0	0	-3	-2	-1	0	0	0	0	0	0	0	5	2	2	0	-2	-1	0	0	0	0
o07	0	0	0	0	0	1	0	1	0	1	0	0	1	0	-1	0	2	0	0	0	-1	0	0	1	0	-2	0	0	0	-1	0	0	-1	0	0
o08	6	1	2	0	0	0	0	0	0	1	-1	0	0	0	0	1	2	0	-1	0	0	1	1	0	1	0	0	1	-1	0	0	-3	0	1	0
o09	-2	-1	0	0	0	0	0	0	-1	-1	-2	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
o10	-2	0	0	0	0	-1	0	0	0	1	0	0	-1	0	0	0	-2	-1	0	1	0	0	0	0	0	-2	-1	0	1	1	-2	2	0	0	0

Source: own research

Table 4. Changes in the order of objects from Table 1 for individual indicators μ_1- μ_7 (using zero unitarization method)

	Indicator μ_1					Indicator μ_2					Indicator μ_3					Indicator μ_4					Indicator μ_5					Indicator μ_6					Indicator μ_7				
	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5
o01	1	0	0	2	-1	3	2	-3	0	0	3	1	0	2	-1	3	1	0	2	-1	3	1	0	0	-1	3	1	0	1	-1	3	1	0	2	-1
o02	-4	-2	0	0	1	-2	-4	-4	0	0	-6	-2	0	-1	2	-6	-2	0	0	2	-5	-2	0	0	2	-7	-3	0	-1	2	-6	-2	0	0	2
o03	-4	-2	-1	-1	1	-7	3	1	0	0	-7	-1	-1	0	1	-7	-1	-1	0	1	-8	1	1	0	1	-7	0	-1	0	1	-7	-1	-2	0	1
o04	5	4	-1	-2	0	0	3	0	1	1	2	3	-1	0	0	2	2	-1	0	0	1	3	-1	0	0	1	3	-1	0	0	2	2	-1	0	0
o05	-1	-1	-3	-1	0	0	-7	1	0	1	0	-4	-4	-1	0	0	-3	-4	0	0	0	-6	-3	0	1	0	-4	-3	0	0	-1	-2	-4	-1	0
o06	-4	0	0	0	0	-6	-2	0	1	1	-5	0	-1	0	0	-5	0	-1	1	0	-5	0	0	1	0	-4	0	-2	1	0	-5	0	0	0	0
o07	2	-1	1	0	0	3	3	2	0	-1	3	1	1	0	0	3	1	1	0	0	3	1	2	0	0	3	1	2	0	0	3	1	1	0	0
o08	1	2	2	2	0	4	4	0	-1	-1	5	3	3	0	-1	4	3	3	-1	-1	4	3	1	-1	-1	5	3	3	-1	-1	4	3	3	0	-1
o09	2	0	0	0	0	2	-4	2	-1	0	2	-2	0	-1	-1	3	-2	0	-2	-1	4	-2	-1	0	-1	3	-2	-1	0	-1	3	-2	0	-2	-1
o10	2	0	2	0	-1	3	2	1	0	-1	3	1	3	1	0	3	1	3	0	0	3	1	1	0	-1	3	1	3	0	0	4	0	3	1	0

Source: own research

SUMMARY

Most often the comparison of effectiveness of corporate units uses data for a single reporting period. It is performed by an automated reporting system (as part of a centralized management information system) which usually utilizes objects defined by (10) as models. This research (based on theoretical data) indicates that a better approach is to choose a dynamic model based on longer time input data (as defined by (11)). Such solution more accurately captures the dynamics of changes to the values of individual variables which constitute a synthetic measure. Our simulations confirm that regardless of the choice of our measures, the differences in rankings of examined objects can still be substantial. Our research needs further verification on empirical samples.

REFERENCES

Aczel A.D. (1989) Complete Business Statistics. Richard D. Irwin Inc. Homewood.

Binderman A. (2006) Klasyfikacja obiektów oparta na dwóch wzorcach. EiOGŻ, Zeszyty Naukowe SGGW, nr 60, Warszawa, 25 – 37.

Binderman Z., Borkowski B., Szczesny W. (2012) Radar coefficient of concentration. Quantitative Methods in Economics, XIII/2, 7-21.

Binderman Z., Borkowski B., Szczesny W. (2013) Synthetic radar measures that factor in weights of features. Quantitative Methods in Economics, XIV/1, 37 – 52.

Breiman L. (1998) Bias-variance, regularization, instability and stabilization. In: C. M. Bishop, red., Neural Networks and Machine Learning, Springer-Verlag, 27 – 56.

Breiman L. (1996) Arcing Classifiers. Technical Report 460, Department of Statistics, University of California, CA, USA.

Breiman L. (1994) Bagging predictors. Technical Report 420, Department of Statistics, University of California, CA, USA.

Gatnar E., Walesiak M. (2009) Statystyczna analiza danych z wykorzystaniem programu R. PWN, Warszawa.

Hair J.F. et al. (1995) Multivariate Data Analysis with Readings. Prentice Hall International, Ltd. London

Hellwig Z. (1968) Zastosowanie metody taksonomicznej do typologicznego podziału krajów ze względu na poziom ich rozwoju oraz zasoby i strukturę kwalifikowanych kadr. Przegląd Stat., 4.

Hwang. C.L., Yoon K. (1981) Multiple attribute decision making: Methods and Applications. Springer-Verlag, New York.

Jackson D.H. (1970) The stability of classifications of binary attribute data. T.R., Cornell University.

Jackson D.H. (1969) Comparison of classification. Academic Press.

Jackson D.H. (1969a) Automatic classifications and information retrieval. Cambridge University.

Kukuła K. (2000) Metoda unitaryzacji zerowanej. PWN, Warszawa.

Mitrinović D. S. (1993) Classical and New Inequalities in Analysis. Springer – Science + Business Media Dordrecht.

Morrison D. F. (1990) Multivariate Statistical Methods. Third Edition. McGraw-Hill,. New York

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII , No. 1, 2017, pp. 18 – 26

Received: 09.03.2017
Accepted: 05.04.2017

TESTING UNCOVERED INTEREST PARITY IN THE PLN/JPY FOREIGN EXCHANGE MARKET: A MARKOV-SWITCHING APPROACH

Katarzyna A. Czech (ORCID: 0000-0002-0660-7124)
Faculty of Economic Sciences
Warsaw University of Life Sciences – SGGW, Poland
e-mail: katarzyna_czech@sggw.pl

Abstract: Uncovered Interest Parity argues that a high-interest-rate currency tends to depreciate and a low-interest-rate currency to appreciate. Many researchers find opposite tendency in foreign exchange market. This puzzling feature of foreign exchange market is known as forward premium puzzle. The aim of the paper is to examine how exchange rate volatility influences the relationship between returns and interest rate differentials. Markov switching model is applied. It is shown that in regime of low volatility, in the PLN/JPY market, forward premium anomaly appears. However, during the time of high volatility the UIP holds.

Keywords: foreign exchange market, uncovered interest rate parity, forward premium anomaly, Markov-switching model

INTRODUCTION

Uncovered Interest Parity (UIP) argues that a high-interest-rate currency tends to depreciate and low-interest-rate currency to appreciate. However, many researchers have usually rejected the theory, pointing the fact that low interest-yielding currencies has a tendency to depreciate rather than appreciate. This puzzling feature of foreign exchange market is one of the robust anomaly in financial economy and it is generally known as *forward premium puzzle* [Fama 1984]. There appears to be many empirical evidence against uncovered interest parity theory. Fama (1984), Froot and Frankel (1989), McCallum (1994), among others, observe UIP deviations in relation between interest rates of two countries and exchange rates between these countries. Many researchers try to understand and tackle the problem of UIP puzzle but there is still no consensus on how to explain it.

DOI: 10.22630/MIBE.2017.18.1.02

JEL: C22, F42

There are two main streams in the literature on international economics which explain forward premium puzzle. The first one is focused on theory of rational expectations. The theory assumes that economic outcomes do not differ sistematically from what the market participants expected them to be. People do not make systematic errors when building forecasts and the deviations from their predictions are only random. Therefore, formed expectations are essencially the same as predictions which are based on available informations and economic models [Muth 1961]. Testing uncovered interest parity involves combining it with the assumption of rational expectations. However, some researchers like Mark and Wu [1998] or Chakraborty and Evans [2008] claim that fuilure of UIP may result from the irrational expectations. Mark and Wu build the model in which both rational and irrational market participants have an impact on the price volatility in foreign exchange market. They believe that irrational traders (noise-traders) contribute to the deviations from UIP conditions for exchange rates. Chakraborty and Evans [2008] build model under assumption of adaptive learning, especially constant-gain learning. This approach explores market participants decision making process within a bounded rationality framework. In adaptive learning approach, market participants are assumed to have limited common knowledge since they estimate their own perceived laws of motion. Moreover, they are assumed to discount past infromation when build their forecasts. Chakraborty and Evans [2008] find that the adaptive learning may lead to deviations from UIP in short time. They claim, however, that even under adaptive learning, in long time, formed expectations will be similar to rational expectations.

The second main stream in literature on uncovered interest rate parity explains forward premium anomaly in respect of the assumption of risk neutrality. It is believed that market participants are not risk-neutral but risk-averse and they require a risk premium when investing in foreign exchange market. Therefore, the fuilure of UIP may result from existence of non-zero risk premium. The problem of risk premium in foreign exchange market is analyzed by many researchers. UIP model with risk premium is built by Domowitz and Hakkio [1985], Jiang and Chiang [2000], Berk and Knot [2001], Serwa [2009], Li et al. [2012], among others. The reasearch results are inconsistent. A non-zero risk premium in foreign exchange market is detected only by some of researchers (Poghosyan et al. [2008], Serwa [2009] and Li at al. [2012]). Estimation results depend on the type of risk premium model, time-horizon, analyzed foreign exchange market etc. McCallum [1994] claims that non-zero risk premium is the main reason leading to deviations from UIP but only over short-term horizon.

The literature provides also other explanation of forward premium puzzle. Alexius [2001] claim that UIP doesn’t hold for short-term interest. However, when you conduct an analysis on the basis of long-term interest rates, UIP holds much better. Bansal and Dahlquist [2000] find that uncovered interest parity performs better in developing compared to developed countries. They claim that country-specific attributes such as per capita income, interest rates, inflation and country risk rating are essential in explaining deviations form uncovered interest parity. Lothian and Wu [2011] study UIP by constructing ultra-long time series. Their results show that UIP may be violated during a particular short period, but it holds much better over the long period.

Flood and Rose [2002] find that UIP works systematically better in the time of crisis, when high price volatility is observed in the financial market. The same results are obtained by Clarida et al. [2009]. They show that forward premium anomaly relates to stable time period, when both exchange and interest rates display consistently lower volatility. This is a starting point for this paper. It is assumed that for high-volatility periods UIP holds. However, in low-volatility and stable periods we can observe forward premium anomaly in foreign exchange market.

Baillie and Chang [2011] claim that UIP deviations may be explained by the existence of carry trade speculation strategies[1]. They assume that exchange rate movement in the direction opposite to that predicted by UIP may result from the growth in carry trade activity. An increase in carry trade activity tends to weaken low interest-yielding currencies and strengthen high interest-yielding currencies, which is contrary to UIP predictions. The most popular funding currency for carry traders is the Japanese yen. Therefore the paper studies the uncovered interest parity in the Polish zloty (PLN) to Japanese yen (JPY) foreign exchange market (PLN/JPY market, where PLN is a quote currency and JPY is a base currency). The aim of the paper is to examine how exchange rate volatility influences direction of relationship between returns and interest rate differentials in Poland and Japan.

METHODOLOGY AND DATA

Uncovered interest parity (UIP) represents the basis parity condition for testing foreign exchange market efficiency. It states that interest rate in quote currency country must be higher (lower) than interest rate in base currency country by an amount equal to the expected depreciation (appreciation) of quote currency. One can assume that quote currency is a domestic currency and base currency is a foreign currency.

Uncovered interest parity describes relationship between interest rates and expected exchange rate changes:

$$\frac{1+r_t}{1+r_t^*}=\frac{E(S_{t+k}|\Omega_t)}{S_t} \tag{1}$$

[1] The motivation behind the carry trade strategy is to exploit profit by applying the combination of low cost of funds in one market and high returns in another. The strategy comprises borrowing funds in a low-interest-rate currency and investing them in high-interest-rate currencies [Fong 2010].

where S_t is the price of base currency in units of quote currency in time t $E(S_{t+k}|\Omega_t)$ is expected spot exchange rate at time $t+k$, based on information known at time t, r_t and r_t^* are interest rates in quote and base currency countries respectively.

Market participants expectations of future spot exchange rates are hardly observable, therefore the UIP is tested jointly with assumption of rational expectations. Under assumption of rational expectations, future value of spot exchange rate is equal to expected spot exchange rate at time $t+k$ plus a white-noise error term which is uncorrelated with information available at time t.

$$S_{t+k} = E_t(S_{t+k}|\Omega_t) + \varepsilon_{t+k} \tag{2}$$

where ε_{t+k} is white-noise error term which is uncorrelated with information available at time t.

Thus assuming that market participants are endowed with rational expectations and risk-neutral, UIP states that realized foreign exchange gain from holding one currency rather than another must be offset by interest rate differential. The baseline econometric model applied to test uncovered interest rate parity is as follows:

$$s_{t+k} - s_t = \alpha + \beta(r_t - r_t^*) + \varepsilon_{t+k} \tag{3}$$

where s_t denotes the logarithm of spot exchange rate at time t, s_{t+k} is the logarithm of spot exchange rate at time $t+k$. Under the UIP parity condition, the slope parameter β in equation (3) should be equal to unity (β = 1) and the coefficient α should be equal to zero (α = 0). Empirical studies based on regression model (3) generally rejects the UIP hypothesis. A well-known empirical regularity is that β is significantly less than one, and in fact very often closer to minus unity than plus unity (Froot and Thaler 1990).

Sarno et al. [2006] claim that deviations from uncovered interest parity condition display significant nonlinearities. In recent years reaserchers apply nonlinear models in explaining relationship between interest rate differentials and change in exchange rates. It is believed that behavior of economic variables depends on different states of the world. Thus, properties of foreign exchange time series are dependent on the regime which prevails at the certain time period. In the article the Markov Switching (MS) model is applied to test uncovered interest rate parity in the PLN/JPY foreign exchange market.

MS model is popularized in economics by Hamilton [1989]. His pioneering work examine a persistency of recessions and booms by applying regime-switching model. The model involves multiple structures that characterize time series in different regimes. Moreover, switching between these structures is permitted. However, a change in regime is not regarded as an outcome of a foreseeable, deterministic event, but rather a change in regime is itself a random variable. Markov switching model includes description of probability law governing the change in regimes. Hamilton uses a two-regime model to explain returns in foreign exchange market. The model is specified as:

$$\begin{aligned} &s_{t+k} - s_t = \alpha_{v_t} + \sigma_{v_t}\varepsilon_{t+k} \\ &where \\ &v_t \in \{1,2\} \end{aligned} \tag{4}$$

where

$\alpha_{v_t}, \sigma_{v_t}$ are the estimated coefficients,

v_t is a random variable that can assume only an integer value $\{1,2,...,M\}$ and evolves according to a first-order Markov process with a transition probability matrix **P**. Process is in regime 1 when v_t equals 1, while the process is in regime 2 when v_t equals 2. Transition probabilities in an $(2x2)$ matrix **P** is presented below:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, p_{ij} \geq 0 \tag{5}$$

The row *j*, column *i* element of **P** is the transition probability p_{ij}. Transition probability p_{ij} gives the probability that state *i* will be followed by state *j*.

In the paper Markov Switching model (6) is applied for analysis of the relationship between change in exchange rates and interest rate differentials:

$$\begin{aligned} &s_{t+k} - s_t = \alpha + \beta_{v_t}(r_t - r_t^*) + \sigma_{v_t}\varepsilon_{t+k} \\ &where \\ &v_t \in \{1,2\} \end{aligned} \tag{6}$$

The MS model (6) assumes that there are simultaneous switches in slope coefficient β and volatility parameter σ. Intercept α is assumed to not switch. It results from the belief that regime switches in exchange rate returns should be interpreted as switches in relationship between exchange rate returs and interest rate differentials rather than just switches in intercept [Ichiue and Koyama 2011].

The study is carried out using end-of-month data over the period from January 2000 to December 2015 with the total of 192 observations. Data covers the Polish zloty to Japenese yen spot exchange rates and monthly interbank interest rates (1M Wibor, 1M Libor JPY) expressed at annual rates. The Polish zloty is assumed to be a domestic (quote) currency, and the Japanese yen is used as a foreign (base) curency. Non-overlapping monthly data with one-month interest rates are analysed in order to avoid possible estimation biases in standard errors arising from overlapping data. All data are from Reuters Datastream.

EMPIRICAL RESULTS

To study uncovered interest parity in the PLN/JPY exchange rate market the Markov Switching model (6) is applied. Table 1 reports the estimation results.

Table 1. Parameters estimates of the regime-switching model (6) in the PLN/JPY market

	PLN/JPY	
	1	2
α	-0.01	
β	1.3*	-0.01
σ	0.08**	0.03**
N	192	
p_{11}, p_{12}	0.82	0.18
p_{21}, p_{22}	0.02	0.98
d_1, d_2	5.42	42.32

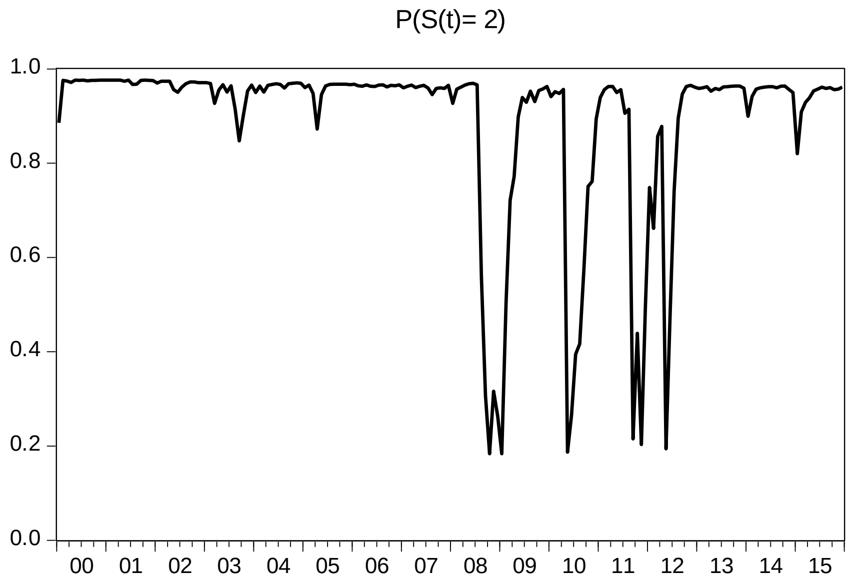
Notes: the values with ** and * are different from 0 at the one and five-percent-significance level, respectively
Source: own calculations based on Eviews 8 econometric software

Table 1 shows that two regimes are distniguished. The first regime is the regime of higher volatility ($\sigma = 0.08$) in which estimated slope coefficient is significant, positive and close to one as uncovered interest rate parity holds. The second regime, however, is the regime of lower volatility ($\sigma = 0.03$) and negative insignificant coefficient β. High-volatility period is associated with the time of high financial turbulences and recession. Low-volatility period is related with economic expansion and good mood among financial market participants. Therefore, the research results are consistent with previous research conducted by Flood and Rose [2002] and Clarida et al. [2009]. It is shown that in regime of low volatility in the PLN/JPY exchange market, forward premium anomaly is observed. However, during the time of high volatility in the foreign exchange market, uncovered interest rate parity holds. It needs to be emphasized that for exchange rates where interest rate differential is higher than for PLN/JPY, coefficient β in low-volatility regime is even closer to -1 than to 0.

Table 1 shows also that value of slope coefficient in regime 1 is higher than the absolute value of coefficient β in regime 2. This suggest that exchange rates move faster when the Japanese yen (low-interest-rate currency) appreciates than when it depreciates against Polish zloty (high-interest-rate currency). With the risk of fast appreciation of low-yielding currency (currency crash risk), market participants may require risk premium for taking short position in that currency. Accoding to Ichiue and Koyama [2011] a time-varying risk premiums may be the reason for forward premium puzzle.

On the basis of estimated transition probabilities we can assume that distinguished regimes are persistent (table 1). The probability of remaining in present state is high, 82% for regime 1 and as much as 98 % for regime 2. Regime 2 is, hovewer, more stable than regime 1. That is, the shift from regime 1 to regime 2 is more likely than that from regime 2. The smoothed probabilities of staying in regime 2 are given in Figure 1 in graphical form.

Figure 1. The smoothed probabilities of staying in regime 2



Source: own preparation

Figure 1 shows that in the time of analysis several regime switches are detected. In the time period from January 2000 to December 2015 the first switch to regime 1 is observed in 2008 when the financial crisis begun and many financial institutions reported huge losses. The financial crisis led to the change in the relationship between the interest rate differentials and the PLN/JPY exchange rate returns. It is woth to emphasized that regime 1 is the regime with higher volatility in which uncovered interest rate parity holds.

Table 1 provides also information about expected duration of each regime (d). The expected duration of regime 1 equals about 5 months and regime 2 about 42 months. The expected duration of regime 1 for which uncovered interest rate parity holds is much lower. It means that the Japanese yen appreciates less frequently against Polish zloty, but once it occurs, the exchange rates move faster than when it depreciates. It may results from huge activity of carry traders. When there is a turmoil in financial market, the risk-aversion increases among investors, then speculators are forced to unwind their carry trade position. They sell higher-yielding currencies and buy Japanese yen to repay a loan. The huge increase in demand for Japanese yen leads to its appreciation. Moreover, Brunnermeier at al. [2009] claim that a reduction in speculators positions increases the exchange rate volatility. It may explain, at least to some degree, the nature of regime 1 with value of slope coefficient close to 1 and high volatility.

SUMMARY

Uncovered interest parity (UIP) assumption is an important building block in many models of open economies. Researchers have usually rejected the theory, indicating a tendency of low interest-yielding currencies to depreciate rather than appreciate as UIP suggests. This puzzling feature of foreign exchange market is generally known as forward premium puzzle. Literature provides several explanations of this phenomenon. In the paper it is assumed that there is a change in the ralationship between interest rate differential and exchange rate return. Moreover, the direction of the relationship depends on the volatility in foreign exchange market. It is believed that there are regimes in which uncovered interest parity holds and regimes in which forward premium anomaly is detected.

A Markow regime-switching model is apllied. The model allows slope coefficient to vary over time. It is shown that in the time of high volatility in the PLN/JPY exchange rate market, uncovered interest parity holds. However, in the time of low volatility, forward premium puzzle appears. The results are consistent with previus studies conducted by Flood and Rose [2002] and Clarida et al. [2009]. Moreover, it is shown that Japanese yen currency appreciates less often than depreciates, but once it occurs, its appreciation is bigger and faster than depreciation.

Accoding to Ichiue and Koyama [2011] fast appreciation of low-yielding currency may lead to existence of non-zero time-varying risk premium in foreign exchange market. The time-varying risk premium, on the other hand, may be a reason for forward premium puzzle.

REFERENCES

Alexius A. (2001) Uncovered Interest Parity Revisited. Review of International Economics, 9 (3), 505-517.

Baillie R. T., Chang S. S. (2011) Carry Trades, Momentum Trading and the Forward Premium Anomaly. Journal of Financial Markets, 14, 441-464.

Bansal R., Dahlquist M. (2000) The forward premium puzzle: different tales from developed and emerging economies. Journal of International Economics, 51, 115-144.

Berk J. M., Knot K. H. W. (2001) Testing for long horizon UIP using PPP-based exchange rate expectations. Journal of Banking & Finance, 25, 377-391.

Brunnermeier M., Nagel S., Pedersen L. (2008) Carry Traders and Currency Crashes. National Bureau of Economic Research Macroeconomics Annual, 23, 313-347.

Chakraborty A., Evans G. W. (2008) Can perpetual learning explain the forward-premium puzzle? Journal of Monetary Economics, 55, 477-490.

Clarida R., Davis J., Pedersen N. (2009) Currency carry trade regimes: Beyond the Fama regression. Journal of International Money and Finance, 28, 1375-1389.

Domowitz I., Hakkio C. S. (1985) Conditional Variance and the Risk Premium in the Foreign Exchange Marke. Journal of International Economics, 19, 47-66.

Fama E. F. (1984) Forward and Spot Exchange Rates. Journal of Monetary Economics, 14, 319-338.

Flood R. P., Rose A. K. (2002) Uncovered Interest Parity in crisis. International Monetary Fund Staff Papers, 49 (2), 252-266.

Fong W. M. (2010) A stochastic dominance analysis of yen carry trade. Journal of Banking and Finance, 34, 1237-1246.

Froot K. A., Frankel J. A. (1989) Forward Discount Bias: Is it an Exchange Risk Premium? The Quarterly Journal of Economics, 104, 139-161.

Froot K. A., Thaler R. H. (1990) Anomalies: Foreign Exchange. The Journal of Economic Perspectives, 4 (3), 179-192.

Hamilton J. D. (1989) A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. Econometrica, 57 (2), 357-384.

Ichiue H., Koyama K. (2011) Regime switches in exchange rate volatility and uncovered interest parity. Journal of International Money and Finance, 30 (7), 1436-1450.

Jiang C., Chiang T. C. (2000) Do foreign exchange risk premiums relate to the volatility in the foreign exchange and equity markets? Applied Financial Economics, 10, 95-104.

Li D., Ghoshray A., Morley B. (2012) Measuring the risk premium in uncovered interest parity using the component GARCH-M model. International Review of Economics and Finance, 24, 167-176.

Lothian J. R., Wu L. (2011) Uncovered Interest-Rate Parity Over the Past Two Centuries. Journal of International Money and Finance, 30, 448-473.

Mark N. C., Wu Y. (1998) Rethinking deviations from uncovered interest parity: the role of covariance risk and noise. Economic Journal, 108, 1686-1706.

McCallum B. T. (1994) A reconsideration of the uncovered interest parity relationship. Journal of Monetary Economics, 33, 105-132.

Muth J. M. [1961], Rational Expectations and the Theory of Price Movements. Econometrica, 29 (3), 315-135.

Poghosyan T., Kocenda E., Zemcik P. (2008) Modeling Foreign Exchange Risk Premium in Armenia. Emerging Markets Finance and Trade, 44 (1), 41-61.

Sarno L., Valente G., Leon H. (2006) Nonlinearity in Deviations from Uncovered Interest Parity: An Explanation of the Forward Bias Puzzle. Review of Finance, 10, 443-482.

Serwa D. (2009) Modele kursu walutowego na rynkach finansowych [in:] Marcinkowska-Lewandowska W. (red.), Rubaszek M., Serwa D., Analiza kursu walutowego. Wydawnictwo C. H. Beck, Warszawa 2009, 130-255.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII , No. 1, 2017, pp. 27 – 37

Received: 07.02.2017
Accepted: 05.04.2017

THE ANALYSIS OF CHANCES OF YOUNG AND MIDDLE-AGED PEOPLE FOR HAVING A JOB USING BAYESIAN LOGISTIC REGRESSION MODEL[1]

Wioletta Grzenda (ORCID: 0000-0002-2226-4563)
Collegium of Economic Analysis
Warsaw School of Economics, Poland
e-mail: wgrzend@sgh.waw.pl

Abstract: The aim of this article is to analyze the chances of having a job using Bayesian logistic regression model. In this study both young and middle-aged people have been considered. The individual characteristics of economically active people have a significant impact on their labour market status. In this research the commonly studied set of features has been extended by adding the following characteristics: marital status, financial situation of the household, health assessment and the fact of living with parents in the case of young people. In this study, Bayesian logistic regression model has been used. The Bayesian approach enabled us to incorporate information from previous studies.

Keywords: employment, logistic regression, Bayesian inference, MCMC

INTRODUCTION

The chances of having a job depend both on the macroeconomic situation, mainly concerning economic situation of the country, and the microeconomic situation in the local labour market. Notwithstanding the economic conditions, the individual characteristics of the economically active persons have a significant impact on the individuals' status on the labour market. Most frequently characteristics such as age, sex, education and place of living are considered in other studies e.g. [Bukowski 2011].

[1] This study has been prepared as part of the project granted by the National Science Centre, Poland entitled "The modeling of parallel family and occupational careers with Bayesian methods" (2015/17/B/HS4/02064).

DOI: 10.22630/MIBE.2017.18.1.03 JEL: C25, J62

Most research on the labour market suggests sex as the main determinant of employment. The situation of women in the labour market is often worse than men, which is reflected in a lower rate of economic activity and a higher level of unemployment among women. According to [CSO 2014] data, in the fourth quarter of 2014, in Poland the rate of economic activity for men was 64.7%, whereas for women 48.5%. In addition, the unemployment rate among women was higher than among men regardless of age. Only among people over 60, there was an inverse relationship. The issues relating to equal opportunities for women and men in the labour market are very complex and concern different aspects, namely economic, social and cultural. However, most studies show that men are more likely to have a job than women e.g. [Bieszk-Stolorz and Markowicz 2013].

Many studies emphasize the impact of education on the individual's situation on the labour market. With the increased share of university graduates among young people, the importance of higher education has been gradually decreasing since 2008. However, research shows that people with higher education are still most likely to find a job [Grzenda 2012]. It is worth noting that the level of education does not only affect the chances of finding work, but also has a significant impact on the stability of unemployment [Núñez and Livanos 2010]. In addition, the report [Ministry of Labour and Social Policy 2012] shows that the duration of unemployment decreases with rising levels of education.

The level of professional activity varies with age. Moreover, it is observed that the effects of determinants on professional activity differ depending on age. According to [CSO 2014] data, the highest unemployment rate is observed among persons aged 24 and under, whereas for the next age group 25-34 it is down by a half, which is still high compared to other age groups. High unemployment among young people is particularly worrying, because it limits their economic independence.

Polish labour market is highly diversified geographically as a consequence of uneven socio-economic development of different regions in Poland. The proportion of jobless people aged 25 and under in the total number of registered unemployed persons in 2012 ranged from 15% in dolnośląskie province to 23.3% in małopolskie province [Ministry of Labour and Social Policy 2012].

Having a job should be considered not only in terms of aspects referring to individuals, but also households they belong to. Moreover, combining work with family life is an important aspect of research in this area [Kotowska et al. 2007]. In this paper, the commonly studied set of features has been extended to include the following characteristics: marital status, financial situation of the household, health assessment and the fact of living with parents in the case of young people.

EMPIRICAL DATA

For the purpose of this study, a data set from the panel survey Generations and Gender Survey (GGS) for Poland conducted under the program Generations and Gender Programme (GGP) has been used. The main objective of this international research program is to obtain information on demographic processes examined in the economic, social and cultural context [Kotowska and Jóźwiak 2011].

The data come from the second half of 2014. In addition, the missing information, which has not changed over time, has been supplemented based on the previous round of research carried out in the years 2010-2011. This study has been conducted among a random sample of respondents aged 18-79. The complete set of data has been divided into two age groups.

The first group included people aged 18-35 with 1960 observations. The second group consisted of people aged over 35 and under 55 with 2740 observations. The proportion of unemployed people among the young amounted to 19.39%, whereas for middle-aged people it was 11.82%. The results indicate a significantly worse situation of the young people on the labour market in Poland compared to the middle-aged people. According to the Labour Force Survey in the fourth quarter of 2015 [CSO 2016], the unemployment rate in Poland was 20.2% for people aged 15-24 and 7.8% for people aged 25-34.

The list of features used in the modelling is presented in Table 1. The initial set of features has been limited in the process of model building. Moreover, each of qualitative variables having k, $k \geq 2$ categories has been introduced into the models using $k - 1$ binary variables. Designations for the new variables have been adopted in accordance with the "Labels of levels" column (Table 1). The variable *marital status* has not been included in the model built for the young people, because it was strongly correlated with the variable *living with birth parents*. In addition, different variables have been used to assess the state of health of young and middle-aged people, due to insufficient numbers of observations in some levels of the variables.

Table 1. The list of characteristics of models

Variable	Names of levels	Labels of levels	Percent	
			Young people	Middle-aged people
Age group for middle-aged people	younger than 40	1	-	29.20
	older than 40 and younger than 44	2		23.03
	older than 44 and younger than 49	3		24.01
	49 years old and older	4		23.76
Age group for young people	younger than 24	1	29.08	-
	older than 24 and younger than 29	2	27.81	
	29 years old and older	3	43.11	
Sex	man	1	49.08	42.85
	woman	2	50.92	57.15

Variable	Names of levels	Labels of levels	Percent	
			Young people	Middle-aged people
Education status	higher	1	25.66	23.47
	post-secondary	2	17.50	11.72
	secondary professional	3	20.92	19.85
	secondary general	4	12.45	7.70
	basic vocational	5	15.66	31.09
	primary school	6	7.81	6.17
Marital status	unmarried, separated or divorced, a widower, a widow,	1	-	25.51
	married	2		74.49
Living with birth parents	no	0	54.29	-
	with at least one parent	1	45.71	
Health problems	no	0	-	76.61
	yes	1		23.39
Health	very good	1	40.56	-
	good	2	49.49	
	so-so and bad or very bad	3	9.95	
Financial situation of household	poor or no response	1	20.97	25.18
	rather poor	2	25.51	27.04
	rather good	3	36.84	34.12
	good and very good	4	16.68	13.65
Region of Poland	central (łódzkie, mazowieckie)	1	16.68	16.02
	south (małopolskie, śląskie)	2	20.05	16.24
	east (lubelskie, podkarpackie, świętokrzyskie, podlaskie)	3	21.17	20.58
	northwest (wielkopolskie, zachodniopomorskie, lubuskie)	4	15.97	17.55
	southwest (dolnośląskie, opolskie)	5	10.71	14.05
	north (kujawsko-pomorskie, warmińsko-mazurskie, pomorskie)	6	15.41	15.55

Source: own analysis of the GGS data 2014

RESEARCH METHOD

The study examines a binary dependent variable describing the fact of having a job. A logit model often called logistic regression is used to model such variable [Finney 1972; Hosmer and Lemeshow 2000].

Let us consider an n-elements random sample and dichotomous dependent variable *Y*. Let $y_i = 1$ mean the occurrence of the test event, and $y_i = 0$ its non-occurrence, for $i = 1, \ldots, n$. Moreover, let p_i be the probability of success i.e.

$y_i = 1$, $p_i = P(y_i = 1)$. Let $\mathbf{x}_i = [1, x_{i1}, \ldots, x_{ik}]^T$ be a vector of independent variables, and $\boldsymbol{\beta} = [\beta_0, \beta_1, \ldots, \beta_k]$ be a vector of regression coefficients.
The logit transformation is defined as follows:

$$\mathrm{logit}(p_i) = \ln\left(\frac{p_i}{1-p_i}\right), \tag{1}$$

where:

$$\mathrm{logit}(p_i) = \boldsymbol{\beta}\mathbf{x}_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_k x_{ki}. \tag{2}$$

In classical logistic regression model [Gruszczyński 2012] p_i is given by:

$$p_i = \frac{\exp(\boldsymbol{\beta}\mathbf{x}_i)}{1+\exp(\boldsymbol{\beta}\mathbf{x}_i)}. \tag{3}$$

In this paper we consider the logistic regression model in Bayesian approach [Albert and Chib 1993; Gelman et al. 2000]. Statistical inference in the Bayesian approach is based on the posterior distribution. The posterior distributions are determined by the prior distributions and the likelihood function (Formula 5). The likelihood function for an n-elements random sample is:

$$\begin{gathered} L(\boldsymbol{\beta} \mid \mathbf{y}) = \prod_{i=1}^{n}\left[(p_i)^{y_i}(1-p_i)^{(1-y_i)}\right] = \\ \prod_{i=1}^{n}\left[\left(\frac{\exp(\boldsymbol{\beta}\mathbf{x}_i)}{1+\exp(\boldsymbol{\beta}\mathbf{x}_i)}\right)^{y_i}\left(1-\frac{\exp(\boldsymbol{\beta}\mathbf{x}_i)}{1+\exp(\boldsymbol{\beta}\mathbf{x}_i)}\right)^{(1-y_i)}\right] \end{gathered} \tag{4}$$

Assuming normal prior distributions for the regression coefficients:

$$\beta_j \sim N(\mu_j, \sigma_j^2) \tag{5}$$

the posterior distribution is given by:

$$\begin{gathered} p(\boldsymbol{\beta} \mid \mathbf{X}, \mathbf{y}) \propto \prod_{i=1}^{n}\left[\left(\frac{\exp(\boldsymbol{\beta}\mathbf{x}_i)}{1+\exp(\boldsymbol{\beta}\mathbf{x}_i)}\right)^{y_i}\left(1-\frac{\exp(\boldsymbol{\beta}\mathbf{x}_i)}{1+\exp(\boldsymbol{\beta}\mathbf{x}_i)}\right)^{(1-y_i)}\right] \cdot \\ \prod_{j=0}^{k}\left[\frac{1}{\sqrt{2\pi}\sigma_j}\exp\left\{-\frac{1}{2}\left(\frac{\beta_j-\mu_j}{\sigma_j}\right)^2\right\}\right] \end{gathered} \tag{6}$$

The Markov Chain Monte Carlo Methods (MCMC) [Casella and George 1992, Gelman et al. 2000] have been used to estimate model parameters. Examples of the estimation of the models of qualitative variables using MCMC method can be found in the works [Marzec 2008a, b].

MODEL ESTIMATION

The estimations of all models have been performed using SAS system. Before administering the Bayesian modelling, potential explanatory variables have been selected and their significance and usefulness in explaining the phenomenon using classical methods verified.

The selection of the model and its evaluation in terms of its relevance for the observed data have been carried out using Akaike information criterion, Bayesian information criterion, deviance statistics and Pearson's chi-square test. The values of the last two of these statistics divided by the number of degrees of freedom have given 0.8987 and 0.9292 for the model for the young people, and 0.7269 and 1.0399 for the model for the middle-aged people. These results show that overdispersion has not occurred. The predictive power of the considered models has been also evaluated with 80% correct decisions predicted by the model. The second part of the modelling has been performed using the Bayesian approach. In Bayesian approach, the deviance information criterion (DIC) [Congdon 2006] has been used for the selection of the model.

In the first stage models have been estimated with the non-informative prior distributions, also referred as flat prior. Therefore, normal prior distributions with the mean 0 and variance 10^6 have been used for all regression parameters. Young people’s situation on the labour market is the subject of many studies, so the model for the young people has been estimated with informative prior distributions based on the paper [Grzenda 2012]. However, for the examined data, the impact of prior distributions on posterior distributions has not been significant, due to large sample. The studies on the unemployment rate among middle-aged people are rarely found in the literature. Moreover, due to a large sample, this model has been estimated with non-informative prior distributions.

The estimated parameters for the young people model have been presented in the Table 2 (Model 1). The results for the middle-aged people model have been provided in the Table 3 (Model 2). Based on the highest probability density interval [Bolstad 2007], all variables for both models are statistically significant.

Table 2. Posterior sample mean and interval statistics

Model 1 for young people					
Parameter	Mean	Standard Deviation	Highest Probability Density Interval (α=0.05)		Exp(Mean)
Intercept	1.7882	0.0110	1.7664	1.8089	5.9787
age_c1	-0.8342	0.00584	-0.8461	-0.8233	0.4342
age_c2	-0.5543	0.00489	-0.5636	-0.5445	0.5745
sex1	0.8991	0.00430	0.8911	0.9079	2.4574

Model 1 for young people					
Parameter	Mean	Standard Deviation	Highest Probability Density Interval (α=0.05)		Exp(Mean)
education1	1.5181	0.00828	1.5027	1.5350	4.5635
education2	0.9823	0.00805	0.9672	0.9986	2.6706
education3	0.4246	0.00736	0.4101	0.4387	1.5290
education4	0.3025	0.00800	0.2874	0.3190	1.3532
education5	0.5164	0.00762	0.5016	0.5313	1.6760
living_parents1	-0.6320	0.00463	-0.6409	-0.6226	0.5315
health_Y2	-0.0448	0.00428	-0.0530	-0.0363	0.9562
health_Y3	-0.3242	0.00642	-0.3365	-0.3114	0.7231
financial_situation1	-1.4179	0.00684	-1.4308	-1.4039	0.2422
financial_situation2	-0.3080	0.00687	-0.3212	-0.2945	0.7349
financial_situation3	0.1033	0.00676	0.0904	0.1170	1.1088
region1	-0.1021	0.00650	-0.1153	-0.0898	0.9029
region2	0.1521	0.00664	0.1392	0.1650	1.1643
region3	-0.5213	0.00683	-0.5346	-0.5078	0.5937
region4	0.1827	0.00736	0.1687	0.1972	1.2005
region5	-0.2869	0.00786	-0.3025	-0.2720	0.7506

Source: own analysis of the GGS data 2014

Table 3. Posterior sample mean and interval statistics

Model 2 for middle-aged people					
Parameter	Mean	Standard Deviation	Highest Probability Density Interval (α=0.05)		Exp(Mean)
Intercept	2.2612	0.0131	2.2342	2.2866	9.5946
age_c1	0.1083	0.0054	0.0980	0.1191	1.1144
age_c2	0.4786	0.0063	0.4664	0.4913	1.6138
age_c3	0.1246	0.0056	0.1140	0.1357	1.1327
sex1	0.6811	0.0043	0.6723	0.6892	1.9761
education1	1.8378	0.0088	1.8210	1.8550	6.2827
education2	1.4598	0.0094	1.4418	1.4785	4.3051
education3	1.2977	0.0079	1.2823	1.3134	3.6609
education4	0.8008	0.0091	0.7830	0.8186	2.2273
education5	0.4044	0.0068	0.3910	0.4175	1.4984
martial1	-0.6000	0.0042	-0.6079	-0.5914	0.5488
health_problems1	-0.1438	0.0046	-0.1531	-0.1349	0.8661
financial_situation1	-2.0734	0.0103	-2.0936	-2.0534	0.1258
financial_situation2	-1.0506	0.0106	-1.0698	-1.0282	0.3497
financial_situation3	-0.3382	0.0109	-0.3596	-0.3168	0.7131
region1	-0.2327	0.0070	-0.2464	-0.2196	0.7924
region2	0.3157	0.00780	0.3010	0.3313	1.3712
region3	-0.6958	0.00701	-0.7100	-0.6825	0.4987
region4	0.0387	0.00736	0.0245	0.0535	1.0395
region5	0.0247	0.00803	0.00980	0.0410	1.0250

Source: own analysis of the GGS data 2014

Geweke test has been used to assess the convergence of Markov chains. This test is based on comparing the mean value for the first part of the chain and the mean value for the last part of the chain. Based on the results for the first model (Table 4) it has been found that there is no indication that Markov chains have converged, at the level of significance α=0.05, for all the parameters of the model.

For the second model, for 2000 burn-in iterations, the convergence of all chains has not been identified, therefore, the burn-in number has been increased to 4000. Then, for the level of significance α=0.05, it has been asserted that there are no grounds to reject the verified hypothesis of convergence of chains. Moreover, Monte Carlo standard errors (MCSE) have been given in Table 4 for all investigated parameters.

Table 4. Geweke convergence diagnostics and MCSE

	Model 1 for young people			Model 2 for middle-aged people		
Parameter	Geveke diagnostics		MCSE	Geveke diagnostics		MCSE
	z	p-value		z	p-value	
Intercept	0.0160	0.9872	0.00069	1.1152	0.2648	0.00095
age_c1	-0.1086	0.9135	0.00016	-0.3681	0.7128	0.00011
age_c2	-0.0728	0.9420	0.00011	-0.2163	0.8288	0.00011
age_c3	-	-	-	-0.0254	0.9797	0.00011
sex1	-0.1207	0.9039	0.00009	0.1076	0.9143	0.00006
education1	-0.0702	0.9440	0.00036	-0.0530	0.9578	0.00028
education2	-1.2906	0.1968	0.00033	0.2847	0.7759	0.00028
education3	-0.2668	0.7897	0.00030	0.0544	0.9566	0.00028
education4	0.0422	0.9664	0.00031	-0.2790	0.7803	0.00028
education5	-0.1460	0.8839	0.00028	0.3230	0.7467	0.00026
living_parents1	-0.1572	0.8751	0.00010	-	-	-
martial1	-	-	-	-1.4990	0.1339	0.00007
health_Y2	-0.3614	0.7178	0.00009	-	-	-
health_Y3	-0.3313	0.7404	0.00011	-	-	-
health_problems1	-	-	-	-0.7781	0.4365	0.00007
financial_situation1	-0.1326	0.8945	0.00031	-1.3700	0.1707	0.00068
financial_situation2	0.0432	0.9656	0.00027	-1.4842	0.1377	0.00068
financial_situation3	-0.4101	0.6817	0.00026	-1.4668	0.1424	0.00066
region1	-0.0630	0.9498	0.00020	0.2659	0.7903	0.00021
region2	0.0160	0.9872	0.00020	-0.4396	0.6602	0.00021
region3	-0.1086	0.9135	0.00020	-0.4891	0.6247	0.00021
region4	-0.0728	0.9420	0.00021	-0.2203	0.8256	0.00021
region5	-0.1207	0.9039	0.00021	0.0805	0.9358	0.00021

Source: own analysis of the GGS data 2014

SUMMARY AND CONCLUSION

This study provided insights into the impact of selected characteristics on the chances of having a job among young and middle-aged people.

For the interpretation of results of the estimated models odds ratio has been used. The odds ratio is the value of $\exp\left(\hat{\beta}_j\right),\ j=1,\ldots,k$ where $\hat{\beta}_j$ is the estimate of model parameter [Gruszczyński 2012]. The values of $\exp\left(\hat{\beta}_j\right)$ have been presented in the Table 2 and 3.

While examining the characteristic *age* for the first model it has been shown that people under the age of 24 have about 56.58% less chance of having a job than those over 29. Moreover, people aged 24-29 have about 42.55% less chance of having a job than people in the oldest group. In the second model it has been indicated that the oldest age group i.e. people aged 49 and older have the lowest chance of having a job. Finally, people aged 40-44 have about 61.38% higher chance of having a job compared to the oldest group.

Some earlier assumptions that men have higher chance of having a job than women have been confirmed. For young people it has been shown that men have about 145.74% higher chance of having a job than women, whereas for the middle-aged people it is 97.61%.

Education is another important determinant widely discussed in various studies. People with primary education have the lowest chance of having a job among young people. People with higher education have about 356.35% higher chance to be employed than people with primary education and people with secondary professional education have about 167.06% higher chance of having a job than people with primary education. Other levels of education result in approximately 35%-53% increase in the chance of having a job compared to the lowest level of education. In the case of older people, these differences are even greater with the exception of professional education for which a similar value has been obtained. However, for higher, post-secondary and secondary professional education in this age group there seem to be 200% bigger odds of having a job for each level of education compared to the results obtained for young people.

Young people living with their parents have about 46.85% less chance of having a job than people who do not live with their parents. Single people in their middle age have about 45.12% less chance of having a job than people who are in a relationship.

Considering the self-assessment of health, it can be concluded that in the case of young people health situation has little effect on the chance of having a job. Those assessing their health as so-so and bad or very bad have about 27.69% less chance of having a job than people who evaluate their health as very good. Similarly, among the middle-aged, people with health problems have about 13.39% less chance of having a job than people who do not have any health problems.

The financial situation of the respondent's household was another investigated characteristic. For the young people it has been indicated that people who evaluated the financial situation of their household as poor or did not answer this question have about 75.78% less chance of having a job than people who evaluated their material situation as good and very good. Those who assessed the situation of their household as rather poor have about 26.51% less chance of having a job, while people who evaluated the situation of their household as rather good have about 10.88% higher chance of having a job than people who admitted that the situation of their household is good and very good. For the middle-aged people similar results have been obtained, except for the level: rather good. People who evaluated the financial situation of the household they belong to as less than good and very good have less chance of having a job. People who described the financial situation of their household as rather good have about 28.69% less chance of having a job than people who evaluated their financial situation as good and very good.

For the characteristics *region* the results are similar both for the young and middle-aged people. Compared to the northern region, young people in the southern region have 16.43% higher chance of having a job and about 20% higher in the northwest region. At the same time middle-aged people have 37.12% higher chance of having a job in the southern region that in the northern region; the figures for the northwest and southwest region are 3.95% and 2.5% respectively. In contrast, people who live in the eastern region have the least chance of having a job, irrespective of their age.

In this paper the impact of selected determinants on the chance of having a job has been considered. This research has provided a comparative analysis of the situation of young and middle-aged people on the labour market. The analysis of two most important determinants of activity: sex and education has indicated that gender has much greater effect on the chance of having a job for young people than for middle-aged people. As for education, the figures suggest that higher and post-secondary education have much greater impact on the chance of having a job for middle-aged people, than for young people. By identifying differences in the effects of selected determinants on the odds of having a job among young and middle-aged people this paper has expanded the results of previous studies on the reasons of high unemployment among young people.

REFERENCES

Albert J. H., Chib S. (1993) Bayesian analysis of binary and polychotomos response data. Journal of the American Statistical Association, 88, 669–679.

Bieszk-Stolorz B., Markowicz I. (2013) Men's and women's economic activity in Poland. Acta Universitatis Lodziensis, Folia Oeconomica, 285, 31–39.

Bolstad W.M. (2007) Introduction to Bayesian statistics. Second Edition, Wiley & Sons, New York.

Bukowski M. (red) (2011) Zatrudnienie w Polsce 2010. Integracja i globalizacja. Instytut Badań Strukturalnych, Warszawa.

Casella G., George E. I. (1992) Explaining the Gibbs sampler. The American Statistician, 46, 167–74.

Congdon P. (2006) Bayesian Statistical Modelling. 2nd ed., John Wiley & Sons Inc., United Kingdom.

Finney D. J. (1972) Probit Analysis, Cambridge University Press, London.

Gelman A., Carlin J.B., Stern H. S., Rubin D. B. (2000) Bayesian data analysis. Chapman &Hall / CRC, London.

Generations and Gender Programme: http://www.ggp-i.org/

Gruszczyński M. (Eds.) (2012) Mikroekonometria. Modele i metody analizy danych indywidualnych. Oficyna a Wolters Kluwer business, Warszawa.

Grzenda W. (2012) Badanie determinant pozostawania bez pracy osób młodych z wykorzystaniem semiparametrycznego modelu Coxa. Przegląd Statystyczny, Numer specjalny 1, 123–139.

GUS (2014) Badanie Aktywności Ekonomicznej Ludności (BAEL).

GUS (2016) Monitoring Rynku Pracy. Kwartalna informacja o rynku pracy, 1–16.

Hosmer D. W., Lemeshow S. (2000) Applied Logistic Regression, Wiley, New York.

Kotowska I. E., Jóźwiak J. (2011) Panelowe badanie przemian relacji między pokoleniami, w rodzinie oraz między kobietami i mężczyznami: generacje, rodziny i płeć kulturowa – GGS-PL. Studia Demograficzne, 1(159), 99–106.

Kotowska I.E., Sztanderska U., Wóycicka I. (Eds.) (2007) Aktywność zawodowa i edukacyjna a obowiązki rodzinne w Polsce w świetle badań empirycznych. Wydawnictwo Naukowe SCHOLAR, Warszawa.

Marzec J. (2008a) Bayesowska analiza i testowanie modeli dwumianowych z rozkładem t Studenta. Przegląd Statystyczny, 55(2), 48–63.

Marzec J. (2008b) Bayesowskie modele zmiennych jakościowych i ograniczonych w badaniach niespłacalności kredytów. Wydawnictwo Uniwersytetu Ekonomicznego w Krakowie, Kraków.

MRPiPS (2012) Sytuacja na rynku pracy osób młodych w 2012 roku, Ministerstwo Pracy i Polityki Społecznej. Departament Rynku Pracy, Wydział Analiz i Statystyki, Warszawa.

Núñez I, Livanos I. (2010) Higher education and unemployment in Europe: an analysis of the academic subject and national effects. Higher Education 59(4), 475–487.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII, No. 1, 2017, pp. 38 – 47

Received: 20.12.2016
Accepted: 05.04.2017

SPATIAL GRAPHIC INTERPRETATION OF THE FOSTER-HART FORMULA

Marcin Halicki
Faculty of Biology and Agriculture, University of Rzeszow, Poland
e-mail: mhalicki@ur.edu.pl
Tadeusz Kwater
Faculty of Mathematics and Natural Sciences, University of Rzeszow, Poland
e-mail: tkwater@univ.rzeszow.pl

Abstract: This article deals with the problem of spatial interpretation of graphical Foster-Hart formulas. The proposed approach allows the assessment of investments with specific expected payouts. This approach may also be, in a certain sense, considered as generalization in relation to the evaluation, as the author has shown how to interpret certain investment cases. It is also important that in a similar way, one can also evaluate all portfolios, which consist not only of financial instruments, but also other investment assets. The paper presents the idea of the Foster-Hart measurement on the basis of the analysis of a hypothetical action, and all simulation tests were carried out in MATLAB programming environment.

Keywords: measure of riskiness, investment, risk, portfolio management, financial model

INTRODUCTION

In the conventional models used within the portfolio management adopted in many different principles, which limit their practical utility by making them tools, whose effectiveness in real conditions is not always high. In literature, one pointed to a number of disadvantages of these models, among which dominates the lack of provisions for possible bankruptcy of the investors, both individual and institutional [Halicki 2016]. It is commonly known that the most well-known and widely used model for the Markowitz [Markowitz 1952] does not satisfy such a condition. Moreover, it assumes that the risk of a single asset forming part of the portfolio should be measured by the standard deviation. These two assumptions

DOI: 10.22630/MIBE.2017.18.1.04 JEL: D81, G11

themselves reduce the usefulness of this model, both in practical and theoretical dimension, because the standard deviation is not an appropriate measure for the risk measurement (not monotone), and the possibility of bankruptcy of the investor is a decisive factor for the preference for a particular asset investment. Their palette, available on the capital market, may contain such instruments that offer on one hand attractive rates of return, with a high risk, however on the other one, increase the likelihood of bankruptcy of the investor. The problem which involves the selection of investment assets and their management, is therefore to measure the level of risk. If it is measured in the wrong way, as is the case with the Markowitz model, the construction of portfolios, taking into account the criterion of the level, will not always be correct from the perspective of diversification.

The effectiveness of portfolio management is assessed through the prism of management skills and methods they use. These skills include identifying attractive assets, and mitigate the risk. Therefore, measuring its level plays a significant role in the management of portfolios. In contrast, the investment models are the support that this measure allows. The result is a trend to develop new risk measures, improving the efficiency of portfolio management. One of them is a measure of the Hart-Foster [Foster & Hart 2009], which the literature does not pay too much attention to. This measure, being monotone, objective and universal [Foster & Hart 2009] also satisfies the condition of considering the possibility of investor's bankruptcy. Presented characteristics suggest that it could raise an interest among investors.

It should also be noted that the measure of the risk of Foster-Hart takes the form of a formula. In modern literature, the articles more often test its basic features [Chudziak & Halicki 2016; Hellmann & Riedel 2015], however, the spatial graphical interpretation of this formula was not carried out,. Such an interpretation may assist investors in the selection of investment assets in terms of the risk reduction. In this light it may seem desirable, and therefore makes an objective of the work. The reason for taking this subject up, is also the fact that the spatial interpretation of graphical formula of the Foster-Hart has also many other advantages. One of them is that they may assist investors in assessing the immediate graphical investments with specific expected payouts. In the present study, we used literature devoted to the Foster-Hart measure, as well as hypothetical data, whose analyses were performed in MATLAB software environment.

THE FOSTER-HART MEASURE AND ITS INTERPRETATION

The measure of Foster-Hart, presented in 2009 [Foster & Hart 2009], is an alternative to other well-known measures, which include among others: measure of risk based on the Markowitz Portfolio Theory, VaR (Value at Risk) and coherent risk measures. This is due to its main features, namely the ability to identify very risky investments which could lead to bankruptcy of the investor. As it is well known, other measures do not have this property. The risk measure of the Foster-Hart for a single investment takes the form of the following formula:

$$E\left[log\left(1+\frac{1}{R(g)}g\right)\right]=0\ , \quad (1)$$

wherein *E[X]* is the expected value of a random variable *X* (the probability value that specifies the expected result of a random experiment), g means income from investments, which can be expected with a certain probability at the end of the investment period, and *R(g)* is a measure of the investment risk and the critical value of the investor's property. The *R(g)* value is calculated not only to determine the level of the risk but also for comparison with the current level of the investor's wealth. In a discrete recording of the random variable *X*, receiving the *value* x_1, x_2, ... x_n with probabilities of, respectively p_1, p_2, ... p_n, the expected value is reduced to the form:

$$E[X]=\sum_{i=1}^{n}x_i\,p_i, \quad (2a)$$

It should be added that the validity of the equation (1) for the selection of investment occurs only when the following conditions are met:

$$\sum_{i=1}^{n}p_i=1\ ,\ \ \sum_{i=1}^{n}p_i g_i>0, \text{ and} \quad (2b)$$

$$g_i<0\ \ then\ \ g_j>0\ , \quad (2c)$$

wherein:

$$i\neq j \text{ and also,} \quad (2d)$$

$$i,j=2,3,\ldots \quad (2e)$$

In the following discussion we will use the equivalent of a formula (1) which, for n components is as follows:

$$\left(1+\frac{g_1}{R(g)}\right)^{p_1}\cdot\left(1+\frac{g_2}{R(g)}\right)^{p_2}\cdot\ \ldots\ \cdot\left(1+\frac{g_n}{R(g)}\right)^{p_n}=1. \quad (3)$$

The idea of the Foster-Hart measure is worth presenting by using the analysis of the hypothetical action for a specific performance, ie. the purchase price and the selling prices of the hypothetical accepted probabilities (Table 1).

Table 1. Hypothetical data for the investor who wants to purchase 1 share of interpretation of the application of the Foster-Hart formula to determine *R(g)*

The purchase price at the beginning of the investment period	1,000 USD			
The hypothetical sales price with probabilities of 25%	USD 685	USD 784	USD 1,279	USD 1,378
The corresponding investment income	USD -315	USD -216	USD 279	USD 378

Source: own study

This approach allows to determine the value of *R(g)* for which the investor is neutral towards the investment. The values of these parameters mean that the presented formula, after surgery logarithmic equation (3), takes the following form:

$$\frac{1}{4}log\left(1+\frac{-315}{R(g)}\right)+\frac{1}{4}log\left(1+\frac{-216}{R(g)}\right)+\frac{1}{4}log\left(1+\frac{279}{R(g)}\right)+\frac{1}{4}log\left(1+\frac{378}{R(g)}\right)=0. \quad (4)$$

With the solution of equation (4) we calculate the value of R(g) of 1,426.65. This means that the investor may purchase shares for USD 1,000 if in addition to this amount, he has a net worth of not less than USD 1,426.65. Therefore, the investor should have at least USD 2,426.65. This will allow him to avoid bankruptcy.

In practice, stock exchange investors may be interested in it even on the grounds that it disregards the preferences, pointing to instruments that would be too risky for them. It should be mentioned that according to a recent study, it has been shown that the optimal portfolios built with its use have high performance [Anand et al. 2016]. This means that the graphic interpretation, unprecedented in literature, may be regarded as interesting, especially if it is used in the process of evaluating various investment assets, including financial instruments, as well as for the construction of portfolios.

GRAPHIC INTERPRETATION OF THE FOSTER-HART FORMULA AND ITS ANALYSIS

An in-depth analysis of the Foster-Hart formula points out that its graphical interpretation requires appropriate mathematical transformations in order to achieve the desired relationship of parameters. For example, the relationship between g_1 and g_2 in the formula (1) or (3) for permanent p_1, p_2 and *R(g)* is determined as follows:

$$g_2 = R \cdot \left[1-\left(1+\frac{g_1}{R(g)}\right)^{\frac{p_1}{p_2}}\right]/(1+\frac{g_1}{R(g)})^{\frac{p_1}{p_2}} \text{, and} \quad (5)$$

$$p_1 + p_2 = 1. \quad (6)$$

where g_1 is the amount of income generated by an investment in the future of probability of p_1, and g_2 is the amount of income generated by an investment in the future of probability of p_2.

The results of the equation (5), (6) for the hypothetical cases of the investment are shown in Figures 2, 4, 6, but they need to accept respective assumptions for the remaining parameters. The basis for this research is the theoretical investment and its two different variants (Table 2). All simulation studies, which were used in the work, has been implemented in MATLAB programming environment.

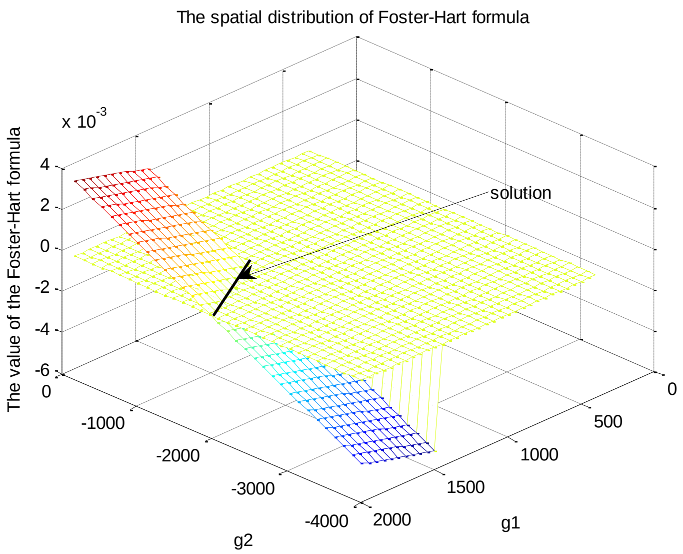
Table 2. Parameters of the sample investment and its analyzed variants

Parameters investments in the basic version	
Parameter	Value
R(g)	250,000
p_1 $(p_2 = 1 - p_1)$	0.5
Range of g_1	(1,500; 2,000)
Range of g_2	(-1,491; -1,984)
Variant number 1	
R(g)	250,000
p_1 $(p_2 = 1 - p_1)$	0.4; 0.5; 0.6; 0.7
Variant number 2	
R(g)	15,000; 20,000; 30,000; 90,000; 900,000
p_1 $(p_2 = 1 - p_1)$	0.5

Source: own calculations

Table 2 contains no g_1, g_2 values for variants 1 and 2, since the figures are based on other adopted parameters of the Foster-Hart formula. Key features differentiating presented cases of the investment is expressed by two-dimensional and three-dimensional charts. The first experiment, carried out for a number of investments in the basic version, the *R(g)*, and the probabilities are fixed, is shown in Figure 1.

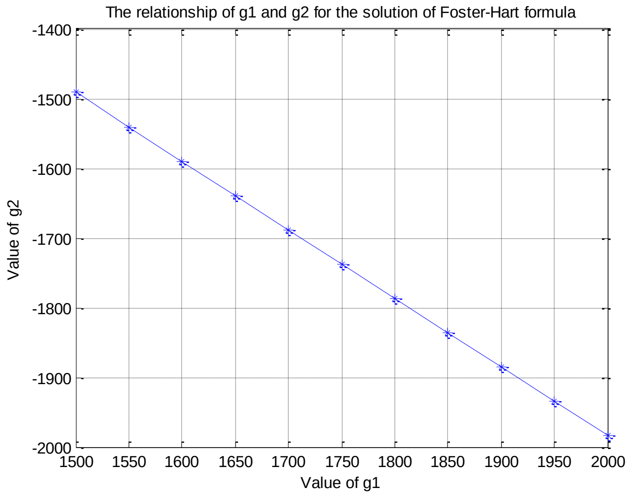
Figure 1. A three-dimensional graph of the relationship between the parameters of the analyzed investment



Source: own study based on the results of the MATLAB program

Figure 1 includes not only the graphical representation of the relationship values of the investment g_1, g_2 but also areas where no solution of the Foster-Hart formula is adopted for $R(g)$. To enhance the presentation, this solution has been selected not only on the presented three-dimensional graph, but also two-dimensional graph (Figure 2).

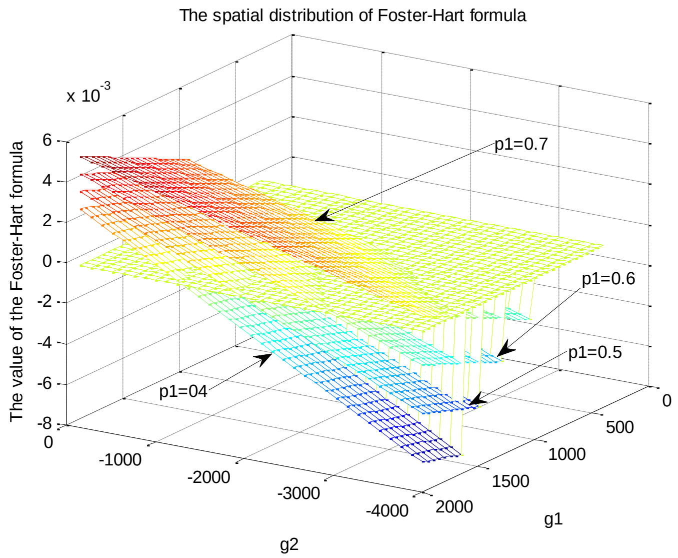
Figure 2. The diagram showing the two-dimensional solution of the Foster-Hart formula depending on the g_1 and g_2 size for the investment in the basic



Source: own study based on the results of the MATLAB program

Figure 2 shows the detailed embodiment of Figure 1 indicated as a line. If one assumes that the probability of positive and negative future revenue are the same, that is 50%, then the pair of these values, where the outcome of the characterized formula reaches a ”0” value , form a line that is depicted in Figure 2. The points not belonging to this line, are not the solution of the Foster-Hart formula, they reflect combinations of the investment, for which the value of this formula is different from zero. Analyzing the pattern (3), one should pay attention to the fact that the Foster-Hart formula is sensitive to the probability values, however the formula may get a solution only with certain pairs of values of future income of the investment. One does not need more to explain that the simulation charts containing the results of numerical experiments with changed probabilities of expected income, can be considered to be cognitively interesting (for the first variant), especially since the results of the Foster-Hart formula take a different graphic form (Figure 3 and Figure 4).

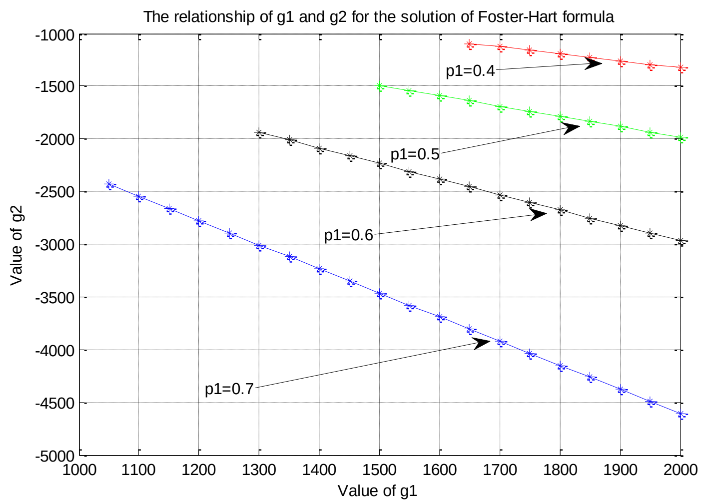
Figure 3. A three-dimensional graph of the relationship between the parameters of the first analyzed variant of the investment



Source: own study based on the results of the MATLAB program

For the accurate interpretation of the Foster-Hart formula in the analysis of the first variant, it is worth correlating the three-dimensional Figure 3 with the two-dimensional Figure 4, and in that drawing the most important conclusions.

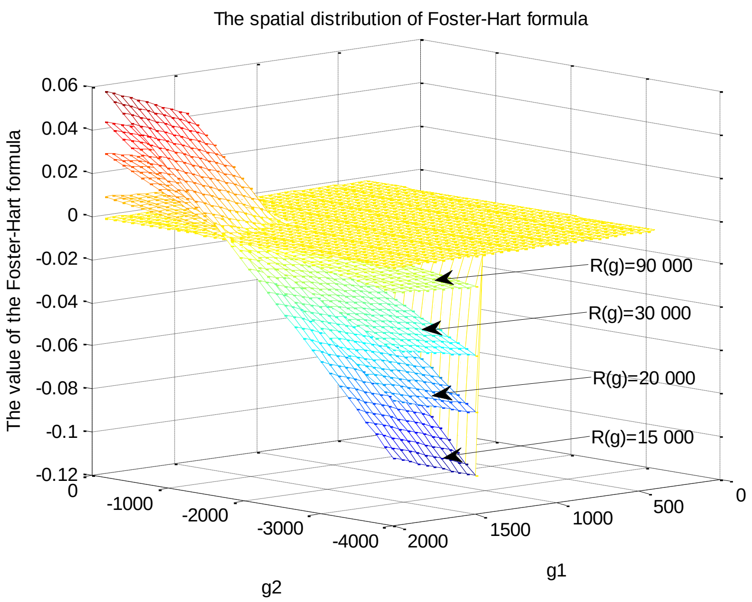
Figure 4. Diagram showing the two-dimensional solution of the Foster-Hart formula depending on the g_1 and g_2 size for the first variant



Source: own study based on the results of the MATLAB program

As it is mentioned, the graphs taking into account the change in probabilities represent a completely different investment situation, expressed even by four angled faces in Figure 3, which however in different places intersect with the horizontal plane, thereby reflecting different combinations of revenue (positive and negative) for providing a zero value of Foster – Hart formula. This application is shown in Figure 4, which summarizes the different solutions reflecting the relationships range of investment income from the specified values of probabilities (p_1 = *0.4, 0.5, 0.6, 0.7* and respectively p_2 = *0.6, 0.5, 0.4, 0.3*). The last investment variant (Table 2) take into account the impact of changes in the value of *R(g)* on the range of expected income (analyzed five possible values). This approach makes the solution for *R(g)* amounting to "900,000" on the three-dimensional graph is imperceptible at the scale adopted at the "*Z*" axis (Figure 5). This is due to the fact that the plane of the case investment is near the horizontal plane.

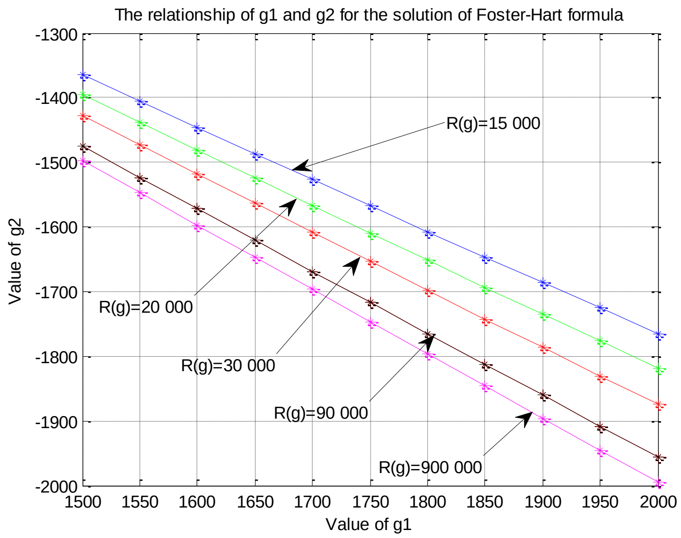
Figure 5. A three-dimensional graph of the relationship between the parameters of the second analyzed variant of the investment



Source: own study based on the results of the MATLAB program

The solution for this investment option is also presented in the form of two-dimensional graph (Figure 6), which already includes the case when *R(g)* is "900,000".

Figure 6. Graph showing the two-dimensional solution of the Foster-Hart formula depending on the value of g_1 and g_2 for the second variant



Source: own study based on the results of the MATLAB program

If the investment is characterized by certain probabilities, which are solid at varying values of *R(g)*, the oblique plane of Figure 5 in different places intersect the horizontal plane. As a result, different combinations of income (positive and negative) provide zero value of the Foster-Hart formula. This case is cognitively interesting for this reason that it allows for an analysis of the selected investments from the point of the investors' assets, whose size can vary considerably. Analyzing the last variant it is easy to see that with a relatively large values of *R(g)*, the ranges of positive and negative income are very little different. Quite different it is in the case of *R(g)* smaller by at least an order of magnitude as the differences of such ranges are already visible.

SUMMARY

The aim of this publication is the spatial graphical interpretation of the Foster-Hart formula. This subject was taken up because of the nature of this formula, which begins to arouse an interest among economists in the world, and which still devotes too little attention. Conducted considerations can be regarded as valuable mainly for this reason that they allow for the graphical assessment of individual investments or even entire portfolios that consist of financial instruments or other investment assets. Though hypothetical data were analyzed, still each investment can be written in a language in which it has been done in the publication.

In theory, there are infinitely many cases of investment, which can be analyzed graphically. Due to changes in the parameters of the Foster-Hart's equation, the choice is limited to three categories. Therefore, attention is paid to the theoretical project and its two variants. In summary, the analysis cites a number of facts indicating that the changes in investor assets are associated with significant changes in potential revenue, generated by the investment, which can be accepted by them without the fear of bankruptcy. Noteworthy is the fact that the probability of received income also affects the range of expected income, which can be accepted with certain assets of the investor. Summing up the above observations and taking into account the characteristics of the Foster-Hart measurement, it is clear that conducted considerations can be regarded as useful for the provision of the type of wealth management and in the preparation of investment products to investors. Moreover, the advantage of the graphical presentation is that one can obtain an immediate estimate of the investment case. This work can also become a base for separate studies related to the analysis of diversified portfolios in terms of Söhnholz, Rieken and Kaiser [Söhnholz & Rieken & Kaiser 2010].

REFERENCES

Anand A., Liy T., Kurosaki T., Kim Y. S. (2016) Foster-Hart Optimal Portfolios. Working Paper: Final Draft, 1-34.

Chudziak J., Halicki M. (2016) On the Foster–Hart Measure of Riskiness under Cumulative Prospect Theory. Acta Physica Polonica A, 129(5), 950-954.

Foster D.P., Hart S. (2009) An Operational Measure of Riskiness. Journal of Political Economy, 117(5), 785-814.

Halicki M. (2016) Foster-Hart measure as a suport element for the portfolio management process. Stan i perspektywy rozwoju nauk o zarządzaniu, Dom Organizatora, Toruń, 594-607.

Hellmann T., Riedel F. (2015) A dynamic extension of the Foster–Hart measure of riskiness. Journal of Mathematical Economics, 59, 66–70.

Markowitz H. (1952) Portfolio Selection. The Journal of Finance, 7(1), 77-91.

Söhnholz D., Rieken S., Kaiser D. G. (2010) Asset Allocation, Risiko-Overlay und Manager-Selektion. Gabler Verlag - Springer Fachmedien, Wiesbaden, 23.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII , No. 1, 2017, pp. 48 – 57

Received: 23.02.2017
Accepted: 05.04.2017

EVALUATION OF INNOVATION ECONOMIES OF THE CENTRAL AND EASTERN EUROPE COMPARED TO OTHER EU COUNTRIES

Grażyna Karmowska (ORCID: 0000-0002-2264-3176)
Faculty of Economics
West Pomeranian University of Technology, Poland
e-mail: gkarmowska@zut.edu.pl
Mirosława Marciniak (ORCID: 0000-0002-7872-4901)
Faculty of Informatics
West Pomeranian University of Technology, Poland
e-mail: mmarciniak@zut.edu.pl

Abstract: The purpopse of this article is the evaluation of innovation economies of Central and Eastern Europe (CEE) compared to other countries of the European Union, based on the aggregate indexes (Global Innovation Index, Innovation Union Scoreboard) and their components. It was found that the CEE countries are still a sizable distance from the "old members" of the European Union. The exceptions are Estonia, Slovenia and the Czech Republic, that owe their position to the effectiveness of the deployment of innovative and relative high expenditure on the development of innovation finance. The weakest proved to be Romania, Bulgaria, Latvia and Poland.

Keywords: Innovation, Central and East Europe, Global Innovation Index, Innovation Union Scoreboard

INTRODUCTION

Innovation sets the development of each country and its processes of transformation, hence the many years of innovation are seen as the main source of competition, economic growth and job creation. Innovation economy is the ability and willingness of operators to continuously seek out and use in business practice the results of research, and of research and development, new concepts, ideas and inventions, improvement and development of the used material and non-material production technology (services), the introduction of new methods and techniques in the

DOI: 10.22630/MIBE.2017.18.1.05 JEL: O11, O19

organisation and management, improve and develop the infrastructure and knowledge resources. In economic science attempts to explain this, observed the functioning of the innovation economy. There's even a new section called innovative economies. The impulse for the creation of the mainstream in economic thought was the entry into NAFTA in 1994, which was a free-trade zone. Parties to the agreement were Canada, Mexico and the United States. The agreement went beyond the traditional zone of free trade formula. Liberalisation also refer to the movement of professional services, which did not bring the expected results. So, innovation economics considers that the factors of production in the economy are documenting innovative social capital, creative capital, intellectual capital and entrepreneurs [Drabińska 2012].

One of the most important EU actions aimed at increasing competitiveness and innovation was announced in 2000, the *Lisbon Strategy*, which set a goal that by 2010, the EU economy has become the most competitive and dynamically developing knowledge-based economy in the world, capable of maintaining sustainable economic growth, create more and better jobs and social cohesion. Unfortunately, this goal could not be achieved, so a new document was developed. - *Europe 2020. A European strategy for smart, sustainable and inclusive growth.* Smart growth means strengthening knowledge and innovation as drivers of future growth, which in practice translates into actions aimed at improving the quality of our education, strengthening research performance, promoting innovation and knowledge transfer throughout the Union.

The objective of this paper was to attempt to answer the questions: What distance is the Central and Eastean Europe countries from other countries of the European Union in terms of the level of innovation? Which components of innovation are strong, and which are the weakest? In an attempt to answer the questions, the authors resort to rankings, which assess the innovation of the individual countries.

MATERIAL AND METHODS

Central and Eastern European Countries is an OECD term for the group of countries comprising of Albania, Bulgaria, Croatia, the Czech Republic, Hungary, Poland, Romania, the Slovak Republic, Slovenia, and the three Baltic States: Estonia, Latvia and Lithuania. The main subject of research work includes only 11 countries belonging to the EU (CEE), and not including Albania.

Innovation and innovation potential is not only difficult, but also supports the measure. This is due to the multi-step and the complexity of the process of the creation and implementation of innovative solutions. Therefore, most commonly for the purpose of diagnosis and international comparisons of innovation, the economies of a number of indicators are to be used in the aggregate indexes. So, in the comparative analysis we used two aggregate indicators of innovation and their sub-indexes: the Global Innovation Index and the Innovation Union Scoreboard. The source material were the thematic reports of the European Commission, data from Eurostat and the Report of the Global Innovation Index, which is co-published by Cornell University, INSEAD, and the World Intellectual Property Organisation (WIPO, an agency of the United Nations). The research applied comparative analysis of methods and descriptive statistics.

AN OVERVIEW OF RESEARCH RESULTS

The Global Innovation Index (GII) project was launched by INSEAD in 2007 with the simple goal of determining how to find metrics and approaches that better capture the richness of innovation in society and go beyond such traditional measures of innovation, as the number of research articles and the level of research and development (R&D) expenditures.

The GII relies on seven pillars. Each pillar is divided into three sub-pillars, and each sub-pillar is composed of two to five individual indicators. Each sub-pillar score is calculated as the weighted average of its individual indicators. Each pillar score is calculated as the weighted average of its sub-pillar scores. The framework of the GII calculation shows Figure 1.

Figure 1. Measurement framework of the Global Innovation Index

Global Innovation Index						
Innovation Efficiency Ratio						
Innovation Potential Input Sub-Index					Innovation Output Sub-Index	
Institutions	Human capital & research	Infrastructure	Market sophistication	Business sophistication	Knowledge &Technology	Creative output
Political environment	Education	ICTs	Credit	Knowledge workers	Knowledge creation	Intangible assets
Regulatory environment	Tertiary education	General infrastructure	Investment	Innovation linkages	Knowledge impact	Creative goods & services
Business environment	Research & development	Ecological sustainbility	Trade & competition	Knowledge absorption	Knowledge diffusion	Online creativity

Source: [The Global Innovation…, 2015]

The Global Innovation Index includes three indices and one ratio:

- The Innovation Input Sub-Index is the average of the first five pillar scores.
- The Innovation Output Sub-Index is the average of the last two pillar scores.
- The Global Innovation Index is the average of the Input and the Output sub-index scores.
- The Innovation Efficiency Ratio is the ratio of the Output sub-index score over the input sub-index score.

Table 1. Rankings of Global Innovation Index and their sub-indexes for members of EU

Countries	Global Innovation Index	Innovation Inputs Sub-index	Innovation Outputs Sub-index	Innovation Efficiency
United Kingdom	2	6	5	18
Sweden	3	7	4	16
Netherlands	4	11	3	8
Finland	6	3	10	41
Ireland	8	14	7	12
Luxembourg	9	20	2	3
Denmark	10	8	12	49
Germany	12	18	8	13
Austria	18	19	18	37
France	21	17	23	51
Estonia	23	26	14	17
Czech Republic	24	27	17	11
Belgium	25	21	28	59
Malta	26	33	13	7
Spain	27	24	29	67
Slovenia	28	30	27	22
Portugal	30	28	33	62
Italy	31	29	32	57
Latvia	33	34	30	26
Cyprus	34	32	43	90
Hungary	35	42	37	35
Slovakia	36	37	38	48
Lithuania	38	35	42	74
Bulgaria	39	49	35	21
Croatia	40	43	41	50
Montenegro	41	50	40	29
Greece	45	38	57	98
Poland	46	39	56	93
Romania	54	57	52	58

Source: own preparation based on [The Global Innovation Index 2015]

Country/economy rankings are provided for indicator, sub-pillar, pillar, and index scores. In 2015 the GII ranking included 141 countries (Table 2).

The head of the rankings are classified, according to Switzerland, and it ranked the three countries belonging to the EU- UK, Sweden and the Netherlands. With a group of CEE countries the best turned out to be Estonia and the Czech Republic (occupying 23rd and 24th place respectively), and the lowest ranked was Poland (46th) and Romania (54th).

The Innovation Efficiency Ratio serves to highlight those economies that have achieved more with less, as well as those that lag behind in terms of fulfilling their innovation potential. In theory, assuming that innovation results go hand in hand with innovation enablers, efficiency ratios should evolve around the number one. This measure thus allows us to complement the GII by providing an insight that should be neutral to the development stages of economies.

Least innovative potential was in Poland and in Lithuania (below 50%), while Estonia and the Czech Republic once again proved to be the best. In turn, Bulgaria is an example of a country, that despite the relatively small potential, intensely deploys innovative solutions. Estonia and the Czech Republic are examples of a country in which have achieved *more with less*. In this comparison the worst economy was Poland which has a large potential for innovative and lowest efficiency of its use. For more information about the components of potential and innovation products for individual EU Member States provide the European Innovation Scoreboard.

The European Innovation Scoreboard (EIS) is used to evaluate and compare the results of innovation of the individual countries, according to the respective indicators. Preparation of this cyclic type of report, is the result of the adopted Lisbon strategy, which is one of the main assumptions about economic growth, which is strongly correlated with the level of innovation. Innovation performance is measured using a composite indicator – **the Summary Innovation Index (SII)** – which summarises the performance of a range of different indicators. The Innovation Union Scoreboard distinguishes between three main types of indicator – *Enablers*, *Firm activities* and *Outputs* – There are 8 innovation dimensions, capturing in total 25 indicators. The Innovation Union Scoreboard 2015, the 14th edition, since the introduction of the European Innovation Scoreboard in 2001, follows the methodology of previous editions. Table 2 summarises the SII obtained by EU countries in 2007 and 2014, this is drawn up on the basis of the rankings. In addition, it has been presented as the SII 2014 in relative to the EU28 (average of the EU countries).

Table 2. Comparison of the Innovation Summary Index by EU countries for 2007 and 2014

Country	Summary Innovation Index (scores)		Ranking of SII		SII 2014 in relative to EU28
	2007	2014	2007	2014	
EU28	**0.519**	**0.555**	–	–	**100%**
Sweden	0.723	0.740	1	1	133%
Finland	0.672	0.676	2	3	122%
Germany	0.650	0.676	3	4	122%
Denmark	0.647	0.736	4	2	133%
Luxembourg	0.640	0.642	5	6	116%
Netherlands	0.573	0.647	6	5	117%
Belgium	0.573	0.619	7	9	112%
Ireland	0.570	0.628	8	8	113%
United Kingdom	0.565	0.636	9	7	115%
Austria	0.557	0.385	10	18	69%
France	0.544	0.391	11	17	70%
Cyprus	0.449	0.445	12	13	80%
Slovenia	0.446	0.534	13	10	96%
Estonia	0.420	0.489	14	11	88%
Spain	0.396	0.385	15	19	69%
Italy	0.393	0.439	16	14	79%
Czech Republic	0.373	0.447	17	12	81%
Portugal	0.365	0.403	18	15	73%
Greece	0.362	0.365	19	21	66%
Hungary	0.336	0.369	20	20	66%
Malta	0.325	0.397	21	16	72%
Slovakia	0.316	0.360	22	22	65%
Croatia	0.296	0.313	23	23	56%
Poland	0.292	0.313	24	24	56%
Lithuania	0.244	0.283	25	25	51%
Romania	0.240	0.204	26	28	37%
Latvia	0.215	0.272	27	26	49%
Bulgaria	0.184	0.229	28	27	41%

Source: own calculation based on Innovation Union Scoreboard 2015

Analysis of the change in ranking positions, showed that the highest increase in the ranking (5 places) recorded were Malta and the Czech Republic, and the largest decline was Austria (8 places) and France (a decrease of 6 places). In the CEE group of five countries has not changed its position and the same amount of rises in the rankings, only Romania has fallen in the ranking by 2 places. In the latest ranking, by states belonging to this group, top ranked Slovenia (10) and Estonia (11) and the list closes with Romania.

As a result, based on Summary Innovation Index, the member states fall into the following performance groups:

1. The group of innovation leaders include Member States in which the innovation performance is well above that of the EU, i.e. more than 20% above the EU average. These are Denmark, Finland, Germany and Sweden, which confirms the top position of these countries as compared with last year's edition of the Innovation Union Scoreboard.
2. The group of innovation followers includes Member States with a performance close to that of the EU average i.e. less than 20% above or more than 90% of the EU average. Austria, Belgium, France, Ireland, Luxembourg, Netherlands, Slovenia and the UK.
3. The group of moderate innovators includes Member States where the innovation performance is below that of the EU average at relative performance rates between 50% and 90% of the EU average. Croatia, Cyprus, the Czech Republic, Estonia, Greece, Hungary, Italy, Lithuania, Malta, Poland, Portugal, Slovakia and Spain.
4. The group of modest innovators includes Member States that show an innovation performance level well below that of the EU average, i.e. less than 50% of the EU average. This group includes Bulgaria, Latvia, and Romania [Innovation…, 2015].

To find the cause of these poor performances from some of the countries of the CEE, a test of comparative analysis of the sub-indicators of the SII (in this model, called the indicator dimensions of innovation). The Innovation outputs are described by the following dimension: *Human Resources, Research Systems, Finance and Support, Firm Investments, Linkages & Entrepreneurship* and *Intellectual Assets*. The last two indicators (the Output dimensions) relate to various aspects of the use of innovative solutions (*Innovators, Economic Effects*). The results of the comparative analysis are presented in Figure 2, where numbers mean relative value of indices to the average of the EU and the symbols reflect the result of a comparative analysis of the value between the analysed countries.

Figure 2. The SII dimension in relative to the EU average for the CEE countries

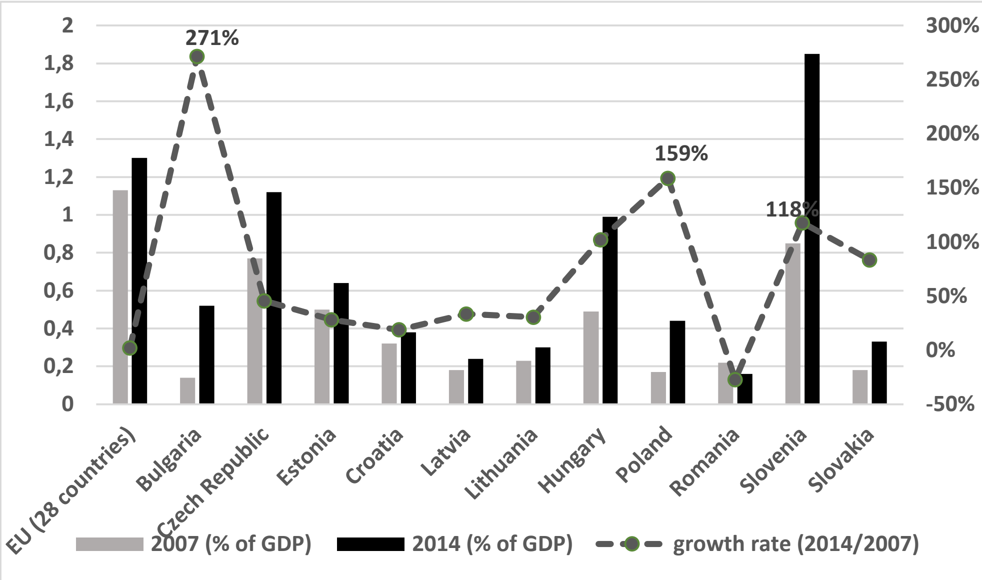
Country	1. Human Resources	2. Research Systems	3. Finance & Support	4. Firm Investments	5. Linkages & Entrepren.	6. Intellectual Assets	7. Innovators	8. Economic Effects
Bulgaria	⇩ 83,1	⇩ 23,2	⇩ 16	⇩ 43,6	⇩ 12,1	⇘ 64,9	⇩ 33,7	⇩ 32,4
Croatia	⇧ 115,4	⇩ 30,1	⇘ 54,7	⇘ 74,9	⇘ 63,2	⇩ 34,9	⇘ 56,8	⇩ 44,8
Czech Republic	⇘ 99,5	⇗ 47,6	⇘ 75,5	⇗ 90,3	⇗ 89,9	⇘ 65,5	⇧ 97,0	⇧ 85,7
Estonia	⇘ 99,8	⇧ 68,3	⇧ 150,5	⇧ 135,2	⇧ 98,3	⇧ 95,5	⇗ 74,3	⇗ 62,2
Hungary	⇩ 82,1	⇘ 37,8	⇘ 62,8	⇗ 85,9	⇘ 38,5	⇘ 55,1	⇗ 64,0	⇧ 92,3
Latvia	⇘ 98,00	⇩ 18,8	⇘ 58,6	⇗ 91,4	⇩ 18,6	⇘ 59,0	⇩ 18,0	⇩ 43,4
Lithuania	⇧ 120,7	⇘ 32,3	⇗ 113,3	⇗ 76,4	⇘ 36,8	⇩ 41,3	⇩ 21,8	⇩ 29,5
Poland	⇘ 96,7	⇩ 23,6	⇘ 65,6	⇗ 79,1	⇩ 14,6	⇘ 67,3	⇘ 49,3	⇘ 53,9
Romania	⇩ 78,80	⇩ 20,8	⇩ 26,4	⇩ 17,6	⇩ 9,1	⇩ 27,4	⇩ 31,5	⇘ 53,6
Slovakia	⇧ 112,9	⇩ 30,8	⇘ 60,6	⇘ 63,2	⇘ 42,3	⇩ 42,9	⇗ 73,7	⇧ 80,5
Slovenia	⇧ 122,2	⇧ 72,5	⇗ 93,9	⇧ 119,8	⇧ 119,2	⇧ 107,9	⇧ 84,8	⇗ 72,0

⇧	up 75%
⇗	between 50% and 75%
⇘	between 25% and 50%
⇩	less 25%

Source: own calculations

The analysed group of countries shows that Slovenia is the best, because effectively uses their innovative potential. The same is true in the case of Estonia. In turn, the Czech Republic and Hungary with less potential for innovation, have reached a relatively high level of the output of innovation (application and economic effects). The poorest in this summary was Romania and Bulgaria, which in all dimensions of innovation clearly deviates from the other Member States. Innovation in the case of the other countries analysed can be described as unbalanced, since it affects the considerable variations among potential factors, innovation and relatively low levels of components of the output products innovation. (for example the Polish economy has the potential for innovation based on human resources, with a very low tendency for innovation and research and development cooperation). Table data show that the large variations between countries occurs in the area of financial support for the development of innovation. How were the expenditure on R&D in individual countries shown graphically on the chart? (Figure 3).

Figure 3. Comparison of the research and development expenditure (business enterprise sector) and the dynamics of changes for CEE countries in 2007 and 2014



Source: own preparation based on EUROSTAT

During the period from 2007 until 2014, countries (except Romania) have had an increase in funding from the R&D sector. The largest increase reported, was for Bulgaria (near three-times) and Poland (c.a. 160%). In 2014 the greatest relative expenditure for the development of innovation finance was in Slovenia and the Czech Republic – they have been highly classified in ranking in the Innovation Summary Index (10^{th} and 12^{th} place). However, in the case of Estonia, with the 11^{th} place in the ranking, the level of expenditure on the R&D is just a bit more than 0.5% of GDP. Of course, the amount of the funding depends on the size of the GDP, so it may not be the basis for the quantitative assessment of inputs, but gives the picture of the trend in development in different countries.

SUMMARY

Over the last programming period (2007-2013), 11 countries of Central and Eastern Europe gained access to almost 176 billion euros of funding from the EU. In this period the amount of allocated EU funds varies by country – the highest budget was allocated to Poland (67,19 EUR billion), which bears the biggest population among the CEE countries. However, EU funds per capita ratio is the highest in the Czech Republic (2,5 EUR), Estonia (2,59 EUR) and Hungary (2,51 EUR). These funds have contributed to the overall development of each economy in many aspects – most visible was in transport infrastructure and environmental protection, both of

which had been neglected during the communist era. Over the 2007-2013 research and development were not enjoyed by the majority of the population, and both the number and value of projects in various countries of Central and Eastern Europe were small. Most of the projects financed by the European Union aimed at improving the quality and alignment of the standard of living among the EU Member States. Not all projects have contributed to building long-term strategic benefits to the economies [EU Founds.., 2015]. So it's no surprise that the level of innovation in the CEE countries significantly differs from the other countries of the European Community.

Among the CEE countries the most innovative were found to be Estonia, Slovenia and the Czech Republic, they owe their position, thanks to the effectiveness of the deployment of innovative and relative high expenditure on the development of innovation finance. Out of the remaining countries belonging to this group, where much work in the area of innovation is required is Romania, Bulgaria, Latvia and Poland. Improving their efficiency requires innovative capital, increasing pressure on the development of research systems and Linkages & Entrepreneurship. These partnerships include, brokerage mechanisms, business linkage initiatives, hybrid commercial and social business models, innovative financing instruments, enhanced enterprise support services, and new types of alliances between companies, trade associations, governments, donors, academic institutions and non-governmental organisations.

REFERENCES

Drabińska D. (2012) Innovation economy in terms of contemporary and historical, Quarterly magazine „Studies&Works", KES, no 2(10)/2012, 11.

The Global Innovation Index 2015: Effective Innovation Policies for Development (2015), Cornell University, INSEAD, and WIPO, Fontainebleau, Geneva.

Innovation Union Scoreboard 2015, European Commission 2015, http://ec.europa.eu/growth/industry/innovation/facts-figures/scoreboards/files/ius-2015_en.pdf (attempt on 13.01.2016)

Measuring innovation output in Europe: towards a new indicator (2013), Brussels, 13.9.2013, COM (2013) 624 final, 4.

EU Funds in Central and Eastern Europe. Progress Report 2007-2014, KPMG,

http://www.kpmg.com/PL/pl/IssuesAndInsights/ArticlesPublications/Documents/2015/EU-Funds-in-Central-and-Eastern-Europe-2007-2014.pdf (attempt on 15.01.2016).

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII , No. 1, 2017, pp. 58 – 67

Received: 14.02.2017
Accepted: 05.04.2017

ON TRANSACTION COSTS IN STOCK TRADING

Marek Andrzej Kociński
Faculty of Applied Informatics and Mathematics
Warsaw University of Life Sciences – SGGW, Poland
e-mail: marek_kocinski@sggw.pl

Abstract: Liquidity is an important characteristic of a stock traded on the stock exchange. The expected value of transaction costs, which takes into account the transaction's volume and duration, may be a considered as an important measure of a liquidity of a traded stock. In this paper the formulas for expected transaction cost, caused by bid-ask spread and market impact are presented. Moreover, in this article, the problem of determining a duration of a transaction of a stock sale which minimizes the transaction cost and takes into account the forecast of the expected stock price on the stock exchange, is considered.

Keywords: liquidity, transaction cost, bid-ask spread, market impact, trade duration

INTRODUCTION

Liquidity is an important issue in stock markets. In fact, a liquidity of a stock traded on the stock exchange is measured by the cost of its trading. For the purposes of market participants, the correct way to view liquidity should imply the possibility of sufficiently accurate forecasting the stock price change caused by the trade initiator and estimating the transaction cost. Transaction costs (trading costs) are widely recognized as an important factor which determines the financial investment performance. Their understanding and assessment is important for economic theory and participants of financial markets. Transaction costs are substantial component of realistic models of the stock market microstructure. In case of the stock transaction between two parties: buyer and seller, transaction costs refer to costs paid by one party of the transaction and not received by the second transaction party. The trading costs can be explicit or implicit. The major sources of transaction costs usually considered in financial investment are:

DOI: 10.22630/MIBE.2017.18.1.06 JEL: G11, G12

commissions (and similar payments), bid-ask spreads and market impact [Elton et al. 1999].

In this article the analysis of the dependence of the transaction cost on the bid-ask spread and the market impact is presented. Moreover, the formula for the average transaction cost is applied to determine the strategy which maximizes the expected amount of money received for selling the stock shares. One of the reason of planning the sale of the shares of the stock may be the forecast of a decrease in the future stock price. In this article, the model of the stock price process is proposed, with the possibility of the shorter duration of the negative drift in the stock price process, than the time for the investor's stock sale. The explicit method for determination of the strategy minimizing the expected transaction cost of selling X shares of the stock, is applied to the numerical computation of the duration of the stock sale which minimizes the cost of trading.

ON SOURCES OF TRANSACTION COSTS

Broker commissions are explicit costs of trading. They are usually easy to evaluate (as percentage of the transaction value) before the start of the trade and therefore they are not the source of the financial risk. In this article, the commissions and similar explicit transaction costs paid by the investor are not taken into account in calculating the average transaction cost of purchase or sale of the shares of the stock.

The bid-ask spread is defined as the difference between the stock's highest bid S^{bid} and the lowest S^{ask} ask prices of one share of the stock in the stock exchange. The average of S^{bid} and S^{ask} may be considered as the market price of the stock. The half of the bid-ask spread is the cost of trading one share of the stock.

Market impact (also called price impact) can be defined as a change in the stock price with respect to a reference price, caused by the transaction. This change is disadvantageous to the initiator of this transaction and therefore the market impact is a source of the trading cost. In theory of finance some distinguish between temporary and permanent market impact. The temporary price impact is considered as the cost of providing lacking liquidity to execute the trade in short time. It affects only a single trade and is assumed not to change the market value of the traded stock. Such impact is caused by supply and demand imbalance. Permanent price impact is considered as the change in the market value of the stock due to the transaction, which remains at least to the completion of this transaction. In case of the stock, a buy transaction signals that the stock may be undervalued and a sell transaction is a signal to the market that the stock may be overvalued. Therefore, the permanent market impact is perceived as a result of an adjustment of the market to the information content of the trade.

MARKET IMPACT FORMULAS AND TRADING COST

In theoretical finance price impact formulas can be used to build more realistic market models and explain the empirical phenomena which seem to contradict the market efficiency [Czekaj et. al. 2001]. In recent years, there has been observed a trend toward the applying of electronic trading algorithms based on a market impact model [Schied and Slynko 2011]. The well accommodated to the real financial market price impact model could help the market participants in pre-trade assessment of the performance of their trading strategies. In practice of financial management it is important to check whether the coefficients and even the functional form of the price impact formula reflect the recent stock market data.

The transaction of the stock purchase or sale, which is the source of market impact, may have a complex structure: it can be fragmented and executed incrementally by the sequences of single financial orders.

A popular formula for market impact, defined as the expected average price return between the beginning and the end of the stock transaction is given as follows:

$$MI = \pm \kappa \sigma \left(\frac{X}{V} \right)^{\delta} \tag{1}$$

where σ is the stock's daily volatility, X is the volume of the executed transaction, V denotes the average daily number of the traded shares of the stock, κ and δ are the numerical constants that can be estimated from the sample of historical transactions. The constant κ is usually described to be of order unity and it seems that it just means that is approximately equal to 1. The constant δ does not exceed 1, is typically found to be approximately 1/2 [Donier J., Bonart J. 2014] and is usually singly estimated for an entire stock market . It is not obvious that κ does not vary stock by stock. The positive and negative sign of MI respectively corresponds to the stock purchase and selling transaction.

The variant of the equation (1) with δ equal to 1, which means that market impact is linear in the traded volume, is used, for example, in [DeMiguel et al. 2014]. The linear dependence of market impact on the transaction volume can be partly justified in financial market microstructure theory by the Kyle model [Bouchaud 2009].

The formula (1) measures, in fact, the difference in the prices of the stock transaction. However, the equation (1) can be used to obtain the average cost of the transaction.

Consider the transaction of purchase of X shares of the stock, which starts at time 0 and ends at time T. Let S_0 denote the market price of one share of the stock just before the transaction and let TC denote the expected transaction cost, per unit of the stock, paid by the initiator of this purchase transaction. The amount of money paid for the stock purchased is $S_0 X(1+TC)$.

It can be calculated that:

$$TC = \frac{s}{2} + \frac{1}{\delta+1}\sigma\left(\frac{X}{V}\right)^{\delta} \tag{2}$$

In case of a transaction of stock selling the amount of money received by the stock seller is $S_0 X(1-TC)$ and the transaction costs TC is also given by the equality (2).

In formula (1) the algorithm of the transaction execution, applied by a market participant , is not taken explicitly into account. In practice there are many types of static and dynamic trading strategies. However, for a given transaction volume, the trading strategy seems to be roughly characterized by the transaction's duration which measures how long the transaction lasts. The duration of the transaction is determined by the speed of execution (trading rate) and the transaction volume. The omitting in (1) of explicit dependence of market impact on the stock trade duration may mean that the influence of the stock trading speed on price impact exists but is significantly smaller than the dependence of the price impact on the number of traded shares of the stock.

In [Almgren et al. 2005] the volume time is defined as the fraction of a stock average day’s volume that has been traded up to clock time t. Under assumption that the stock's daily volume is independent of a trading day and the speed of trading of the stock's daily volume in the market is constant during a trading day the volume time coincides with physical time.

In [Almgren et al. 2005] the two variables I and J are defined. I denotes the permanent market impact and J is the transaction cost per unit of the stock. Let $\bar{I}$ and $\bar{J}$ respectively denote the average values of I and J. The formulas for $\bar{I}$ and $\bar{J}$ can be calculated as follows:

$$\bar{I} = \gamma\sigma\frac{X}{V}\left(\frac{\Theta}{V}\right)^{1/4} \tag{3}$$

and

$$\bar{J} = \frac{I}{2} + \operatorname{sgn}(X)\eta\sigma\left|\frac{X}{VT}\right|^{3/5} \tag{4}$$

where γ and η are numerical constants, X denotes the number of traded shares of the stock, V is the average daily volume of the stock, Θ denotes the number of outstanding stock’s shares, T is a trade duration and σ is the daily volatility of the stock.

If J is negative, it means that the it was calculated for the stock sale transaction and then $|J|$ is the cost of selling per unit of the stock. The values of γ and η were determined in [Almgren et al. 2005] by linear regression: $\gamma = 0{,}314 \pm 0{,}041$ and $\eta = 0{,}142 \pm 0{,}0062$.

The price impact model described in [Almgren et al. 2005], refers to the transactions completed by uniform rate of trading over their volume time interval and takes into account the trade duration, which means the investor can apply this model in analysing the effect of the speed of his trading on the price impact. In model presented in [Almgren et al. 2005], the bid-ask spread cost is a part of the transaction cost.

THE STRATEGY OF SELLING STOCK WITH TRANSACTION COSTS

Consider the following formula for the expected transaction cost implied by the bid-ask spread and the market impact, which includes the stock's transaction duration and volume (as fraction of the average daily number of the traded shares of the stock):

$$\sigma a \theta + \sigma b \left(\frac{\theta}{T} \right)^c \tag{5}$$

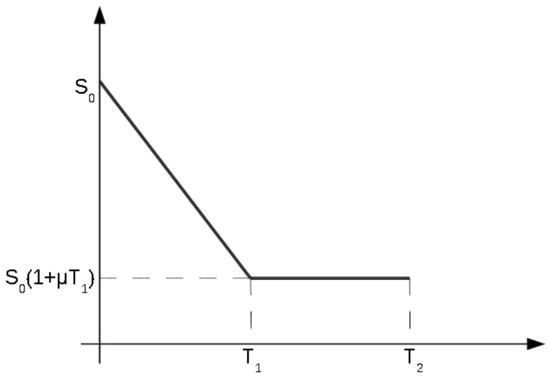
where σ is the stock's daily volatility, θ denotes the volume of the investor's stock and T is the trade duration. Moreover, a, b and c denote the numerical coefficients and $0 < c < 1$.

Consider an investor who, at time 0, holds X units of the stock and expects that the trading activity of other market participants in time interval $[0, T_1]$ will cause a linear, negative drift μ in the process of the stock's market price. He wants to sell his stock's shares up to time T_2 where $T_2 \geq T_1$. The drift μ can be interpreted as the reaction of the market on the information announced at time 0, concerning the financial forecast of the company which issued the stock. For example, the negative trend μ might be generated by the information that the stock dividend would be less than the market participants expected. It can be assumed that the investor assesses the trend duration on the basis of his past experiences with the market reactions on the announcements concerning the financial situation of this company or other companies which issued stocks traded on the stock exchange. Since the investor does not forecast a direction of the stock price movements, which are independent of his trading, after time T_1 therefore he assumes a drift equal to 0 in the market price of the stock, in the interval $[T_1, T_2]$.

Let S_t denote the expected price of the stock at time t. For $[0,T_1]$, S_t is determined by the equality:

$$S_t = \begin{cases} (1+\mu t)S_0 \text{ if } t < T_1 \\ (1+\mu T_1)S_0 \quad \text{if } t \in [\mathrm{T}_1, T_2] \end{cases} \tag{6}$$

Figure 1. The example of investor's views of the expected market price of the stock's share up to time T_2, due to the stock trading of other market participants



Source: own preparation

Under assumption of the constant speed of selling the investor's stock in the interval $[0,T]$, it can be calculated that the average transaction cost caused by the drift is given by the formula:

$$TC(\mu) = \begin{cases} \frac{1}{2}\mu T \text{ if } T < T_1 \\ \left(1 - \frac{T_1}{2T}\right)\mu T_1 \text{ if } t \in [T_1, T_2] \end{cases} \tag{7}$$

The objective of the investor is to maximize the amount of money received from the sale of his X shares of the stock. In the choice of the duration T^* which maximizes the amount of money obtained for selling the stock's volume X, the investor takes into account that decreasing the speed of the execution of selling the stock reduces the trading cost of market impact but also, in time interval $[0,T_1]$, increases the cost caused by the drift μ.

By (5) and (7) the expected cost of the investor's transaction is

$$TC(T)=\begin{cases}\sigma a\theta+\sigma b\left(\dfrac{\theta}{T}\right)^{c}+\dfrac{1}{2}\mu T \text{ for } T\le T_1\\ \sigma a\theta+\sigma b\left(\dfrac{\theta}{T}\right)^{c}+\left(1-\dfrac{T_1}{2T}\right)\mu T_1 \quad \text{for } T_1<T\le T_2\end{cases} \tag{8}$$

For $T \le T_1$ the derivative of (8) with respect to T equals 0 for $T=\left(\dfrac{2\sigma bc\theta^{c}}{\mu}\right)^{\frac{1}{1+c}}$ and $TC(T)$ is convex on the interval $(0,T_1]$. Thus, the minimum of (8) on the set $(0,T_1]$ is obtained for

$$T^{*}_{(0,T_1]}=\begin{cases}\left(\dfrac{2\sigma bc\theta^{c}}{\mu}\right)^{\frac{1}{1+c}} \text{ if } \left(\dfrac{2\sigma bc\theta^{c}}{\mu}\right)^{\frac{1}{1+c}}\le T_1\\ T_1 \quad \text{if } \left(\dfrac{2\sigma bc\theta^{c}}{\mu}\right)^{\frac{1}{1+c}}>T_1\end{cases} \tag{9}$$

For $0<c<1$, $TC(T)$ is concave on the interval $[T_1,T_2]$. Therefore, the minimum of (8) on the set $[T_1,T_2]$ is obtained for

$$T^{*}_{[T_1,T_2]}=\begin{cases}T_1 \text{ if } TC(T_1)\le TC(T_2)\\ T_2 \quad \text{if } TC(T_1)>TC(T_2)\end{cases} \tag{10}$$

In consequence, by (9) and (10) it follows that

$$T^{*}=\begin{cases}T^{*}_{(0,T_1]} \text{ if } TC\left(T^{*}_{[0,T_1]}\right)\le TC\left(T^{*}_{[T_1,T_2]}\right)\\ T^{*}_{[T_1,T_2]} \text{ if } TC\left(T^{*}_{[0,T_1]}\right)>TC\left(T^{*}_{[T_1,T_2]}\right)\end{cases} \tag{11}$$

Numerical example

Consider an investor who intends to sell $\theta = 25\%$ of the stock's average daily number of the traded shares of the stock. In this numerical example of the formula (5) application, the values of a is set to 1, b is set to 0.142 and c is set to 0.6. For these values of the coefficients a, b and c, the dependence of the average transaction cost on the transaction volume in (5) is the same as the dependence of the expected cost of trading on the number of stock's shares in the linear version of (1) and the dependence of the average transaction cost on the trading rate in (5) is such as the dependence of the average transaction cost on the speed of trading in (4). Moreover, the values of T_1 and T_2 are set to 0.5 and 1, respectively.

Table 1 presents the values of the trade duration T^* as the function of the stock's price drift and the average daily volatility of the stock price.

Table 1. T^* as function of μ and σ

		σ						
		2.50%	**7.50%**	**12.50%**	**17.50%**	**22.50%**	**27.50%**	**32.50%**
μ	**2.00%**	0.226	0.449	1.000	1.000	1.000	1.000	1.000
	4.00%	0.147	0.291	0.401	1.000	1.000	1.000	1.000
	6.00%	0.114	0.226	0.311	0.384	0.449	1.000	1.000
	8.00%	0.095	0.189	0.260	0.321	0.375	0.426	1.000
	10.00%	0.083	0.164	0.226	0.279	0.327	0.370	0.411
	12.00%	0.074	0.147	0.202	0.249	0.291	0.330	0.367
	14.00%	0.067	0.133	0.183	0.226	0.265	0.300	0.333
	16.00%	0.062	0.123	0.169	0.208	0.243	0.276	0.306
	18.00%	0.057	0.114	0.157	0.193	0.226	0.256	0.285
	20.00%	0.054	0.107	0.147	0.181	0.212	0.240	0.266

Source: own computation

Table 2 show dependence of the value of the expected transaction cost on the drift of the stock and the stock's price.

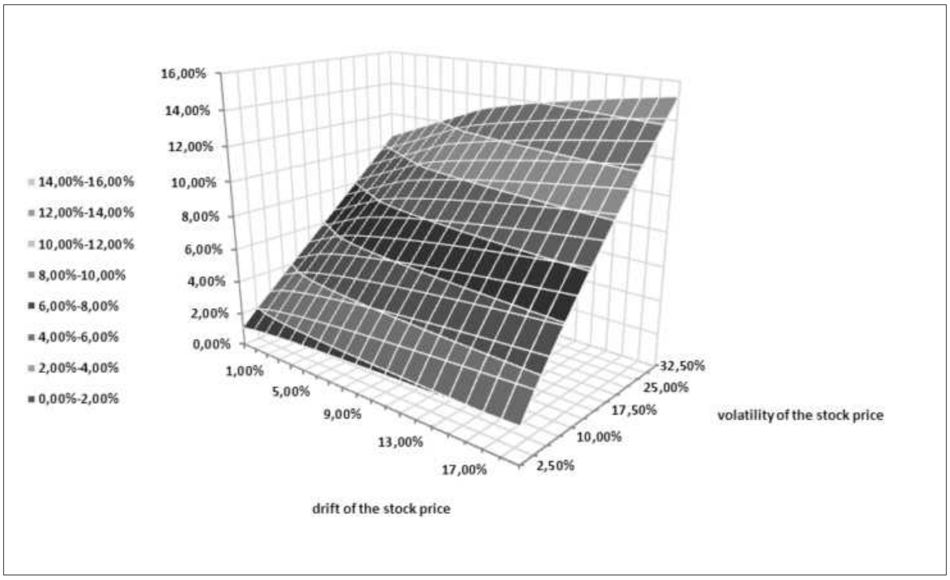
Table 2. The value of average transaction cost depending on the on μ and σ

		σ						
		2.50%	**7.50%**	**12.50%**	**17.50%**	**22.50%**	**27.50%**	**32.50%**
μ	**2.00%**	1.23%	3.07%	4.65%	6.21%	7.77%	9.32%	10.88%
	4.00%	1.41%	3.43%	5.26%	6.96%	8.52%	10.07%	11.63%
	6.00%	1.54%	3.68%	5.62%	7.45%	9.22%	10.82%	12.38%
	8.00%	1.64%	3.89%	5.90%	7.80%	9.63%	11.42%	13.13%
	10.00%	1.73%	4.07%	6.14%	8.10%	9.98%	11.81%	13.60%
	12.00%	1.81%	4.22%	6.35%	8.36%	10.29%	12.16%	13.99%
	14.00%	1.88%	4.36%	6.55%	8.60%	10.57%	12.48%	14.34%
	16.00%	1.94%	4.49%	6.72%	8.81%	10.82%	12.76%	14.66%
	18.00%	2.00%	4.61%	6.88%	9.01%	11.05%	13.03%	14.96%
	20.00%	2.06%	4.72%	7.04%	9.20%	11.27%	13.28%	15.23%

Source: own computation

Figure 2 shows graphically how the expected transaction cost depends on the stock's price drift μ and the average daily volatility σ of the stock.

Figure 2. The example of the average cost of selling the stock as the function of μ and σ



Source: own preparation

CONCLUSION

In this article the problem of determining the trading rate which minimizes the average cost of selling the investor's stock is explicitly solved in a framework of the model ,which can be applied in the case when the negative drift in the stock price lasts less than the investor's stock selling. The numerical example included in the paper shows that the solution of the problem of the expected transaction costs minimization in case of selling the stock, may be significantly affected by the volatility of the stock and the drift in the stock price process.

REFERENCES

Almgren R., Thum C., Hauptmann E., Li H. (2005) Direct Estimation of Equity Market Impact Risk. 18, 58–62.

Bouchaud J. P. (2009) Price Impact, https://arxiv.org/pdf/0903.2428.pdf.

Czekaj J., Woś M., Żarnowski, J. (2001) Efektywność giełdowego rynku akcji w Polsce. Z perspektywy dziesięciolecia, Warszawa: Wydawnictwo Naukowe PWN.

DeMiguel V., Martín-Utrera A., Nogales F. J. (2014) Parameter uncertainty in multiperiod portfolio optimization with transaction costs. http://faculty.london.edu/avmiguel/DMN-2014-01-09.pdf.

Donier J., Bonart J. (2014) A million metaorder analysis of market impact on the Bitcoin. https://arxiv.org/pdf/1412.4503.pdf.

Elton R. J., Gruber M. J., Brown S. J., Goetzmann W. N. (2010) Modern Portfolio Theory and Investment Analysis, John Wiley & Sons, Hoboken.

Kociński M. (2015) Trade Duration and Market Impact. Quantitative Methods in Economics XVI/1, 137–146.

Schied A., Slynko A. (2011) [in:] Blath J., Imkeller P., Roelly S. (eds.) Surveys in Stochastic Processes, European Mathematical Society Publishing House, Zürich.

Tóth B., Lempérière Y., Deremble C., de Lataillade J., Kockelkoren J., Bouchaud J.-P. (2011) Anomalous Price Impact and the Critical Nature of Liquidity in Financial Markets. http://journals.aps.org/prx/pdf/10.1103/PhysRevX.1.021006.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII, No.1, 2017, pp. 68 – 77

Received: 25.01.2017
Accepted: 05.04.2017

COMPARING PROPORTIONS OF SENSITIVE ITEMS IN TWO POPULATIONS WHEN USING POISSON AND NEGATIVE BINOMIAL ITEM COUNT TECHNIQUES

Barbara Kowalczyk (ORCID: 0000-0002-5407-3438)
Collegium of Economic Analysis
Warsaw School of Economics – SGH, Poland
e-mail: bkowal@sgh.waw.pl
Robert Wieczorkowski
Central Statistical Office of Poland, GUS, Poland
e-mail: R.Wieczorkowski@stat.gov.pl

Abstract: Sensitive attributes are extremely difficult to be measured directly. Recently new indirect methods of questioning, called Poisson and negative binomial item count techniques, have been proposed by [Tian et al. 2014]. This paper focuses on important problem of comparing proportions of sensitive items in two populations when using new indirect method. Proper statistical theory is introduced, including tests for equality of two sensitive proportions followed by derivation of their asymptotic power functions. Simulation studies are conducted to illustrate the problem.

Keywords: sensitive questions, Poisson and negative binomial item count techniques, latent data, two populations, asymptotic test

INTRODUCTION

Illegal and socially stigmatized behaviors, like tax evasion, addictions, sexual risk activities etc. are usually impossible to be measured via direct questioning. Therefore social science and mathematical statistics have developed some indirect methods of questioning, i.e. methods in which the sensitive question is not asked directly and therefore individual answer to this particular question cannot be recognized. Such a procedure guarantees privacy and allows truthful answers. One of the most popular methods of dealing with sensitive questions nowadays are various item count techniques [e.g. Gonzales-Ocantos et. al 2012, Kuha and Jackson 2013, Wolter and Laier 2014], which started with [Miller 1984] and are is still being

DOI: 10.22630/MIBE.2017.18.1.07 JEL: C15, C63

extensively developed [Imai 2011, Hussain 2012, Glynn 2013, Kuha and Jackson 2013, Tian et al. 2014].

Recent proposition by [Tian et al. 2014] assumes a new approach to item count procedure and introduces Poisson and negative binomial item count techniques. Authors propose to randomly assign respondents into control and treatment groups. In a control group respondents are asked one neutral question independent of the sensitive one with possible outcomes 0,1,2,… , which can be modeled by a counting variable X, e.g. "How many times have you been to the cinema last month?" In a treatment group respondents are presented with two questions: one neutral and exactly the same as in control group, X, and the other one sensitive, Z, with possible outcomes 0 or 1. e.g. "1. How many times have you been to the cinema last month?, 2. Have you bribed a police officer during last year? Assign 1 if yes and 0 if not. Please report only the sum of your answers." Thus respondents in a control group report X, and respondents in a treatment group report $Y=X+Z$. Sensitive variable Z is a latent one and is not directly observable. Because by definition X is a counting variable, two basic models are considered by [Tian et al. 2014] for X, Poisson and negative binomial. Selection of the proper model is possible after the survey is done on the basis of a control group by standard methods. In a control group of n_1 elements we observe vector $(X_{1,}, \dots, X_{n_1})$, and in a treatment group of n_2 elements we observe $(Y_{1,}, \dots, Y_{n_2})$, where $Y_j = X_{n_1+j} + Z_j$.

In the present paper we extend [Tian et al. 2014] method into two populations. The problem is crucial in statistical practice and to our best knowledge has not been developed yet. We introduce statistical tests based on asymptotic normality of unbiased estimators suitable for testing equality of sensitive proportions in two populations when using Poisson and negative binomial ICTs. We also provide asymptotic power of the proposed tests together with numerical illustration. Next to classic approach we also consider an alternative one. Finally we conduct a Monte Carlo simulation study to illustrate tests and EM algorithm performances.

CLASSIC POISSON MODEL

Preliminaries

We presume that two independent surveys are conducted in two populations with different control questions. We assume that in both cases control variable follows Poisson distribution. Therefore in population I we have a control variable $X_1 \sim Poisson(\lambda_1)$ independent of the sensitive variable $Z_1 \sim Bernoulli(\pi_1)$. Two independent random samples are observable: $(X_{11,}, \dots, X_{1n_{11}})$, $(Y_{11}, \dots, Y_{1n_{12}})$, where $Y_{1j} = X_{1n_{11}+j} + Z_{1j}$ and $n_1 = n_{11} + n_{12}$ is a total sample size. Analogously, in population II we have a control variable $X_2 \sim Poisson(\lambda_2)$ independent of the sensitive variable $Z_2 \sim Bernoulli(\pi_2)$, and two independent random samples of observable variables: $(X_{21,}, \dots, X_{2n_{21}})$ and $(Y_{21}, \dots, Y_{2n_{22}})$, $Y_{2j} = X_{2n_{22}+j} + Z_{2j}$,

$n_2 = n_{21} + n_{22}$. Unknown parameters λ_1, λ_2 can be assessed by $\bar{X}_1$ and $\bar{X}_2$ respectively. Unknown sensitive proportions π_1 and π_2 can be assessed by unbiased method of moments MM estimators $\hat{\pi}_1 = (\bar{Y}_1 - \bar{X}_1)$ and $\hat{\pi}_2 = (\bar{Y}_2 - \bar{X}_2)$ respectively. Therefore natural unbiased estimator of $\pi_2 - \pi_1$ is $\hat{\pi}_2 - \hat{\pi}_1 = (\bar{Y}_2 - \bar{X}_2) - (\bar{Y}_1 - \bar{X}_1)$ with variance:

$$D^2(\hat{\pi}_2 - \hat{\pi}_1) = \left(\frac{\lambda_1}{n_{11}} + \frac{\lambda_1 + \pi_1(1-\pi_1)}{n_{12}}\right) + \left(\frac{\lambda_2}{n_{21}} + \frac{\lambda_2 + \pi_2(1-\pi_2)}{n_{22}}\right). \quad (1)$$

Hypothesis testing

For the sake of definiteness let us focus on two sided test. Hypothesis testing problem of interest is H_0: $\pi_1 = \pi_2$ versus H_1: $\pi_1 \neq \pi_2$. Introduced test is based on asymptotic normality of unbiased estimator $\hat{\pi}_2 - \hat{\pi}_1$. Two-sided (restricted) test of size α is to reject H_0 if:

$$\frac{|\hat{\pi}_2 - \hat{\pi}_1|}{\sqrt{\left(\frac{\hat{\lambda}_1}{n_{11}} + \frac{\hat{\lambda}_1}{n_{12}}\right) + \left(\frac{\hat{\lambda}_2}{n_{21}} + \frac{\hat{\lambda}_2}{n_{22}}\right) + \hat{\pi}(1-\hat{\pi})\left(\frac{1}{n_{12}} + \frac{1}{n_{22}}\right)}} > z_{1-\frac{\alpha}{2}}, \quad (2)$$

where $z_{1-\frac{\alpha}{2}}$ is the $\left(1 - \frac{\alpha}{2}\right)$th quantile of standard normal distribution, $\hat{\lambda}_1, \hat{\lambda}_2$ are control group sample means, and $\hat{\pi} = w_1\hat{\pi}_1 + (1 - w_1)\hat{\pi}_2$ is a restricted estimator of the joint sensitive proportion with

$$w_1 = \left(\frac{\hat{\lambda}_2}{n_{21}} + \frac{\hat{\lambda}_2}{n_{22}} + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_{22}}\right) \Big/ \left(\frac{\hat{\lambda}_1}{n_{11}} + \frac{\hat{\lambda}_1}{n_{12}} + \frac{\hat{\pi}_1(1-\hat{\pi}_1)}{n_{12}} + \frac{\hat{\lambda}_2}{n_{21}} + \frac{\hat{\lambda}_2}{n_{22}} + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_{22}}\right) \quad (3)$$

If alternative $\hat{\pi}_2 \neq \hat{\pi}_1$ is true, asymptotic power of the proposed test is:

$$1 - \Phi\left(z_{1-\frac{\alpha}{2}} \frac{\sqrt{\delta\lambda_1(1+k_1) + \lambda_2(1+k_2) + (\delta+1)(v_1\pi_1 + v_2\pi_2)(1 - v_1\pi_1 - v_2\pi_2)}}{\sqrt{\delta\lambda_1(1+k_1) + \lambda_2(1+k_2) + \delta\pi_1(1-\pi_1) + \pi_2(1-\pi_2)}} - \right.$$
$$\left. - \frac{|\pi_2 - \pi_1|\sqrt{n_{22}}}{\sqrt{\delta\lambda_1(1+k_1) + \lambda_2(1+k_2) + \delta\pi_1(1-\pi_1) + \pi_2(1-\pi_2)}}\right) \quad (4)$$

where $\delta = n_{22}/n_{12}$, $k_1 = n_{12}/n_{11}$, $k_2 = n_{22}/n_{21}$, $v_2 = 1 - v_1$ and

$$v_1 = \frac{\lambda_2(1+k_2) + \pi_2(1-\pi_2)}{\delta[\lambda_1(1+k_1) + \pi_1(1-\pi_1)] + \lambda_2(1+k_2) + \pi_2(1-\pi_2)} \quad (5)$$

General remarks

Theory presented in this section gives some important directions for practitioners. Although it is clear that indirect questioning demands much larger sample sizes, only exact mathematical formulas allow for definite analysis of the problem and proper survey design. Therefore below we present several numerical examples concerning power of test (2) calculated on the basis of formula (4). All examples are obtained for balanced samples, i.e. for $n_{11} = n_{12} = n_{21} = n_{22}$.

Table 1. Asymptotic power of test (2) for $\alpha = 0.05$ and different model parameters

	$\pi_1 = 0.10$ $\pi_2 = 0.20$			$\pi_1 = 0.10$ $\pi_2 = 0.25$		
sample size n_1=n_2	$\lambda_1 = 0.9$ $\lambda_2 = 1.1$	$\lambda_1 = 1.9$ $\lambda_2 = 2.1$	$\lambda_1 = 2.9$ $\lambda_2 = 3.1$	$\lambda_1 = 0.9$ $\lambda_2 = 1.1$	$\lambda_1 = 1.9$ $\lambda_2 = 2.1$	$\lambda_1 = 2.9$ $\lambda_2 = 3.1$
500	0.117	0.079	0.066	0.208	0.128	0.100
1000	0.191	0.119	0.093	0.368	0.213	0.158
2000	0.335	0.195	0.145	0.631	0.378	0.272

Source: own calculations

ALTERNATIVE POISSON MODEL

Preliminaries

Here we consider a situation when comparing two sensitive proportions is the main goal of the survey. Therefore the same control question is asked in two populations, to which the answer is independent of the population and can be modeled by a Poisson distribution (with the same parameter in two populations). In this case it is reasonable to resign from control samples to increase precision of difference between sensitive proportions estimation. Thus whole samples of n_1 and n_2 elements from population I and II respectively are allocated to treatment groups, where $Y_1 = X + Z_1$ and $Y_2 = X + Z_2$ are observable. Mathematical model is the following: $X \sim Poisson(\lambda)$ is independent of $Z_1 \sim Bernoulli\,(\pi_1)$ and $Z_2 \sim Bernoulli\,(\pi_2)$. Two independent samples of observable variables from two populations are available: $(Y_{11}, \dots, Y_{1n_1})$ and $(Y_{21}, \dots, Y_{2n_2})$. Unbiased MM estimator of $\pi_2 - \pi_1$ is $\hat{d} = \bar{Y}_2 - \bar{Y}_1$ with variance:

$$D^2(\hat{d}) = \frac{\lambda + \pi_1(1-\pi_1)}{n_1} + \frac{\lambda + \pi_2(1-\pi_2)}{n_2} \tag{6}$$

Hypothesis testing

Hypothesis testing problem of interest is H_0: $\pi_1 = \pi_2$ versus H_1: $\pi_1 \neq \pi_2$. Introduced test is based on asymptotic normality of unbiased estimator $\hat{d}$. Null hypothesis in this model implies equality of variances $D^2Y_1 = D^2Y_2$. Two sided test (restricted) of size α is to reject H_0 if:

$$\frac{|\bar{y}_2 - \bar{y}_1|}{\sqrt{S^2 \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} > z_{1-\frac{\alpha}{2}}, \tag{7}$$

where $S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2}$ is a pooled sample variance and $S_i^2 = \frac{1}{n_i}\sum_{k=1}^{n_i}(Y_{ik} - \bar{Y}_i)^2$, $i = 1{,}2$. Let us further denote $n_2 = tn_1$. If alternative $\hat{\pi}_2 \neq \hat{\pi}_1$ is true, asymptotic power of the proposed test is:

$$1-\Phi\left(z_{1-\frac{\alpha}{2}}\frac{\sqrt{\lambda(t+1)+(\pi_1+\pi_2\mathrm{t})\left(1-\frac{\pi_1+\pi_2\mathrm{t}}{\mathrm{t}+1}\right)}}{\sqrt{[\lambda+\pi_1(1-\pi_1)]t+\lambda+\pi_2(1-\pi_2)}}-\frac{|\pi_2-\pi_1|\sqrt{\mathrm{n}_2}}{\sqrt{[\lambda+\pi_1(1-\pi_1)]t+\lambda+\pi_2(1-\pi_2)}}\right) \quad (8)$$

Although formula (7) takes familiar form, due to existence of control variable, implications for practitioners are not straightforward. In table 2 we present asymptotic power of test (7) obtained for selected model parameters and the same sample sizes $n_1 = n_2$.

Table 2. Asymptotic power of test (7) for $\alpha = 0.05$ and different model parameters

	$\pi_1 = 0.10$ $\pi_2 = 0.20$			$\pi_1 = 0.10$ $\pi_1 = 0.25$		
sample size	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$
500	0.319	0.190	0.143	0.602	0.367	0.267
1000	0.558	0.335	0.243	0.881	0.630	0.473
2000	0.846	0.582	0.432	0.993	0.900	0.763

Source: own calculations

It is clear that asymptotic power of test (7) in alternative model is substantially larger, under similar parameters, than the one for classic model. But in alternative approach comparing proportions of sensitive items constitutes the main aim of the survey and no information about sensitive proportions in each population separately is available through MM estimation. To address this issue below we analyze ML estimation via EM algorithm, that allows for estimation all model parameters, including sensitive proportions in each population separately. The working version of ML estimation via EM algorithm for this particular model is discussed later in a simulation study.

ML estimation via EM algorithm

Likelihood function based on complete data in the analyzed model is:

$$L(\pi_1,\pi_2,\lambda;\boldsymbol{y}_1,\boldsymbol{y}_2,\boldsymbol{z}_1,\boldsymbol{z}_2)=$$

$$\prod_{i=1}^{n_1}\frac{e^{-\lambda}\lambda^{y_{1i}-z_{1i}}}{(y_{1i}-z_{1i})!}\pi_1^{z_{1i}}(1-\pi_1)^{1-z_{1i}}\prod_{j=1}^{n_2}\frac{e^{-\lambda}\lambda^{y_{2j}-z_{2j}}}{(y_{2j}-z_{2j})!}\pi_2^{z_{2j}}(1-\pi_2)^{1-z_{2j}} \quad (9)$$

M step of EM algorithm results in:

$$\hat{\lambda}_{ML}=\frac{1}{n_1+n_2}\left(\sum_{i=1}^{n_1}(y_{1i}-z_{1i})+\sum_{j=1}^{n_2}(y_{2j}-z_{2j})\right), \quad (10)$$

$$\hat{\pi}_{1ML}=\frac{1}{n_1}\sum_{i=1}^{n_1}z_{1i}\,,\ \hat{\pi}_{2ML}=\frac{1}{n_2}\sum_{j=1}^{n_2}z_{2j}. \quad (11)$$

In E step values $\{z_{1i}\}_{i=1}^{n_1}$ and $\{z_{2j}\}_{j=1}^{n_2}$ are replaced by conditional expectations:

$$E(Z_1|Y_1;\pi_1,\pi_2,\lambda)=\frac{\pi_1 y_{1i}}{\pi_1 y_{1i}+\lambda(1-\pi_1)}\ ,i=1,\dots,n_1 \quad (12)$$

$$E(Z_2|Y_2;\pi_1,\pi_2,\lambda)=\frac{\pi_2 y_{2j}}{\pi_2 y_{2j}+\lambda(1-\pi_2)},j=1,\dots,n_2 \quad (13)$$

NEGATIVE BINOMIAL MODEL

Having presented foundation for Poisson model, obtaining analogous theory for negative binomial distribution is quite straightforward. Therefore we give here only selected closing formulas. Notation is exactly the same as in previous sections, the only difference is that here in classic negative binomial model we have $X_1 \sim NB(r_1, p_1)$, $X_2 \sim NB(r_2, p_2)$ and in alternative negative binomial model with the same control question in two populations $X \sim NB(r, p)$. In classic approach unbiased MM estimator of $\pi_2 - \pi_1$ is also $\hat{\pi}_2 - \hat{\pi}_1$, but now its variance is:

$$\frac{r_1 p_1}{(1-p_1)^2}\left(\frac{1}{n_{11}} + \frac{1}{n_{12}}\right) + \frac{\pi_1(1-\pi_1)}{n_{12}} + \frac{r_2 p_2}{(1-p_2)^2}\left(\frac{1}{n_{21}} + \frac{1}{n_{22}}\right) + \frac{\pi_2(1-\pi_2)}{n_{22}}. \qquad (14)$$

Hypothesis testing problem of interest is H_0: $\pi_1 = \pi_2$ versus H_1: $\pi_1 \neq \pi_2$. Two-sided test (restricted) of size α based on asymptotic normality of unbiased estimator $\hat{\pi}_2 - \hat{\pi}_1$ is to reject H_0 if:

$$\frac{|\hat{\pi}_2 - \hat{\pi}_1|}{\sqrt{\frac{\hat{r}_1 \hat{p}_1}{(1-\hat{p}_1)^2}\left(\frac{1}{n_{11}} + \frac{1}{n_{12}}\right) + \frac{\hat{r}_2 \hat{p}_2}{(1-\hat{p}_2)^2}\left(\frac{1}{n_{21}} + \frac{1}{n_{22}}\right) + \hat{\pi}(1-\hat{\pi})\left(\frac{1}{n_{12}} + \frac{1}{n_{22}}\right)}} > z_{1-\frac{\alpha}{2}} \qquad (15)$$

where $\hat{r}_1, \hat{p}_1$ and $\hat{r}_2, \hat{p}_2$ are either MM or ML estimators of r_1, p_1 and r_2, p_2 based on control groups from two populations, $\hat{\pi} = w_1 \hat{\pi}_1 + (1 - w_1)\hat{\pi}_2$ is a restricted estimator of the joint sensitive proportion with:

$$w_1 = \left(\frac{\hat{r}_2 \hat{p}_2}{(1-\hat{p}_2)^2}\left(\frac{1}{n_{21}} + \frac{1}{n_{22}}\right) + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_{22}}\right) /$$
$$/\left(\frac{\hat{r}_1 \hat{p}_1}{(1-\hat{p}_1)^2}\left(\frac{1}{n_{11}} + \frac{1}{n_{12}}\right) + \frac{\hat{\pi}_1(1-\hat{\pi}_1)}{n_{12}} + \frac{\hat{r}_2 \hat{p}_2}{(1-\hat{p}_2)^2}\left(\frac{1}{n_{21}} + \frac{1}{n_{22}}\right) + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_{22}}\right) \qquad (16)$$

If alternative $\hat{\pi}_2 \neq \hat{\pi}_1$ is true, asymptotic power of the proposed test is:

$$1 - \Phi\left(z_{1-\frac{\alpha}{2}} \frac{\sqrt{A}}{\sqrt{B}} - \frac{|\pi_2 - \pi_1|\sqrt{\mathrm{n}_{22}}}{\sqrt{B}}\right), \qquad (17)$$

where

$$A = \delta \frac{r_1 p_1}{(1-p_1)^2}(1 + k_1) + \frac{r_2 p_2}{(1-p_2)^2}(1 + k_2) +$$
$$+(\delta + 1)(a_1\pi_1 + a_2\pi_2)(1 - a_1\pi_1 - a_2\pi_2), \qquad (18)$$

$$a_1 = \frac{\frac{r_2 p_2}{(1-p_2)^2}(1+k_2) + \pi_2(1-\pi_2)}{\delta\left[\frac{r_1 p_1}{(1-p_1)^2}(1+k_1) + \pi_1(1-\pi_1)\right] + \frac{r_2 p_2}{(1-p_2)^2}(1+k_2) + \pi_2(1-\pi_2)}, \qquad (19)$$

$$B = \delta \frac{r_1 p_1}{(1-p_1)^2}(1 + k_1) + \frac{r_2 p_2}{(1-p_2)^2}(1 + k_2) + \delta\pi_1(1 - \pi_1) + \pi_2(1 - \pi_2) \qquad (20)$$

and $a_2 = 1 - a_1$. In alternative approach, assuming that the same control variable X follows $NB(r, p)$, two sided test (restricted) of size α is exactly the same as the one defined in formula (7). If alternative $\hat{\pi}_2 \neq \hat{\pi}_1$ is true, asymptotic power of the proposed test is:

$$1-\Phi\left(z_{1-\frac{\alpha}{2}}\frac{\sqrt{\frac{rp}{(1-p)^2}(1+\mathrm{t})+(\pi_1+\pi_2\mathrm{t})\left(1-\frac{\pi_1+\pi_2\mathrm{t}}{\mathrm{t}+1}\right)}}{\sqrt{\left[\frac{rp}{(1-p)^2}+\pi_1(1-\pi_1)\right]t+\frac{rp}{(1-p)^2}+\pi_2(1-\pi_2)}}-\right.$$
$$\left.-\frac{|\pi_2-\pi_1|\sqrt{\mathrm{n}_2}}{\sqrt{\left[\frac{rp}{(1-p)^2}+\pi_1(1-\pi_1)\right]t+\frac{rp}{(1-p)^2}+\pi_2(1-\pi_2)}}\right) \tag{21}$$

Additionally, in classic approach also mixed model is possible, where $X_1 \sim Poisson(\lambda)$ and $X_2 \sim NB(r,p)$. Asymptotic test for equality of two sensitive proportions can be constructed analogously to the ones presented above.

SIMULATION STUDIES

First, a series of Monte Carlo simulation studies is conducted to assess asymptotic tests performances for Poisson classic and alternative models. 50 000 replications are used for every single set of model parameters. Summarized simulation results for type I error rates are given in Table 3. Both tests control Type I error satisfactory good for relatively small sample sizes with a tendency to minutely exceed the nominal $\alpha = 0.05$.

Table 3.Type I error rate for different Poisson model parameters, balanced designs and nominal $\alpha = 0.05$

sample size	parameters of control variables	$\pi_1 = \pi_2$			
		0.05	0.10	0.20	0.30
Classic model – Test (2)					
200	$\lambda_1 = 1, \lambda_2 = 2$	0.051	0.050	0.050	0.050
200	$\lambda_1 = 1, \lambda_2 = 3$	0.051	0.051	0.049	0.053
200	$\lambda_1 = 2, \lambda_2 = 3$	0.051	0.050	0.051	0.052
Alternative model – Test (7)					
200	λ =1	0.052	0.052	0.051	0.051
200	λ =2	0.051	0.051	0.051	0.051
200	λ =3	0.052	0.053	0.051	0.051

Source: own calculations

Next, empirical powers of the considered tests are obtained, i.e. the proportion of cases out of 50 000 where the null hypothesis is correctly rejected. In Table 4 empirical (E) and asymptotical theoretical (T) powers are juxtaposed together for selected model parameters and $\pi_1 = 0.10$, $\pi_2 = 0.20$. For each set of model parameters, absolute difference between achieved empirical power and asymptotical theoretical one is a decreasing function of a sample size, with some minor exception for each test, which is typical for simulations. For large sample size empirical power is very close to the asymptotical theoretical one.

Table 4. Empirical (E) and asymptotical theoretical (T) powers of tests (2) and (7) for $\alpha = 0.05$, $\pi_1 = 0.10$, $\pi_2 = 0.20$ and different Poisson model parameters

sample size	E	T	E	T	E	T
Classic model – Test (2)						
	$\lambda_1 = 1, \lambda_2 = 2$		$\lambda_1 = 1, \lambda_2 = 3$		$\lambda_1 = 1, \lambda_2 = 4$	
200	0.073	0.060	0.067	0.054	0.064	0.050
500	0.099	0.093	0.088	0.080	0.082	0.072
1000	0.145	0.144	0.123	0.119	0.112	0.104
2000	0.246	0.244	0.198	0.196	0.169	0.166
Alternative model –Test (6)						
	λ =1		λ =2		λ =3	
200	0.159	0.154	0.107	0.101	0.089	0.081
500	0.321	0.319	0.191	0.190	0.149	0.143
1000	0.556	0.558	0.332	0.335	0.243	0.243
2000	0.847	0.846	0.584	0.582	0.433	0.432

Source: own calculations

In the case of classic negative binomial model, rate of convergence to the limit distribution will depend on applied estimators of r and p. For estimating procedures see [e.g. Lloyd-Smith 2007]. As the problem goes beyond the purpose of this paper, here we give only exemplary simulation results for alternative negative binomial model. In Table 5 empirical power is juxtaposed with asymptotical theoretical one given in (21). For each set of model parameters 50000 replications are used.

Table 5. Empirical (E) and asymptotical theoretical (T) powers of test (7) for $\alpha = 0.05$, $\pi_1 = 0.10$, $\pi_2 = 0.20$ and different negative binomial model parameters

sample size	E	T	E	T	E	T
Alternative model – Test (6)						
	$r = 2, p = 0.2$		$r = 2, p = 0.3$		$r = 2, p = 0.4$	
200	0.216	0.209	0.140	0.135	0.103	0.095
500	0.447	0.445	0.278	0.274	0.177	0.176
1000	0.736	0.732	0.485	0.485	0.312	0.308
2000	0.955	0.954	0.779	0.776	0.545	0.541

Source: own calculations

Subsequently, a series of Monte Carlo simulation studies is conducted for Poisson alternative model to illustrate ML estimation via EM algorithm described in formulas (9)-(13). Simulation results are of particular importance in this case because no theoretical formulas for variances of estimators are available for EM algorithm. Moreover, in alternative model estimating each sensitive proportion separately is not even possible through MM estimation, thus no reference point is available. For every set of model parameters 10 000 replications are used. Summarized results for estimating difference $\pi_2 - \pi_1$ in alternative model are given in Table 6. Results indicate that the smaller sample size is (and the higher λ) the larger gain in efficiency is achieved from using ML estimation via EM algorithm as compared to simple MM estimation.

Table 6. Simulation MSE of MM and ML (via EM algorithm) estimators of difference $\pi_2 - \pi_1$ between two sensitive proportions for different Poisson model parameters

	$\pi_1 = 0.05$ $\pi_2 = 0.05$		$\pi_1 = 0.05$ $\pi_2 = 0.15$		$\pi_1 = 0.05$ $\pi_2 = 0.30$		$\pi_1 = 0{,}30$ $\pi_2 = 0{,}30$	
	MM	ML	MM	ML	MM	ML	MM	ML
λ	n=200							
$\lambda = 3$	0.0303	0.0272	0.0310	0.0273	0.0319	0.0276	0.0319	0.0294
$\lambda = 2$	0.0203	0.0183	0.0203	0.0181	0.0214	0.0182	0.0223	0.0204
$\lambda = 1$	0.0104	0.0093	0.0108	0.0096	0.0113	0.0096	0.0121	0.0111
n	$\lambda = 2$							
200	0.0203	0.0183	0.0203	0.0181	0.0214	0.0182	0.0223	0.0204
500	0.0081	0.0076	0.0084	0.0077	0.0084	0.0075	0.0087	0.0082
1000	0.0040	0.0039	0.0042	0.0039	0.0042	0.0038	0.0044	0.0042
2000	0.0021	0.0020	0.0020	0.0019	0.0021	0.0019	0.0022	0.0021

Source: own calculations

Table 7. Simulation MSE of ML (via EM algorithm) estimator of π_1 for different model parameters

π_1 π_2	0.05 0.05	0.15 0.15	0.30 0.30	0.05 0.15	0.15 0.05	0.05 0.30	0.30 0.05
λ	n=200						
$\lambda = 3$	0.0787	0.0575	0.0528	0.0632	0.0642	0.0505	0.0497
$\lambda = 2$	0.0508	0.0369	0.0373	0.0406	0.0408	0.0291	0.0303
$\lambda = 1$	0.0233	0.0182	0.0211	0.0176	0.0183	0.0131	0.0134
n	$\lambda = 2$						
200	0.0508	0.0369	0.0373	0.0406	0.0408	0.0291	0.0303
500	0.0288	0.0209	0.0237	0.0217	0.0225	0.0150	0.0152
1000	0.0193	0.0140	0.0151	0.0137	0.0141	0.0088	0.0089
2000	0.0124	0.0093	0.0082	0.0086	0.0086	0.0054	0.0052

Source: own calculations

In Table 7 results of simulation studies are presented for estimating π_1 via EM algorithm. MSE of a single proportion estimator is much higher as compared to MSE of a difference between two proportions estimator. Situation here is thus reversed to the classic one. Although alternative model is in favor when estimating difference between two sensitive proportions, the same does not apply for estimating individual sensitive proportions. MSE of ML (via EM algorithm) estimator of π_1 is very high.

SUMMARY

In the paper we have provided an extensive theory for testing equality of two sensitive proportions for two populations when using Poisson and negative binomial item count techniques, introduced earlier for a single population case in a seminal paper by [Tian et al. 2014]. To give practitioners some directions we have illustrated theoretical results by numerical calculations and simulation studies. All simulation results are consistent with theoretical models. Considering not only classic approach for two population problem, but also the alternative one, brought about some interesting results. In our opinion a compromised approach should be closely explored in a future research on multipurpose surveys concerning sensitive items.

REFERENCES

Hussain Z., Shab E.A., Shabbir J. (2012) An alternative item count technique in sensitive surveys. Revista Colombiana de Estadistica, 35, 39 – 54.

Gonzalez-Ocantos, E., de Jonge, C.K., Meléndez, C., Osorio, J., Nickerson, D.W. (2012) Vote buying and social desirability bias: experimental evidence from Nicaragua. American Journal of Political Science, 56(1), 202 – 217.

Glynn A.N. (2013) What can we learn with statistical truth serum? Design and analysis of the list experiment. Public Opinion Quarterly, 77, 159 – 172.

Imai K. (2011) Multivariate regression analysis for the item count technique. Journal of American Statistical Association, 206, 407 – 416.

Kuha J., Jackson J. (2014) The item count method for sensitive survey questions: modeling criminal behavior. Journal of the Royal Statistical Society: Series C, 63(2), 321 – 341.

Lloyd-Smith J.O. (2007) Maximum Likelihood Estimation of the Negative Binomial Dispersion Parameter for Highly Overdispersed Data, with Applications to Infectious Diseases. PLOS One, 2 (2), DOI: 10.1371/journal.pone.0000180.

Miller, J.D. (1984) A new survey technique for studying deviant behavior. PhD diss, The George Washington University.

Tian G-L., Tang M-L., Wu Q., Liu Y. (2014) Poisson and negative binomial item count techniques for surveys with sensitive question. Statistical Methods in Medical Research, Prepublished on December 16, 2014 as DOI: 10.1177/0962280214563345.

Wolter, F., Laier B. (2014) The Effectiveness of the Item Count Technique in Eliciting Valid Answers to Sensitive Questions. An Evaluation in the Context of Self-Reported Delinquency. Survey Research Methods, 8 (3), 153 – 168.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII , No. 1, 2017, pp. 78 – 87

Received: 05.02.2017
Accepted: 05.04.2017

IMPACT OF THE FINANCING STRUCTURE ON EFFICIENCY OF HEALTHCARE SYSTEMS IN THE FORMER EASTERN BLOC COUNTRIES

Justyna Kujawska (ORCID: 0000-0002-0416-3048)
Faculty of Management and Economics
Gdańsk University of Technology, Poland
e-mail: Justyna.Kujawska@zie.pg.gda.pl

Abstract: Out-of-pocket expenditures are a significant barrier in accessing health services. This paper aims to analyse the structure of financing system in the context of the performance indicators of healthcare systems. The study was conducted for the 28 countries of the former Eastern bloc in the years 2000 and 2013, based on data from the World Health Organization. In the DEA-CCR input-oriented model, inputs are the percentage share of private spending in the total expenditure on healthcare and the percentage share of out-of-pocket patient spending in total private spending. The outputs are life expectancy and mortality rate. A ranking of the countries was created and the differences between the two study periods, as well the desired directions of changes in the financing structure were pointed out.

Keywords: healthcare system, private expenditure, out-of-pocket spending, Data Envelopment Analysis

INTRODUCTION

All countries regardless of their level of economic development endeavour to improve the quality and accessibility of health services, which requires objective and reliable assessment of the functioning of their healthcare systems. The policy makers and the public expect the best possible effects due to the relatively high cost [González et al. 2010]. International comparative studies often use, among other resources, healthcare spending measured as the share of gross domestic product (GDP) or per capita. [Anell, Willis 2000]. Control of the healthcare financing system is a priority aspect of public policies design — especially in

DOI: 10.22630/MIBE.2017.18.1.08 JEL: C61, H51

recent years, due to soaring budget deficits and public debt caused by the economic crisis [de Cos, Moral-Benito 2014].

The access to medical care is affected by a number of factors, the most important of which are, according to the subject-matter literature: scope of access to healthcare, unmet needs related to medical care, out-of-pocket private medical expenses, geographic distribution of physicians and the time of waiting for planned treatment [OECD 2015].

An illness can cause worsening of economic security both directly and indirectly. For those without health insurance or with partial health insurance, medical expenses can be catastrophic, leading to debt or opting out of treatment at the expense of worsening health in the future. However, health insurance may cover different options and even the insured individuals may incur high costs, paying directly for some services or medicines [Stiglitz et al. 2009].

The purpose of this article is to find the relation between the share of private spending on healthcare (in particular the costs borne directly by the patient) and the results of the functioning of healthcare systems in countries of the former Eastern bloc. These are countries that at the beginning of the twenty-first century have to make radical changes in their health care systems. The study was conducted for 28 countries for the years 2000 and 2013, using Data Envelopment Analysis (DEA) method.

HEALTHCARE FINANCING

Health systems are usually funded from multiple sources, such as taxes, social insurance contribution and private insurance contributions or patients' out-of-pocket payments [Wagstaff et al. 1992]. The percentage of healthcare financing from public funds is used as an indicator enabling the assessment of the role of the state in this area. The strong role of the state, reflected by a high level of funding from the budget, points to better cost control and reduction of inequalities in access to medical services. On the other hand, the percentage of out-of-pocket patient payments or private insurance allows for the assessment of the financial burden imposed on society in the event of necessity to use health services. The high level of out-of-pocket expenses generally increases the difficulty of obtaining medical assistance for people with lower income and inferior health status [Wendt 2014]. The countries with a low share of public expenditure should aim at reducing the level of out-of-pocket payments in favour of prepaid private insurance. This way, the public could finance health services in a more predictable manner, without facing the problematic, sudden necessity to find the funds to pay for treatment in case of an unforeseen illness [Xu et al. 2005].

The financial security of patients provided by public or private health insurance substantially reduces the number of individuals paying for medical care directly, however in some countries the burden of out-of-pocket spending can create barriers in access to healthcare and in many cases prevent availing of it. The households that encounter difficulties in paying medical bills may delay or even abandon the necessary healthcare [OECD 2015]. The large share of out-of-pocket payments in case of the poorer social groups exacerbates the risk of the so-called catastrophic spending, leading to impoverishment or abandonment of often necessary medical services [Xu et al. 2003; Xu et al. 2007]. Spending is defined as catastrophic if a household's share in financing healthcare exceeds 40% of the income remaining after satisfying the everyday needs [Xu et al. 2003]. Moving away from the out-of-pocket patients' payments towards mechanisms of prepaid private insurance is the key to reducing the possibility of a financial catastrophe [Xu et al. 2007].

RESEARCH ON THE EFFICIENCY OF HEALTHCARE SYSTEMS

The DEA method is widely used in testing the efficiency of healthcare systems at practically all levels, ranging from physicians (both primary and specialist care), through providers of medical services (hospitals, emergency assistance etc.), to global, country-level assessments. Depending on the purpose and scope of research, the models can have a more diverse structure. One of the fundamental difficulties indicated by many authors is providing the definition of the outcomes of healthcare systems [e.g. Retzlaff-Roberts et al. 2004; Afonso, Aubyn 2005; González et al. 2010; Hadad et al. 2013; Papanicolas, Smith 2013]. The main outcome of the system is the improvement of the health of society, however measuring such a parameter is difficult. It is much easier to define the inputs, which, when used properly, determine the overall efficiency. Usually the resource approach is used, based on quantifiable inputs such as the number of physicians or available infrastructure (e.g. number of beds, diagnostic equipment, financial resources etc.). It is also common practice to base models on variables indirectly reflecting outputs and inputs (proxies), which is a consequence of the availability of relevant data. Most often the public statistics are used. Institutions such as the World Health Organization (WHO), OECD and Eurostat, improve their data collection procedures, which increases the reliability of analyses.

Given the purpose of the article, the review of the literature focuses on the studies of the efficiency of health systems conducted in the world, treating expenditure and its structure as inputs.

The share of public spending in total healthcare expenditure [Or et al. 2005] was included as one of the inputs in the study of differences in physicians' efficiency of improving public health in OECD countries. In addition, the analysis takes into account the number of physicians, the level of GDP per capita, the level of education of the society, as well as the environmental variables: the consumption of alcohol and smoking. The outputs were based on the life expectancy at birth and at 65 years of age and the number of years of life lost due to heart diseases (for men and women separately), as well as mortality. These variables are commonly used as the outputs of healthcare systems.

The analysis carried out for the 165 countries for which data were available in the WHO database shows that the share of public healthcare spending and the size of healthcare spending in public budgets are two factors positively related to the functioning of healthcare systems [González et al. 2010]. A modified DEA model was used, allowing for the introduction of weight restrictions, which increases the discriminatory strength of the method. Two kinds of input, the total expenditure on health per capita and the expected length of education (as an environmental factor), as well as two outputs — healthy life expectancy and the disability adjusted life years — were taken into account. The level of public financing reached 64% in the most efficient countries from the sample, whereas in the least efficient ones the public funding did not exceed 50%. It can be said that in the countries whose governments show commitment to the development and financing of healthcare systems the available resources are used more efficiently, while allowing for achieving adequate health outcomes.

A similar approach to creating models of technical efficiency of healthcare systems can be found in other publications. In the case of OECD countries, a study of the efficiency of healthcare resources usage, measured by such parameters as the number of physicians, the number of beds per 1 000 inhabitants, the number of units of magnetic resonance imaging (MRI) per million inhabitants or healthcare spending as the percentage of GDP was conducted [Retzlaff-Roberts et al. 2004]. The authors adopted infant mortality rate and life expectancy at birth as the outputs. In the second stage, the analysis takes into account also the social and environmental factors, such as the Gini coefficient, the expected length of education or smoking. In another study of the same group of countries [Hadad et al. 2013] the authors built two models, which used life expectancy and infant mortality as outputs. The inputs in the first model were parameters considered controllable by healthcare systems, such as the number of physicians and hospital beds per 1 000 inhabitants, whereas the second one was based on inputs which cannot be controlled by healthcare systems, i.e. the GDP per capita and environmental factors such as the consumption of fruit and vegetables per capita. Both models also included the total expenditure on health per capita.

THE PROPOSED MODEL AND THE UTILISED DATA

'Efficiency', as used in this article, should be interpreted as technical efficiency, which evaluate by how much input quantity can be proportionally reduced without changing the output quantities [Afonso, Aubyn 2005].

The proposed model is based on two inputs: PRIV – the percentage share of private spending in the total expenditure on healthcare and OOP – the percentage share of out-of-pocket patient spending in total private spending (the remaining part of private spending is financed with prepaid health insurance). The overall health status of population is generally operationalized by indicators of longevity such as life expectancy, healthy life expectancy, overall mortality [Tchouaket et al.

2012]. In the opinion of OECD life expectancy at age 60 include advances in medical care combined with greater access to health care, healthier lifestyles and improved living conditions before and after people reach age 60. Increased life expectancy does not necessarily mean that the extra years lived are in good health [OECD 2015]. So the outputs are reflected by five variables: LE_F and LE_M – life expectancy at age 60 for men and women; HLE_F and HLE_M - healthy life expectancy at birth for men and women and MORT - mortality of adults aged 15–60 years per 1 000 people, which is the unwanted output and was included in the model as the difference 1 000-MORT. The assumptions are met that increased input reduces efficiency, whilst increased output increases efficiency [Dyson et al. 2001; Guzik 2009]. The basic descriptive statistics of variables for years 2000 and 2013 are presented in Table 1. The two last rows shows the differences between the mean and median values of the variables (2013–2000). The mean and median share of private spending did change in a small extent, however the mean and median share of patients' out-of-pocket expenditure decreased by 2.6 percentage points and 5.0 percentage points respectively, which is a proof of small development of the pre-paid health insurance. All outputs have improved: the mean value of LE_F and LE_M increased by about 7–8%, while the mean of remaining parameters increased by about 4–5%. The median value of all outputs grow up about 10%.

Table 1. The basic descriptive statistics of variables for years 2000 and 2013

Year	Statistics	PRIV	OOP	MORT	LE_F	LE_M	HLE_F	HLE_M
2000	Mean	41.2	91.2	813.0	19.5	15.8	64.6	58.3
	Stand. error	21.7	13.8	57.1	1.8	1.6	3.5	3.6
	Max	83.0	100.0	881.0	23.0	19.0	69.0	63.0
	Min	9.7	44.1	688.0	16.0	12.0	57.0	51.0
	Median	39.6	98.95	825.5	19.5	16.0	66.0	59.0
2013	Mean	41.2	88.6	848.7	21.1	17.0	67.3	61.1
	Stand. error	16.2	13.2	49.8	2.3	1.9	3.2	3.4
	Max	79.2	100.0	918.0	26.0	21.0	72.0	66.0
	Min	16.7	42.7	710.0	17.0	13.0	59.0	53.0
	Median	38.8	94.0	859.5	21.5	17.0	68.0	61.5
Change in the mean		**0,0**	**-2.6**	**35.7**	**1.6**	**1.1**	**2.6**	**2.9**
Change in median		**-0,8**	**-5.0**	**34.0**	**2.0**	**1.0**	**2.0**	**2.5**

Source: own computation

The non-parametric DEA method allows for assessing the relative efficiency of the compared objects, called Decision Making Units (DMUs), described by multiple inputs and multiple outputs. It is not necessary to know a functional relationship between the inputs and the outputs. The evaluation of the efficiency involves determining the DMUs creating the ‘best practice’ frontier and comparing them to other objects [Cooper et al. 2011].

The CCR (Charnes-Cooper-Rhodes) model, with constant returns to scale, was chosen as suitable when the set of evaluated objects is homogeneous [Eilat et al. 2008]. Since only the inputs are controllable by the decision-makers shaping the health policy, an input-oriented model was adopted. In an input orientation improvement of efficiency is possible through proportional reduction of inputs. The efficiency score θ_o^* of DMU_o $(o = 1, \dots, n)$ is calculated for given amounts of outputs y_{rj}, $r = 1, \dots, s$ and inputs y_{ij}, $i = 1, \dots, m$, where $j = 1, \dots, n$. The input-oriented CCR model is shown below. [Cooper et al. 2011]:

$$\theta_o^* = min\, \theta_o \tag{1}$$

for the conditions:

$$\sum_{j=1}^{n} x_{ij}\lambda_{jo} \leq \theta_o x_{io} \quad i = 1,2, \dots, m \tag{2}$$

$$\sum_{j=1}^{n} y_{rj}\lambda_{jo} \geq y_{ro} \quad r = 1,2, \dots, s$$

$$\lambda_{jo}, \theta_o \geq 0 \quad j = 1,2, \dots, n \tag{3}$$

where λ_j are intensity variables [Guzik 2009].

Using the above-described model, the 28 countries of the former Eastern bloc were analysed. Data from the years 2000 and 2013 from the WHO database were used. The calculations were carried out by means of the DEA-Solver-LV (3) software by Saitech.

THE RESULTS AND THEIR INTERPRETATION

The results of computation are shown in Table 2.

Table 2. The results of efficiency computation for the years 2000 and 2013

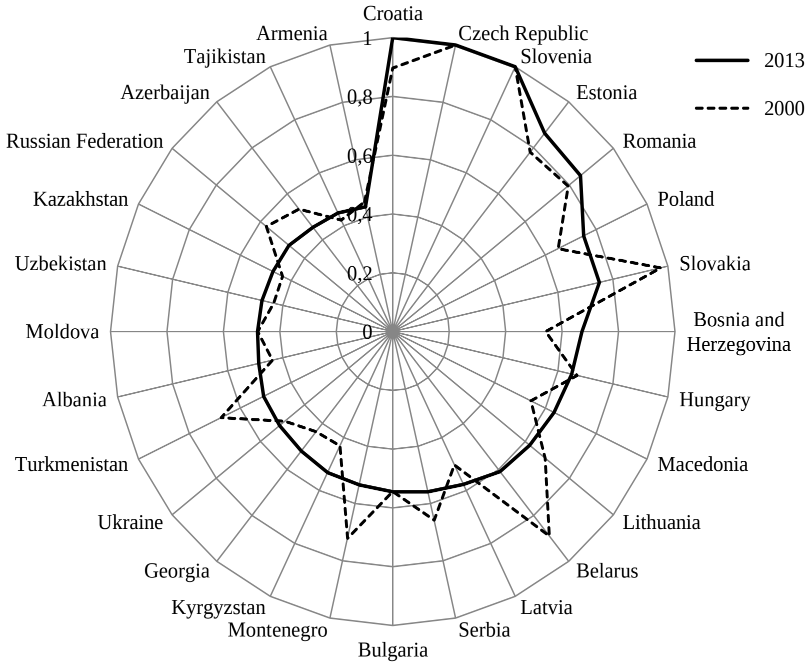
Country	2000		2013		Country	2000		2013	
	Ef	R	Ef	R		Ef	R	Ef	R
Albania	0.44	23	0.49	21	Lithuania	0.69	9	0.62	11
Armenia	0.45	22	0.44	28	Macedonia	0.54	16	0.63	10
Azerbaijan	0.53	18	0.45	26	Moldova	0.48	21	0.48	22
Belarus	0.89	5	0.61	12	Montenegro	0.72	8	0.53	16
Bosnia and Herzegovina	0.54	17	0.67	8	Poland	0.65	13	0.75	6
Bulgaria	0.54	15	0.55	15	Russian Federation	0.57	14	0.47	25
Croatia	0.90	4	**1.00**	1	Romania	0.79	6	0.85	5
Czech Republic	**1.00**	1	**1.00**	1	Serbia	0.66	12	0.56	14
Estonia	0.78	7	0.86	4	Slovakia	0.97	3	0.75	7
Georgia	0.44	24	0.52	18	Slovenia	**1.00**	1	**1.00**	1
Hungary	0.67	11	0.65	9	Tajikistan	0.42	28	0.45	27
Kazakhstan	0.43	25	0.47	24	Turkmenistan	0.67	10	0.51	20
Kyrgyzstan	0.43	27	0.53	17	Ukraine	0.49	20	0.51	19
Latvia	0.50	19	0.58	13	Uzbekistan	0.43	26	0.48	23

Source: own computation

Column "Ef" contains the efficiency score and column "R" the position in the ranking. In the year 2000 the full efficiency was achieved by the Czech Republic and Slovenia, which were among the best also in 2013. The full efficiency in 2013 was also reached by Croatia. These countries also had the best structure of spending in relation to the achieved results that were included in the model.

The figure below shows the efficiency scores in descending order, allowing for the analysis of the direction and magnitude of change.

Figure 1. Comparison of the efficiency scores in the years 2000 and 2013



Source: own elaboration

Table 3 contains the source data for selected countries. The Czech Republic and Slovenia, which are the leaders, improved all the outputs. The Czech Republic had the lowest value of PRIV (9.7% and 16.7%) of all the countries, with OOP equal to 100% in 2000 and 94% in 2013. Slovenia has private expenditure at the levels of 26.0% and 28.4% respectively, however OOP was equal to 44.1% and 42.7%. In 2000, Croatia had PRIV equal to 13.9%, whereas in 2013 it reached 20%. However, the share of OOP decreased from 100% in 2000 to 62.4% in 2013. Just as in the Czech Republic and Slovenia, all the outputs improved.

Table 3. Data from selected countries

Country	Year	PRIV	OOP	MORT	LE_F	LE_M	HLE_F	HL_M
Czech Republic	2000	9.7	100.0	875	21	17	69	63
	2013	16.7	94.1	907	24	19	71	66
Slovenia	2000	26.0	44.1	878	23	18	69	63
	2013	28.4	42.7	918	26	21	72	66
Croatia	2000	13.9	100.0	870	21	17	68	62
	2013	20.0	62.4	903	24	19	70	65
Belarus	2000	24.5	57.1	758	20	14	65	55
	2013	34.6	92.0	801	22	14	68	57
Slovakia	2000	10.6	100.0	853	21	16	68	60
	2013	30.0	73.9	882	23	18	70	63
Bosnia and Herzegovina	2000	43.3	100.0	881	21	18	68	63
	2013	30.0	96.9	899	22	19	70	66
Poland	2000	30.0	100.0	848	22	17	68	61
	2013	30.4	75.0	871	24	19	71	63

Source: own computation

The further analysis was based on countries which recorded the greatest increase or decrease of efficiency in the analysed period. The two countries which recorded the highest decline in efficiency, i.e. Belarus and Slovakia, had generally lower outputs than in the case of the leaders. Moreover, their improvement in the analysed period was lower than in the case of the best countries. The structure of expenditure deteriorated significantly in Belarus, PRIV increased from 24.5% to 34.5%, while OOP rose from 57.1% to 92.0%. On the other hand, in Slovakia PRIV increased from 10.6% to 30.0% but there was a decrease in OOP from 100% to 73.9%, but this is still near two times greater than minimum value 44,1%.

The two countries, which recorded the highest increase in efficiency (except Croatia), i.e. Bosnia and Herzegovina and Poland, had outputs similar to those of the leaders. In Bosnia and Herzegovina PRIV decreased from 43.3% to 30.0%, with almost constant OOP (100% and 96.9%). In Poland, on the other hand, PRIV reached 30.0% in both years, while OOP decreased from 100% to 75%.

The above analysis allows for indicating several typical situations. Achieving better health outcomes is observed in countries with a low level of private spending, such as the Czech Republic — in such circumstances the role of a large share of patients' out-of-pocket expenses is insignificant. Another situation is the example of Slovenia and Croatia, where the share of private spending is higher, while the out-of-pocket expenditure is low or decreasing. Increasing private spending in the context of a large share of out-of-pocket expenditure negatively affects the achieved health outcomes, especially in Belarus. On the other hand, reducing the share of private expenditure in the context of a constant share of out-of-pocket expenditure (Bosnia and Herzegovina) or maintaining the share of private spending while reducing the out-of-pocket expenditure (Poland) results in the improvement of health outcomes.

The share of private expenditure in the total expenditure (PRIV) on healthcare and the share of patients' out-of-pocket expenses (OOP) are the variables which indirectly characterize the barriers in access to healthcare services. Of course, the obtained results should not be interpreted as meaning that a change in the financing structure has a direct impact on the improvement of health outcomes. However, the indirect effect has been demonstrated, which confirms the results of other authors dealing with research on the availability of medical services for patients, signalled at the beginning of the article.

The next stage of the analysis shows the possibilities of the model used as far as formulating recommendations for the inefficient countries is concerned. In order to achieve full efficiency, these countries should change the structure of financing — for example Belarus should reduce PRIV to 21.1% and OOP to 56.0%, Slovakia should reduce PRIV to 22.5% and OOP to 55.5%, Bosnia and Herzegovina should reduce PRIV to 20.1% and OOP to 56.1% and Poland should reduce PRIV to 22.9% and OOP to 56.3% (actual values are provided in Table 3).

SUMMARY AND CONCLUSIONS

All the analysed countries have made radical changes in their healthcare systems as a result of the political changes at the beginning of the XXI century. They were introduced in various ways but limited public funds were a common feature, which resulted in varying degrees of shifting the costs to the patients. Most of these countries have poorly developed systems of private health insurance. Such insurance is indicated in the literature as a good way to protect patients against catastrophic healthcare expenditure, often resulting in the resignation from the necessary medical services, which is reflected afterwards in outcomes related to public health. The purpose of this article was to find a relationship between the share of private spending, which indirectly determines the availability of medical services, and outcomes related to health. It must be emphasized once again that health outcomes depend on a number of other factors but the proposed model focuses on financial barriers related to access to medical services.

REFERENCES

Afonso A., Aubyn M.S.T. (2005) Non-parametric approaches to education and health efficiency in OECD countries. Journal of Applied Economics, VIII/2, 227-246.

Anell A., Willis M. (2000) International comparison of health care systems using resource profiles. Bulletin of the World Health Organization, 78(6), 770-778.

Cooper W. W., Seiford L. M., Zhu J. (2011) Handbook on Data Envelopment Analysis. Springer, New York.

de Cos P.H., Moral-Benito E. (2014) Determinants of health-system efficiency: evidence from OECD countries. International Journal of Health Care Finance and Economics, 14, 69–93.

Dyson R.G., Allen R., Camanho A.S., Podinovski V.V., Sarrico S.C., Shale E.A. (2001) Pitfalls and protocols in DEA. European Journal of Operational Research, 132, 245–259.

Eilat H., Golany B., Shtub A. (2008) R&D project evaluation: An integrated DEA and balanced scorecard approach. Omega - International Journal of Management Science, 36, 895–912.

González E., Cárcaba A., Ventura J. (2010) Value efficiency analysis of health systems: does public financing play a role? Journal of Public Health, 18, 337–350.

Guzik B. (2009) Podstawowe modele DEA w badaniu efektywności gospodarczej i społecznej. Wydawnictwo Uniwersytetu Ekonomicznego w Poznaniu, Poznań.

Hadad S., Hadad Y., Simon-Tuval T. (2013) Determinants of healthcare system's efficiency in OECD countries. European Journal of Health Economics, 14 (2), 253–265.

OECD (2015) Health at a Glance 2015: OECD Indicators, OECD Publishing, Paris.

Or Z., Wang J., Jamison D. (2005) International differences in the impact of doctors on health: a multilevel analysis of OECD countries. Journal of Health Economics, 24, 531–560.

Papanicolas I., Smith P.C. (2013) Health system performance comparison. An agenda for policy, information and research. McGraw-Hill Education, Open University Press, New York.

Retzlaff-Roberts D., Chang C.F., Rubin R.M. (2004) Technical efficiency in the use of health care resources: a comparison of OECD countries. Health Policy, 69, 55–72.

Stiglitz J.E., Sen A., Fitoussi J.P. (2009) Report by the commission on the measurement of economic performance and social progress. Paris, http://www.stiglitz-senfitoussi.fr/en/index.htm (dostęp 16.02.2016).

Tchouaket E.N., Lamarche P.A., Goulet L., Contandriopoulos A.P. (2012) Health care system performance of 27 OECD countries. The International Journal of Health Planning and Management, 27, 104–109.

Wagstaff A., van Doorslaer E., Calonge S., Christiansen T., Gerfin M., Gottschalk P., Janssen R., Lachaud C., Leu R.E., Nolan B. et al. (1992) Equity in the finance of health care: Some international comparisons. Journal of Health Economics, 11, 361–387.

Wendt C. (2014) Changing Healthcare System Types. Social Policy & Administration, 48, (7), 864–882.

Xu, K., Evans, D.B., Carrin, G., Aguilar-Rivera, A. M. (2005) Designing health financing systems to reduce catastrophic health expenditure. Technical brief for policy-makers, no. 2/2005, World Health Organization.

Xu K., Evans D.B., Carrin G., Aguilar-Rivera A.M., Musgrove P., Evans T. (2007) Protecting Households From Catastrophic Health Spending. Health Affairs, 26 (4), 972–983.

Xu K., Evans D.B., Kawabata K., Zeramdini R., Klavus J., Murray C.J.L. (2003) Household catastrophic health expenditure: a multi-country analysis. The Lancet, 362, 111–117.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII, No. 1, 2017, pp. 88 – 98

Received: 25.12.2016
Accepted: 05.04.2017

TESTING FOR TRADING-DAY EFFECTS IN PRODUCTION IN INDUSTRY: A BAYESIAN APPROACH

Łukasz Lenart
Faculty of Finance and Law
Cracow University of Economics, Poland
e-mail: lenartl@uek.krakow.pl

Abstract: The aim of this paper is to construct a parametric method in a Bayesian framework to identify trading-day frequency for monthly data. The well-known visual spectral test (implemented, for example, in X-12-ARIMA) is a popular tool in the literature. In the article's proposed method, the assumption concerning the almost periodicity of the mean function plays a central role. We use a set of frequencies that corresponds to the trading-day effect for monthly data. As an illustration, we examine this effect in production in industry in European economies for data adjusted by working days and for gross data.

Keywords: trading-day effect, production in industry, almost periodic function, AR model

INTRODUCTION

The trading-day effect (or calendar effect) in both monthly and quarterly macroeconomic time series is well known [Cleveland et al. 1980, Cleveland et al. 1982, Bell et al. 1983, Dagum et al. 1993, Bell et al. 2004, Soukup et al. 1999, Ladiray 2012]. The work of [Ladiray 2012] describes the present state of advances in this field.

The calendar effect is caused by different numbers of working days during months or quarters. For example, each February (in non-leap-years) has four weeks, which means that in the month we have four Mondays, Tuesdays, Wednesdays, etc. For other months, the number of days is not a multiple of 7, which means that the number of working days (from Monday to Friday) varies from month to month. This periodic phenomenon in numbers of working days in

DOI: 10.22630/MIBE.2017.18.1.09

JEL: C11, C22

months and quarters can be a source of additional variability for macroeconomic time series called the "trading-day effect" or the "calendar effect".

Let f(t) denote the deterministic function defining the number of working days (or Mondays, Tuesdays, etc.) in month t. As the most popular Georgian calendar is periodic with periods equal to 400 years, the function f can be represented by a Fourier series with a known theoretical set of frequencies. However, the number of such frequencies is quite large. In [Ladiray 2012], the periodograms of the number of weekdays (from Monday to Friday) were evaluated using a theoretical path that covers 400 years. This theoretical set of frequencies contains two frequencies of 2.18733 and 2.71093 with dominating amplitude and many others with much lower amplitude. In practice, the real frequencies corresponding to the calendar effect must be estimated because the length of the sample is much shorter than that used in theoretical evaluation. Therefore, to address the estimation problem in this paper, we use the idea of almost periodic (*ap*) functions. An almost periodic function is a generalization of a well-known periodic case.

Note that for such macroeconomic time series as production in industry or GDP, the calendar adjustment is a preliminary step (beside seasonal adjustment) in real data analysis. The estimation of trading-day effects is possible using regression variables (see implementation in X-13-ARIMA-SEATS and other procedures).

The fundamental problem connected with calendar adjustment is a diagnostic to determine whether this effect is present in the data set. One of the popular methods for examining the trading-day effects is the so-called "visual test" (implemented for example in X-12-ARIMA). This method is based on a graphical observation of the usual periodogram (statistics). Note that a detailed interpretation of the periodogram depends on the assumptions. First, it is an inconsistent estimator of spectral density function (under zero mean assumption; see: Priestley 1981; Hamilton 1994), and second, any point from the interval $[0,2\pi)$ can be interpreted as the estimator of the magnitude of the Fourier coefficient (for the Fourier representation of an almost periodic mean function [Lenart et al. 2013a, Lenart et al. 2013b, Lenart 2013, Lenart 2015, and Lenart et al. 2016b].

In this paper, we consider the autoregressive model with an almost periodic mean function, introduced in [Lenart et al. 2016a]. Under standard prior distributions for the parameters, the posterior distribution for vectors of Fourier frequencies in Fourier representation of the mean function can be explicitly evaluated (by simple integration). Based on this distribution, we evaluate the mass of the probability concentrated around main calendar frequencies. The closer these masses are to one around some frequency, the stronger the data support this frequency. We find that, for a large majority of year over year (in short YOY) production in industry (monthly data) in the period 2001–2014, the frequency 2.18733 is predominant over other calendar frequencies in the case of gross data. The corresponding data adjusted by working days was also examined. We found that for some economies in these data sets, the frequency 2.18733 is still predominant. We use the datasets published by Eurostat.

METHODOLOGY

Almost periodic in mean time series - basics

The class of the almost periodic function on an integer line is well known in the literature [Corduneanu 1989]. Economic applications of almost periodic functions in the first or second moments of time series have been included in [Mazur et al. 2012, Lenart et al. 2013a, Lenart et al. 2013b, Lenart 2013, Lenart 2015 and others]. Generally, in the case of the Almost Periodically Correlated (APC) time series, the mean function and the autocovariance function have Fourier representation:

$$\mu(t) \sim \sum_{\psi \in \Psi} m(\psi) e^{i\psi t}, B(t,\tau) \sim \sum_{\lambda \in \Lambda_\tau} a(\lambda,\tau) e^{i\lambda t}, \text{ for any } \tau \in Z,$$

where the Fourier coefficients $m(\psi)$ and $a(\lambda,\tau)$ are given by the limits:

$$m(\psi) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \mu(t) e^{-i\psi t}, \qquad a(\lambda,\tau) = \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} B(j,\tau) e^{-i\lambda j}$$

[Hurd 1989, Hurd 1991, Dehay et al. 1994].

The sets $\Psi = \{\psi \in [0,2\pi): m_X(\psi) \neq 0\}$ and $\Lambda_\tau = \{\lambda \in [0,2\pi): a(\lambda,\tau) \neq 0\}$ are countable. Note that, for any vector of frequencies $\boldsymbol{\psi} = (\psi_1, \psi_2, \ldots, \psi_F) \in (0,\pi]^F$ there is a corresponding vector of Fourier coefficients $m = (m(\psi_2), m(\psi_2), \ldots, m(\psi_F)) \in \mathbb{C}^F$. Note that $|m(\psi)| \neq 0 \Leftrightarrow \psi \in \Psi$, which means that statistical inference concerning the frequencies in the set Ψ can be based equivalently on statistical inference for $|m(\psi)|$.

"Visual test" for calendar frequencies

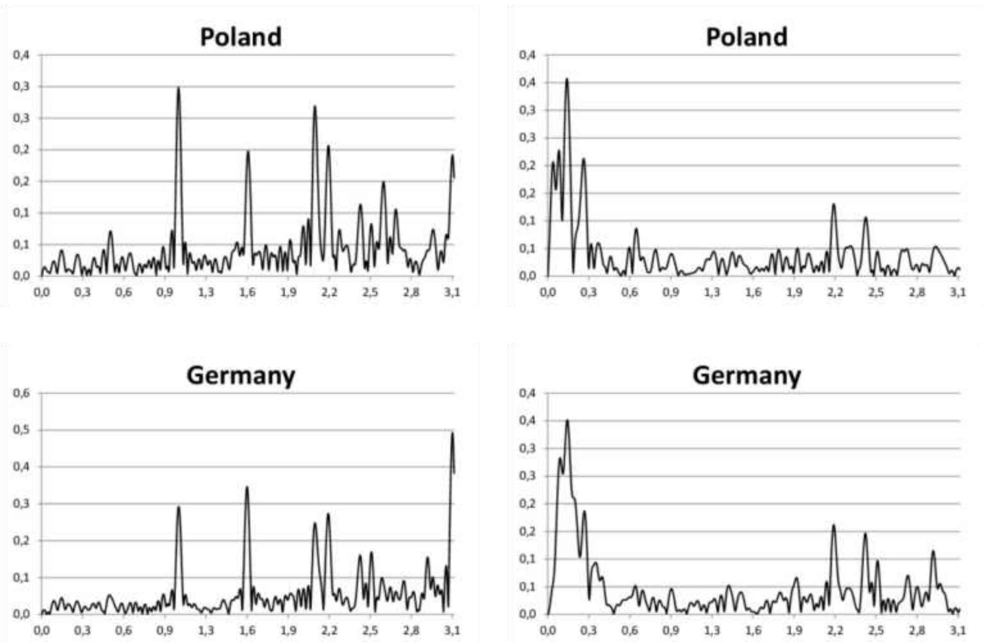
In a non-parametric approach, the natural estimator of the magnitude of Fourier coefficients $|m(\psi)|$based on sample $\{X_1, X_2, \ldots, X_n\}$ has the following form

$$|\widehat{m}_n(\psi)| = |\tfrac{1}{n} \textstyle\sum_{j=1}^{n} X_j e^{-ij\psi}|$$

where $\psi \in [0,2\pi)$. As shown in [Lenart 2013], this estimator (after appropriate normalizing) is asymptotically normally distributed with known asymptotic variance-covariance matrix that depends on a spectral density function. Note that the statistic $|\widehat{m}_n(\psi)|$ is a usual periodogram function used in practical applications to examine the existence of calendar effects. The peak on the periodogram at frequency ψ_0 means the data support a periodic phenomenon connected with this frequency (in first or second moment). This simple tool is often used in applications. Let us consider an illustrative example of the properties of the periodogram.

We analyze the periodograms for percentage change over the previous period (MOM; gross data) and percentage change compared to the same period of the previous year (YOY; gross data) for production in industry (B-D: mining and quarrying; manufacturing; electricity, gas, steam and air conditioning supply) in Poland and Germany from Jan. 2000 to Dec. 2014. We use gross data in this example.

Figure 1. Estimate of the magnitude of Fourier transform (or periodogram): $|\widehat{m}_n(\psi)|$. The case of mining and quarrying; manufacturing; electricity, gas, steam and air conditioning supply (from Jan. 2000 to Dec. 2014) for MOM (on the left) and YOY (on the right).



Source: own preparation

In the case of MOM data, the frequency corresponding to seasonal fluctuations is clearly observed, whereas in the case of YOY, the mass on the periodograms mainly concentrates near the interval that corresponds to business cycle fluctuations. Note that for both cases, the mass is also concentrated near the frequency of approximately 2.19, which corresponds to predominant trading-day effect frequency.

Bayesian inference for the frequencies

Following [Lenart et al. 2016a], we consider a usual autoregressive model of order p:

$$L(B)(y_t-\mu(t))=\varepsilon_t,$$

where $L(B)=1-\eta_1 B-\eta_2 B^2-\cdots-\eta_p B^p$ is a lag polynomial,

$$\varepsilon_t \sim N(0,\tau^{-1}),$$

$$\mu(t)=\delta_0+\sum_{f=1}^{F}\left[a_f\sin(t\psi_f)+b_f\cos(t\psi_f)\right].$$

For the parameters, we assume: $\delta_0\in\mathbb{R}$, $\mathbf{a}=(a_1,a_2,\ldots,a_F)\in\mathbb{R}^F$, $\mathbf{b}=(b_1,b_2,\ldots,b_F)\in\mathbb{R}^F$, and $\boldsymbol{\psi}=(\psi_1,\psi_2,\ldots,\psi_F)\in(0,\pi]^F$. We assume the following prior structure:

$$p(\mathbf{a},\mathbf{b},\tau,\boldsymbol{\psi})=p(\mathbf{a},\mathbf{b},\tau)p(\boldsymbol{\varphi})=p(\mathbf{a},\mathbf{b}|\tau)p(\tau)p(\boldsymbol{\varphi}),$$

with uniform distribution on $(0,\pi]^F$ for frequency vector $\boldsymbol{\psi}$**,** $(\mathbf{a},\mathbf{b})|\,\tau\sim N(\mathbf{0},(\tau\mathbf{B})^{-1})$ and $\tau\sim G\left(\frac{n_0}{2},\frac{s_0}{2}\right)$, where $N(\mathbf{0},(\tau\mathbf{B})^{-1})$ denotes the Normal distribution (with hyperparameter $\mathbf{B}$) and $G\left(\frac{n_0}{2},\frac{s_0}{2}\right)$ denotes the Gamma distribution with hyperparameters s_0 and n_0. Under such standard prior distribution we obtain the following form of the posterior distribution for frequency vector $\boldsymbol{\psi}$:

$$p(\boldsymbol{\psi}|\mathbf{y})\propto(\det(\mathbf{X}'\mathbf{X}+\mathbf{B}))^{-1/2}(\mathbf{y}'[\mathbf{I}-\mathbf{X}(\mathbf{X}'\mathbf{X}+\mathbf{B})^{-1}\mathbf{X}']\mathbf{y}+s_0)^{-\frac{n+n_0}{2}},$$

where $\mathbf{y}$ is a vector of observations and $\mathbf{X}$ is a matrix that depends on vector $\boldsymbol{\psi}$ (see details in [Lenart et al. 2016a]).

Posterior distribution in examination of calendar frequency

Using the above posterior distribution, the mass concentration for frequencies can be examined (for different orders of an autoregressive part). Note that under assumption $\boldsymbol{\psi}\in(0,\pi]^F$, the data may strongly support the frequencies that correspond to business cycle fluctuations (see illustrative periodograms in previous section). Therefore, we restrict the support for frequencies by considering only the set $(\frac{2\pi}{T1.5},\pi]^F$ (where T is a number of observation during the year), which excludes the fluctuations that correspond to fluctuations longer than 1.5 years. Summing up, we consider the mass location for the distribution related to the following kernel:

$$p(\boldsymbol{\psi}|\mathbf{y})1\{\boldsymbol{\psi} \in (2\pi/(T1.5), \pi]^F\}$$

where $1\{A\}$ is the indicator of the event A. Note that under this restriction, the only autoregressive part (in considered model) is allowed to model fluctuations identified with business fluctuations and other fluctuations with longer period.

In this paper, we propose to observe the mass concentration in the posterior distribution around the dominant calendar frequency, approximately 2.18733. Now we assume that $F = 1$, which means that we consider only one frequency (in theoretical specification). The more general case $F > 1$ can also be taken into consideration.

We propose the following methodology and interpretation strategy. For a fixed p (order of autoregressive part), we calculate the posterior probability on the interval (ball)

$$S_\gamma = [2.18733 - \gamma; 2.18733 + \gamma].$$

The mass concentration on this interval means that the data support the existence of trading-day effects related to this frequency. If this probability is comparative with fraction $2\gamma/(\pi - \frac{2\pi}{T1.5})$ it means that the data does not strongly support fluctuations connected with this frequency. In such a case (for gross data), another frequency can be supported more strongly.

EMPIRICAL ANALYSIS

Data description and existing empirical results

We consider production in industry (*mining and quarrying; manufacturing; electricity, gas, steam and air conditioning supply*, percentage change compared to the same period of previous year, YOY) for thirty European countries (Belgium, Bulgaria, Czech Republic, Denmark, Germany (until 1990 former territory of the FRG), Estonia, Greece, Spain, France, Croatia, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Malta, Netherlands, Austria, Poland, Portugal, Romania, Slovenia, Slovakia, Finland, Sweden, United Kingdom, Norway, Former Yugoslav Republic of Macedonia, and Serbia) from Jan. 2001 to Dec. 2014. In this case $T = 12$. The same data set was analyzed in [Lenart's 2015] paper using different methodologies, and we formulate the main thesis concerning calendar frequencies based on this paper. As a first step, the usual periodogram (related to "visual test") for both the gross data and the data adjusted by working day was examined on the interval $(0, \pi]$. The majority of the periodograms clearly show peaks near the frequency 2.18733 for gross data. In addition, based on the nonparametric test used in [Lenart 2015], the nonexistence of this frequency in the true set of frequencies was rejected (at significance level 5%) for most of the data sets. This frequency was also estimated using the contraction method (CM) proposed by [Li et al. 2002]. Note that for data adjusted by working days, the mass concentration around the frequency of approximately 2.19 was not observed in the periodograms.

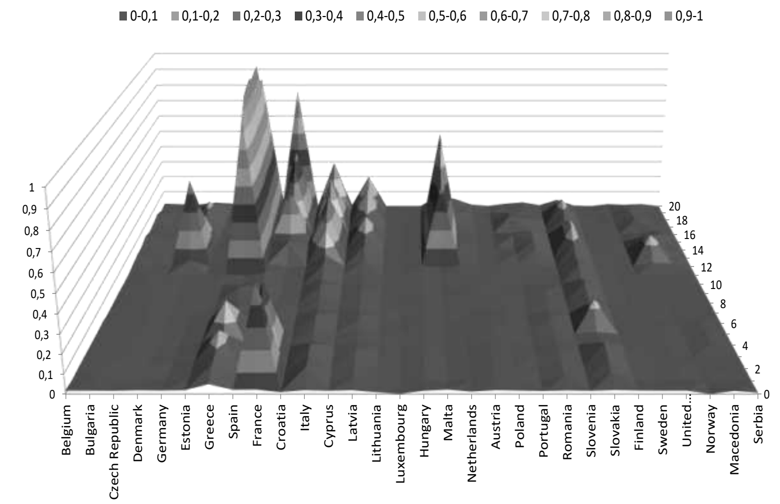
Results obtained in the proposed Bayesian approach

We take $S_\gamma = [2.18733 - \gamma; 2.18733 + \gamma]$ with $\gamma = 0.005$, such that the interval S_γ covers approximately $\frac{1}{50}$ of the length of whole support $\left(\frac{2\pi}{12*1.5}, \pi\right]$. We take $\gamma = 0.005$ by simple observation (graphical) of the posterior mass concentration. For each $p = 0,1,2, \dots 20$ (the order of the autoregressive model), the posterior probability of the set S_γ was individually calculated. Table 1 contains this posterior probability for the gross data, while Table 2 (see also Figure 2) shows the posterior probability for the data adjusted by working days.

The results concerning the gross data confirm that the frequency 2.18733 is predominant in the trading-day effect in the analyzed set of data. Only in the case of Lithuania (maximum posteriori probability on the set S_γ equals 0.045), Malta (maximum posteriori probability on the set S_γ equals 0.05) and Macedonia (maximum posteriori probability on the set S_γ equals 0.054) this frequency was not supported by the data. For other countries, this maximum probability exceeds levels of 0.3 (the lowest probability for Slovakia); 0.8 (the lowest probability for Luxemburg); and 0.9 for other countries.

In the case of the data adjusted by working-days, the results show that in a few cases the frequency 2.18733 is still supported by the data for some orders p. In the cases of Greece, France and Malta, calculated posterior probability exceeds 0.5 at least one time (see Table 2 or the same results on Figure 2). Hence, it can be concluded that, for these countries, the data strongly support the frequency 2.18733. However, the amplitude connected with this frequency is not high enough to say that there is a problem with the adjustment procedure. It means only that on the interval $\left(\frac{2\pi}{12*1.5}, \pi\right]$, the frequency 2.18733 is still strongly supported by the data.

Figure 2. Posterior probability on the set $S_{0.005}$ for considered countries (vertical axis) and different AR orders (horizontal axis)



Source: own preparation

SUMMARY

This paper proposed a new method for detecting calendar frequencies. The method is based on a Bayesian framework and an autoregressive model with an almost periodic mean function and an unknown set of frequencies. The mass location for posterior distribution for the frequency is analyzed in the proposed methodology for a single frequency. However, this methodology can be easily generalized to any set of frequencies (for example: two or more calendar frequencies simultaneously), and this is a topic of the author's future research.

Empirical analysis shows that this tool can clearly detect predominant calendar frequency (2.18733) in gross data for production in industry. In the case of data adjusted by working-days, this method also detects the same predominant frequency (2.18733) in a few cases, but the amplitude of this fluctuation was not analyzed in detail. This is a second point of the author's intensive research.

Table 1. Posterior probability of the set $S_{0.005}$ (in columns order p)

Order p →	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Belgium	0.72	1.00	1.00	0.96	0.43	0.47	0.99	0.66	0.07	0.13	0.14	0.25	1.00	1.00	0.99	0.99	0.69	0.51	0.47	0.02	0.13
Bulgaria	0.04	0.94	0.86	0.61	0.16	0.15	0.24	0.18	0.38	0.54	0.17	0.29	0.45	0.94	0.96	0.98	0.44	0.61	0.61	0.31	0.37
Czech Republic	0.42	1.00	1.00	1.00	0.82	0.36	0.98	0.87	0.84	0.91	0.36	0.38	0.80	0.99	0.98	1.00	0.83	0.81	0.99	0.73	0.77
Denmark	0.87	1.00	1.00	0.31	0.05	0.57	0.84	0.66	0.46	0.65	0.33	0.43	1.00	1.00	1.00	0.79	0.23	0.26	0.04	0.03	0.03
Germany	0.31	0.70	1.00	0.98	0.54	0.16	0.64	0.83	0.61	0.76	0.28	0.30	1.00	1.00	1.00	1.00	0.85	0.90	0.92	0.57	0.18
Estonia	0.06	0.99	0.98	0.97	0.25	0.07	0.72	0.15	0.01	0.02	0.02	0.05	0.71	0.56	0.66	0.62	0.20	0.25	0.81	0.15	0.17
Greece	0.06	0.01	0.28	0.04	0.04	0.33	0.09	0.13	0.15	0.04	0.01	0.00	0.94	0.99	0.99	0.99	0.93	0.91	0.91	0.84	0.74
Spain	0.71	1.00	1.00	0.70	0.63	0.28	0.84	0.77	0.44	0.47	0.18	0.17	0.87	0.78	0.54	0.46	0.51	0.43	0.33	0.21	0.21
France	0.91	1.00	1.00	1.00	0.95	0.65	1.00	1.00	0.99	0.99	0.91	0.96	1.00	1.00	1.00	1.00	0.98	0.98	0.98	0.61	0.72
Croatia	0.12	0.00	0.43	0.09	0.05	0.84	0.86	0.83	0.83	0.26	0.02	0.04	0.57	0.99	0.87	0.75	0.69	0.25	0.18	0.02	0.01
Italy	0.62	1.00	1.00	1.00	0.96	0.80	1.00	0.99	0.99	0.99	0.91	0.87	0.94	0.98	0.97	0.99	0.69	0.36	0.27	0.04	0.13
Cyprus	0.07	0.90	0.81	0.16	0.11	0.02	0.12	0.16	0.11	0.08	0.03	0.44	0.94	1.00	0.99	0.99	0.98	0.95	0.98	0.96	0.97
Latvia	0.07	0.98	0.98	0.88	0.19	0.18	0.59	0.13	0.06	0.05	0.06	0.06	0.30	0.69	0.76	0.83	0.52	0.40	0.89	0.54	0.50
Lithuania	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.04	0.05	0.03
Luxembourg	0.01	0.75	0.80	0.47	0.10	0.12	0.13	0.08	0.08	0.08	0.08	0.12	0.10	0.86	0.53	0.32	0.01	0.00	0.06	0.01	0.01
Hungary	0.10	1.00	0.99	0.99	0.81	0.72	0.96	0.32	0.40	0.56	0.14	0.32	0.67	0.94	0.88	0.85	0.22	0.23	0.17	0.01	0.06
Malta	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.04	0.05	0.01	0.01	0.01	0.01	0.01
Netherlands	0.51	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.68	0.59	1.00	1.00	1.00	1.00	0.97	0.97	0.99	0.92	0.93
Austria	0.97	1.00	1.00	1.00	0.99	0.98	1.00	0.98	0.89	0.96	0.07	0.31	0.96	0.99	0.99	0.97	0.72	0.61	0.76	0.18	0.40
Poland	0.24	0.90	0.97	0.85	0.35	0.22	0.59	0.19	0.03	0.12	0.06	0.10	0.84	0.52	0.57	0.53	0.21	0.33	0.33	0.04	0.09
Portugal	0.64	0.99	1.00	0.99	0.95	0.98	0.98	0.97	0.97	0.92	0.25	0.20	0.97	0.99	0.99	1.00	0.95	0.95	0.95	0.48	0.94
Romania	0.28	1.00	1.00	0.95	0.51	0.34	0.91	0.16	0.13	0.12	0.05	0.15	0.65	0.86	0.84	0.77	0.04	0.05	0.03	0.01	0.02
Slovenia	0.33	1.00	1.00	0.99	0.42	0.51	0.91	0.01	0.02	0.05	0.04	0.07	0.04	0.94	0.93	0.94	0.46	0.45	0.37	0.17	0.35
Slovakia	0.02	0.07	0.10	0.12	0.02	0.02	0.23	0.07	0.08	0.12	0.05	0.11	0.10	0.19	0.31	0.29	0.08	0.15	0.22	0.05	0.05
Finland	0.13	1.00	1.00	0.98	0.48	0.57	0.98	0.84	0.81	0.75	0.78	0.93	0.96	0.98	0.87	0.50	0.30	0.23	0.51	0.22	0.36
Sweden	0.25	1.00	1.00	1.00	0.79	0.45	0.79	0.72	0.53	0.67	0.26	0.47	0.86	1.00	0.98	0.96	0.71	0.54	0.50	0.25	0.25
United Kingdom	1.00	1.00	1.00	1.00	0.99	0.91	0.99	0.94	0.75	0.73	0.01	0.00	0.98	0.93	1.00	0.84	0.99	0.96	1.00	0.83	0.66
Norway	0.01	0.07	0.05	0.02	0.01	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.93	0.99	0.97	0.46	0.26	0.63	0.71	0.79	0.66
Macedonia	0.02	0.00	0.00	0.00	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.04	0.05	0.03	0.02	0.01	0.01	0.01
Serbia	0.01	0.08	0.07	0.04	0.04	0.05	0.05	0.03	0.03	0.01	0.00	0.00	0.01	0.80	0.91	0.90	0.94	0.93	0.89	0.81	0.93

Source: own calculations

Table 2. Posterior probability of the set $S_{0.005}$ (in columns order p)

Order $p \rightarrow$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Belgium	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.02	0.02	0.04	0.02	0.03	0.04	0.01	0.02
Bulgaria	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.02	0.01	0.02	0.01	0.01	0.01
Czech Republic	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01
Denmark	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.47	0.32	0.15	0.07	0.08	0.12	0.06	0.02
Germany	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
Estonia	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.00	0.00	0.00	0.01	0.02	0.02	0.02	0.02	0.01	0.02	0.02	0.02	0.02
Greece	0.05	0.09	0.09	0.14	0.06	0.19	0.04	0.03	0.02	0.04	0.00	0.00	0.86	0.97	0.99	0.99	0.97	0.98	0.98	0.86	0.84
Spain	0.02	0.01	0.02	0.02	0.02	0.01	0.02	0.01	0.02	0.03	0.02	0.02	0.02	0.04	0.07	0.06	0.03	0.04	0.03	0.02	0.02
France	0.02	0.50	0.29	0.40	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.10	0.09	0.31	0.18	0.38	0.45	0.15	0.76
Croatia	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.06	0.02	0.00	0.00	0.00	0.00	0.00
Italy	0.02	0.08	0.06	0.07	0.02	0.04	0.05	0.03	0.03	0.05	0.04	0.07	0.07	0.18	0.26	0.29	0.14	0.31	0.22	0.09	0.29
Cyprus	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.01	0.02	0.04	0.02	0.01
Latvia	0.02	0.03	0.03	0.03	0.02	0.02	0.03	0.02	0.02	0.02	0.01	0.01	0.04	0.08	0.10	0.16	0.07	0.11	0.20	0.12	0.20
Lithuania	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Luxembourg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hungary	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Malta	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.50	0.67	0.11	0.12	0.12	0.05	0.07
Netherlands	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.01	0.02	0.03	0.03	0.03	0.02	0.04	0.06	0.05	0.05
Austria	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Poland	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.00	0.00	0.00
Portugal	0.02	0.00	0.01	0.02	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.01	0.05	0.01	0.07	0.05	0.00	0.02
Romania	0.02	0.03	0.03	0.03	0.02	0.02	0.03	0.01	0.02	0.02	0.00	0.00	0.01	0.05	0.06	0.08	0.02	0.06	0.02	0.01	0.03
Slovenia	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Slovakia	0.02	0.05	0.06	0.08	0.02	0.01	0.13	0.04	0.05	0.08	0.04	0.07	0.07	0.09	0.15	0.15	0.06	0.08	0.14	0.04	0.04
Finland	0.02	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Sweden	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
United Kingdom	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01
Norway	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.14	0.09	0.03	0.01	0.05	0.04	0.04	0.01
Macedonia	0.02	0.00	0.00	0.00	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.04	0.05	0.03	0.02	0.01	0.01	0.01
Serbia	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00

Source: own calculations

REFERENCES

Bell W.R., Hillmer S.C. (1983) Modeling Time Series With Calendar Variation. Journal of the American Statistical Association, 78, 526 – 534.

Bell W.R., Martin D.E. (2004) Modeling time-varying trading day effects in monthly time series. In: ASA Proceedings of the Joint Statistical Meetings: Alexandria. American Statistical Association.

Cleveland W.S., Devlin S.J. (1980) Calendar Effect in Monthly Time Series: Detection by Spectrum Analysis and Graphical Methods. Journal of the American Statistical Association, 75, 487 – 496.

Cleveland W.S., Devlin S.J. (1982) Calendar Effects in Monthly Time Series: Modeling and Adjustment. Journal of the American Statistical Association, 77(379), 520 – 528.

Dagum E., Quenneville B., Sutradhar B. (1993) Trading-day variations multiple regression models with random parameters. International Statistical Review, 60, 57 – 73.

Hamilton J.D. (1994) Time Series Analysis. Princeton University Press, New Jersey.

Hurd H. (1989). Nonparametric time series analysis for periodically correlated processes. IEEE Trans. Inf. Theory, 35(2), 350 – 359.

Hurd H. (1991) Correlation theory of almost periodically correlated processes. Journal of Multivariate Analysis, 37, 24 – 45.

Ladiray D. (2012) Theoretical and real trading-day frequencies. In: Bell W.R., Holan S.H. and McErloy T.S. (ed.), Economic time Series: Modeling and Seasonality, 255 – 279.

Lenart Ł. (2013) Non-parametric frequency identification and estimation in mean function for almost periodically correlated time series. Journal of Multivariate Analysis, 115, 252 – 269.

Lenart Ł. (2015) Discrete spectral analysis: the case of industrial production in selected European countries. Dynamic Econometric Models, 15, 27 – 47.

Lenart Ł., Mazur B. (2016a) On Bayesian estimation of almost periodic in mean autoregressive models. Statistical Review, 63(3), 255 – 271.

Lenart Ł., Pipień M. (2013a) Almost periodically correlated time series in business fluctuations analysis. Acta Physica Polonica A, 123(3), 567 – 583.

Lenart Ł., Pipień M. (2013b) Seasonality revisited - statistical testing for almost periodically correlated processes. Central European Journal of Economic Modelling and Econometrics, 5, 85 – 102.

Lenart Ł., Pipień M. (2016b) Testing the common length of the business cycles with discrete spectral analysis and subsampling approach. Submitted to Argumenta Oeconomica.

Li T.H., Song K.S. (2002) Asymptotic analysis of a fast algorithm for efficient multiple frequency estimation. IEEE Transactions on Information Theory, 48(10), 2709 – 2720.

Priestley M.B. (1981) Spectral Analysis and Time Series. Academic Press, London.

Soukup R., Findley D. (1999) On the Spectrum Diagnostics Used by X-12-ARIMA to Indicate the Presence of Trading Day Effects After Modeling or Adjustment. Proceedings of the American Statistical Association, Business and Economic Statistics Section, 144 – 149.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII , No. 1, 2017, pp. 99 – 109

Received: 19.03.2017
Accepted: 05.04.2017

MACROECONOMIC DETERMINANTS OF INVESTMENT IN AGRICULTURE IN POLAND – DEMATEL METHOD

Danuta Miłaszewicz (ORCID: 0000-0001-6429-7331)
Kesra Nermend (ORCID: 0000-0002-4309-7434)
Faculty of Economics and Management
University of Szczecin, Poland
e-mail: dmilasz@wneiz.pl, kesra@wneiz.pl

Abstract: Investments in agriculture have a direct impact on the sector as well as on the economy in general. These effects are determined by many internal (microeconomic) and external (macroeconomic) factors. In the literature there are many studies on the influence of microeconomic factors on decisions regarding investments in agriculture holdings. Few authors, however, have dealt with macroeconomic conditionality for such decisions. The paper presents the possibility to apply the DEcision MAking Trial and Evaluation Laboratory (DEMATEL) method in examining the causal links between macroeconomic factors and investment in rural areas. Basing on the three independent experts' opinions referring to the analyzed relationships, we reveal direct and indirect links between the investigated variables.

Keywords: agriculture, investment, macroeconomic factors, causal relations, DEMATEL

INTRODUCTION

The challenge facing modern agriculture is to balance sustainable development with its increased effectiveness [Jain 2012]. Responding to this challenge, that requires considerable restructuring and investments in agriculture, may encourage the growth of businesses operating in this economic sector [Józwiak 2010]. Despite the fact that the share of agriculture in elementary macroeconomic categories is decreasing in relative terms, the sector, being one of the parts of food production and agribusiness, is of fundamental importance for the whole economy as it ensures the national food security [Babuchowska and Marks-Bielska 2015], [Kowalski 2009, 2010].

DOI: 10.22630/MIBE.2017.18.1.10 JEL: C44, O13

The range and character of agricultural investments give direction to the development trends in this sector and largely determine its economic situation, the changes in agricultural production and the transformations of economic units within the sector. Similarly to investing in other parts of the economy, agricultural investments trigger multiplier effects that enhance production in the economy in general. Therefore, the impact of investments in agriculture goes beyond the limits of the sector as such.

Rich literature discussing investments in Polish agriculture confirms their inherent importance to the growth of agricultural businesses. Their effects are mostly visible in production. They ensure technological advancement of farms, which, in turn, determines their economic situation [Kocira 2008; Zając 2012], they facilitate the development and modernisation of farms [Sobczyński 2011; Wójcicki and Rudeńska 2015], enhance their productivity, competitiveness and market power [Kisiel and Babuchowska 2013; Dziwulski 2013]. Along with savings and external transfers, the investments give a ground for the changes in technical relationships and in the productivity of agricultural production [Bezet-Jastrzębska and Rembisz 2015]. Moreover, the investments in agriculture bring effects outside the production as they improve animal welfare, environmental protection as well as food and work safety [Wasąg 2009; Grzelak 2015].

Similarly to the economy in general, the conditions for investments in agriculture are varied. They can be of internal character or result from the external situation [Gołębiowska 2010]. This means that investments are determined by a number of factors on the part of farmers (internal, endogenous, microeconomic) [Poczta and Siemiński 2009] and by factors that occur independently from them (external, exogenous, macroeconomic).

According to the literature, there are the following external determinant factors of agricultural investments:

- natural environment in which a given farm is operating and where its production is located [Zając 2012];
- the volume of disposable income, the supply of subsidised loans, commercial interest rates and the accessibility of EU funding [Sulewski 2005];
- market conditions (e.g. demand for new agricultural products, price stability on the produce market, stability and flexibility of the produce market institutions, prospects of the produce market), technological conditions (e.g. productivity and the quality of effects of new technological solutions) and financial conditions (e.g. credit restrictions or the financial market policies and institutions) [Kataria, Curttiss and Balmann 2012];
- macroeconomic variables, market conditions and financial standing of farms [Bórawski 2014];
- stages of the economic cycle, legal regulations concerning business activity, competitiveness and economy globalisation [Filipiak 2014];

- factors related with the macroeconomic and political situation, demographic pressure, institutional solutions and legal regulations [Kusz, Gędek, Ruda and Zając 2014];
- factors related with the demand for a given produce, expected and current prices (of produce), supply conditionality (costs to incur, the availability and cost of production factors), present and projected economic trends, systemic solutions (financial, economic and institutional), economic policies (agricultural, fiscal and monetary in particular), inflation and interest rates that determine capital costs, the level of economic openness, legal regulations, insurance and consulting organisations and institutions, and, finally, the requirements concerning environmental protection and animal welfare [Thijssen 1996, Kusz 2012, Kusz and Gędek, Kata 2015];
- a pro-investment impulse provided by the Common Agricultural Policy funds whose importance has been growing since the Poland's pre-accession period [Domańska and Felczak 2014, Czubak 2015], and which offer investment support from public funds [Kusz and Gędek 2015].

Some of the above listed external factors are the ones that are specific for agriculture, but the majority are the general macroeconomic determinants of investment [Miłaszewicz 2007, p. 64-79].

The investment decisions made by farmers are a resultant of both exogenous and endogenous factors. The present article discusses the latter as its purpose is to present the use of the DEcision MAking Trial and Evaluation Laboratory (DEMATEL) method to investigate the cause-and-effect relationship between agricultural investments and their macroeconomic determinant factors. The proposed algorithm of a multi-criteria assessment and ordering of the interplay among the examined variables helps to define the structure and hierarchy of the factors that are built according to the independent experts' opinions.

MATERIAL AND METHODS

The multi-criterial method referred to as DEMATEL (DEcision MAking Trial and Evaluation Laboratory) was proposed by Gabus and Fontela [1972] in the 1970s with a view to detect the causal relationships between the global and regional economic and social problems. As such it can be used for determining the relevance of factors determining investments in agriculture. It is a method consisting of several steps, the first of which is the selection of factors to be analyzed [Wawrzynek 2014].

The factors used in the DEMATEL-based analysis were selected on the basis of theoretical deliberations in the previous part of this paper. The above mentioned factors not only determine the internal determinants of agricultural investments, but they are interrelated as well. Out of numerous external determinants described in the literature (W1) the authors chose 13 potential variables: W2 – the possibility to change the use of land, W 3 – domestic economic situation (in general and in agricultural industry), W4 – the price of land; W5 – inflation, W 6 – unemployment rate, W7 – legal regulations (referring to trading in agricultural land, environment protection, etc.), W8 – monetary concessions (e.g. credit facilities for farmers), W9 – fiscal concessions (e.g. tax credits, tax exemptions, paying tax in instalments or fuel subsidies), W10 – EU subsidies, W11 – profitability of agricultural production, W12 – available markets, W13 – support institutions, W14 – land supply. The use of the DEMATEL method allows their mutual influence and a cause-and-effect relationship between the variables and agricultural investments to be shown.

Another step in the DEMATEL method is the evaluation of the examined factors. The experts' opinions on the relations between these factors are collected. In the subsequent step, on the basis of the experts' opinions a graph illustrating these cause and effect relationships serves as a starting point for calculations. The arrows in the graph show the relations between factors or events, simultaneously indicating the direction of impact. There is a variety of scales to define power this relations. In this paper the following scale is applied [Kobryń 2014]:

1. Non-influence.
2. Low influence.
3. High influence.
4. Very high influence.

On the basis of this graph a matrix of direct impact B is produced. It is a square matrix where all the entries on the main diagonal always equal zero. They denote the influence of a given factor on itself. The other entries are derived from the graph. The element with an index where i is the influence received and j the influence given represents the power of this influence. The direct influence matrix is normalised [Ginda and Maślak 2012]:

$$B' = \frac{1}{\lambda}B, \tag{1}$$

then [Ginda and Maślak 2012]:

$$\lambda = \max\left\{\max_j \sum_{i=1}^{n} b_{i,j}\,; \max_i \sum_{j=1}^{n} b_{i,j}\right\}. \tag{2}$$

where $b_{i,j}$ is an element of the direct influence matrix and *n* is its size.

The influence matrix B' describes only the direct influence. However, the relationships between the factors can be of indirect nature as well. The matrix B' does not contain information about such influence. This information is delivered by the total influence matrix T that is a sum of the matrix B' and the indirect influence matrix $\hat{B}$ [Kobryń 2014]:

$$T = B' + \hat{B}. \tag{3}$$

The indirect influence matrix $\hat{B}$ is produced by summing up direct influence sub-matrices B' raised to *i*-th power, where *i* =1, 2, 3,… [Ginda and Maślak 2012]:

$$\hat{B} = B'^2 + B'^3 + \cdots = \sum_{i=2}^{\infty} B'^i. \quad (4)$$

Substituting $\hat{B}$ to Formula (3):

$$T = B' + B'^2 + B'^3 + \cdots = \sum_{i=1}^{\infty} B'^i, \quad (5)$$

which equals (Ginda and Maślak 2012):

$$T = B'(I - B')^{-1}, \quad (6)$$

where I is an identity matrix.

The indirect influence can be derived from the Formula [Kobryń 2014]:

$$\hat{B} = B'^2(I - B')^{-1}. \quad (7)$$

When the collective analysis of all influences is necessary, for each of the factors we calculate the significance indices [Kobryń 2014]:

$$t_i^{+} = \sum_{j=1}^{n} t_{i,j} + \sum_{j=1}^{n} t_{j,i}\,, \quad (8)$$

and the influence indices (Kobryń 2014):

$$t_i^{-} = \sum_{j=1}^{n} t_{i,j} - \sum_{j=1}^{n} t_{j,i}. \quad (9)$$

The significance index describes the general participation of the object in a network of influence. The higher the index, the stronger a given factor influences the remaining factors and/or the stronger the remaining factors influence this factor. The influence index shows if the influence given by a factor is stronger than the influence it receives.

RESULTS

For the purpose of the study into the influence of macroeconomic factors on investments, three experts in various fields of economic science, such as management, finances and economics, were asked to share their opinions. The respondents' opinions on the influence power of individual factors were expressed on the scale from 0 to 3. For each expert a sub-matrix of direct influence between macroeconomic factors was produced (Table 1).

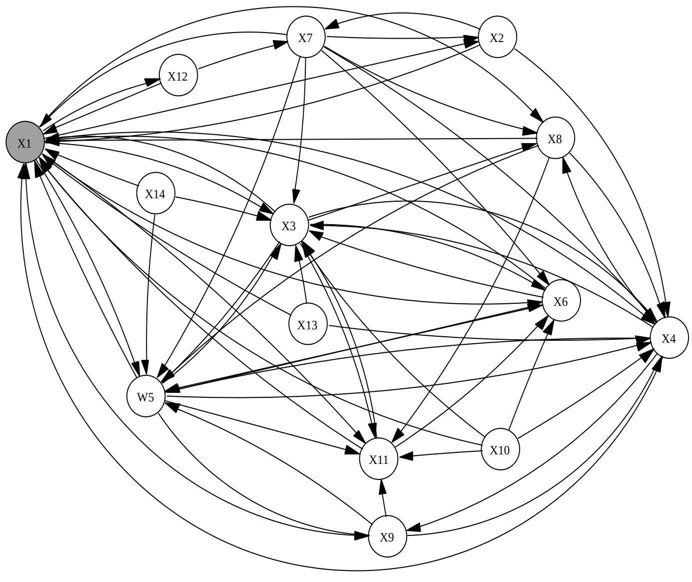
First, a partial matrix of the direct impact of the analysed factors was made for each expert. Then, the median defining the impact power was calculated on the basis of the matrix values. In the next step, the matrix of direct impact was constructed whose representation is a cause-and-effect diagram (Figure 1).

Table 1. Sub-matrix of direct influence

Varia-bles	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	W_{11}	W_{12}	W_{13}	W_{14}
W_1	0	2	3	2	1	3	0	1	0	0	2	1	0	0
W_2	2	0	0	3	0	0	3	0	0	0	0	0	0	0
W_3	3	0	0	2	2	2	0	1	0	0	2	0	0	0
W_4	2	0	1	0	3	0	0	2	1	0	0	0	0	0
W_5	2	0	2	3	0	2	0	0	2	0	2	0	0	0
W_6	2	0	3	0	2	0	1	0	0	0	0	0	0	0
W_7	3	3	2	3	1	1	0	2	0	0	0	0	0	0
W_8	3	0	0	2	1	0	0	1	0	0	1	0	0	0
W_9	2	0	0	1	2	0	0	0	0	0	1	0	0	0
W_{10}	2	0	2	2	0	1	0	0	0	0	2	0	0	0
W_{11}	3	0	2	0	0	1	0	0	0	0	0	0	0	0
W_{12}	3	0	0	0	0	0	1	0	0	0	0	0	0	0
W_{13}	1	0	1	1	0	0	0	0	0	0	0	0	0	0
W_{14}	3	0	1	0	1	0	0	0	0	0	0	0	0	0

Source: own analysis

Figure 1. Cause-and-effect diagram of analysed factors' impact on investments



Source: own analysis

Having normalised the matrix of direct impact and the constructed on that basis the matrix of indirect impact, a normalised matrix of the total impact of the analysed factors was made (Table 2).

Table 2. The normalized total influence matrix

Varia -bles	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	W_{11}	W_{12}	W_{13}	W_{14}
W_1	0.05	0.07	0.13	0.09	0.06	0.12	0.01	0.05	0.01	0	0.08	0.03	0	0
W_2	0.09	0.02	0.02	0.12	0.02	0.02	0.10	0.02	0.01	0	0.01	0	0	0
W_3	0.13	0.01	0.04	0.09	0.09	0.09	0	0.05	0.01	0	0.08	0	0	0
W_4	0.09	0.01	0.05	0.03	0.11	0.02	0	0.07	0.04	0	0.02	0	0	0
W_5	0.10	0.01	0.09	0.12	0.03	0.09	0	0.01	0.07	0	0.08	0	0	0
W_6	0.09	0.01	0.12	0.03	0.08	0.02	0.03	0.01	0.01	0	0.02	0	0	0
W_7	0.14	0.11	0.09	0.14	0.06	0.06	0.01	0.08	0.01	0	0.02	0	0	0
W_8	0.12	0.01	0.02	0.08	0.05	0.02	0	0.04	0.01	0	0.05	0	0	0
W_9	0.08	0.01	0.02	0.05	0.07	0.02	0	0.01	0.01	0	0.04	0	0	0
W_{10}	0.09	0.01	0.09	0.08	0.02	0.05	0	0.01	0	0	0.08	0	0	0
W_{11}	0.11	0.01	0.08	0.02	0.01	0.05	0	0.01	0	0	0.01	0	0	0
W_{12}	0.11	0.01	0.02	0.01	0.01	0.01	0.03	0.01	0	0	0.01	0	0	0
W_{13}	0.04	0	0.04	0.04	0.01	0.01	0	0.01	0	0	0.01	0	0	0
W_{14}	0.11	0.01	0.05	0.02	0.04	0.02	0	0.01	0	0	0.01	0	0	0

Source: own analysis

Basing on the matrix of total impact a conclusion can be drawn that the agricultural investments (W1) are mostly affected by legal regulations (concerning trading in agricultural land, environmental protection, etc.) (W7), domestic economic situation (in general and in agricultural industry) (W3) and monetary concessions (e.g. credit facilities for farmers) (W8). The majority of the analysed factors have a relatively strong impact on agricultural investments (W1) with the exception of support institutions (W13). Agricultural investments (W1) strongly affect unemployment rates (W6). Similarly, legal regulations (referring to trading in agricultural land, environment protection, etc.) (W7) have a considerable influence on the price of land (W4). Moreover, unemployment rates (W6) significantly influence domestic economic situation (in general and in agricultural industry) (W3), while inflation (W5) and the possibility to change the use of land (W2) have a strong effect on the price of land (W4).

The aforementioned calculations have revealed a weak indirect impact of available markets (W12) and land supply (W14) on agricultural investments (W1) despite the fact that the experts indicated a strong relationship among these variables. A similar discrepancy occurs in reference to the impact of legal regulations (concerning trading in agricultural land, environmental protection, etc.) (W7), the possibility to change the use of land (W2) and the price of land (W4) on inflation (W5), as well as to the relationship between the possibility to change the use of land (W2) and legal regulations (W7). The experts considered these impacts weak. The study, however, showed a strong indirect relationship between legal regulations (W7) and agricultural investments (W1) and land prices (W4). The matrix of indirect impact also reveals that there is a strong indirect impact of inflation (W5) on agricultural investments (W1).

Basing on the total influence matrix a matrix of significance indices $\boldsymbol{T}^{+}$ and influence indices $\boldsymbol{T}^{-}$ of the analysed variables are determined (Table 3).

Table 3. Significance indices $\boldsymbol{T}^{+}$ and influence indices $\boldsymbol{T}^{-}$

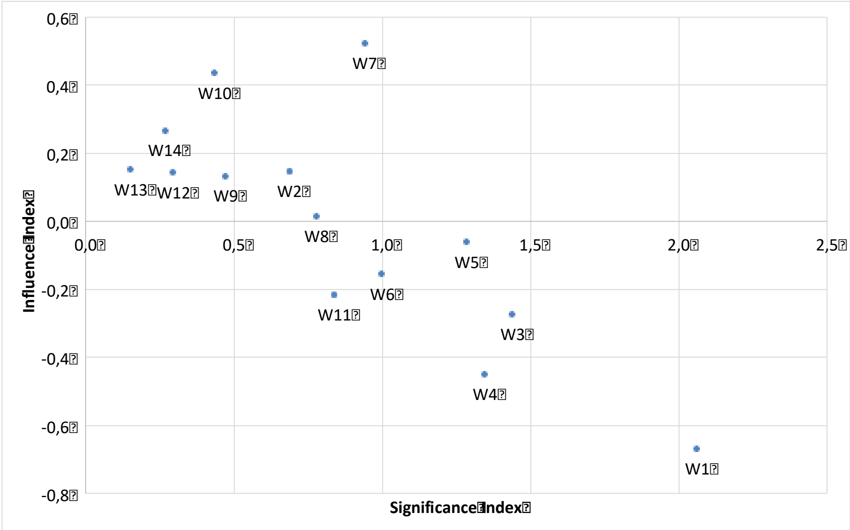
Factors	$\boldsymbol{T}^{+}$	$\boldsymbol{T}^{-}$
W_1	2.06	-0.67
W_2	0.69	0.15
W_3	1.44	-0.27
W_4	1.35	-0.45
W_5	1.28	-0.06
W_6	1.00	-0.15
W_7	0.94	0.52
W_8	0.78	0.02
W_9	0.47	0.13
W_{10}	0.44	0.44
W_{11}	0.84	-0.21
W_{12}	0.30	0.14
W_{13}	0.15	0.15
W_{14}	0.27	0.27

Source: own analysis

The above results give ground for the conclusion that there is a strong interrelation between the investments in agriculture and other factors. The value T_1^- equal -0.67 implies that mainly the analysed factors have an impact on the agricultural investments. High values of T_i^- for legal regulations (concerning trading in agricultural land, environmental protection, etc.) (W7) and EU subsidies (W10) mean that these two factors have stronger influence on the remaining variables that the remaining variables have on them. Moreover, high values of T_i^+ for agricultural investments (W1), domestic economic situation (in general and in agricultural industry) (W3), the price of land (W4) and inflation (W5) indicate strong relationships between these factors and the remaining ones.

The above findings are confirmed by the map of the total impact (Figure 2) among the analysed factors. A conclusion can be made that the position of W1 in the lower part of the diagram (a negative value of the impact indicator at -0.67) means that the impact of the remaining factors on the agricultural investments is much stronger than the reverse relationship. Because the value of the importance indicator is high (2.06), the interaction between W1 and the remaining determinants of agricultural investments is very strong.

Figure 2. Map of impact of analysed variables – final analysis results



Source: own analysis

CONCLUSION

In theoretical considerations about the economy it is possible to eliminate certain phenomena (factors) and relationships by adopting the rule of *ceteris paribus*. In the real economic world none of the phenomena occur in isolation and it is their specific feature that they influence one another in a cause-and-effect relationship. This complexity of the economic reality hinders the observation of individual phenomena, their investigation and analysis as well as impedes their modification by means of economic policies. The obstruction to the analysis can be overcome if we have adequate tools at our disposal. One of them is DEMATEL - an uncomplicated method based on simple mathematical transformations which is an effective tool for identifying the cause-and-effect relationships among selected main factors of direct and indirect impact within a certain process or phenomenon.

The results of applying this method in the analysis of the impact of exogenous factors on agricultural investments confirm the research value of the DEMATEL method for economics. The fact that it has been based on the opinions of experts from different areas of economics makes it possible to include factors that are evaluated from different points of view. Based on the relevance of the cause-and-effect relationships among any number of factors, the findings obtained by means of the DEMATEL method can be used in decision-making processes in the analysed economic sector, when creating economic policies influencing these decisions as well as in other areas of theoretical and applied economics.

REFERENCES

Babuchowska K., Marks-Bielska R. (2015). Rola czynnika ludzkiego w unowocześnianiu polskiego rolnictwa na przykładzie producentów mleka. Acta Universitatis Lodziensis, Folia Geographica Socio-oeconomica, 20, 45–57.

Bezat-Jarzębowska A., Rembisz W. (2015). Wprowadzenie do analizy inwestycji, produktywności, efektywności i zmian technicznych w rolnictwie. Instytut Ekonomiki Rolnictwa i Gospodarki Żywnościowej – Państwowy Instytut Badawczy, Warszawa.

Bórawski P. (2014) Zróżnicowanie inwestycji w gospodarstwach mlecznych, Stowarzyszenie Ekonomistów Rolnictwa i Agrobiznesu, Roczniki Naukowe, 16 (2), 27–32.

Czubak W. (2015) Nakłady inwestycyjne w rolnictwie polskim w kontekście wdrażania Wspólnej Polityki Rolnej Unii Europejskiej. [In:] A. Czyżewski, B. Klepacki (Eds) Problemy rozwoju rolnictwa i gospodarki żywnościowej w pierwszej dekadzie członkostwa Polski w Unii Europejskiej. PTR, Warszawa.

Domańska T., Felczak T. (2014) Czynniki różnicujące wydatki inwestycyjne w gospodarstwach rolniczych. Polityki Europejskie, Finanse i Marketing, 11 (60), 46–55.

Dziwulski M. (2013). Aktywność inwestycyjna gospodarstw rolnych w Polsce w 2010 roku z uwzględnieniem ich wielkości ekonomicznej. Zeszyty Naukowe Uniwersytetu Szczecińskiego, Finanse. Rynki finansowe. Ubezpieczenia, 59, 481–490.

Filipiak T. (2014) Nakłady inwestycyjne a wyniki ekonomiczne gospodarstw warzywnych. Stowarzyszenie Ekonomistów Rolnictwa i Agrobiznesu, Roczniki Naukowe, 16 (6), 109-114.

Gołębiewska B. (2010) Kierunki podejmowanych działań inwestycyjnych w gospodarstwach rolnych o zróżnicowanych powiązaniach z otoczeniem. Roczniki Naukowe Rolnictwa, 97 (4), 60-68.

Grzelak A. (2015) Determinanty zasobowe procesów reprodukcji majątku gospodarstw rolnych prowadzących rachunkowość rolną (FADN). Stowarzyszenie Ekonomistów Rolnictwa i Agrobiznesu, Roczniki Naukowe, 17 (2), 69-74.

Jain H.K. (2012) Transition to Twenty-First Century Agriculture: Change of Direction. Agricultural Research, 1 (1), 12–17.

Józwiak W. (2010) Polskie gospodarstwa rolne w pierwszych latach członkostwa kwestie efektywności i konkurencyjności, IERiGż, Warszawa.

Kataria K., Curtiss J., Balmann A. (2012) Drivers of Agricultural Physical Capital Development, Theoretical Framework and Hypotheses. Factor Markets Working Paper, no. 18.

Kisiel R., Bauchowska K. (2013) Nakłady inwestycyjne w gospodarstwach rolnych – ujęcie regionalne. Roczniki Ekonomii Rolnictwa i Rozwoju Obszarów Wiejskich, 100 (1), 62-69.

Kocira S. (2008) Wpływ technicznego uzbrojenia procesu pracy na nadwyżkę bezpośrednią w gospodarstwach rodzinnych. Inżynieria Rolnicza, 4 (102), 375–380.

Kowalski A. (2009) Czynniki wpływające na kierunki rozwoju rolnictwa w zmieniającym się świecie. [In:] Przyszłość sektora rolno-spożywczego i obszarów wiejskich. I Kongres Nauk Rolniczych Nauka – Praktyce.

Kowalski A. (2010) Wstęp. [In:] Kowalski A. (red.), Ekonomiczne i społeczne uwarunkowania rozwoju polskiej gospodarki żywnościowej po wstąpieniu Polski do Unii Europejskiej (Synteza). IERiGŻ, Warszawa.

Kusz D. (2012) Egzogeniczne i endogeniczne uwarunkowania procesu modernizacji rolnictwa. Roczniki Ekonomii Rolnictwa i Rozwoju Obszarów Wiejskich, 99 (2), 53–67.

Kusz D., Gędek S. (2015) Egzogeniczne i endogeniczne uwarunkowania inwestycji w rolnictwie w Polsce. Stowarzyszenie Ekonomistów Rolnictwa i Agrobiznesu, Roczniki Naukowe, 17 (3), 237–242.

Kusz D., Gędek S., Kata R. (2015) Egzogeniczne uwarunkowania inwestycji w rolnictwie polskim. [In:] A. Czyżewski, B. Klepacki (Eds), Problemy rozwoju rolnictwa i gospodarki żywnościowej w pierwszej dekadzie członkostwa Polski w Unii Europejskiej. Polskie Towarzystwo Ekonomiczne, Warszawa.

Kusz D., Gędek S., Ruda M., Zając S. (2014) Endogenous Determinants of Investments in Farms of Selected Countries of Central and Eastern Europe. Scientific Papers, series Management, Economic Engineering in Agriculture and Rural Development, 14 (3), 110–119.

Miłaszewicz D. (2007) Problem wyboru narzędzi realizacji roli państwa w procesach inwestycyjnych w Polsce w okresie przemian. Wydawnictwo Naukowe Uniwersytetu Szczecińskiego, Szczecin.

Poczta W., Siemiński P. (2009) Sytuacja ekonomiczna gospodarstw rolnych w warunkach WPR UE – próba prognozy do 2013 roku. Journal of Agribusiness and Rural Development, 3 (13), 174-175.

Sobczyński T. (2011) Ocena możliwości rozwojowych gospodarstw rolniczych Unii Europejskiej na podstawie inwestycji netto w latach 1998-2008. Folia Pomeranae Universitatis Technologiae Stetinensis, Oeconomica, 65, 145-156.

Sulewski P. (2005) Inwestycje a wyniki ekonomiczne gospodarstw indywidualnych. Roczniki Naukowe SERiA, 7 (1), 233–238.

Thijssen G. (1996) Farmers' Investment Behavior: An Empirical Assessment of Two Specifications of Expectations. American Journal of Agricultural Economics, 78 (1), 166–174.

Wasąg Z. (2009) Wpływ dofinansowania unijnego na modernizacje techniczną gospodarstw rolnych w Polsce. Inżynieria Rolnicza, 8 (17), 267–273.

Wójcicki Z., Rudeńska B. (20150 Kierunki modernizacji wybranych gospodarstw rodzinnych. Problemy Inżynierii Rolniczej, 2 (88), 37–46.

Zając D. (2012) Inwestycje jako czynnik modernizacji gospodarstw rolnych z działalnością pozarolniczą. Nierówności Społeczne a Wzrost Gospodarczy, 26, 284–204.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII, No. 1, 2017, pp. 110 – 119
Received: 11.03.2017
Accepted: 05.04.2017

GERMAN FISCAL AUSTERITY EFFECTS ON INVESTMENTS AND EXPORTS IN THE CENTRAL AND EASTERN EUROPEAN COUNTRIES

Victor Shevchuk (ORCID: 0000-0002-5937-3333)
Faculty of Physics, Mathematics and Computer Science
Cracow University of Technology, Poland
e-mail: vshevchuk@pk.edu.pl
Roman Kopych
Department of International Economic Relations
Ivan Franko Lviv National University, Ukraine
e-mail: kopych_r@bigmir.net

Abstract: Fiscal austerity in Germany used to be blamed for a stagnant growth in the European countries. However, it seems not to be the case for the Central and Eastern European (CEE) countries. As it is established on the basis of vector error-correction model (VECM) estimates for quarterly series over the 2002–2015 period, a positive non-Keynesian spillover from the fiscal austerity in Germany to output in the CEE countries is realized mainly through an increase in investments, with the export channel being rather ambiguous across countries. Following an improvement in the German budget balance, there is a decrease in exports measured as percent of GDP in 6 out of 10 CEE countries. Depreciation of the real exchange rate (RER) is found for countries with exchange rate flexibility, while the opposite effect of RER appreciation is observed in countries with a fixed exchange rate arrangement. It is possible to argue that spillover effects of German austerity on the CEE countries are dominated by capital flows or confidence measures while foreign demand or relative price channels are rather weak.

Keywords: fiscal spillovers, Germany, Central and Eastern European countries, investments, exports

DOI: 10.22630/MIBE.2017.18.1.11
JEL: C32, O24

INTRODUCTION

It is quite common to explain recent economic problems in the European countries by the deficit reduction policy (usually referred to as fiscal “austerity”) in Germany, as the budget surplus had been substantial over the 2011–2015 period. For example, Bellofiore [2013] claims that “the German self-defeating obsession for fiscal austerity decisively drove the area into a double-dip recession”. For industrial countries, there is an argument that an increasing propensity to save combined with a decline in propensity to invest leads to the so-called secular stagnation, with a declining equilibrium real interest rate, a tendency for lower bounds on interest rates, and a consequent persistence of inadequate demand leading to slow growth, sub-target inflation, and excessive unemployment [Eggertson et al. 2016]. On the other hand, fiscal austerity is justified on the grounds of lowering risk in a public debt-ridden environment [Müller 2014].

Both theoretical and empirical arguments are rather ambiguous. Although the predictions of conventional restrictionary fiscal austerity spillovers seem to prevail in empirical studies for the European countries [Alesina et al. 2015; Beetsma et al. 2006; Ivanova and Weber 2011], there is no evidence that it is attained due to a significant RER appreciation and a crowding out of net exports, as it is implied by the Keynesian concept of a cross-border fiscal spillover. For the CEE countries, Crespo Cuaresma et al. [2011] obtain that the budget deficit in Germany has an expansionary effect on output in Hungary and Poland, while being restrictionary for the Czech Republic, Slovakia and Slovenia. However, Shevchuk and Kopych [2016] find that fiscal austerity in Germany contributes to output growth in seven CEE countries. Both studies do not answer the question of what are the mechanisms of cross-border fiscal spillovers in general and reasons for likely cross-country differences in response to foreign fiscal shocks in particular.

The objective of our paper is to provide empirical evidence on the relative price, investment and export channels for German fiscal austerity spillovers to 10 CEE countries, with a focus on potential differences between exchange rate regimes. The remainder of the paper proceeds as follows. Section 2 provides a brief review of the theoretical issues regarding international fiscal spillovers. In Section 3, data and statistical methodology are presented. Section 4 contains the econometric estimates of the German budget balance effects upon the RER, investments and exports of ten CEE economies. Section 5 concludes.

THEORETICAL ISSUES

Traditional analysis based on a two-country Mundell–Fleming model, for instance McCallum [1996], implies that fiscal austerity at home leads to a decrease in the aggregate demand and a negative fiscal spillover. However, a likely decrease in the interest rate, at least in the short-run, can bring about capital outflows and influence aggregate demand in the acceptor country through movements in the exchange rate or money supply. As a likely exchange rate appreciation under a floating exchange rate regime is expected to be restrictionary due to the foreign trade channel, monetization of capital inflows under a fixed exchange rate regime can be expansionary abroad. There is empirical evidence that the fiscal multiplier is larger under a fixed exchange rate system [Born et al. 2013]. Boughton [2001] demonstrates that the negative impact of the RER appreciation on output is neutralized if capital inflows contribute directly to investments.

More recent open economy models emphasize the role of expectations, supply-side effects and intertemporal optimization. In the New Keynesian models with forward-looking expectations, fiscal austerity is associated with such a mix of price and wage cuts that increase output abroad despite unfavourable relative price developments in the context of a monetary union [Barbier-Gaucard et al. 2015]. Using a two-good, two-country real business cycle model, Corsetti et al. [2010] find that a decrease in government spending brings about an increase in private consumption, investment, exports and output abroad, but these effects are reversed in the case of anticipated spending reversal. Considering demand- and supply-side effects of fiscal policies in a two-country model with the Phillips curve, Bénassy-Quéré [2006] obtains that fiscal austerity spillovers are generally positive if the central bank does not accommodate the fiscal shock..

Besides investment content of capital inflows or structural effects, fiscal austerity can be justified when public debt and sovereign risk are high [Müller 2014]. Indeed, there is evidence that fiscal multipliers are negative in the high-debt countries [Ilzetzki et al. 2013]. If austerity in Germany reduces uncertainties related to the sovereign debt in the Euro area, it can decrease sovereign borrowing costs and thus stimulate aggregate demand.

Recently, the same contradictory arguments have emerged in respect to the zero lower bound (ZLB) on the nominal interest rates. Within the framework of a New Keynesian model with endogenous capital accumulation, Johannsen [2014] argues that uncertainty about expansionary fiscal policy can cause large declines in consumption, investment, and output under ZLB. It confronts the arguments by Christiano et al. [2011] that the government-spending multiplier can be much larger than one when the ZLB on the nominal interest rate binds.

Using a textbook IS-MP model, Eggertson et al. [2016] demonstrate that secular stagnation resulting from a situation in which the desired savings at full employment outpace desired investment could be transmitted abroad, as capital inflows bring about the RER appreciation and 'crowding out' of exports. A substantial increase in the budget deficit abroad (i.e. in Germany) eliminates the possibility of a secular stagnation through an increase in the interest rate and subsequent reverse in the capital flows thus yielding positive externalities for trading partners. The effect is strengthened by the RER appreciation abroad.

Regardless of risk and ZLB considerations, there is an issue of the balance-of-payments adjustment following changes in the fiscal policy. Bellafiore [2013] compares present situation in the Eurozone with the neo-mercantilist model of the late 1940s and the persistent German surpluses. It is suggested that the policy of temporary but substantial increases in ‘productive’ government deficits financed by new money should be implemented in order to achieve a higher level of productivity within the system. However, it is not ruled out that the German trade surpluses would be shrunk by fiscal austerity, not fiscal expansion, as it is implied by a New Keynesian model with endogenous terms of trade and habit persistence in consumption [Cardi and Müller 2011]. If so, it is likely that German fiscal austerity would be associated with a higher demand for imports thus leading to an increase in exports abroad.

DATA AND STATISTICAL METHODOLOGY

Our empirical model is estimated for ten CEE economies, using variables of the cyclically adjusted German budget balance (in percent of GDP), BG_t, the real nominal effective exchange rate (index, 2010=100), RER_t, investments and exports (in percent of GDP), I_t and X_t, respectively. The use of cyclically adjusted budget balance in Germany is motivated by a purpose to filter out effects of the business cycle. A real depreciation means that RER_t goes up, while a real exchange rate appreciation means that RER_t goes down. The crisis dummy, $CRISIS_t$, controls for crisis developments of the 2008–2009 period, taking the value 1 from 2008Q3 to 2009Q4 and 0 otherwise.

All data come from the Eurostat and IMF *International Financial Statistics* online databases. The estimation samples for the individual economies are as follows: 2002Q1 to 2014Q4 for Bulgaria, the Czech Republic, Hungary, Poland, Romania, Slovakia, 2002Q1 to 2009Q4 for Slovenia, 2002Q1 to 2011Q4 for Estonia and Lithuania, 2002Q1 to 2013Q4 for Latvia, depending upon availability of time series data on RER. The Czech Republic, Hungary, Poland and Romania can be classified as countries with substantial exchange rate flexibility, while Bulgaria, Slovakia and the Baltic States have been maintaining different kind of fixed exchange rate arrangements.

The augmented Dickey-Fuller (ADF) unit root test is used for checking the orders of integration of the three times series, with a trend being included for all time series and autoregressive lags being chosen according to the Akaike information criterion in the corresponding VAR model. The ADF tests show that all the variables included in the analysis are I(1), although it is only at the 5% significance level for the RER for Bulgaria, Slovakia, Latvia and Lithuania and at the 10% level for Estonia (Table 1). In order to investigate the cointegration rank of two separate autoregressive models, with investments and exports respectively, the Johansen Trace test is used. The hypothesis of at least one cointegration equation cannot be rejected at the 5% level for all countries, while $r = 1$ is likely to be the case in models with either investments, or exports.

Table 1. Test of data characteristics

Country	RER_t		I_t		X_t	
	level	FD	level	FD	level	FD
(a) unit root test (augmented Dickey-Fuller test)						
Czech Republic	–1.90	–4.68***	–1.73	–7.33***	–0.11	–6.84***
Hungary	–0.92	–6.81***	–1.45	–7.33***	–0.18	–6.81***
Poland	0.55	–5.99***	–1.21	–6.18***	–1.81	–6.07***
Romania	–1.80	–6.48***	–1.64	–7.01***	–0.88	–7.81***
Slovenia	0.42	–6.32***	0.06	–5.80***	–2.26	–4.89***
Bulgaria	–2.02	–3.41**	–1.63	–6.78***	–1.64	–5.88***
Slovakia	–0.45	–5.27**	–2.79	–8.54***	–1.76	–5.94***
Estonia	–1.65*	–2.45*	–1.38	–8.65***	–2.30	–7.83***
Latvia	–0.90	–6.20**	–1.60	–6.61***	–1.78	–4.97***
Lithuania	–2.04	–7.25**	–1.37	–7.38***	–1.79	–6.22***
	$r \le 0$		$r \le 1$		$r \le 2$	
(b) test for co-integration (Trace test)						
Czech Republic	56.31** (54.08)		29.66* (35.19)		14.83 (20.26)	
Hungary	61.98*** (54.08)		33.41* (35.19)		13.26 (20.26)	
Poland	80.40*** (63.67)		51.94*** (42.92)		25.05* (25.87)	
Romania	64.85*** (54.08)		30.73 (35.19**)		13.10 (20.26)	
Slovenia	56.35** (54.08)		34.49* (35.19)		18.69* (20.26)	
Bulgaria	48.90*** (40.17)		21.12 (24.28)		4.10 (12.52)	
Slovakia	75.97*** (47.85)		43.50*** (29.79)		20.37*** (16.49)	
Estonia	65.69** (54.08)		35.17* (35.19)		18.74* (20.26)	
Latvia	75.06*** (54.08)		32.20* (35.19)		20.26 (20.26)	
Lithuania	65.99*** (47.85)		29.33* (29.79)		8.42 (16.49)	

Note: numbers in parentheses are critical values for relevant r (rank); *, **, *** imply statistical significance at the 10, 5 and 1% level, respectively; FD is for first differences.
Source: own calculations

In order to account for the cointegration properties of the variables, the VECM with cointegration rank $r < k$ and p lags is used as follows:

$$\Delta \mathbf{y}_t = \alpha\beta' \mathbf{y}_{t-1} + \Gamma_j \Delta \mathbf{y}_{t-p+1} + \mathbf{D}_t + \varepsilon_t, \tag{1}$$

where $\mathbf{y}_t$ is a $k \times 1$ vector of endogenous variables, α is a $k \times r$ matrix of adjustment coefficients, β is a $k \times r$ contegration matrix, Γ_j is a $k \times k$ short run coefficient matrix for $j = 1, ..., p-1$, $\mathbf{D}_t$ is a vector of exogenous variables, ε_t is a white noise error vector.

In the analysis a four variable VECM $y_t = (BG_t, RER_t, I_t, X_t)'$ is used. Assuming that there is a VEC($p-1$) representation for a VAR(p) process containing cointegrated variables, the order p is chosen so that no residual autocorrelation is left in the corresponding VAR model. The resulting lag lengths amount to 2 in the case of Bulgaria, the Czech Republic, Slovenia and Estonia, to 3 for Hungary, Poland, Lithuania, and to 4 for Slovakia and Latvia. Most of models include a constant and a linear trend as deterministic terms. Besides a dummy $CRISIS_t$, the London Interbank Offer Rate (LIBOR) is used as an independent variable to control for the international financial market conditions.

Unlike study of German fiscal shock spillovers by Crespo Cuaresma et al. [2011] that allow for cointegration in a seven variable VAR in levels form, our approach has several advantages by consistent estimation of cointegration relations in a four variable VECM. As demonstrated by Phillips [1998], the VEC specification significantly improves impulse responses even for short horizons. The cointegration relations provide identification restrictions that potentially make it easier to distinguish between permanent and transitory effects.

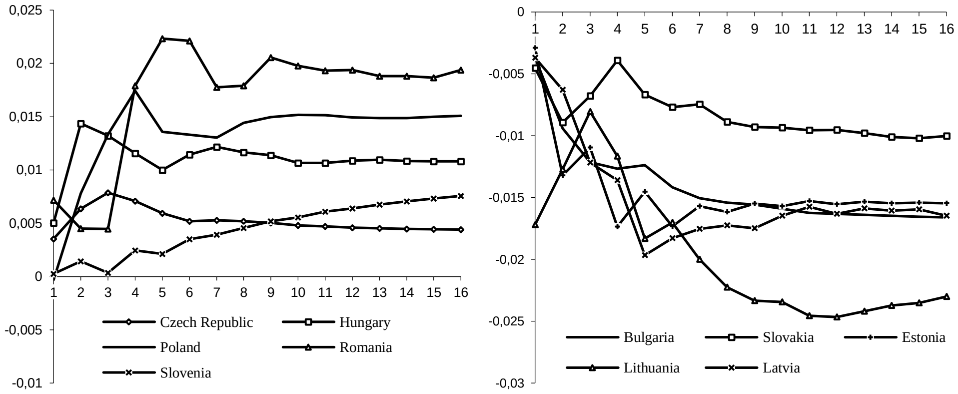
EMPIRICAL RESULTS

The impulse responses for the VECM regarding dynamic effects of an exogenous increase in the Germany's budget balance upon the RER, investments and exports of ten CEE countries are presented in Figure 2 to 4, respectively (estimation results can be obtained on request from the authors). On the vertical axes, the RER is measured in log-level (Figure 1), while investments (Figure 2) and exports (Figure 3) are measured in percent of GDP. The horizontal axe measures time in quarter units.

The response of relative prices to the German austerity shock is asymmetrical across exchange rate regimes (Figure 1). Depreciation of the RER is found for countries with exchange rate flexibility (or floaters), while the opposite effect of RER appreciation is observed for countries with a fixed exchange rate arrangements (or peggers). Our results for floaters contradict predictions of the Mundell-Fleming model, providing a hint on monetization of capital inflows in efforts to avoid a currency appreciation. Alternative explanations are provided by the New Keynesian models [Barbier-Gaucard et al. 2015]. For a fixed exchange rate regime, an increase in the money supply can lead to higher prices and the RER appreciation, as it is found empirically. The fraction of BG_t in the forecast error variance decomposition (FEVD) of RER is much lower on average for floaters if compared with peggers (Table 2). However, the results for Estonia and Lithuania are comparable to that for the countries with exchange rate flexibility.

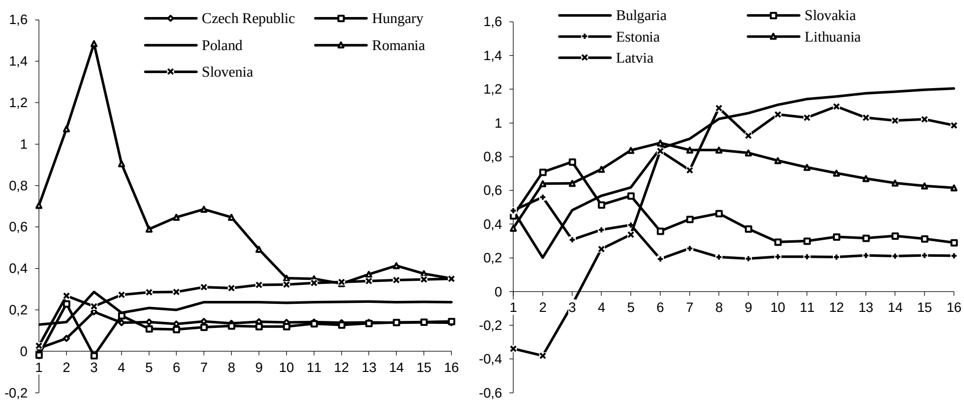
Regardless of the exchange rate arrangements, fiscal austerity in Germany unambiguously contributes to investments, although the results seem to be stronger under a fixed exchange rate regime (Figure 2). Improvement of the budget balance in Germany with an initial size of 1% of GDP results in a cumulative expansion in investment between 0.6% and 1.2% of GDP for Bulgaria, Lithuania and Latvia, while being at a lower level of around 0.3% of GDP for Estonia and Slovakia. Under a floating exchange rate regime, the strongest stimulating effect is obtained for Romania on impact at 1.5% of GDP but it is gradually phased out. For other countries in that group, the estimates imply an increase in investments by 0.3% to 0.4% of GDP.

Figure 1. German budget balance effects on the RER



Source: the author's calculations

Figure 2. German budget balance effects on investments

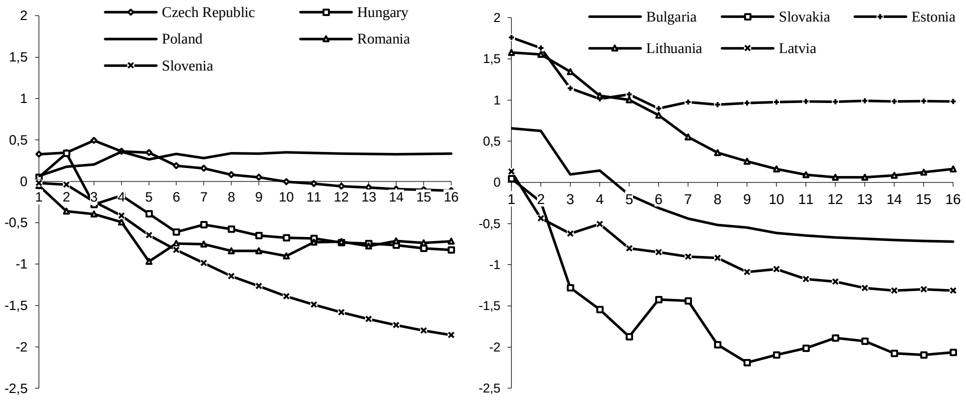


Source: the author's calculations

Except Poland and Estonia, no persistent stimulating effect of German fiscal shock upon exports is found (Figure 3). For the Czech Republic and Lithuania, there is a short-lived positive effect on impact only and this result is further supported by a small fraction of BG_t in the FEVD of exports, especially for the former. Other countries demonstrate a decrease in the fraction of exports in GDP, with no particular exchange rate regime-specific features either. The largest drop in exports is observed in Slovenia and Slovakia, with the fraction of BG_t in changes of exports at its maximum at 20% and 58%, respectively. Impulse responses for Romania and Hungary are similar in the long run and indicate a decrease in the exports/GDP relationship of about 0.6 percentage points in response to a 1%

German fiscal shock, but this effect is not very important according to the FEVD for the latter (Table 2). Results for Bulgaria are similar to those ones found for Hungary. The impulse response negative effect on exports is twice as strong for Latvia, with a fraction of BG_t in the FEVD gradually increasing from 13% to as high as 74%.

Figure 3. German budget balance effects on exports



Source: the author's calculations

Our results indicate that the direct trade channel in transmission of a German fiscal shock is quite heterogeneous, although a gradual decline in the share of exports in GDP clearly dominates. As the inverse link between fiscal austerity in Germany and exports abroad can be easily explained by insufficient foreign demand, the positive response of CEE exports to BG_t, as in Poland, Estonia, the Czech Republic and Lithuania to some extent, implies a more complicated interplay of indirect transmission mechanisms. For example, it is likely that a capital inflows-driven increase in investments creates a sort of crowding out effect for the local exporters due to higher returns on the domestic market-oriented activities. Obviously, a further research is needed in order to clarify mechanisms of German fiscal austerity spillovers for the CEE countries.

CONCLUSIONS

There is evidence that the positive link between the German austerity and output growth in the CEE countries, as obtained by Shevchuk and Kopych [2016], is achieved through an increase in investments which is much stronger for peggers, while other channels are not so homogenous across countries. There is a uniform RER depreciation in response to a German austerity shock under floating, while the opposite outcome of RER appreciation is observed for all peggers. For 6 out of 10 CEE countries, there is a negative link between German fiscal austerity and exports, thus confirming presence of a standard trade channel in transmission of the European cross-border fiscal shocks.

Table 2. Forecast error variance decomposition

Responses of	Innovations to	Country	Forecast horizons			
			4	8	12	16
Real exchange rate (*RER*)	*BG*	Czech Republic	4	4	3	2
		Hungary	12	15	16	17
		Poland	8	12	14	15
		Romania	7	14	15	15
		Slovenia	1	3	5	7
		Bulgaria	26	32	35	36
		Slovakia	23	35	40	36
		Estonia	6	7	8	8
		Lithuania	12	13	13	13
		Latvia	47	68	75	77
Investments (*I*)	*BG*	Czech Republic	5	7	8	9
		Hungary	4	4	4	4
		Poland	27	26	24	24
		Romania	22	16	11	9
		Slovenia	7	9	11	11
		Bulgaria	7	16	23	26
		Slovakia	37	33	30	31
		Estonia	20	12	8	7
		Lithuania	9	10	10	9
		Latvia	9	26	32	33
Exports (*X*)	*BG*	Czech Republic	3	2	1	1
		Hungary	1	1	2	3
		Poland	11	12	15	16
		Romania	7	17	22	23
		Slovenia	2	11	17	20
		Bulgaria	2	2	3	4
		Slovakia	20	42	57	58
		Estonia	9	6	5	5
		Lithuania	16	10	9	6
		Latvia	13	51	68	75

Source: the author's calculations

REFERENCES

Alesina A., Barbiero O., Giavazzi F., Paradisi M. (2015) Austerity in 2009–2013. Economic Policy, 30 (83), 385–437.

Barbier-Gaucard A., Betti T., Diana G. (2015) Spillover effects in a monetary union: Why fiscal policy instruments matter? Working Papers of BETA 2015-02, Strasbourg: Bureau d'Economie Théorique et Appliquée.

Beetsma R., Giuliodori M., Klaassen F. (2006) Trade Spillovers of Fiscal Policy in the European Union: A Panel Analysis. Economic Policy, 21 (48), 640–687.

Bellofiore R. (2013) ‘Two or three things I know about her’: Europe in the global crisis and heterodox economics. Cambridge Journal of Economics, 37 (3), 497–512.

Bénassy-Quéré A. (2006) Short-Term Fiscal Spillovers in a Monetary Union, CEPII Working Paper No. 2006–13, Paris: Centre D’Etudes Prospectives Et D’Informations Internationales.

Born B., Juessen B., Müller G. (2013) Exchange rate regimes and fiscal multipliers. Journal of Economic Dynamics and Control, 37 (2), 446–465.

Boughton J. (2001) Different Strokes? Common and Uncommon Responses to Financial Crisis, IMF Working Paper No. WP/01/12, Washington: International Monetary Fund.

Cardi O., Müller G. (2011) Habit formation and fiscal transmission in open economies, Journal of International Economics, 85 (2), 256–267.

Christiano L., Eichenbaum M., Rebelo S. (2011) When is the government spending multiplier large? Journal of Political Economy, 119 (1), 78–121.

Corsetti G., Meier A., Müller G. (2010) Cross-Border Spillovers from Fiscal Stimulus, International Journal of Central Banking, 6 (1), 5–37.

Crespo Cuaresma J., Eller M., Mehrotra A. (2011) The Economic Transmission of Fiscal Policy Shocks from Western to Eastern Europe. Focus on European Economic Integration, 2, 44–68.

Eggertson G., Mehrotra N., Summers L. (2016) Secular Stagnation in the Open Economy. American Economic Review: Papers & Proceedings 2016, 106 (5), 503–507.

Ivanova A., Weber S. (2011) Do Fiscal Spillovers Matter?. IMF Working Paper No. WP/11/211, Washington: International Monetary Fund.

Ilzetzki, E., Mendoza E., Végh C. (2013) How big (small?) are fiscal multipliers?. Journal of Monetary Economics, 60 (2), 239–254.

Johannsen B. (2014) When are the Effects of Fiscal Policy Uncertainty Large?. Finance and Economics Discussion Series, No. 2014–40.

McCallum B. (1996) International Monetary Economics. Oxford, Oxford University Press.

Müller G. (2014) Fiscal Austerity and the Multiplier in Times of Crisis. German Economic Review, 15 (2), 243–258.

Phillips P. (1998) Impulse response and forecast error variance asymptotics in nonstationary VARs. Journal of Econometrics, 83 (1), 21–56.

Shevchuk V., Kopych R. (2016) German fiscal austerity and its spillovers on the Central and East European Countries, Proceedings of the 10th Professor Aleksander Zelias International Conference on Modelling and Forecasting of Socio-Economic Phenomena (May 12-15, 2014, Zakopane, Poland), M. Papiez and S. Smiech (eds.), Cracow: Cracow University of Economics, 195–204.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII , No. 1, 2017, pp. 120 – 133

Received: 05.02.2017
Accepted: 05.04.2017

IMPROVING GLOBAL ELASTICITY OF BONUS-MALUS SYSTEM

Marcin Topolewski
Michał Bernardelli (ORCID: 0000-0002-5504-257X)
Collegium of Economic Analysis
Warsaw School of Economics - SGH, Poland
e-mail: mtopol@sgh.waw.pl, mbernard@sgh.waw.pl

Abstract: We optimize transition rules of bonus-malus system to achieve possibly best premium elasticity as defined by Loimaranta [1972] and later generalized as global elasticity by De Pril [1978]. We use premium scale given by Norberg [1976]. This issue constitutes a nonlinear nonconvex discrete optimization problem. To solve this problem, we apply improved greedy optimization algorithm, similar to one proposed by Morlock [1984]. We analyse systems of different size for portfolios characterized by inverse Gaussian risk structure function with various parameters. We also propose alternative measures of global elasticity.

Keywords: bonus-malus system, transition rules, optimization, premium elasticity, automobile insurance

INTRODUCTION

Bonus-malus systems (BMS) are used as a tools of a posteriori premiums differentiation in risk assessment process in automobile insurance. While tools of systems analysis and premium calculation criteria are well-described in the literature, relatively little space is devoted to the optimization of transition rules between classes of a bonus-malus system. We try to optimize transition rules in order to address two issues described below.

Goal of the research

Bonus-malus systems have been criticised because:
- bonus-malus systems have low premium elasticity,
- policyholders tend to cluster in 'better classes'.

Considering above disadvantages of bonus-malus systems our research question is:

DOI: 10.22630/MIBE.2017.18.1.12 JEL: C51, G22

- Can we eliminate these disadvantages by optimizing transition rules in order to achieve higher premium elasticity?

RISK

Risk process is modelled typically for this kind of problems, so we assume that:

- claim amount and number of claims are independent,
- expected claim amount equals 1 (claim rate λ is a measure of risk of a single insured),
- policyholders form a heterogeneous portfolio (insured differ by claim rate λ) with overdispersion,
- there is no bonus hunger.

We distinguish two random variables:

K – number of claims ~ Poisson(λ),

Λ – claim rate ~ Inverse Gaussian IG(μ, θ),

furthermore

$u(\lambda)$ – is probability density function of Λ, so called risk structure function.

Conditional probability of reporting *k* claims in unitary period (one year)

$$P_k(\lambda) = P(K = k|\Lambda = \lambda) = \frac{e^{-\lambda}\lambda^k}{k!} \tag{1}$$

Unconditional probability of *k* claims in unitary period (one year)

$$P_k = P(K = k) = \int_0^\infty \frac{e^{-\lambda}\lambda^k}{k!} u(\lambda) d\lambda = \int_0^\infty \frac{e^{-\lambda}\lambda^k}{k!} dU(\lambda) \tag{2}$$

With above assumptions we have expected value of number of claims and its variance given by:

$$EK = \mu, \ \ VarK = \mu + \mu\theta \tag{3}$$

and expected claim rate and variance of claim rate given by:

$$E\Lambda = \mu, \ \ Var\Lambda = \mu\theta \tag{4}$$

BONUS-MALUS SYSTEM (BMS)

We assume that bonus-malus system consists of [Lemaire 1985]:

- finite number of classes $i \epsilon S, S = \{1, 2, \dots, s\}$ such as insured belongs to one and only one class in unitary period and the class in the next period depends only on the class and the number of claims reported in the current period according to transition rules,
- premiums b_i specified for each class,

- specified starting class for those who insure for the first time (unnecessary condition for stationary state analysis).

Additionally, we assume that:

- the best class is class number 1 (best class means class with the lowest premium and the most favourable transition rules),
- the worst class is class number *s*.

Transition rules can be represented by a transition table or transition matrix $\boldsymbol{T} = [t_{ik}]$, which shows to which class insured passes after reporting *k* claims in class *i*.

Example of transition table

	k =	0	1	2	3+
class	1	1	2	3	5
	2	1	3	5	5
	3	2	5	6	6
	4	3	6	6	7
	5	4	6	7	7
	6	5	7	7	8
	7	6	7	8	8
	8	7	8	8	9
	9	8	9	9	10
	10	9	10	10	10

Example of transition matrix

$$\boldsymbol{T} = [t_{ik}] = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 1 & 3 & 5 & 5 \\ 2 & 5 & 6 & 6 \\ 3 & 6 & 6 & 7 \\ 4 & 6 & 7 & 7 \\ 5 & 7 & 7 & 8 \\ 6 & 7 & 8 & 8 \\ 7 & 8 & 8 & 9 \\ 8 & 9 & 9 & 10 \\ 9 & 10 & 10 & 10 \end{bmatrix}$$

MODEL OF A BONUS-MALUS SYSTEM

As bonus-malus system possess Markov property (class in the next period depends only on the class and the number of claims in the previous period) it is usually modelled by suitable Markov chain [Lemaire 1985, 1995].

Transformation matrix is a matrix $\boldsymbol{T}_k = [t_{ij}(k)]$, where:

$$t_{ij}(k) = \begin{cases} 1 \text{ for } t_{ik} = j \\ 0 \text{ for } t_{ik} \neq j \end{cases} \tag{5}$$

Probability of transition from class *i* to class *j* (depending on claim rate λ)

$$p_{ij}(\lambda) = \sum_{k=0}^{\infty} p_k(\lambda) t_{ij}(k) \tag{6}$$

The transition probability matrix of Markov chain

$$\boldsymbol{P}(\lambda) = [p_{ij}(\lambda)] = \sum_{k=0}^{\infty} p_k(\lambda) \boldsymbol{T}_k \tag{7}$$

For regular transition probability matrix, after sufficient time the chain tends to stationary state [Kemeny 1976] with stationary distribution:

$$\boldsymbol{e}(\lambda) = [e_1(\lambda), \dots, e_s(\lambda)] \tag{8}$$

$$\begin{cases} \boldsymbol{e}(\lambda)\boldsymbol{P}(\lambda) = \boldsymbol{e}(\lambda) \\ \boldsymbol{e}(\lambda)\mathbf{1} = 1 \end{cases} \tag{9}$$

Unconditional stationary distribution is given by

$$\boldsymbol{e} = [e_1, \dots, e_s] = [\int_0^\infty e_1(\lambda)u(\lambda)d\lambda, \dots, \int_0^\infty e_s(\lambda)u(\lambda)d\lambda] \tag{10}$$

PERMISSIBLE SYSTEMS

We limit ourselves to systems which fulfil below conditions:

- elements in rows of transition matrix $\boldsymbol{T}$ are non-decreasing (weak monotonicity in rows) - in each class penalty* for more claims is no less than for fewer claims
- elements in columns of transition matrix $\boldsymbol{T}$ are non-decreasing (weak monotonicity in columns) - penalty[1] for the same number of claims in the worse class cannot be less than in the better class (with the exception of the worst class)
- systems are irreducible (are modelled by an irreducible Markov chain) - none of elements of stationary distribution equal zero
- systems are ergodic (are modelled by an ergodic Markov chain) - stationary distribution does not depend on starting class

Systems which fulfil above conditions are called permissible systems.

PREMIUMS

We use Norberg criterion of premiums calculation [Norberg 1976]

$$Q(\boldsymbol{b}) = \int_0^\infty \sum_{j=1}^s (b_j - \lambda)^2 e_j(\lambda)u(\lambda)d\lambda \to min \tag{11}$$

Which gives so called Q-optimal premiums (where b_j is a premium to be paid in class j)

$$b_j = \frac{\int_0^\infty \lambda e_j(\lambda)u(\lambda)d\lambda}{\int_0^\infty e_j(\lambda)u(\lambda)d\lambda} = \frac{\int_0^\infty \lambda e_j(\lambda)u(\lambda)d\lambda}{e_j} \tag{12}$$

For Q-optimal premiums, system is financially balanced, that is stationary premium equals expected claim rate for the portfolio (and equals μ for IG risk structure function)

$$\sum_{j=1}^s e_j b_j = E\Lambda = EK = \mu \tag{13}$$

[1] Penalty is understood in terms of transition to a worse class, we do not address premiums yet.

CHARACTERISTICS OF A BONUS-MALUS SYSTEM

In order to monitor performance of bonus-malus systems we use characteristics which describe quality of particular system over different dimensions (different aspects).

Stationary premium [Loimaranta 1972]

$$b_e = \sum_{j=1}^{s} e_j b_j \tag{14}$$

shows expected premium after sufficient number of periods. Can be interpreted as average income from one policy in stationary state. It is further used in many other measures of quality of BMS.

Volatility coefficient of the stationary premium [Lemaire 1985, 1995]

$$V_{b_e} = \frac{\sqrt{\sum_{j=1}^{s} (b_j - b_e)^2 e_j}}{b_e} \tag{15}$$

shows how on average stationary premium differs for randomly chosen policyholder. Can be interpreted as a measure of financial toughness of BMS. Higher values show that relatively high part of the risk is transferred to policyholder. Low values show relatively low system ability to risk differentiation. Some authors [Lemaire, Zi 1994] indicate that values higher than 1 can be hard to accept by customers. To compromise, most preferred values are close to 1.

RSAL – Relative stationary average level [Lemaire 1985, 1995]

$$RSAL = \frac{b_e - b_1}{b_s - b_1} \tag{16}$$

takes values from 0 to 1 and indicates position of stationary premium over the distance between the lowest and the highest possible premium. Values closer to 0 suggest clustering of policyholders in better (cheaper) classes.

Elasticity of the stationary premium [Loimaranta 1972]

$$\eta(\lambda) = \frac{\partial b_e}{b_e} \Big/ \frac{\partial \lambda}{\lambda} = \frac{\partial b_e}{\partial \lambda} \frac{\lambda}{b_e} \tag{17}$$

also called point elasticity, shows reaction of stationary premium for the change of claim rate λ. Namely, 1% change in claim rate is associated with $\eta(\lambda)\%$ change in stationary premium. Ideal value of elasticity is 1, which means that stationary premium reacts exactly proportionally for the change in the risk.

Global elasticity of the stationary premium [De Pril 1978]

$$\eta = \int_0^{\infty} \eta(\lambda) u(\lambda) d\lambda \tag{18}$$

can be interpreted as portfolio elasticity, that is elasticity weighted by the risk structure function.

As for majority of systems global elasticity takes values much lower than one, global elasticity becomes main point of our interest and farther on we try to arrange transition rules of bonus-malus system in the way that would lift up global elasticity.

Measure of goodness of risk assessment [Topolewski & Bernardelli 2015]

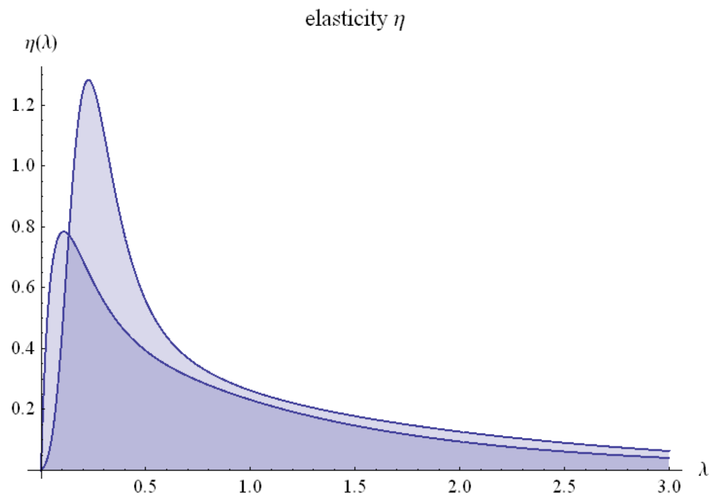
$$QN = \frac{\sum_{j=1}^{s} e_j b_j^2 - E^2\Lambda}{E\Lambda^2 - E^2\Lambda} \tag{19}$$

This measure is adequate only for systems with premiums given by (20). It is normalized measure that takes values from 0 to 1 and shows goodness of risk assessment of system with Norberg premiums for particular portfolio given by risk structure function. Values closer to 1 reflect better fit.

RESEARCH

Simple maximization of global elasticity may lead to spurious results, as lifting up its value may be achieved by lifting point elasticity too high, that is producing system which overreacts. Though we may get system with higher global elasticity, its point elasticity may be too high and we only substitute one imperfection with another. see figure 1.

Figure 2. Examples of too low and too high elasticity of stationary premium with respect to claim ratio λ



Source: own preparation

To overcome this problem, we propose optimization criteria that allow to keep global elasticity of BMS possibly close to one, namely we minimize distance between $\eta(\lambda)$ and 1, weighted by risk structure function.

Our objective function becomes (respectively):

- to maximize global elasticity η (typical approach – may give overreacting system)

$$\eta = \int_0^\infty \eta(\lambda)u(\lambda)d\lambda \to max \tag{I}$$

- to minimize mean absolute error (mean absolute distance from 1)

$$MAE = \int_0^\infty |1-\eta(\lambda)|u(\lambda)d\lambda \to min \tag{II}$$

- to minimize root square mean error (root mean square distance from 1)

$$RMSE = \sqrt{\int_0^\infty [1-\eta(\lambda)]^2 u(\lambda)d\lambda} \to min \tag{III}$$

Optimization of transition rules with respect to above functions is nonlinear and nonconvex discrete optimization problem. To solve this problem, we have to use adequate algorithm.

The algorithm

We use greedy algorithm similar to one used by [Morlock 1985] but with some alteration:

- We consider stationary state (stationary distribution)
- We impose weak monotonicity conditions, both in rows and in columns in the table of bonus-malus system (permissible systems)
- We limit ourselves to irreducible and ergodic systems
- We use different directions of optimization (rows, columns, diagonals)

Subsequently for each element t_{ik} of transition matrix $\boldsymbol{T}$ we change its value (taking into account the conditions for irreducibility, ergodicity and monotonicity of the system), for each value of t_{ik} we calculate premiums $\boldsymbol{b}$ and global elasticity and we choose t_{ik} which optimizes global elasticity. After optimization of all elements of $\boldsymbol{T}$ matrix procedure is repeated and we compare the results with the previous iteration. If in two subsequent iterative steps algorithm shows the same solution, we stop the procedure. We apply above algorithm in three ways, changing values of t_{ik} elements in rows, columns and by 'diagonals' starting from different initial systems (different $\boldsymbol{T}$ matrices). Solutions may differ – this is a greedy algorithm and may not always give globally optimal solution for each way.

Portfolios

We study systems of 10 classes that count up to 3 claims (more than 3 is treated as 3) and operate on different portfolios. Portfolios differ by parameters of risk structure function, IG(μ, θ), to screen portfolios with low and high claim rate and claim variance. We designate nine portfolios (nine sets of parameters) that reflect portfolios which can be meet in practice. Values of parameters have been chosen in the way, that they are close to parameters of claims distributions from Willmot 1987. Parameters of portfolios can be seen in Table 1.

Table 1. Portfolios characterised by parameters

Portfolio 1	Portfolio 2	Portfolio 3
$\mu = 0.05$	$\mu = 0.05$	$\mu = 0.05$
$\theta = 0.01$	$\theta = 0.05$	$\theta = 0.15$

Portfolio 4	Portfolio 5	Portfolio 6
$\mu = 0.15$	$\mu = 0.15$	$\mu = 0.15$
$\theta = 0.01$	$\theta = 0.05$	$\theta = 0.15$

Portfolio 7	Portfolio 8	Portfolio 9
$\mu = 0.30$	$\mu = 0.30$	$\mu = 0.30$
$\theta = 0.01$	$\theta = 0.05$	$\theta = 0.15$

Source: own preparation

It is worth notice, that majority of real portfolios would be more like portfolios 1 to 6 from Table 1 (will have average claim rate closer to 0.05 – 0.15), than like portfolios 7 to 9 (having very high average claim rate 0.3). But to have more complete portfolio review we decided to include also high claim rate portfolios.

RESULTS

Transition rules of systems given by algorithm as optimal for different portfolios and subsequent optimisation criteria are shown respectively in Tables 2, 3 and 4.

Table 2. Systems given by the algorithm as optimal by criterion (I) η → max

η → max											
Portfolio 1				Portfolio 2				Portfolio 3			
1	8	10	10	1	5	10	10	1	6	10	10
1	10	10	10	1	10	10	10	1	10	10	10
2	10	10	10	2	10	10	10	2	10	10	10
3	10	10	10	3	10	10	10	3	10	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 4				Portfolio 5				Portfolio 6			
1	9	10	10	1	5	9	10	1	3	7	9
1	10	10	10	1	9	10	10	1	7	9	10
2	10	10	10	2	9	10	10	2	7	9	10
3	10	10	10	3	9	10	10	3	9	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 7				Portfolio 8				Portfolio 9			
1	9	10	10	1	5	9	10	1	2	7	9
1	10	10	10	1	9	10	10	1	7	9	9
2	10	10	10	2	10	10	10	2	9	9	10
3	10	10	10	3	10	10	10	3	9	9	10
4	10	10	10	4	10	10	10	4	9	9	10
5	10	10	10	5	10	10	10	5	9	10	10
6	10	10	10	6	10	10	10	6	9	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10

Source: own preparation

Table 3. Systems given by the algorithm as optimal by criterion (II) MAE → min

MAE → min											
Portfolio 1				Portfolio 2				Portfolio 3			
1	8	10	10	1	5	10	10	1	6	10	10
1	10	10	10	1	10	10	10	1	10	10	10
2	10	10	10	2	10	10	10	2	10	10	10
3	10	10	10	3	10	10	10	3	10	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 4				Portfolio 5				Portfolio 6			
1	9	10	10	1	5	9	10	1	3	7	9
1	10	10	10	1	9	10	10	1	7	9	10
2	10	10	10	2	9	10	10	2	7	9	10
3	10	10	10	3	9	10	10	3	9	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 7				Portfolio 8				Portfolio 9			
1	10	10	10	1	6	10	10	1	4	9	10
1	10	10	10	1	10	10	10	1	9	9	10
2	10	10	10	2	10	10	10	2	9	9	10
3	10	10	10	3	10	10	10	3	9	9	10
4	10	10	10	4	10	10	10	4	9	9	10
5	10	10	10	5	10	10	10	5	9	9	10
6	10	10	10	6	10	10	10	6	9	9	10
7	10	10	10	7	10	10	10	7	9	10	10
8	10	10	10	8	10	10	10	8	9	10	10
9	10	10	10	9	10	10	10	9	10	10	10

Source: own preparation

Table 4. Systems given by the algorithm as optimal by criterion (III) RMSE → min

RMSE → min											
Portfolio 1				Portfolio 2				Portfolio 3			
1	10	10	10	1	6	10	10	1	6	10	10
1	10	10	10	1	10	10	10	1	10	10	10
2	10	10	10	2	10	10	10	2	10	10	10
3	10	10	10	3	10	10	10	3	10	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 4				Portfolio 5				Portfolio 6			
1	10	10	10	1	8	10	10	1	4	9	9
1	10	10	10	1	10	10	10	1	9	9	10
2	10	10	10	2	10	10	10	2	9	9	10
3	10	10	10	3	10	10	10	3	9	9	10
4	10	10	10	4	10	10	10	4	9	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 7				Portfolio 8				Portfolio 9			
1	10	10	10	1	9	10	10	1	4	9	10
1	10	10	10	1	10	10	10	1	9	9	10
2	10	10	10	2	10	10	10	2	9	9	10
3	10	10	10	3	10	10	10	3	9	9	10
4	10	10	10	4	10	10	10	4	9	9	10
5	10	10	10	5	10	10	10	5	9	9	10
6	10	10	10	6	10	10	10	6	9	9	10
7	10	10	10	7	10	10	10	7	9	10	10
8	10	10	10	8	10	10	10	8	9	10	10
9	10	10	10	9	10	10	10	9	10	10	10

Source: own preparation

We can observe that for majority of portfolios optimal systems are rather tough in terms of rules (sending policyholder to the worst or almost worst class for any reported claim).

To monitor properties of systems given as optimal for subsequent optimization criteria we calculate system's characteristics which are given in Tables 5, 6, 7. Systems are ranked according to the values of underling criterion.

Table 5. Ranking of optimal systems by criterion (I) η → max

Portfolio	*μ*	*θ*	*QN*	V_{be}	*RSAL*	***η***	*ME*	*MAE*	*RMSE*
8	0.3	0.05	0.321419	1.388710	0.201847	**0.599605**	0.400395	0.430068	0.528553
7	0.3	0.01	0.201232	2.457020	0.106629	**0.596285**	0.403715	0.438937	0.533067
9	0.3	0.15	0.466695	0.966121	0.226400	**0.576430**	0.423570	0.479767	0.574299
4	0.15	0.01	0.309714	2.155390	0.100166	**0.510879**	0.489121	0.489121	0.565899
5	0.15	0.05	0.427321	1.132240	0.164996	**0.487513**	0.512487	0.512487	0.590434
6	0.15	0.15	0.450998	0.671564	0.191713	**0.426207**	0.573793	0.573793	0.634645
1	0.05	0.01	0.389539	1.395600	0.083622	**0.355104**	0.644896	0.644896	0.687815
2	0.05	0.05	0.269839	0.519460	0.108728	**0.230867**	0.769133	0.769133	0.786754
3	0.05	0.15	0.117212	0.197663	0.135810	**0.112706**	0.887294	0.887294	0.889157

Source: own preparation

Table 6. Ranking of optimal systems by criterion (II) MAE → min

Portfolio	*μ*	*θ*	*QN*	V_{be}	*RSAL*	*η*	*ME*	***MAE***	*RMSE*
7	0.3	0.01	0.170241	2.259920	0.124114	0.594975	0.405025	**0.405025**	0.452337
8	0.3	0.05	0.307132	1.357500	0.210503	0.595962	0.404038	**0.407535**	0.506089
9	0.3	0.15	0.515548	1.015430	0.170585	0.561343	0.438657	**0.438657**	0.483195
4	0.15	0.01	0.309714	2.155390	0.100166	0.510879	0.489121	**0.489121**	0.565899
5	0.15	0.05	0.427321	1.132240	0.164996	0.487513	0.512487	**0.512487**	0.590434
6	0.15	0.15	0.450998	0.671564	0.191713	0.426207	0.573793	**0.573793**	0.634645
1	0.05	0.01	0.389539	1.39560	0.0836215	0.355104	0.644896	**0.644896**	0.687815
2	0.05	0.05	0.269839	0.51946	0.1087280	0.230867	0.769133	**0.769133**	0.786754
3	0.05	0.15	0.117212	0.197663	0.1358100	0.112706	0.887294	**0.887294**	0.889157

Source: own preparation

Table 7. Ranking of optimal systems by criterion (III) RMSE → min

Portfolio	μ	θ	*QN*	V_{be}	*RSAL*	η	*ME*	*MAE*	***RMSE***
7	0.3	0.01	0.170241	2.259920	0.124114	0.594975	0.405025	0.405025	**0.452337**
9	0.3	0.15	0.515548	1.015430	0.170585	0.561343	0.438657	0.438657	**0.483195**
8	0.3	0.05	0.291295	1.322030	0.215832	0.559857	0.440143	0.440143	**0.485149**
4	0.15	0.01	0.254498	1.953830	0.121405	0.510505	0.489495	0.489495	**0.524757**
5	0.15	0.05	0.355297	1.032420	0.198442	0.462336	0.537664	0.537664	**0.568946**
6	0.15	0.15	0.401352	0.633524	0.209115	0.414519	0.585481	0.585481	**0.617525**
1	0.05	0.01	0.285054	1.193850	0.125651	0.347099	0.652901	0.652901	**0.672276**
2	0.05	0.05	0.25877	0.508695	0.118548	0.230662	0.769338	0.769338	**0.783442**
3	0.05	0.15	0.117212	0.197663	0.135810	0.112706	0.887294	0.887294	**0.889157**

Source: own preparation

Analysing Tables 5, 6, 7 we can clearly see that for portfolios with low and medium claim rate (portfolio 1 to portfolio 6) the highest possible level of premium elasticity is rather low for any optimization criterion.

CONCLUSIONS

- Level of global elasticity depends on portfolio (claim rate and claim variance).
- For portfolios with low and medium claim rate (most typical portfolios) we have lower values of global elasticity.
- Systems optimal in sense of elasticity are tough in terms of transition rules
- For particular claim rate, systems with high elasticity are financially tough – rather high volatility coefficient V_{b_e}
- For particular claim rate, high elasticity tends to go together with concentration in better classes – low RSAL
- Better elasticity does not go together with better risk assessment – see QN

It is easier to achieve higher elasticity (for any optimization criterion) for portfolios with higher claim rate, but high elasticity generally requires a tough system. Considering other characteristics of BMS, optimization of elasticity does not make them necessarily better.

For most typical portfolios elasticity has rather low level and for majority of portfolios policyholders tend to cluster in better (cheaper) classes.

REFERENCES

Borgan O., Hoem J.M., Norberg R. (1981) A Nonasymptotic Criterion for Evaluation of Automobile Bonus System. Scandinavian Actuarial Journal, 165-178.

De Pril N. (1978) The Efficiency of a Bonus-Malus System. ASTIN Bulletin, 10, Part 1, 59-72.

Gilde V., Sundt B. (1989) On Bonus Systems with Credibility Scale. Scandinavian Actuarial Journal, 13-22.

Lemaire J. (1985) Automobile Insurance: Actuarial Models. Kluwer Nijhoff, Boston.

Lemaire J. (1995) Bonus-malus Systems in Automobile Insurance. Kluwer Nijhoff, Boston.

Lemaire J., Zi H. (1994) High deductibles instead of bonus-malus. Can it work? Astin Bulletin, 24 (1), 75–86.

Loimaranta K. (1972) Some Asymptotic Properties of Bonus Systems. ASTIN Bulletin, VI, Part 3, 233–245.

Morlock M. (1985) Aspects of optimization in automobile insurance. Contributions to Operations Research, Springer-Verlag, 240, 131–141.

Norberg R. (1976) A credibility theory for automobile bonus systems. Scandinavian Actuarial Journal, 92–107.

Podgórska M., Cieślik B., Kryszeń B., Niemiec M., Topolewski M. (2006) System bonus-malus sprawiedliwy w sensie przejść między klasami. Instytut Ekonometrii SGH, Warszawa.

Kemeny J., Snell J. (1976) Finite Markov Chains. Springer-Verlag.

Topolewski M., Bernardelli M. (2015) Optymalizacja reguł przejścia systemu bonus-malus o składkach Q-optymalnych. Roczniki Kolegium Analiz Ekonomicznych, 37, 229-252.

Willmot G. (1987) The Poisson-Inverse Gaussian Distribution as an Alternative to the Negative Binomial. Scandinavian Actuarial Journal, 113–127.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII , No. 1, 2017, pp. 134 – 142

Received: 24.12.2016
Accepted: 05.04.2017

THE VARIABILITY OF TURKEY LIVESTOCK PRICE AND ITS RELATION WITH THE PRICE OF CHICKENS, PORK AND BEEF IN 2006-2015

Katarzyna Utnik-Banaś (ORCID: 0000-0002-5008-9301)
Faculty of Agriculture and Economics
University of Agriculture in Krakow, Poland
e-mail: rrbanas@cyf-kr.edu.pl

Abstract: The work determines the range of variability in the price of turkey livestock and its relation to prices of chickens, pork and beef in 2006-2015. In 2006-2011, the variability coefficient for turkey livestock was 5 to 10%, whereas in 2012-2015 it dropped below 5%. The prices were most stable out of prices of chickens, pork or beef. The biggest influence on the variability of the turkey livestock price was the long-term trend and cyclic fluctuations (82% of the total variability on average). Seasonal fluctuations had lower amplitude (between 7 and 4%) than in the case of prices of pork and chickens, and their input in the total variability amounted to 14% on average. The price of turkey livestock presented the strongest correlation with the price of beef (r=0,851) and chickens (0,837), and was the least correlated with pork (0,681). In 2006-2015 the price of turkey livestock increased by 68%, beef by 57%, chickens by 29% and pork by 13%.

Keywords: turkeys, price, variability, time series, seasonality, cyclic fluctuations

INTRODUCTION

In the dynamically developing poultry business, turkeys are placed second (after broilers) in the poultry species structure. The production of turkeys was 450 thousand tons (in live weight) in 2015, constituting 15.4% of the total poultry livestock and increased by 15.4% in comparison to the previous year [Biegański 2016]. A big increase in the production of turkeys in last years has been positively influenced by, among others, a stable level of buying-in prices of livestock [Świetlik 2016].

DOI: 10.22630/MIBE.2017.18.1.13 JEL: C22, Q11

The price variability is a key aspect of price risk for all market members: producers, processors, as well as consumers [Figiel et al. 2012]. The price levels of agricultural raw materials are mainly influenced by: the biological-technical character of agricultural production, low short-term elasticity of supply, inter-market relations and relations to world prices [Hamulczuk and Stańko 2011]. Price variability is inevitable, however, it is crucial to know the causes lying behind the variability, which may allow foreseeing or preventing sudden changes in price levels. Characteristic elements of the price variability in agriculture include annual seasonal fluctuations or longer, periodically repetitive cyclic fluctuations. Best known are pig cycles in pork production [Szymańska 2012]. Despite numerous studies and a relatively well described mechanism, the occurrence of "pig cycles" has not been eliminated. The level of prices in livestock production is also influenced by the presence of supplementarity and relations between prices of pork, poultry and beef [Rembeza 2007]. In literature on price variability of different kind of livestock or meat [Hamulczuk 2009, Idzik 2009, Olszańska 2012] poultry is generally treated without a division into species: chickens, turkeys, geese and ducks. Given the big share of turkeys in the structure of the produced poultry [Dybowski 2014] and sizeable differences in supply-and-need conditions, it is essential to know more about elements shaping the level of turkey livestock prices.

The aim of the paper is to present the type and the range of variability of turkey livestock prices and their relations with livestock prices for chickens, pork and beef in 2006-2015.

RESEARCH MATERIAL AND METHODS

The research material was monthly time series for prices of turkeys, broilers, pork and beef livestock in 2006-2015. The prices of turkeys and chickens came from the Integrated System of Agricultural Market Information [2016], whereas the prices of pork and beef livestock were obtained from the Central Statistical Office [Prices of agricultural products 2006-2015].

The range of price variability in a year was presented with a variability coefficient, minimal value and maximal value (interval), maximal monthly change in price (increase or decrease) and the change in price indicator (in %). The analysis of the price variability for turkey livestock was conducted with a price time series decomposition. A time series includes the following elements [Dittmann 2008]:

- Developmental tendency – trend (T) – it shows the long-term tendency for one-way changes (increase or decrease) of the price. It is understood as the effect of the influence of a constant set of factors,
- Cyclic fluctuations (C) – they are formed as long-term, rhythmically repetitive price fluctuations around the developmental tendency in time intervals longer than one year,

- Seasonal fluctuations (S) – are price fluctuations of the observed variable (price) around the developmental tendency and repeat in a time interval not longer than one year.
- Random fluctuations – random element – (I).

Given the mutual relation between the long-term trend (T) and cyclic fluctuations (C) formed by similar factors, the elements of the time series are treated in the paper as a whole trend-cycle element (T_tC_t). To describe the time series for turkey livestock prices, a multiplicative model was used in the form of the following formula [Stańko 2013]:

$$Y_t = T_t C_t S_t I_t$$

where:

Y_t – livestock price in time t,
T_tC_t – long-term trend and cyclic fluctuations,
S_t – seasonal fluctuations,
I_t – random fluctuations.

The Census II/X11 [Idzik 2009] method was used to determine the seasonality of indicators. The advantage of Census II/X11 is, among others, the ability to calculate seasonal fluctuations for each year separately, which allows for an analysis of possible changes in seasonality models in longer periods of time. In order to check the relevance of the seasonality indicators, a variance analysis was carried out for indicator values in particular months using the F test.

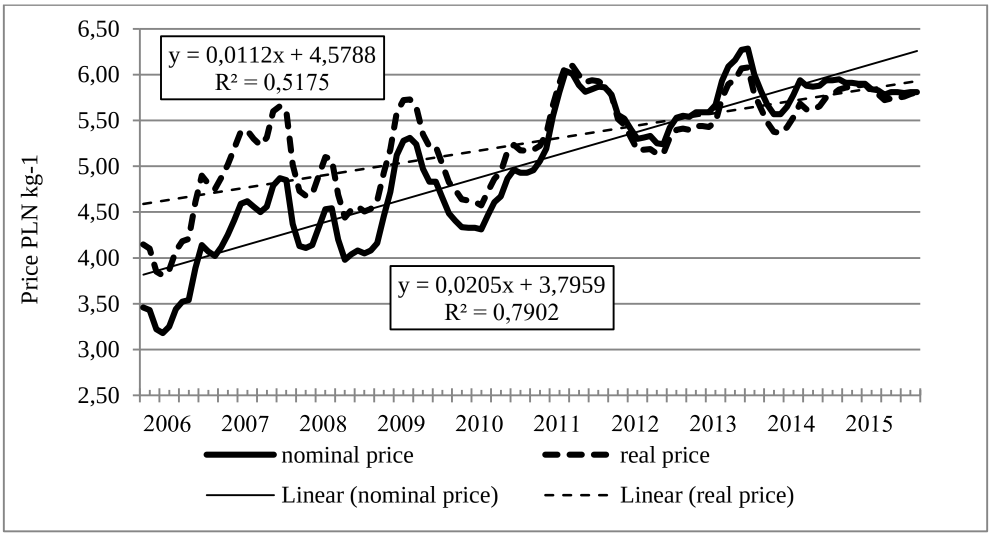
The influence of particular components of the time series, such as: seasonality (S), random fluctuation (I) and developmental tendency (TC) on the general variability of broiler livestock prices was determined in relation to the duration of changes. To this end, the share of variances for particular components of the series in the total price variance was analyzed. The calculations were carried out with a forecasting and time series analysis packet included in the computer program Statistica 9.0 [Kot et al. 2011].

RESULTS

Over ten years, the nominal price of turkey livestock increased from 3.46 PLN/kg in January 2006 to 5.81 PLN/kg in December 2015 (Figure 1). The biggest increase (9.8%) took place in 2006, in which the price rose from 3.54 PLN/kg in August to 3.89 PLN/kg in September. The biggest drop, however, took place in 2007, in which the price decreased from 4.85 PLN/kg to 4.37 PLN/kg in December.

In 2006-2011 the price variability coefficient in a year was ranging between 4.5 and 9.6%, whereas in 2012 prices in a year were subjected to lower fluctuations, and the variability coefficient was under 5% (Table 1).

Figure. 1. Level of real and nominal prices of turkey livestock in 2006-2015



Source: own study based on: Integrated System of Agricultural Market Information, 2016

Table 1. Variability of nominal prices of broiler livestock in 2006-2015

Year	Nominal price							
	Average	min	max	Variability coeficient%	Max monthly %		Index of change	
	PLN kg^{-1}				decrease	increase	yearly	2006=1
2006	3.60	3.18	4.14	9.56	-6.12	9.89	1.16	1.16
2007	4.54	4.12	4.87	5.08	-9.90	5.04	1.06	1.26
2008	4.19	3.98	4.54	4.46	-7.49	4.83	0.99	1.18
2009	4.84	4.16	5.31	7.50	-5.15	8.47	1.08	1.30
2010	4.60	4.31	4.96	5.78	-1.77	4.20	1.12	1.42
2011	5.66	4.96	6.04	6.69	-2.00	6.90	1.18	1.69
2012	5.43	5.24	5.78	2.95	-3.81	3.44	0.96	1.60
2013	5.88	5.54	6.29	4.79	-4.55	4.59	1.05	1.68
2014	5.80	5.57	5.95	2.57	-2.58	2.82	1.05	1.72
2015	5.84	5.78	5.91	0.83	-1.02	0.52	0.98	1.68

Source: own study based on: Integrated System of Agricultural Market Information, 2016

Real prices in 2006 were 20% higher than nominal prices. The difference was gradually decreasing, and in 2012-2014 real prices were 2-4% lower than nominal prices. The long-term linear trend of real prices (having eliminated the influence of inflation) in 2006-2015 indicates their decrease by 0.011 PLN/ month (R^2 = 0.52) on average.

Maximal monthly changes in prices of turkey livestock did not exceed 10% and were comparable to maximal monthly decreases (increase) in prices of broilers (Table 1). The price of beef livestock in a monthly horizon of changes were more steady (maximal decrease in prices -6.8%, increase +8.4%), whereas monthly changes in prices of pork were much bigger than turkey livestock and were between -11.6% to 16.2% (Table 2). In the yearly horizon of changes in prices of turkey livestock (-4 to 18%) were also comparable to changes in prices of chickens and much lower than changes in prices of pork, which reached -18 to +50% in a year. Throughout ten years (2006-2015) (nominal) prices of livestock increased for: turkeys by 68%, beef by 57%, chickens by 29% and pork by 13%. Prices of the compared livestock kinds were significantly related with prices of turkey livestock being most correlated with the price of beef (r = 0.851) and chickens (0.837), and least with pork (0.681).

Table 2. Changes in prices of turkey, chicken, pork and beef livestock in 2006-2015

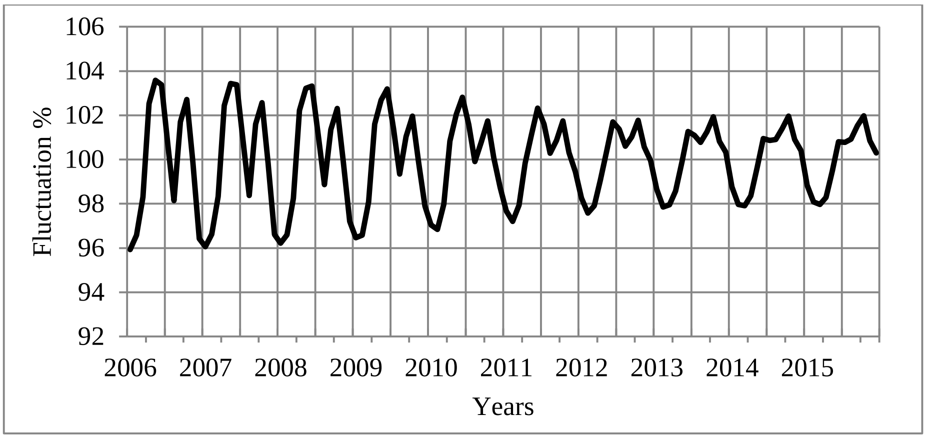
Horizon of changes	Maximal change	Livestock price			
		Turkeys	Chickens	Pork	Beef
Monthly	increase %	-9.9	-9.0	-11.6	-6.8
	decrease %	9.9	10.6	16.2	8.4
Yearly[1])	increase %	96	93	82	93
	decrease %	118	115	150	121
change indicator 2006-2015[2]) %		168	129	113	157
Correlation coefficient[3])		1	0.837	0.689	0.851

[1]) previous year = 100%; [2])2006 =100%; [3]) correlation with the price of turkey livestock $p < 0.001$;

Source: own study based on: Integrated System of Agricultural Market Information. 2016. Prices of agricultural products 2006-2015 Central Statistical Office

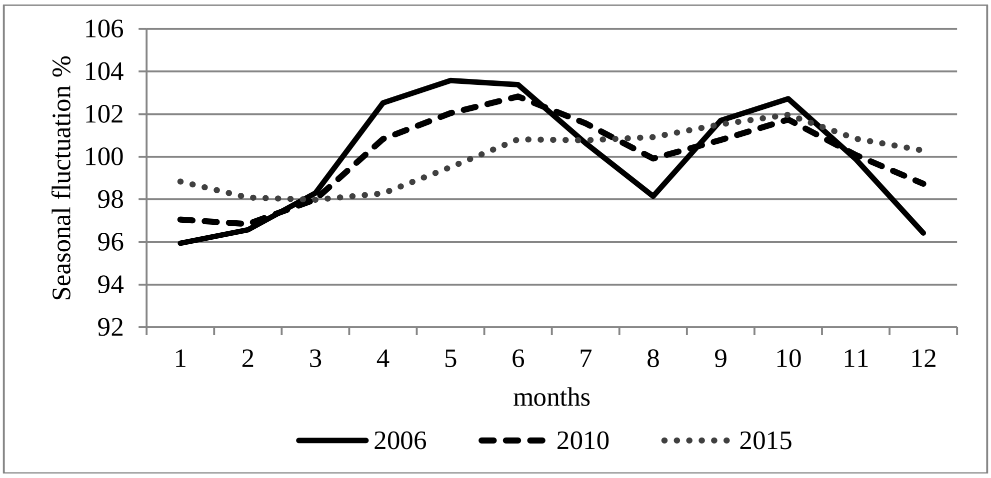
The decomposition of the time series for prices of turkeys indicates regular seasonal and cyclic fluctuations as well as irregular random fluctuations. The steady seasonality test results proved that the seasonal variability of prices of turkeys is statistically significant ($p < 0.001$. statistics value $F = 9.769$). In the analyzed period there was a noticeable change in the model seasonality and a decrease in the amplitude of seasonal fluctuations. In 2006 turkey livestock was cheapest (96%) in winter months (December-February). Two periods of increase in the price were also characteristic: spring with its peak in May (103.6%) and autumn with its slightly lower peak (102.7%) in October (Figures 2 and 3). The amplitude of seasonal fluctuations amounted to 7.7%.

Figure. 2. Seasonal fluctuation of real prices of turkey livestock in 2006-2015



Source: own study based on: Integrated System of Agricultural Market Information. 2016

Figure 3. Changes in the price seasonality model of turkey livestock in 2006-2015



Source: own study based on: Integrated System of Agricultural Market Information. 2016

In the following years. there was a gradual decrease in the fluctuation amplitude and the spring increase in prices. In 2015. lowest prices of turkey livestock (98% of the annual average) were in February and March. whereas the highest (102%) in October. The share of seasonal fluctuations in a monthly horizon amounted to 30% of the total price variability. in three-month horizon it was 22%. and the share dropped under 10% in a horizon longer than half a year (Table 3).

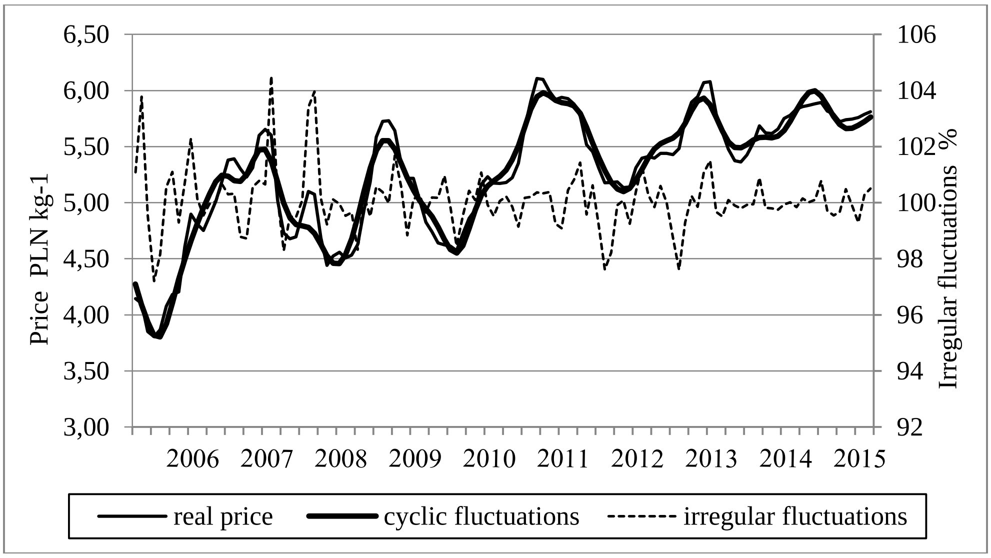
Changes in prices of turkey livestock in 2006 – 2015 were characterized by a noticeable cyclicity of 2 to 3 year-long cycles (Figure 4).

Table 3. Share of seasonal. cyclic and irregular changes in the total price variability of turkey livestock in 2006-2015

Horizon of changes (months)	Changes %		
	irregular	cyclic	seasonal
1	16.1	53.5	30.4
2	8.0	65.0	27.0
3	4.4	73.6	22.0
4	2.8	80.3	16.9
5	2.2	84.7	13.1
6	1.5	87.6	10.9
7	1.0	89.9	9.1
9	1.0	92.9	6.1
11	0.8	97.6	1.6
12	0.9	99.0	0.1
Average	3.9	82.4	13.7

Source: own study based on: Integrated System of Agricultural Market Information. 2016

Figure 4. Results of the decomposition of the time series for real prices of turkey livestock



Source: own study based on: Integrated System of Agricultural Market Information. 2016

Bottom turning-points occurred in: June 2006. August 2008. May 2010. August 2012 and March 2014. Top turning-points (peaks) occurred in: October

2006. June 2009. July 2011 and October 2013. The value of MCD = 4.25 indicates that after five months of one-way changes a new cycle occurs. Cyclic fluctuations constituted the major share in the total price variability of turkey livestock: in a month horizon of changes they amounted to 53.5% and in a 4 month period their share constituted 80% of the total variability (Table 3). Irregular fluctuations in a month horizon of changes amounted to 16% of the total variability. whereas in a three month horizon their share was under 5%. Annually. on average. cyclic fluctuations amounted to 82.4%. seasonal fluctuations 13.7% and random 3.9% of the total price variability for turkey livestock.

DISCUSSION

Price variability is characteristic of the free market. functioning based on the rule of balance between supply and demand [Figiel et al. 2012]. Significant price fluctuations frequent in agriculture are rooted in a relatively poorly flexible in price demand influenced by slow changes and practically fixed supply in short-term. often influenced by quite rapid changes. Figiel [2002] points out that the range of price fluctuations depends greatly on the price efficiency of a given market. expressed as the ability to set a price quickly. objectively reflecting the demand-and-supply situation both at the present moment and in future determined for the given market.

The price variability of turkey livestock expressed in the most general measurement. which is variability coefficient. was on average 5% in a year and was small in comparison to other livestock types. Rembeza [2007] states that in 1996-2007 the highest price variability was characteristic for beef livestock (around 19%). lower for pork (15%). and the lowest for poultry (10%).

The type of variability that is most associated with agriculture is seasonal fluctuations [Hamulczuk i Stańko 2011]. Price seasonality of turkeys is characterized by a small amplitude (5% in 2011). comparable to seasonal fluctuations of beef (7%). [Idzik 2009] and significantly lower than chickens (16% in 2011. [Utnik-Banaś 2011] or pork (18%) [Olszańska 2012]. The share of seasonal fluctuations in the total price variability of turkey or beef livestock (14%) is also small (14% on average) in comparison to chickens or pork. where the share of this type of variability amounts 36 and 46% respectively. In recent years (2010-2015) the seasonality model of turkey livestock prices has changed with its peak in autumn (October) and differs from the seasonality model of chickens. which peak in summer (August). The highest influence on the price variability of turkey livestock came from the long-term trend and cyclic fluctuations (82% of the total variability on average). As a comparison with other kinds of livestock. this type of price variability amounted to 75% for beef. 55% for chickens and 46% for pork. The influence of random fluctuations on the price variability of turkey livestock was much lower (4% on average) than for pork (8%). chickens (9%) and beef (11%). Figiel [2002] points out the fact that the decrease in random fluctuations of prices having no justification in real supply-and-demand relations is characteristic of an increase in the price efficiency of a given market. The results of the paper indicate tighter relations between prices of turkey and beef livestock than chickens and pork.

REFERENCES

Biegański M. (2016) Produkcja indyków w Polsce w 2015r. Polskie Drobiarstwo, 4, 54-58.

Ceny produktów rolnych (2006-2015) Główny Urząd Statystyczny, Warszawa.

Dittmann P. (2008) Prognozowanie w przedsiębiorstwie. Metody i ich zastosowanie. Oficyna Wolters Kluwer Business, Kraków.

Dybowski G. (2014) Podstawy konkurencyjności polskiej branży drobiarskiej. Studia i Monografie, 160. IERiGŻ−PIB. Warszawa.

Figiel S. (2002) Cenowa efektywność rynku towarowego na przykładzie zbóż w Polsce. Wydawnictwo Uniwersytetu Warmińsko−Mazurskiego. Olsztyn.

Figiel S., Hamulczuk M., Klimkowski C. (2012) Metodyczne aspekty analizy zmienności cen oraz pomiaru ryzyka cenowego na towarowych rynkach rolnych. Komunikaty. Raporty. Ekspertyzy, 559. IERiGŻ−PIB. Warszawa.

Hamulczuk M. (2009) Analiza, prognozowanie i zarządzanie ryzykiem cenowym na podstawowych rynkach rolnych – możliwości stabilizowania dochodów producentów rolnych. IERiGŻ−PIB. Warszawa, 149. 1-68.

Hamulczuk M., Stańko S. (2011) Prognozowanie cen surowców rolnych - uwarunkowania i metody. Komunikaty. raporty. ekspertyzy nr 547. IERiGŻ-PIB. Warszawa, 5 – 43.

Idzik M. (2009) Analiza struktury szeregów czasowych cen produktów rolnych. [In:] Zarządzanie ryzykiem cenowym a możliwości stabilizowania dochodów producentów rolnych − aspekty poznawcze i aplikacyjne. Red. M. Hamulczuk. S. Stańko. IERiGŻ-PIB. Warszawa, 148. 15 – 47.

Kot S. M., Jakubowski J., Sokołowski A. (2011) Statystyka. Difin, Warszawa, 335 – 357.

Olszańska A. (2012) Rynek żywca w Polsce (1955−2010) – zmiany strukturalne. koncentracja produkcji i wahania podaży. Monografie i opracowania nr 214. Wyd. UE we Wrocławiu.

Rembeza J. (2007) Transmisja cen na rynku mięsa. [W:] Ewolucja rynku mięsnego i jej wpływ na proces transmisji cen. IERiGŻ-PIB., 73, 183 – 213, Warszawa.

Stańko S. (red.) (2013) Prognozowanie w agrobiznesie. teoria i przykłady zastosowania. Wydawnictwo SGGW. Warszawa.

Szymańska E. (2012) Zmienność koniunktury na rynku trzody chlewnej w Polsce. Roczn. Nauk. SERiA. XIV/1, 524 − 528

Świetlik K. (2016) Zmienność i zróżnicowanie cen żywca drobiowego w 2015r. Polskie Drobiarstwo, 4, 2-7.

Utnik−Banaś K. (2012) Analiza szeregu czasowego cen żywca brojlerów w latach 1991−2011. Metody Ilościowe w Badaniach Ekonomicznych. Wydawnictwo SGGW. Warszawa. XIII/1, 224 – 233.

Zintegrowany System Rolniczej Informacji Rynkowej (2016) http://www.minrol.gov.pl/pol/ Rynki-rolne [12.04.2016]

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII, No. 1, 2017, pp. 143 – 156

Received: 04.03.2017
Accepted: 05.04.2017

THE LONG TERM MODELING OF RESIDENTIAL PROPERTY PRICES IN POLAND

Rafał Zbyrowski (ORCID: 0000-0001-8499-8899)
Faculty of Management, Warsaw University, Poland
e-mail: rzbyrowski@wz.uw.edu.pl

Abstract: The main purpose of this article is to describe a dependence between prices of flats and index of creditworthiness in Poland. In the empirical part of this paper the author tests mentioned relations according to Engle-Granger's procedure. Moreover the long time relation had been verified by Johansen's procedure and a VAR model. This case leads to the examination and estimation cointegration with testing lags between very important variables on real estate market in Poland. The database used in the research contains monthly observations from the middle of 2010 to the beginning of 2014.

Keywords: Econometric modeling, VAR, VEC, real estate, residential real estate.

INTRODUCTION

The activation of government programs related to support housing construction in Poland had been carried out since the nineties. Old tax incentives have been replaced by much less effective programs "Family on its own" (Rodzina na swoim) and then "Apartment for the young" (Mieszkanie dla młodych). Over the last 25 years the real estate market in Poland passed tumultuous changes. Systematically the market had become free of large-scale government campaigns. The older government campaigns had been aimed at the activation of the housing sector (in favor of an increase the role of macroeconomic variables, among which a special role is played by factors affecting the creditworthiness of households). The creditworthiness is dependent on many determinants such as incomes, credit periods, the currency of the loan, interest rates or the number of members constituting the household. Of course, the most important is the borrower's income. Average wages in 2005-2013 had increased by 53,5%, while inflation during this

DOI: 10.22630/MIBE.2017.18.1.14 JEL: C32, R31

period amounted to 45,45%. It shows that real incomes had increased by 8%. Thus, in theoretical terms creditworthiness should had been increased. Unfortunately, it did not happen because in the period 12.2007-12.2013 the creditworthiness decreased by 15% (Figure 1). The real wage growth has not caused an increase of the creditworthiness for apartment buyers. An important factor which determine the creditworthiness is also the interest rate. It depends on the WIBOR 3M for credit in PLN (in rare cases the 3M EURIBOR for the euro). This indicator is a derivative with respect to the rate of inflation. In the years 2005-2012 the 3M WIBOR amounted to approx. 5%. The maximum of 6.41% had been noted in 2008, while the world crisis had been reaching Poland. In 2013 the 3M WIBOR fell by 76% compared to 2012. At the present time, its value at the level of 1.73% is almost the lowest in history. According to Home Broker (Figure 1) the index of creditworthiness for a 3-person family with a net income of 5000 zł increased in the period from 12.2012 to 04.2014 by 10.6%. Thus, the average family could receive an average of 387 000 zł credit in April 2014 year instead of 350 000 zł in December 2012. The increase in creditworthiness increases the demand. Consequently, it causes an increase in real estate prices. This economic dependence is reflected in graphs of the time series (Figure 1) in the context of which raise the following questions:

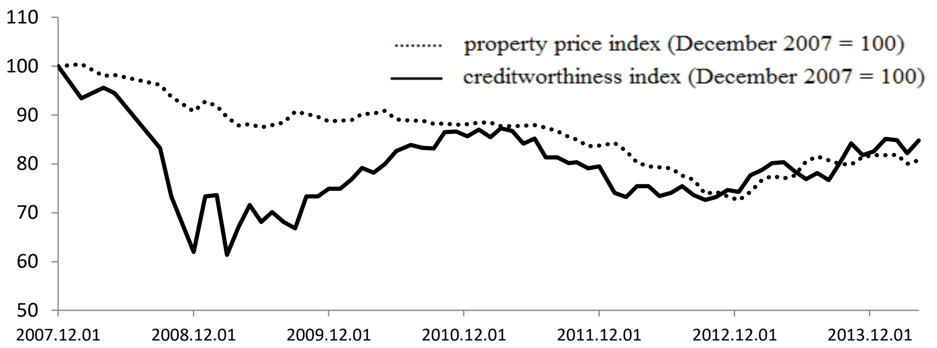
Is there a long-term relationship between the level of prices in the housing market and the credit worthiness in Poland? What is the real strength of this relationship?

On these questions the author tries to answer on the basis of analysis carried out in cointegration Engle-Granger's and Johansen's procedures.

THE LEVEL OF PROPERTY PRICES AND THE AVERAGE CREDITWORTHINESS

The years from 2008 to 2012 are considered to be the period of crisis in the polish economy. The following chart 1 shows quickly declining creditworthiness of consumers and how this affected the price of real estate in Poland. As we can see a lot of fluctuations in the market were associated with activities of banks and government institutions.

Figure 1. The level of property prices and the average creditworthiness



Source: portal www.egospodarka.pl

Observed in the years 2008 - 2012 the fluctuation in housing prices has progressed at different stages with varying dynamics. In the period from January 2008 to mid-2009, housing prices fell quicker. The reason for this was limited crediting by banks caused by high credit spreads, decline in the creditworthiness of consumers and increase the number of negative credit decisions. Above all the most important had been the limitation of foreign currency credits - especially in CHF. In subsequent months, the credit market gradually began to affect interest rate cuts, which reduced the cost of financing. Important was also raising the price limits in the "Family on its own" government program. According to higher price limits even relatively expensive housing could be subsidizing. In 2010 and 2011 there has been a stabilization of housing prices despite the rapidly increasing supply in the primary market. The higher supply was caused by the liberalization of the credit policy of commercial banks. At the beginning of 2011, there were increases in interest rates. Moreover in 2011 government institution (KNF) tightened requirements on the availability of credit. According to regulations of the financial supervision (KNF) the amount of credit installments could not exceed 50% of net income for citizens earning below the national average and 65% of income for the others. This resulted in a gradual decline in the purchasing power of housing buyers. The limitation for buyers was also a decrease the availability of the government program "Rodzina na swoim" from August 2011. From this moment a much lower price limits had eligibled for housing subsidies in most cities. The program caused stimulating the competition between the supply side of the market. At the beginning of 2012 KNF entered into force the revised recommendation "S", which limited the demand for residential properties a few months later. New regulations had hindered the access to mortgage loans in foreign currencies, and also had changed the method of calculating creditworthiness. From that moment, regardless of the mortgage duration, banks evaluated the ability of credit as if it was made on the 25 years. Enacted legislation caused a decrease in the index of housing prices. It can be said that the situation in the credit market had a very significant impact on the dynamics of real estate prices. During the five-year depreciation of housing values we can see moments when it accelerate with decreasing availability of the mortgage financing. Since the end of 2012 to today, the ability to credit and property prices are rising as shown in the Figure 2.

THE RESEARCH METHOD

Using the Engle-Granger's procedure, we can define and estimate cointegration on the basis of economic theory [Charemza 1997]. This procedure shows the dependence in a form of a regression equation:

$$\ln Y_t = \beta \cdot \ln X_t + c + \xi_t \tag{1}$$

The regression expresses a long-term equilibrium relation between the variables X and Y [Enders, 2003]. In the empirical part of this article we can find an example of an application of Engle-Granger's procedure for cointegration modeling. This procedure comprises the following steps:

1. Definition for the variables (as dependent and independent variable on the basis of theory).
2. The examination of a stationary for variables Xt and Yt in the equation: $\ln Y_t = \beta \cdot \ln X_t + c + \xi_t$ (both variables are integrated at the first difference in the most common case).
3. The estimation of the regression $\ln Y_t = \beta \cdot \ln X_t + c + \xi_t$ by LSM.

4. Testing stationary for residuals previously created from the regression. Stationary residuals indicates the existence of the cointegration vector, which describes a long-term relationship between variables X_t and Y_t. The correct relationship $\ln Y_t = \beta \cdot \ln X_t + c + \xi_t$ creates the opportunity to build a short-term error correction model ECM: $\Delta \ln Y_t = \alpha \cdot \Delta \ln X_t - \gamma \cdot \xi_{t-1} + c_1$

Fundamentals of methodological procedures for the Engle-Granger's approach limit the research to identifying at most one cointegration vector (Gajda 2004). The specified cointegration vector may be only one of many such vectors.

THE RELATIONSHIP BETWEEN REAL ESTATE PRICES AND THE CREDITWORTHINESS

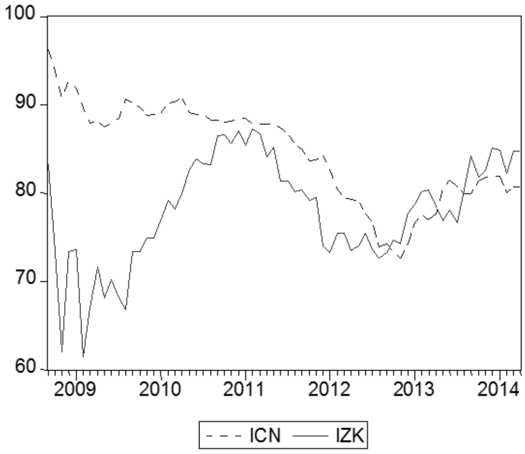
Time series shown in the Figure 2 have a monthly frequency since 2009. Property price indexes (ICN_t[1], December 2007 =100) and creditworthiness (IZK_t[2],

---

1 ICN_t – published monthly by the Home Broker and Open Finance index of housing prices formed on the basis of transactions in 16 cities (Bialystok, Zielona Góra, Bielsko-Biala, Torun, Bydgoszcz, Krakow, Lublin, Gdansk, Gdynia, Olsztyn, Katowice, Lodz, Poznań, Szczecin, Wrocław and Warsaw). The index is calculated according to the formula: $Index = C_1 \cdot \left(\frac{I_1}{St}\right) + C_2 \cdot \left(\frac{I_2}{St}\right) + C_3 \cdot \left(\frac{I_3}{St}\right) + C_4 \cdot \left(\frac{I_4}{St}\right) + \ldots + C_{16} \cdot \left(\frac{I_{16}}{St}\right)$

December 2007 =100) have been prepared by Lion'sBank (based on data from banks, mortgage lenders and the companies of Home Broker and Open Finance). Index values for the initial sample size (especially the last months of 2008) are illustrative purpose only because of the irregular frequency.

Figure 2. Indexes of the real estate prices and the creditworthiness in the years 2008-2014



Source: own preparation based on www.egospodarka.pl

The Figure 2 during the period 2008.09 - 2014.04 reveals considerable divergence of indexes during the global crisis. We can see a common trajectory of the time series from 2010.09 to 2014.04. Fluctuations of balance in the years 2008 - 2010 were so strong that they caused a complete reversal depending based on economic theory between ICN_t and IZK_t. Since the last months of 2010 up to now we can observe the presence of a positive relationship between the two variables. The Figure 2 of time series suggests a relationship with a delay. it seems to be true because of the specificity of variables. As it have been mentioned before in the first part of the sample there was irregular frequency of data combined with the effect of the global crisis. Thus the study have been focused on the time period from 2010.09 to 2014.04.

---

where:

l_1, l_2, l_3, l_4… l_{16}– the number of transactions carried out in each city, *St*– the sum of transactions made in all cities,

C_1, C_2, C_3, C_4…C_{16}– the price of one square meter of housing in individual cities, calculated according to the formula:

$Cena = (m_1 \cdot 0{,}5) + (m_2 \cdot 0{,}3) + (m_3 \cdot 0{,}2)$

where:

m_1 - median value of one square meter in the last calendar month,

m_2 - median value of one square meter in a month (m_1– 1),

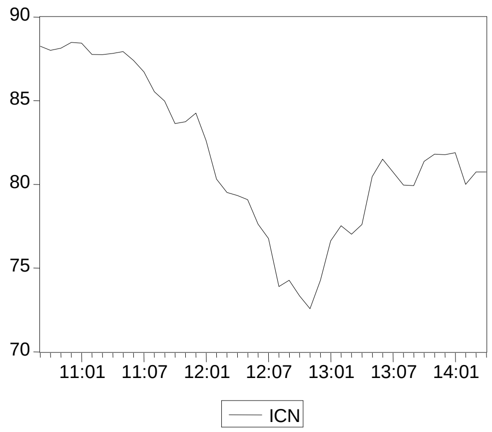
m_3 - median value of one square meter in a month (m_1 – 2).

2 IZK_t – the index built on the basis of data containing median of creditworthiness for a family (2 + 1) with an income of 5 thousand zł net collected by the Lion's House. Credit for 30 years, buying real estate in the city with a population of 150 thousand.

The examination of the stationarity for ICN_t and IZK_t

The integration testing of the time series was made on the basis of the Dickey-Fuller'a test and autocorrelation function (Majsterek 2014) in Eviews. Both time series occur to be stationary on the first difference and are integrated I(1).

Figure 3. The time series ICN in the periods 2010.09 – 2014.04



Source: own preparations

Table 1. ADF test for the ICN_t in the periods 2010.09 – 2014.04

ADF Test Statistic	-1.537272	1% Critical Value*	-3.5930
		5% Critical Value	-2.9320
		10% Critical Value	-2.6039
*MacKinnon critical values for rejection of hypothesis of a unit root.			

Source: own calculations in program Eviews

Time series of the index of real estate prices ICN_t is nonstationary on level. The value of the augmented Dickey-Fuller test ADF (-1.5372), exceeds the critical values for the low level of significance (Table 1).

Table 2. ADF test for the first difference of ICN_t in the periods 2010.09 – 2014.04

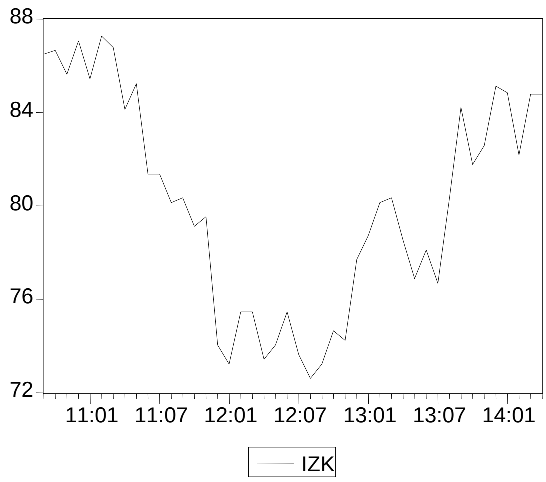
ADF Test Statistic	-4.040223	1% Critical Value*	-3.597
		5% Critical Value	-2.934
		10% Critical Value	-2.605
*MacKinnon critical values for rejection of hypothesis of a unit root.			

Source: own calculations in program Eviews

The time series of ICN_t seems to be integrated on the first difference because the value of the ADF test (-4.0402) is less than the critical value (Table 2).

Analysis of the autocorrelation function for levels and first difference of ICN_t conducts to the same conclusions. Autocorrelation function takes a sinusoidal shape for level of ICN_t, while for the first difference the autocorrelation function is fading.

Figure 4. The time series IZK in the periods 2010.09 – 2014.04



Source: own preparations in program Eviews

The time series of creditworthiness IZK has a similar course in time to the ICN in the considered period. Testing for integration has been carried out below:

Table 3. ADF test for the IZK_t in the periods 2010.09 – 2014.04

ADF Test Statistic	-1.475296	1% Critical Value*	-3.593
		5% Critical Value	-2.932
		10% Critical Value	-2.604
*MacKinnon critical values for rejection of hypothesis of a unit root.			

Source: own calculations in program Eviews

A study for the level of IZKt confirms nonstationarity. The value of the ADF test (-1.4753) does not allow to reject the null hypothesis of the presence of unit root (Table 3).

Table 4. ADF test for the first difference of IZK_t in the periods 2010.09 – 2014.04

ADF Test Statistic	-5.162493	1%	Critical Value*	-3.597
		5%	Critical Value	-2.934
		10%	Critical Value	-2.605
*MacKinnon critical values for rejection of hypothesis of a unit root.				

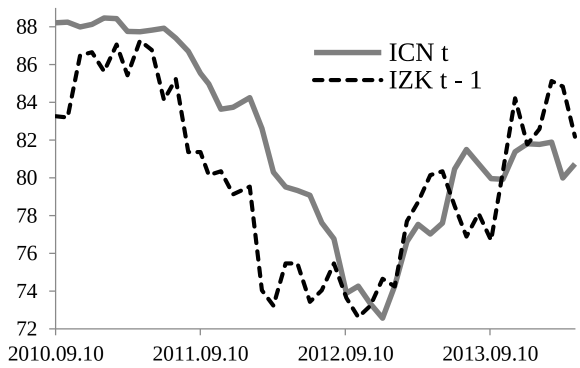
Source: own calculations in program Eviews

Due to the augmented Dickey-Fuller test (Table 4) the time series IZKt seems to be integrated I(1) in the first difference. The integration of the two series in the first difference with the previously cited theory and charts has led the author to verify the cointegrating relationship. Other recommended in the econometrics literature tests like KPSS or integrated test of Durbin-Watson lead to the same conclusions in this case. Further analysis will be based on the Engle-Granger's and Johansen's procedures.

The long-term regression between variables ICN_t i IZK_t

The regression model posibly describing the long-term equilibrium was carried out in the program Eviews. Due to the specificities of the exogenous variable, IZK was lagged.

Figure 5. Indexes of the real estate prices and the 1 month lagged creditworthiness



Source: own preparations

After a long modeling the long-term relation the author decided that the optimal lag for the variable IZKt is one month. This means that the change of index IZKt causes the adjustment of ICNt after one month. Models built for other lags showed worse statistical properties (both smaller R-squared and values of t-Student statistics for parameters). It seems that the dependence between the index of creditworthiness IZK and the index of real estate prices ICN without lags is unrealistic. The real estate market requires time to react to changes of the demand.

The index of creditworthiness is undoubtedly an important factor in creating the demand for housing. Although the impulse of change is "recognize by the market," with an average of 1 month delay. It is a quite quick response, but still delayed.

Table 5. The long-term relation between IZK_t and ICN_t in the period 2010.09 – 2014.04

Dependent Variable: ICN
Method: LeastSquares
Date: 05/10/16 Time: 14:44
Sample: 2010M09 2014M04
Includedobservations: 44

Variable	Coefficient	Std. Error	t-Statistic	Prob.
IZK(-1)	0.667301	0.072611	9.190123	0.0000
C	35.93121	6.285064	5.716919	0.0000
T	-0.172769	0.025951	-6.657564	0.0000

R-squared	0.823165	Mean dependent var	81.80936
Adjusted R-squared	0.814538	S.D. dependent var	4.784893
S.E. of regression	2.060626	Akaike info criterion	4.349643
Sum squaredresid	174.0934	Schwarz criterion	4.471292
Log likelihood	-92.69215	Hannan-Quinn criter.	4.394756
F-statistic	95.42699	Durbin-Watson stat	0.602274
Prob(F-statistic)	0.000000		

Source: own calculations in program Eviews

The estimated model (Table 5) is characterized by fairly high R-squared value. The relationship describing the potential long-term relationship can be written as:

$$ICN_t = 0.667\ IZK_{t-1} + 35.931 - 0.173\ t$$

The increase in the index of creditworthiness IZKt by one point in the past period t-1 (a month earlier) results in an average increase in the price index of real estate ICNt about 0.667 point, assuming ceteris paribus. This regression (Table 5) is characterized by quite high fit to the empirical data (Adjusted R-squared: 0.8145). We can say that more than 81% of the variation of the ICNt have been explained by the variation of the IZKt and a variable showing a deterministic trend [Borkowski 2007].

The next stage of the research will become the verification of previously built long-term relationships with the ADF test for residuals. According to the Engle-Granger's procedure, the correct long-term relationship should generate stationary residual [Maddala 2006]. Stationarity of residual is an evidence of a linear combination of nonstationary variables which represents the cointegrating vector based on the theory of economy.

The examination of the stationarity for residuals of the long-term relation

The examination of the stationarity for apropriate residuals performed using the augmented unit root test ADF (Augmented Dickey Fuller) and the autocorrelation function. The stationarity testing of residuals can answer to the question of the existence of a long-term relationship [Syczewska 1999]. In case of the nonstationarity of residuals the long-term relationship turns out to be completely untrue.

Table 6. ADF test for residual series from the long-term regression

Null Hypothesis: RESZTY has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=9)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.706089	0.0080
Test criticalvalues:	1% level	-2.619851	
	5% level	-1.948686	
	10% level	-1.612036	

*MacKinnon (1996) one-sided p-values.

Source: own calculations in program Eviews

Table 6 indicates that the residuals generated by the built model are stationary. Thus, the relationship between the index of creditworthiness IZKt and the index of real estate prices ICNt is essentially a long-term dependency. Examined variables ICN_t and IZK_t are cointegrated. In this situation, the long-term parameter 0.667 can be interpreted as the elasticity of the long-term relation (Table 5). The alternative approach for modeling nonstationary time series are models VAR with the Johansen's cointegration test.

The VAR relation between ICN_t and IZK_t

The VAR model was built with 1 month lag. VAR with higher lag showed a total lack of statistically significant coefficients for variables delayed more than 1 month. Moreover statistically insignificant were constant and deterministic trend. Coefficients from the VAR model are consistent with the theory of economy. Noteworthy is a very strong influence of changes in real estate prices ICN from the previous month to its current value.

Table 7. Model VAR for variables ICN_t i IZK_t

VectorAutoregressionEstimates
Date: 05/10/16 Time: 10:53
Sample: 2010M09 2014M04
Includedobservations: 44
Standard errors in ( ) & t-statistics in [ ]

	ICN	IZK
ICN(-1)	0.826582	0.017442
	(0.04075)	(0.08914)
	[20.2853]	[0.19567]
IZK(-1)	0.175419	0.982276
	(0.04176)	(0.09136)
	[4.20050]	[10.7519]
R-squared	0.962724	0.808747
Adj. R-squared	0.961837	0.804193
Sum sq. resids	36.69769	175.6247
S.E. equation	0.934748	2.044881
F-statistic	1084.740	177.6042
Log likelihood	-58.44083	-92.88481
Akaike AIC	2.747310	4.312946
Schwarz SC	2.828410	4.394045
Mean dependent	81.80936	80.04843
S.D. dependent	4.784893	4.621194
Determinant resid covariance (dof adj.)		3.618250
Determinant residcovariance		3.296794
Log likelihood		-151.1115
Akaikeinformationcriterion		7.050523
Schwarz criterion		7.212722

Source: own calculations in program Eviews

The analytical formula of the built VAR model is shown below:
$ICN_t = 0.827 * ICN_{(t-1)} + 0.175 * IZK_{(t-1)}$
$IZK_t = 0.017 * ICN_{(t-1)} + 0.982 * IZK_{(t-1)}$

The first equation of the VAR model is consistent with the assumptions of the theory of economy (ICN is dependent on changes in real estate prices and the changes in the creditworthiness IZK in the previous month). The second equation seems to be contrary to theory of economy. The variables are nonstationary on levels and integrated in the first differences, hence VEC model was built. The VEC model has correct adjustment coefficients (negative sign), but in many cases the large standard errors of short-term flexibility coefficients. The upper part of the

Table 8 shows the scalar of cointegration equal to 1.014. Moreover the value 1.014 has a very small standard error of the estimation (0.0098), hence quite high the absolute value of the t-Student statistic (-103.453). On this basis, the estimation of the long-term relationship between ICN and IZK seems to be statistically significant.

Table 8. Model VEC for variables ICN_t i IZK_t

CointegratingEq	CointEq1	
ICN(-1)	1.000000	
IZK(-1)	-1.014261	
	(0.00980)	
	(-103.453)	
Error Correction:	D(ICN)	D(IZK)
CointEq1	-0.175698	-0.033616
	(0.04661)	(0.10461)
	(-3.76943)	(-0.32136)
D(ICN(-1))	0.195882	-0.136484
	(0.13057)	(0.29304)
	(1.50015)	(-0.46575)
D(IZK(-1))	-0.059036	-0.144421
	(0.08124)	(0.18231)
	(-0.72672)	(-0.79216)
R-squared	0.346156	0.019215
Adj. R-squared	0.314261	-0.028628
Sum sq. resids	34.22215	172.3617
S.E. equation	0.913612	2.050352
F-statistic	10.85304	0.401632
Log likelihood	-56.90433	-92.47221
Akaike AIC	2.722924	4.339646
Schwarz SC	2.844573	4.461295

Source: own calculations in program Eviews

In addition, the number of cointegration vectors has been tested on the basis of the Johansen's test (Kusideł 2000).

Table 9. The Johansen's cointegration test for variables ICN_t i IZK_t.

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.257429	13.09698	12.32090	0.0370
At most 1	2.22E-05	0.000975	4.129906	0.9814

Trace test indicates 1 cointegratingeqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Source: own calculations in program Eviews

Table 9 shows that there is only one vector cointegration, thus in the long term is correct relationship consistent with the theory of economy. It has been estimated the value of the long-term parameter equal to 1.014 (Table 8) in Eviews program. This is the higher value than the apropriate coefficient nfrom the Engle-Granger's procedure. However, the sign of coefficient is correct from both procedures (reffering the sign to the market's mechanisms and the theory of economy).

SUMMARY

1. In the period from 2010.09 to 2014.04, we can confirm the thesis about the existence of cointegration relationship between the real estate price index (ICNt) and the index of creditworthiness (IZKt).
2. The dependence of both indexes requires apropriate lags. In the case of the real estate market it turned out that the "effect of adjustment" will take 1 month (when the change of the creditworthiness led to a correction in prices).
3. The VAR model and the Johansen's procedure led to estimation of the long-term flexibility equal to 1.014. Thus, an increase in the index of creditworthiness IZKt by one percentage point causes an average increase in the price index of real estate ICNt about 1.014 percentage point with a 1 month lag.

The value more than 1 of coefficient for the long-term relation may indicate a process conducive to speculative bubbles in the real estate market (the price index grows faster than the index of creditworthiness). On the other hand, the coefficient of long-term relation is less than one on the basis of the Engle-Granger's procedure. The Johansen's procedure based on the VAR modeling is considered to be more reliable tool in empirical researches [Welfe 2009].

REFERENCES

Borkowski B., Dudek H., Szczesny W. (2007) Ekonometria, wybrane zagadnienia. PWN.

Bryx M. (2006). Wybrane aspekty finansowania i organizacji rynku nieruchomości. SGH Oficyna Wydawnicza.

Charemza W., Deadman D. (1997) Nowa ekonometria. PWE.

Enders W. (2003) Applied Econometric Time Series. John Wiley & Sons.

Franses P. H., De Groot B. (2013) Do commercial real estate prices have predictive content for GDP? Taylor & Francis.

Gajda J. (2004) Ekonometria. C.H. BECK.

Kusideł E. (2000) Modele wektorowo-autoregresyjne VAR metodologia i zastosowania. Absolwent.

Maddala G.S. (2006) Ekonometria. Warszawa, PWN.

Majsterek M. (2014) Modelowanie systemów skointegrowanych. Aspekty teoretyczne. Bank i Kredyt 45(5),.

Syczewska E. (1999) Analiza relacji długookresowych: estymacja i weryfikacja. Szkoła Główna Handlowa.

Welfe A. (2009) Ekonometria. Polskie Wydawnictwo Ekonomiczne.

QUANTITATIVE METHODS IN ECONOMICS
Volume XVIII , No. 1, 2017, pp. 157 – 166

Received: 15.03.2017
Accepted: 05.04.2017

TO IMPROVE THE EMPLOYEE ASSESSMENT PROCEDURES – DEVELOPMENT OF RATIO WITH THE USE OF IRT MODELS

Anna Zięba (ORCID: 0000-0002-0575-8318)
Faculty of Economic Sciences
Wrocław University of Economics, Poland
e-mail: anna.zieba@ue.wroc.pl

Abstract: Taking into account the trends in selecting and evaluating employees connected with the use of more and more objective tools, as well as more and more crucial human resources issues in an enterprise, it seems necessary to develop more and more innovative methods and tools. The objective of this paper is to develop a ratio to assess the employee on the basis of several traits at the same time as well as the achievement of specific targets in the company growth. Such ratios are rarely discussed in the literature, however, the analysis can provide a lot of useful information about a specific issue.

Keywords: GRM, IRT models, ratio, employee assessment, periodic assessment

INTRODUCTION

The thesis that the employee is the main factor determining the competitiveness and the human capital is one of the most crucial resources of the company has already become a canon in the theories of human resources management and organization psychology. The activities supporting the employee development in the scope of formal as well as interpersonal qualifications are obvious yet often underappreciated. The organizations which realize the value provided by the motivated and committed employee tend to "diagnose" their possibilities and weaknesses as precisely as possible, and the decisions regarding their career paths are corrected regularly on the basis of various kinds of employee assessments [Verbruggen 2010]. Those employee assessments are made for the purpose of promotion, career path development, trainings or in connection with the company reorganization. Such assessments also provide useful information.

DOI: 10.22630/MIBE.2017.18.1.15

JEL: C25, C51

Knowledge and information are the two factors which at present determine the life of the organization. That is why the problem which often arises is how to make assessments so that they are an effective tool in the organization management process.

A lot of ratios, such as absenteeism, productivity, efficiency are used in the employee assessment process. All of the ratios are important, however, it should considered how to distinguish one employee from another on the basis of those ratios. Is it enough to say that their assessment score is the same (e.g. in regard of efficiency) which would presumably mean that they are equally “good” and they should get the same bonus? The most common system relies on that very assumption. However, the employee’s attitude to work (their motivation), commitment to all kinds of additional activities, cooperation with other workers and the general work discipline are also worth evaluating. So in effect a problem emerges – namely what tool should be applied to be able to assess the employees, comparing several traits at the same time (or evaluating the execution of several targets at the same time).

So far a lot of attention has been paid in the literature to the issue of employee assessment. Different assessment methods have been discussed [Sidor-Rządkowska 2000, Jędrzejczak 2000]. The authors agree that it is difficult to indicate a solution which would replace the employee assessment and that is why the question which is asked now in the subject literature is not “why assess?” but “how to assess?” What is stressed is the need to use assessments in the process of developing the company growth strategies, in controlling the achievement of set targets [Juchnowicz 2003]. However, it is known that in spite of the growing interest in the problem of employee assessment little attention has been paid so far to that issue from the perspective of the impact of latent traits on the assessment results. The most frequent ratios suggested in the literature do not include the differences between employees caused by different level of intensity of latent traits.

That is why the objective of this paper is to develop a ratio which would support the decision making process and could be employed to assess the employee in respect of several traits at the same time as well as the achievement of specific targets of the company development. It is important then to develop the measuring tools to formulate correct conclusions, affect and facilitate decision making.

Based on the issues mentioned above the following hypothesis was put forward:

Hypothesis: a ratio developed with the use of IRT models is a tool that can be applied to assess the employee in respect of several traits at the same time and the achievement of specific targets of the company development.

EMPLOYEE ASSMESSMENT

Employee assessment is a process to evaluate personal traits, attitudes, behaviors and the assigned task completion level. The assessment results are the

basis for planning trainings and employee development as well as remuneration schemes, making decisions on promotions, awards, pay rises or dismissals.
As a result of employee assessment it is possible to provide their employers and the employees themselves with information on the results, on how they are perceived by other employees, on the possibilities for growth in a given position.
It is important why assessments are made as well as what objectives of the assessment are. The objectives of periodic assessments should be then the starting point [Listwa 1999, Szałkowski 2000].

The assessment criteria should correspond to the question: What are we going to assess? All applicable assessment criteria are usually divided into four groups [Ludwiczyński 2014]: qualification criteria, effectiveness criteria, behavioral criteria and personality criteria. Most controversy is caused by the personality criteria which regard the traits of a given employee which determine their behavior and attitude at work. Some examples of personality criteria used in practice include responsibility, creativity, assertiveness, resistance to stress. Some also mention intelligence, talents and temperament [Pawlak 2003, Pocztowski 2003]. The discussions over that criterion regard the issue of dependence between human personality and work results.

A reliable and accurate employee assessment is difficult. What are the problems connected with the employee assessment process? The easiest thing to assess is the employee effectiveness on the basis of the results e.g. sales performance. It is more difficult to make assessment on the basis of behaviors. Most organizations apply the following ratios: employee productivity (the ratio of the number of manufactured products to the number of hours worked by the employees), completion of the production plan (the number of manufactured finished products on a shift, whole day in comparison to the number assumed in the plan), absenteeism (absences in hours or days in comparison to the whole amount of time in hours or days.) These are only examples because the type and number of the ratios should correspond to the company needs.

To sum up the above discussion it is worth repeating the question asked in the introduction: How to distinguish one employee from another? Is it enough to say that their assessment score is the same, which would mean that they are equally "good?" The following sections of the paper explain the claim that the influence of other traits which are not directly observable on the results should be taken into account and the results should be differentiated.

GRM – KEY INFORMATION

The most obvious reason for the development of multi-category models of responses is the fact that multi-category questions are most often used in various kinds of studies. Depending on whether the categories are ordinal or not, there are various types of models.

Samejima initially proposed Graded Response Model used for the analysis of multi-category questions. Each question no. j in this model (GRM) is characterized by two kinds of parameters: parameter β_j describing the item discrimination parameter and by parameters known as item location thresholds α_{jm}, where m=1,2,…,M means the number of categories [Samejima 1997]. For instance: in the case of a question with three categories of responses there are two threshold values: α_{j2} - threshold separating the first from the second category and threshold α_{j3} - threshold separating the second from the third category. Parameters α_{jm} indicate the latent trait level that is necessary to provide a response above that threshold value.

Each question no. j includes K_j possible response categories. The respondent chooses one of the categories (the possibility of choosing several categories within one question is a different issue). The probability of response to question j can be defined in each of the categories for person i and question j with K_j response categories. That probability is designated as π_{ijk}, $k = 0,...,K_j - 1$. These probabilities within each question sum up to 1.

The Samejima's model is based on the accumulated probability. The function describing the probability of providing response to question j in category k was defined as follows:

$$\log\left(\frac{\pi_{ijk}}{1-\pi_{ijk}}\right) = \theta_i - \alpha_{jk} \tag{1}$$

θ_i – parameter related to respondent i, indicating the degree of intensity of the analyzed latent trait.

The probability of choosing the k or higher category and the (k+1) or higher category, in the case of question j, is defined as follows (Samejima 1997):

$$P(X_{ij} \geq k) = \frac{\exp((\theta_i - \alpha_{jk})\beta_j)}{1 + \exp((\theta_i - \alpha_{jk})\beta_j)} \tag{2}$$

and

$$P(X_{ij} \geq k+1) = \frac{\exp((\theta_i - \alpha_{j(k+1)})\beta_j)}{1 + \exp((\theta_i - \alpha_{j(k+1)})\beta_j)} \tag{3}$$

In order to avoid the situation in which

$P(X_{ij} = k) = P(X_{ij} \geq k) - P(X_{ij} \geq k+1) < 0$ it is assumed that $\alpha_{jk} < \alpha_{j(k+1)}$.

DEVELOPMENT OF THE RATIO – RESEARCH METHOD

The assessment ratios are an important element affecting the implementation of a strategy assumed in an organization. With the use of the ratios it is easier to systematically monitor the completion of the objectives and take actions eliminating the inadmissible (too big) deviations of the ratios from the assumed target values (norms.)

In the case of employee assessment it is necessary to analyze many different situations and many different traits. The assessment is made by comparing the traits, qualifications, behaviors of one employee towards other employees or against a set standard. In effect then the assessment should have some point of reference [Ludwiczyński 2014]. It is difficult, however, to compare employees and at the same time analyze all studied traits. What is needed then is the ratios with which it would be possible to evaluate at the same time the completion of several goals by the employees.

The ratio suggested here should be calculated on the basis of selected traits (most crucial for a given position from the point of view of the employer.) The ratio shall be developed in several stages.

Stage I

All studied traits should be comparable. The most common approach is to change all indicators into stimulants. If, however, the analyzed values are expressed in different units of measure, they should be standardized.

Stage II

The level of intensity of the studied traits is estimated with the use of the latent trait models. The directly unobservable traits (latent traits) are measured with the useful tool called IRT models (*Item Response Theory.*) With the use of IRT models it is possible to evaluate the relationship between the responses to the questions and the level of intensity of the analyzed traits (see e.g. [Ayala 2009, Wilson 2004]). One of the features of those models is the use of observable behaviors to estimate the level of intensity of the latent trait which is studied.

Stage III

The value which is the most desirable from the point of view of the observer is identified from among all values of the selected trait. Usually this is the maximum value from among all values of a given trait.

Stage IV

Most often the similarity of observations is determined with the use of the distances between the observations. Large distances mean a small probability and the other way around.

The most common methods of determining the distances are based on the following metrics: the Minkowski distance, the Czebyshev distance, the Manhattan distance, the Euclidean distance.

The ratio will be based on the Euclidean distance.

$$d_{ij} = \left[\sum_{k=1}^{p} \left(x_{ik} - x_{jk} \right)^2 \right]^{0,5} = \left[\left(\mathbf{x}_i - \mathbf{x}_j \right)^T \left(\mathbf{x}_i - \mathbf{x}_j \right) \right]^{0,5} \tag{4}$$

We have *n* employees, each of them with *k* traits (each of them is evaluated in respect of *k* traits.)

Let's define:

$x_j^* = \max_i x_{ij}$ where *j=1,2,….k* – number of comparable traits,

i=1,2,…n – number of employees.

It can be then claimed that a model value is selected in every analyzed category (for each trait). Then:

x_1^* - model value for trait no. 1

x_2^* - model value for trait no. 2

.

.

x_k^* - model value for trait no. k

The objective of the assessment is to compare the employees in respect of several traits as the same time. By indicating the maximum value of each of the traits a certain model of employee x^* is developed with the desired model values of the traits: $x^* = \left(x_1^*; x_2^*, \ldots, x_k^* \right)$.

Formula (4) looks as follows:

$$d_i = \left[\sum_{j=1}^{k} \left(x_{ij} - x_j^* \right)^2 \right]^{0,5} \tag{5}$$

i – number, *i=1,2,…,n*

That value indicates the distance of the *i*-th employee from the comparable (model) employee, taking into account the studied and compared traits.

Stage V

The ratio is ultimately determined as:

$$W = \frac{d_i}{\max\limits_i d_i} \tag{6}$$

After such a transformation the ratio assumes the value from the range [0,1].

In the last stage, the values of ratio W can be used as the basis for the development of ranking of the employees.

The interpretation of the measure developed this way is as follow: the closer the value of the ratio to 1, the more different from the model (comparable) a given employee is. The best employees will be characterized by the value of the calculated ratio which is the closest to 0 – meaning that a given employee is ranked the closest to the model.
Consequently, the lower values of the suggested ratio shall mean the degree of completion of a specific objective is higher.
Furthermore, it should be noted that with the use of the information about the distance between the employee and the model in a base year, it is possible to evaluate the degree of completion of the objectives in the following years of monitoring the results.

APPLICATION

Study participants

The practical application of the ratio was presented with the data collected from a study on a sample of 500 employees employed as workers in a mining sector company in Poland.

Tools

The "Your Job" questionnaire with 32 questions diagnosing 4 aspects of work was used in the study. The questionnaire is a translation of the American tool *Job Content Questionnaire – JCQ* by Robert Karasek which has been recently adapted to Poland conditions by Żołnierczyk-Zreda and Bedyńska. The questions used in the questionnaire regard: assessment of demands, decision latitude that is the feeling that the employees can meet the requirements, job insecurity and the superior's and co-workers' support. The responses were coded as follows: 1 - I completely disagree, 2 – I don't agree, 3 – I agree 4 – I completely agree.
As a result of the study the traits mentioned above were measured. All calculations were made with the use of the ltm package in *R* program [Rizopoulos 2006].

Results

The possibilities of use of the ratio suggested above were presented with the selected 100 employees who were diagnosed by comparing their level of the following traits: job insecurity, decision latitude and co-workers' support.

Table 1 presents only examples of estimated (with the use of R program) levels of analyzed traits in a group of 10 selected employees.

Table 1. Estimated levels of analyzed traits in a group of 10 employees

Employee	Job insecurity	Support level	Decision latitude
1	1.878	-2.830	-2.497
2	1.833	-2.382	-2.317
3	1.648	-2.056	-1.301
4	1.451	-1.501	-1.708
5	1.740	-0.389	-1.122
6	1.552	1.466	-3.042
7	1.633	-2.574	-1.430
8	1.440	-2.155	-1.393
9	1.228	-2.268	-0.835
10	1.201	-1.790	-0.175

Source: own calculation

Trait: job insecurity was changed into a stimulant by multiplying the initially estimated values by -1. Next, the maximum value was selected from among 100 employees selected for assessment for each of the comparable traits. The following values were received:

$x_{1,\max} = 1.878$ – maximum value of the trait: job insecurity,

$x_{2,\max} = 2.329$ – maximum value of the trait: co-workers' support,

$x_{3,\max} = 1.410$ – maximum value of the trait: decision latitude.

In the next stage, the employees were compared in respect of all three traits at the same time, calculating the value of ratio *W* presented for each of them. Table 2 presents the values of the ratio for the group of 10 example employees.

Table 2. Values of ratio *W* presented for 10 example employees

Employee	1	2	3	4	5	6	7	8	9	10
ratio *W*	1	0.928	0.797	0.765	0.574	0.702	0.876	0.819	0.796	0.689

Source: own calculation

The results obtained were the basis for the development of the ranking of 100 employees. The employees were ranked from the smallest to the biggest distance from the model (value of the ratio 1).

The conducted analysis provided the following information about the group of the assessed employees:

- in the group of 100 employees, only 10 of them demonstrated ratio *W* below 0.5. That means that only 10 employees met the employer's requirements/expectations at the highest degree defined as the model,
- 16 employees in the conducted assessment are the least similar to the defined model in respect of the studied traits; they met the requirements regarding the desired values of the comparable traits set by the model to the lowest degree. The value of the ratio calculated for them is higher than 0.7,
- the other employees demonstrated value of the ratio in the range [0.5; 0.7].

With the use of the ratio it is possible to select from a group of employees those who, in regard of the studied traits, hold the highest or the lowest positions. On the basis of their analysis it is possible to indicate those who change their positions over time – indicate the direction of change: positive or negative.

CONCLUSIONS AND APPLICATION IN PRACTICE

Contemporary organizations operate in a fast-changing environment [Krupski 2005] which affects the changes in the management process. There is no universal way of managing a contemporary organization. The suggested assessment ratio facilitates the introduction of a method of monitoring the completion of specific objectives in the strategy of organization growth. The objective of monitoring is to provide a possibility of implementing actions to eliminate inadmissible deviations of the ratios from their assumed target values, to systematically monitor the completion of set objectives and to determine whether their further completion is not in danger. The suggested ratio can be used as a tool to control whether the set objectives have been achieved, to measure, analyze and evaluate work performance for specific employees. It can help organizations in achieving their long-term goals.

The potential benefits for the organizations of the use of the ratio to make assessments include the following:

- easier development of different kinds of teams of employees,
- selection of the best employees and planning their individual career paths,
- selection of the weakest employees,
- support in making decisions on how to connect the employee remunered ratio with their work performance (pay rises, bonuses, awards),
- development of a database to facilitate assessment of progress in achieving goals set by individual employees,
- monitoring changes in case of big deviations from the required ratio values.

The assessment with the use of that kind of ratio can be used to provide the employers or managers with additional feedback information on the quality of work of their employees and facilitate the process of identification of the factors which affect their professional growth.

REFERENCES

Ayala R.J. (2009) The Theory and Practice of Item Response Theory, Springer Street, New York.

Jędrzejczak J. (2000) Oceny Okresowe. Zarządzanie przez ocenianie, Ośrodek Doradztwa I Doskonalenia Kadr, Gdańsk.

Karasek R. (1979) Job demands, job decision latitude and mental strain: Implication for job redesign, Administrative Science Quarterly, 24, 285-308.

Krupski R. (2005) Zarządzanie przedsiębiorstwem w turbulentnym otoczeniu, Polskie Wydawnictwo Ekonomiczne, Warszawa.

Juchnowicz M. (2003) Narzędzia i praktyka zarządzania zasobami ludzkimi, Wydawnictwo Poltext, Warszawa.

Listwa T. (1999) Zarządzanie kadrami. Podstawy teoretyczne i ćwiczenia, Wydawnictwo AE im. Oskara Lanego, Wrocław.

Ludwiczyński A. (2014) Zarządzanie Zasobami Ludzkimi. Tworzenie kapitału ludzkiego organizacji, Wydawnictwo Naukowe PWN, Warszawa.

Pawlak Z. (2003) Personalna funkcja firmy. Procesy i procedury kadrowe, Poltext, Warsaw.

Pocztowski A. (2003) Zarządzanie zasobami ludzkimi, PWE, Warszawa.

Rizopoulos D. (2006) An R Package for Latent Variable Modeling and Item Response Theory Analyses, Journal of Statistical Software, 175.

Samejima F. (1997) Graded Response Model, Handbook of modern item response theory, Springer, New York.

Sidor-Rządkowska M. (2000) Kształtowanie nowoczesnych systemów ocen pracowniczych, Oficyna Ekonomiczna, Kraków.

Szałkowski A. (2000) Wprowadzenie do zarządzania personelem, Wydawnictwo Akademii Ekonomicznej w Krakowie, Kraków.

Verbruggen M. (2010) Career Counseling in the New Career Era, Review of Business and Economics LV (1), 2-22.

Wilson M., de Boeck P. (2004) Explanatory Item Response Models, Springer-Verlag, New York.

Żołnierczyk – Zreda D., Bedyńska S. (2014) Psychometric properties of the Polish version of Karasek's Job Content Questionnaire, International Journal of Occupational Safety and Ergonomics, 20 (4), 583-593.