METODY ILOŚCIOWE
W BADANIACH EKONOMICZNYCH

QUANTITATIVE METHODS
IN ECONOMICS

Vol. XIX, No. 1

Warsaw University of Life Sciences – SGGW
Faculty of Applied Informatics and Mathematics
Department of Econometrics and Statistics

QUANTITATIVE METHODS IN ECONOMICS

METODY ILOŚCIOWE W BADANIACH EKONOMICZNYCH

Volume XIX, No. 1

Warsaw 2018

ISSN 2082-792X
e-ISSN 2543-8565

Warsaw 2018, Volume XIX, No.1

The original version is the paper version
Journal homepage: qme.sggw.pl
Published by Warsaw University of Life Sciences Press

CONTENTS

QUANTITATIVE METHODS IN ECONOMICS
Volume XIX, No. 1, 2018, pp. 1 – 11 Received: 22.02.2018
Accepted: 05.04.2018

HOW DEMOGRAPHY AFFECTS THE ECONOMY – IMPACT OF POPULATION AGEING ON INFLATION

Paulina Broniatowska (ORCID: 0000-0002-4919-9191)
Collegium of Economic Analysis
Warsaw School of Economics, Poland
e-mail: paulina.broniatowska@gmail.com

Abstract: The motivation of this paper is to check whether inflation is linked to the population age structure. To check this hypothesis, a panel data model is used. We regress the changes in CPI on a set of macroeconomic variables. The results of the estimations suggest that there may be a relation between demography and low-frequency inflation. A larger old-age dependency ratio is correlated with lower inflation. This may confirm some of the previous empirical findings that ageing is deflationary when related to increased life expectancy.

Keywords: population ageing, inflation, demography

JEL classification: C23, J10

This is a preliminary draft. This paper was prepared for the 18th Conference of Quantitative Methods in Economics organised by Department of Econometrics and Statistics, Faculty of Applied Informatics and Mathematics, Warsaw University of Life Sciences. This work is part of on-going research supported by the National Science Center under Grant 2015/19/N/HS4/00363.

INTRODUCTION

In the nearest future many advanced economies will face the demographic change. Not only are we experiencing a slowing population growth due to decreasing fertility rates. A deterioration in fertility rates among increased longevity leads also to population ageing. In almost all advenced economies the ratio of elderly people in the population increases. The pace of this increase is also getting quicker. According to OECD population projections, over the next 50 years

DOI: 10.22630/MIBE.2018.19.1.1

ageing will be rapid, with old-age dependecy ratios more than doubling in many developed countries.

At the same time, many of these economies are facing a very low inflation level. Inflation that was high in most countries in the 1970s, is now chronically low. A conventional approach would suggest that population ageing as a slow-moving trend is not connected to inflation, which is a monetary phenomenon. However, recently in the public debate the opinion appeared that low inflation levels may be linked to changing demographic population structures (see eg. Shirakawa 2011a,b, 2012, 2013; Bullard et al. 2012). Therefore, a larger share of the elderly in advanced economies may make it more difficult to exit the low inflation trap. If there really exists a link between demography and inflation, it may also cause significant implications for the conduct of monetary policy.

Therefore a motivation of this paper is to check whether changes in the age structure of population can impact inflation, especially whether the rise of the old-age dependency ratio is correlated with lower inflation rates. In order to achieve this, firstly, the character and strength of this relation will be analysed. Secondly, it will be checked whether one can distinguish between the effects on inflation caused by two different dependent groups, namely changes in the shares of young (aged 0-15) and elderly (aged 65 and over) population. The main focus is however set on elderly population.

The reminder of this paper is organized as follows. In the Literature review the hitherto literature is being presented and discussed. The second section describes the data and the methodology used in the empirical analysis. It also presents key facts about population ageing in the analysed countries. Finally, the last section investigates empirically the link between inflation and demography in selected economies and concludes.

LITERATURE REVIEW

To the best of the author's knowledge there is still little evidence on the hypothesis on impact of population ageing on inflation. Moreover, few empirical studies that have been devoted to this topic remain non-conclusive on the sign of the impact of demographic changes on inflation.

The existence of a link between inflation and age-structure of population was regularly mentioned by Shirakawa [2011a, 2011b, 2012, 2013]. The former Governor of the Bank of Japan has repetedly stated that an ageing population could lead to an increase in deflationary pressures, primarily due to expectations of a slowdown in economic growth. In addition, it may cause a reduction in the size of consumer demand and investment. Looking at the hitherto publications on inflation and demography, two contradictory streams of research can be distinguished.

The more popular and traditional view emerges from the life-cycle hypothesis. As the median age of population increases, more households finance their consumption from before accumulated savings and do not directly produce added value. Therefore the discrepancy between aggregate demand and output in the economy rises and demand-driven inflationary pressure appears. Simultaneously, as the labour supply is shrinking, wages are being pushed up, which increases inflation through the cost channel.

In line with this theory, McMillan & Baesel [1990] confirm the forecasting power of demographics for low-frequency inflation. They use correlation between demographics and inflation in the United States to predict the moderation of inflation in the 1990s. Lindh and Malmberg [2000] describe the impact of demographics on the existence of low-frequency inflation using a panel model. They estimate the relation between inflation and age structure on annual OECD data 1960–1994 for 20 countries. According to their results increases in the population of net savers dampen inflation, whereas especially the younger retirees fan inflation as they start consuming out of accumulated pension claims.

More recently, Juselius and Takáts [2015], who performed a panel data analysis on 22 advanced economies over the 1955-2010 period, suggest that population ageing could lead to increased inflationary pressures. Their estimates show that demography accounts for 1/3 of the variation in inflation in the analysed period. They find a stable and significant relationship between the age structure of a population and low-frequency inflation. In their following work, Juselius and Takáts [2016] confirm that the age-structure of population is a systematic driver of inflation. According to their research, in the US this age-structure effect accounts for about 6.5 percentage points of disinflation between 1975 and 2016.

Recently new views appeared in the literature and another outlook on the link between low inflation and ageing gained on popularity. One of the arguments are the demand-side effects of population ageing. Changing consumption preferences would lead to reduced aggregated demand and lower inflation. Analyzing lifecycle consumption and saving patterns (see eg. Ando and Modigliani 1963) suggests that net consumers cohorts (dependents) drive up the real equilibrium interest rate. This trend was analyzed by Anderson et al. [2014], who by using the IMF GIMF Model finds deflationary pressures from ageing, stemming mainly from declining GDP growth and falling land prices.

Yoon et al. [2014] conduct a panel data analysis to prove that population ageing has economically and statistically significant impact on key macroeconomic variables. They find that while population growth is inflationary, in the long run dependant cohorts appear to have negative inflationary pressures.Their estimation proves that in the long run societies with larger dependant age groups and smaller working age population face a statistically significant decline in hours worked, real rates, savings and investment and higher inflation.

Konishi and Ueda [2013] argue that ageing could be more deflationary when caused by increased life expectancy. This is because the government is motivated to appease older voters by supporting the income of the elderly by increasing income tax rates. In general, rising ageing-related government spending can be financed either by income tax or by producing inflation. Rising income tax increases the burden for younger generations who are economically active. Producing inflation is however costly mostly for bond-holders, who are in general the older generation. Therefore, when there are more elderly people in the society, their political influence rises. Also empirical research conducted for Germany by Faik [2012] and for a sample of OECD countries by Gajewski [2016] show that demographic ageing exerts downward pressure on prices.

Effects of ageing may also depend on its causes. According to Katagiri et al. [2014] ageing is deflationary when caused by an increase in longevity but it is inflationary when caused by a decline in birth rate. Using a OLG model, they proved that over the past 40 years ageing caused yearly deflation of about 0.6 percentage points in Japan. Also Konishi and Ueda [2013] show that the direction in which the ageing of the population affects the inflation rate depends on the roots of this process. They state that population ageing stemming from a decline in the birth rate generates inflation by shrinking the tax base and raising fiscal expenditure.

DATA AND METHODOLOGY

In the study the largest possible available sample of OECD countries is included. The sample covers 32 economies: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Latvia, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, and United States.

As to show the age structure of population we use three different variables. First, the dependency ratio (denoted as $dep_{i,j}$, where i=1,…,N is a country index and j=1,…,T is a time index) captures the share of the non-active age population, which is economically dependent. It is the number of the young (aged 0–14) and the old (aged 65 and more) population divided by the working age population (aged 15-64). Therefore

$$dep_{i,j} = (n_{i,j}^{young} + n_{i,j}^{old})/n_{i,j}^{working\ age}.$$

Another variable of interest is the youth dependency ratio that covers the number of the young population (aged 0-14) divided by the working age population (aged 15–64). It is denoted as

$$ydep_{i,j} = n_{i,j}^{young}/n_{i,j}^{working\ age}.$$

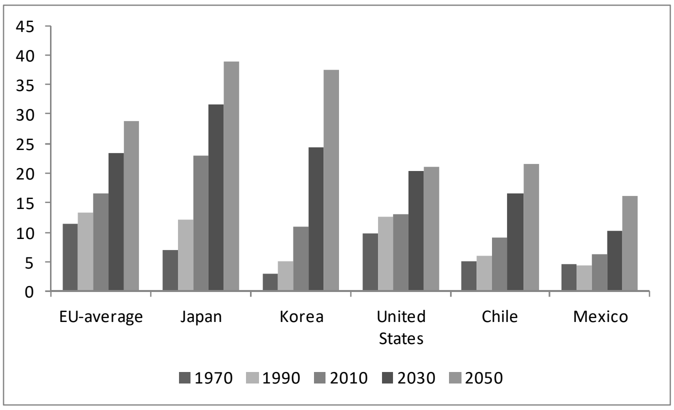
Third demographic variable is the old-age dependency ratio, which shows the proportion of elderly people to the working age population and is denoted as follows:

$$olddep_{i,j} = n_{i,j}^{old}/n_{i,j}^{working\ age}.$$

As mentioned in the Introduction, the developed world is currently experiencing a shift in the age composition of populations. Fertility rates are decreasing and as the so-called baby-boomer generation marches through working age, the workforce is ageing. Furthermore, due to gains in longevity, the share of the elderly in population is rising.

These demographic changes have already begun in some of the analysed countries (such as Japan). In other – eg. Mexico - this change is occuring more slowly and population is still relatively young. Nevertheless, in the whole sample in the analyzed period the average share of people aged 65 and more in the society rose from 10% in 1971 to 16.8% in 2015. Forecasts predict that in the future this share will be growing further, reaching as much as 27% in the year 2050.

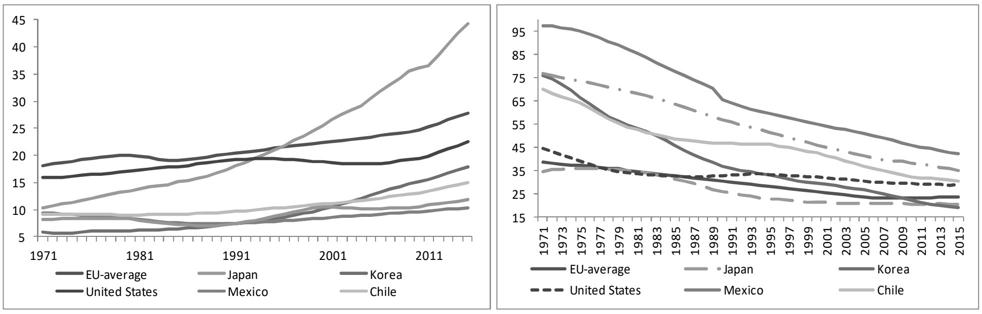
Figure 1. Elderly population (65 and more) as % of total population in selected OECD countries



Source: OECD database

Looking at the development of the old-age dependency ratio over the analyzed period, one can notice it has been growing steady in most countries in the sample, with the noticeable exception of Japan, where the pace of population ageing is faster than in other economies (see Figure 2, left panel). At the same time we are also experiencing a declining youth dependency ratio (see Figure 2, right panel). In the analyzed time period its mean value in the countries in the sample dropped from 45.5% to 25.6%. According to OECD forecasts, it will stabilize at 20–25% till 2050.

Figure 2. Old-age dependency ratio in selected OECD economies, 1971–2015 (left panel),
Youth dependency ratio in selected OECD economies, 1971–2015 (right panel)



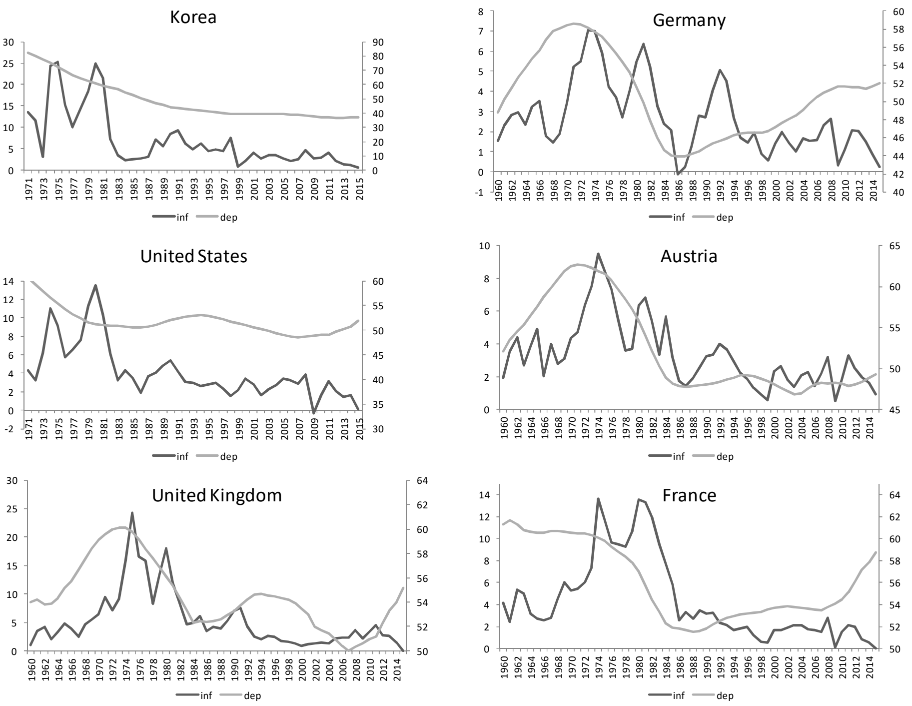
Source: OECD database

As mentioned in the Introduction, these demographic developments coexist with another economic trend, which has been lately observed in several ageing countries, namely historically low inflation. Since 1971 the average inflation rate in the analyzed countries dropped from 7.13% to 0.41%. In recent years in some of these countries inflation rates became even negative. Although for most of the analyzed period there is a visible heterogeneity between countries in the sample, in the 2000s years inflation rates have moderated and decreased in all countries.

In order to include inflation in the model we take the yearly inflation rate, obtained from OECD database. As low-frequency inflation dynamics are analyzed, yearly data are sufficient. The inflation rate is denoted as $inf_{i,j}$, where i=1,…,N is a country index and j=1,…,T is a time index. Following Gajewski [2016] the sample has been truncated from above at an inflation rate of 25% in order to exclude periods of sharp macroeconomic instability. Leaving those variables could create serious bias in estimation results.

Following Juselius and Takáts [2015] we begin with a simple graphical comparison of two variables – inflation and dependency ratio as a common measure of the demographic change. In order to maintain the clarity of the text, Figure 3 shows this comparison only in chosen six of the analysed economies. A first look at the data does reveal that there may be some relationship between inflation and demography. In the long run they seem to correlate.

Figure 3. Inflation rate (lhs, %) and dependency ratio (rhs, %) in selected economies



Source: OECD database

This relation may be however purely coincidental. Inflation may be driven by some common factors across countries (as before the late 1980s, it was strongly driven by such factors as oil price shocks). Also these variables, especially dependency ratio have been more or less similar across countries. Therefore, this relationship recquired a more careful analysis that considers other variables.

The general empirical model is given by equation (1). We index country by i, where i=1,2,3,…,N and year by j, where j=1,2,3,…,T.

$$inf_{i,j} = \alpha_0 + \alpha_1 \cdot dep_{i,j} + \alpha_2 \cdot tot_{i,j} + \alpha_3 \cdot m_{i,j} + \alpha_4 \cdot ggdp_{i,j} + \alpha_5 \cdot budbal_{i,j} + \varepsilon_{i,j} \qquad (1)$$

We regress inflation on dependency ratio as the demographic variable, as well as other, control variables. They have been added in order to better capture relations between inflation and demography. The choice of control variables is based mainly on Yoon et al. [2014], who have analyzed how different demographic variables (such as population growth, shares of specific age groups or life expectancy) influence macroeconomic variables – economic growth, inflation, savings and investment and fiscal balances.

Among the control variables, there are:
$tot_{i,j}$ – which denotes the yearly change in the terms of trade index (Source: OECD database)
$ggdp_{i,j}$ – denotes the annual growth rate of real GDP (Source: OECD database)
$m_{i,j}$ – denotes the base money growth rate (Source: IFS)
$budbal_{i,j}$ – which denotes the annual change of general government deficit (Source: OECD database).

In the next step, in order to check the hypothesis whether the impact of different dependent age groups on inflation is different, variable *dep* is divided into two categories: young dependency ratio ($ydep_{i,j}$) and old-age dependency ratio ($olddep_{i,j}$). Therefore, equation (2) looks as follows:

$$inf_{i,j} = \alpha_0 + \alpha_1 \cdot ydep_{i,j} + \alpha_2 \cdot olddep_{i,j} + \alpha_3 \cdot tot_{i,j} + \alpha_4 \cdot m_{i,j} + \alpha_5 \cdot ggdp_{i,j} + \alpha_6 \cdot budbal_{i,j} + \varepsilon_{i,j} \quad (2)$$

Table 1. Descriptive statistics for the model variables

Description	Obs.	Mean	Std. Dev.	Min	Max
CPI rate (annual, %)	1301	7.25	7.11	-4.48	25
(population aged 0-14 and population aged 65 and more)/population aged 15-64	1440	52.68	8.40	36.79	107.05
population aged 0-14/population aged 15-64	1440	33.39	11.61	19.54	97.60
population aged 65 and more/population aged 15-64	1440	19.28	5.52	5.62	42.52
base money growth rate (annual, %)	1099	14.81	16.43	-25.42	144.80
growth rate of real GDP (annual, %)	1274	2.90	3.26	-14.72	26.28
change in the terms of trade index (annual, %)	1273	0.01	5.63	-100	49.28
change of general government deficit (annual, %)	726	-2.02	4.40	-32.12	18.70

Source: own calculations

Table 1 presents descriptive statistics for the variables used in the model. Only for demographic variables there is no missing data in the analyzed period. The data for recent decades is the most complete in the sense of having less missing values. This results mainly from the changes in political and economic systems in many European countries.

RESULTS AND CONCLUSIONS

The results of estimations are presented in Table 2. The regression equation parameters are initially estimated using OLS. In the first form of the model, with dependency ratio as the only demographic variable, a positive and significant impact on inflation has been observed. A growth in dependency ratio of 1 percent leads to 0.101 change in inflation. The relationship between inflation and demography cannot therefore be rejected. To further check this hypothesis, we perform the regression equation using fixed effects (FE) and random effects (RE). In the fixed effects model there also seems to be a positive relationship between inflation and dependency ratio. The coefficient by variable $dep_{i,j}$ is positive (0.297) and significant at the 1% level. We apply a modified Wald statistics for groupwise heteroscedasticity in the residuals and the Woolridge test for serial correlation, following Gajewski [2016]. These tests show that both problems exist in the sample and should be controlled for as the FE estimator may be inefficient and lead to biased standard errors. As the Wooldrigde test is significant and rejects the null hypothesis, indicating the presence of serial correlation, the model is estimated using the GLS method, which analyses panel-data linear models by using feasible generalized least squares. This allows to estimate in the presence of AR(1) autocorrelation within panels and cross-sectional correlation and heteroskedasticity across panels. Again, the parameter by the variable $dep_{i,j}$ is positive, albeit this impact is expected to be smaller. A growth in dependency ratio of 1 percent led to a positive change in infation (0.065).

Table 2. Estimation results

1971-2015	OLS	FE	RE	GLS	OLS	FE	RE	GLS
dep	0,101*	0,297***	0,160***	0,065*				
	(-0,039)	-0,031	-0,028	-0,029				
ydep					0,114**	0,269***	0,137***	0,118***
					-0,036	-0,029	-0,025	-0,029
olddep					-0,173***	-0,285***	-0,164***	-0,249***
					-0,032	-0,068	-0,038	-0,033
m	0,222***	0,141***	0,193***	0,071***	0,172***	0,093***	0,161***	0,039**
	(-0,032)	-0,019	-0,019	-0,012	-0,027	-0,019	-0,019	-0,012
ggdp	-0,1	-0,256***	-0,159**	-0,072**	-0,225***	- 0,306***	-0,227***	-0,105***
	(-0,069)	-0,051	-0,051	-0,028	-0,065	-0,047	-0,049	-0,026
tot	-0,128**	-0,129***	-0,127***	-0,103***	-0,119***	-0,115***	-0,121***	-0,102***
	(-0,041)	-0,031	-0,033	-0,016	-0,034	-0,029	-0,031	-0,014
budbal	-0,102**	0	-0,099*	-0,077***	-0,107***	0,04	-0,106**	-0,069***
	(-0,034)	-0,052	-0,04	-0,018	-0,032	-0,048	-0,035	-0,014
obs.	526	526	526	526	526	526	526	526
F-test	12,82	42,34			31,87	56,7		
	0	0			0	0		
Wald chi²			175,46	100,91			308,67	456,2
			0	0			0	0
R^2	0,26				0,39			
R^2 within		0,3	0,27			0,41	0,36	
R^2 overall		0,15	0,24			0,32	0,39	

Notes:
Significance at the 1%, 5% and 10% levels are denoted respectively by ***, **, *.
Std. errors in parentheses.
In GLS panel-specific autocorrelation AR(1) imposed.

Source: own calculations

The next step is to divide dependency ratio into its components (young dependency ratio and old-age dependency ratio) in order to check the hypothesis whether the impact on inflation of old dependents may differ from the impact of young dependents. Indeed, different results have been obtained. In each specification the negative and significant effect of increasing old-age dependency ratio has been confirmed. An increase of old-age dependency ratio of one percentage translates into a 0.29 to 0.16 percent decrease in the average inflation rate. This may confirm the hypothesis that ageing is deflationary. Also, regardless of the specification a significant and positive effect of an increase in young dependency ratio has been observed. The results are robust to different time periods, control variables and estimation techniques.

The results of this empirical analysis add to the ongoing discussion on the relationship between demography and inflation rate. They suggest that demographic changes may have deflationary impact in the next years, particularly in those economies, where significant population ageing is currently experienced or expected. There is indeed a relationship between demography and inflation – while old-age dependents are deflationary, young dependents seem to be rather inflationary. This result should motivate further research.

This area is still underexploited and needs further research. Not only the impact of demographic variables on inflation rate is to be analyzed. Another still not enough addressed problem is the impact of ageing on the conduct of monetary policy, as demography as a driver of inflation may be relevant for monetary policy makers in the near future. The macroeconomic policy framework may therefore need to be revisited in the future. Demographic changes are not only one of the most important long-term challenges for the economy. They also can be relatively well predicted. This may be the reason why the demographic impact of inflation probably could be taken into account in monetary policy decisions.

REFERENCES

Anderson D., Botman D., Hunt B. (2014) Is Japan's Population aging Deflationary? IMF Working Paper 14/139, August.

Ando A., Modigliani F. (1963) The 'Life-cycle' Hypothesis of Saving: Aggregate Implications and Tests. American Economic Review, 53(1), 55-84.

Bloom D. E., Canning D. (2006) Demographic Challenges, Fiscal Sustainability and Economic Growth. PGDA Working Papers 0806, Program on the Global Demography of Aging.

Bullard J., Garriga C., Walker C. J. (2012) Demographics, Redistribution, and Optimal Inflation. Federal Reserve Bank of St. Louis Review, November/December 2012, 94(6), 419-39.

Faik J. (2012) Impacts of an Ageing Society on Macroeconomics and Income Inequality – TGE Case of Germany since the 1980s. Society for the Study of Economic Inequality Working Paper Series No. 2012-272, Society for the Study of Economic Inequality, Verona.

Gajewski P. (2016) Is Ageing Deflationary? Some Evidence from OECD Countries. Applied Economics Letters, 22(11), 916-919.

Juselius M., Takáts E. (2015) Can Demography Affect Inflation and Monetary Policy? BIS Working Papers, 485.

Juselius M., Takáts E. (2016) The Age-structure-inflation Puzzle. Bank of Finland Research Discussion Paper 4/2016.

Katagiri, M., Konishi H., Ueda K. (2014). Aging and Deflation from a Fiscal Perspective. Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute, Working Paper, 218.

Konishi H., Ueda K. (2013) Aging and Deflation from a Fiscal Perspective. IMES Discussion Paper Series, 2013-E-13.

Kotowska I. E., Jóźwiak J. (2012) Nowa demografia Europy a rodzina. Roczniki Kolegium Analiz Ekonomicznych, 28/2012, 9-33 (in Polish).

Lindh T., Malmberg B. (2000) Can Age Structure Forecast Inflation Trends? Journal of Economics and Business, 52, 31-49.

Macura M., MacDonald A. L., Haug W. (eds.) (2005) The New Demographic Regime. Population Challenges and Policy Responses. United Nations, New York and Geneva.

McMillan H., Baesel J. (1990) The Macroeconomic Impact of the Baby Boom Generation. Journal of Macroeconomics 12, 167-195.

Shirakawa M. (2011a) Bubbles, Demographic Change and Natural Disasters. Opening Speech at 2011 Annual International Conference hosted by the Institute for Monetary and Economic Studies, the Bank of Japan, June 1.

Shirakawa M. (2011b) Globalization and Population Aging: Challenges Facing Japan. Speech to the Board of Councillors of Nippon Keidanren, December 22.

Shirakawa M. (2012) Demographic Changes and Macroeconomic Performance: Japanese Experiences. Opening Remark at 2012 BOJ-IMES Conference hosted by the Institute for Monetary and Economic Studies, the Bank of Japan, May 30.

Shirakawa M. (2013) Toward Strengthening the Competitiveness and Growth Potential of Japan’s Economy. Speech at the Executive Member Meeting of the Policy Board of Nippon Keidanren, Tokyo, 28 February.

Yoon J.-W., Kim J., Lee J. (2014) Impact of Demographic Changes on Inflation and the Macroeconomy. IMF Working Paper 14/210, November.

QUANTITATIVE METHODS IN ECONOMICS
Volume XIX, No. 1, 2018, pp. 12 – 27

Received: 25.03.2018
Accepted: 05.04.2018

IMAGE PATTERN ANALYSIS WITH IMAGE POTENTIAL TRANSFORM

Oleg Butusov (ORCID: 0000-0003-1361-2121)
Faculty of Basic Competences
Moscow Polytechnic University, Moscow, Russia
e-mail: butusov-1@mail.ru
Vasily Dikusar
Russian Academy of Sciences, Moscow, Russia
e-mail: dikussar@yandex.ru

Abstract: Pattern analysis with image transform based on potential calculation was considered. Initial gray-scale image is sliced into equidistant levels and resulting binary image was prepared by joining of some levels to one binary image. Binary image was transformed under assumption that white pixels in it may be considered as electric charges or spins. Using this assumption Ising model and Coulomb model interaction between white pixels was used for image potential transform. The transform was calculated using moving window. The resulting gray-scale image was again transformed to binary image using the thresholding on 0.5 level. Further binary images were analyzed using statistical indices (average, standard deviation, skewness, kurtosis) and geometric signatures: area, eccentricity, Euler number, orientation and perimeter. It was found that the most suitable geometric signature for pattern configuration analysis of Ising potential transform (IPT) and Coulomb potential transform (CPT) is area value. Similarly the most suitable statistics is distance statistics between white pixels.

Keywords: binary image transform, distance and potential transform, statistical indices, geometric signatures, pattern analysis, pattern recognition

JEL classification: C52, C69

DOI: 10.22630/MIBE.2018.19.1.2

INTRODUCTION

Pattern analysis and recognition [1], data mining [2], classification [3] and clustering [4] are the most known problems in image processing. In some cases, image may show the multifractal properties and as a result fractal dimensions may be used as important characteristics of image patterns. So, fractal dimensions may be used for image classification or clustering. The advances in image fractal property study are widely used in different fields such as materials science [5], medicine [6-8], remote sensing [9,10] et al. Frequently objects on image are fuzzy and have fuzzy boundaries [11].

Different methods were developed for analysis of hidden pattern in images: stochastic methods and Markov random fields [12], morphological image processing [13], border detection [14], Fourier transform and wavelets [15,16], threshold or slicing binarization [17], texture analysis [18], genetic algorithms [19] et al. It should be noted that slicing binarization may be used to project complex structure of image into several pixel configurations which sometimes reflect peculiarities of inner patterns. In binarization the question of slicing levels is important. Several approaches may be used. Automatic thresholding was in details considered in [14]. The local adaptive thresholding was proposed by Bernsen [14]. For thresholding Bernsen used moving window and got threshold as average between maximum and minimum pixel values in the window. In [20] probabilities were used to find threshold between two pixels classes. Maximum entropy method is enough effective to calculate global threshold of gray-scale image [14, 21]. In case the histogram of gray-scale image has several modes the border between modes may be used as slicing measure.

After binarization binary images are often analyzed using mathematical morphology operations [22, 23] to discover hidden patterns. For example, in [23] binary image was received using water network mask. Further it was segmented using morphological calculation to three classes: core pixels, islet pixels and connector pixels.

A separate group of image processing algorithms comprise ones which are called distance transform (DT) algorithms [24]. There are many different methods and distance measures which are used in DT calculations. Euclidian distance DT (EDT) is the one of the popular distance measure for using in DT transform algorithms [25]. The problem of sparse object representation in discrete geometry was considered in [26]. The DT algorithm was also used in [27] for automatic pattern recognition. The problem of DT transform algorithm complexity was considered in [28]. It is well-known that EDT calculation is rather time-consuming operation. To solve this problem several effective algorithms were developed [28]: Linear-time Legendre transform (LLT) algorithm, the parabolic envelope (PE) algorithm and non-expansive proximal mapping (NEP) algorithm. It was shown in [28] that these algorithms have linear complexity and so may be effectively used for DT processing of binary images. Modern efficient means of parallel computing and computing with GPU are often used for EDT calculation [29]. DT proved to be useful in many practical applications. In medical imaging DT is one of best means for discovering the similarity between images. DT image transform is important for 3D study of inner organs using slice-by-slice method [30]. Good results were obtained using together watershed algorithm and DT for blood cell image segmentation [31]. Watershed algorithm needs grayscale images. So, DT transform may be used to transform binary image to gray-scale. In [31] watershed and distance transform algorithm were used together with four distances measures: EDT, city-block, chessboard and quasi-Euclidean. It was found in [31] that chess board DT measure has better results in watershed segmentation then Euclidean, city block and quasi-Euclidean DT measures [31].

In our present work we considered another three kinds of DT:

- Ising potential DT;
- Coulomb potential DT using white foreground pixels as positive charges;
- Coulomb potential DT using both white foreground pixels as positive charges and black background pixels as negative charges.

The proposed models of DT were used in present work for pattern recognition.

BINARIZATION

One of the popular method for detecting hidden patterns in image is simple binarization [20-23]. Sometimes binarization can produce good results after several successive binarizations of different kinds. Sometimes patterns show itself after using the union ('OR-operation') of several different binary slices. Let us call it nonlinear binarization.

Let us consider as working example the fragment (200x200 pixels) of microphotography of quartz glass (silicon dioxide). The original image and its histogram is shown in Figure1. To create binary image, one may use great variety of algorithms. One of the most simple algorithm is equal step quantization (ESQ) $\Delta b = (b_{\max} - b_{\min})/N$, where N – number of steps and N+1 – number of levels. Using ESQ one may take additional choices. It is possible to use for gray-scale image digitization the round, floor or ceil operations or their nonlinear variants. For example, one may use asymmetric rounding algorithm $k = \begin{cases} ceil(v), if\,(v - floor(v) \geq s) \\ floor(v),\ othervise \end{cases}$,where v – pixel value, s – threshold. Sometimes it is useful to join several binary slicing levels in one joined level (union with 'OR-operation'). This operation may be called '*nonlinear binarization*'. The operation may produce more complex patterns then simple slicing. Let us consider asymmetric binarization example in which original image (Figure 1) is splinted into 6-levels. The results are shown in Figure 2.

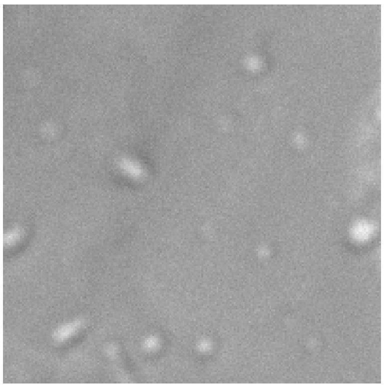
White points in Figure 2 are pixels containing ones ("one" or foreground-points) and black point are pixels – containing zeros ("zero" or background-points). Binary image is (0-1) matrix. Images in Figure 2 are the result of nonlinear binarization by joining (2,4,5)-slicing levels into one binary image.

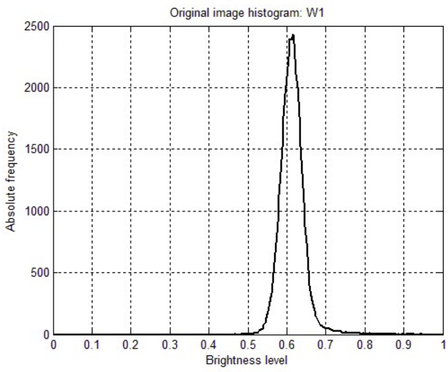
Another one of the well-known binarization algorithms is the density algorithm. This algorithm is binary to binary transform algorithm and is based on using moving window (MW). Central pixel of MW is filled with ones if the density of white points inside moving window exceeds the specified threshold $t = \frac{n_b}{(2d+1)^2} \geq h$, where h – threshold, n_b – number of white pixels inside MW, d – half-width of moving window. In our calculation we used d = 3 or 5 pixels MW and h = 0.3 or 0.1. The size of resulting density image is (200-2d) x (200-2d) pixels. Figure 3 shows the result of Figure 2a density image transform.

DISTANCE TRANSFORM FOR PATTERN ANALYSIS

Distance transform algorithm is often used in none fuzzy object recognition as border detection means. This algorithm transforms a binary image to gray scale image (binary-to-gray scale algorithm). The foreground pixels in binary image are marked by "ones" (white points) and background pixels by "zero" (black points). DT-algorithm calculates distance from every foreground pixel to the nearest background pixel and assigns this value to the central pixel. Similarly, it calculates distance from background pixel to the nearest foreground pixel and assigns this value to the central pixel [24-31].

Figure 1. Original grayscale image and it histogram

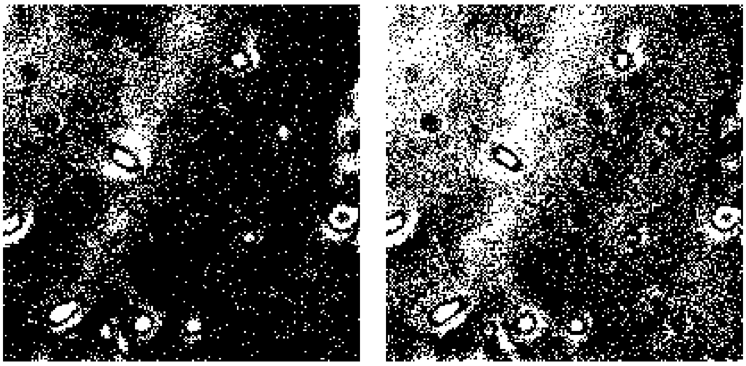




Source: own preparation

Figure 2. Asymmetric binarization of original image into 6-levels and joining up levels with numbers (2,4,5): (a) using threshold s = 0.2; (b) using threshold s = 0.4

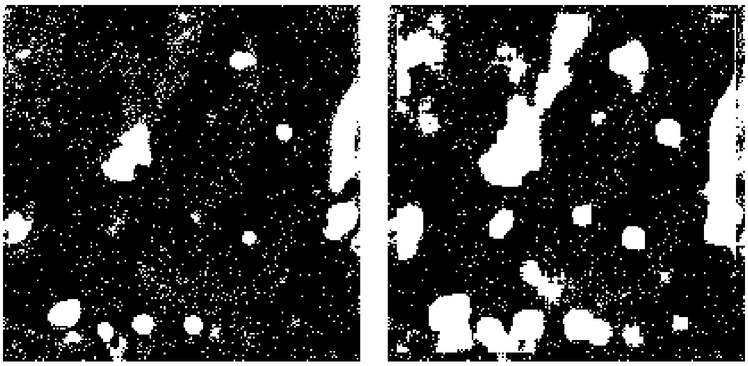
a b



Source: own preparation

Figure 3. Binary images after density transform: (a) d = 3; h = 0.3; (b) d = 5; h = 0.3

a b



Source: own preparation

At present time some DT-algorithms were generalized to three and more dimensions. It is rather important for medical image processing as medical images often are three dimensional or consist of many two-dimensional slices of three dimensional organs [30].

DISTANCE TRANSFORM CALCULATION

In case of fuzzy foreground different forms of distance measures and distance transform may be used for hidden patterns analysis. For example, one may use moving window and calculate sum or average of all distances between white points in MW. The resulting value may be assigned to the central pixel. This

algorithm may be called “moving window distance transform (MWDT)”. Let us consider distance measures used for MWDT transform of image in Figure 3a:
Squared Euclidian distance (SED):

$$SED(k,p)=\sum_{G}(i_1-i_2)^2+(j_1-j_2)^2\,. \tag{1}$$

City-block distance (CBD):

$$CBD(k,p)=\sum_{G}|i_1-i_2|+|j_1-j_2|\,. \tag{2}$$

Chessboard distance (ChBD):

$$ChBD(k,p)=\sum_{G}\max(|i_1-i_2|,|j_1-j_2|)\,. \tag{3}$$

Quasi-euclidean distance (QED):

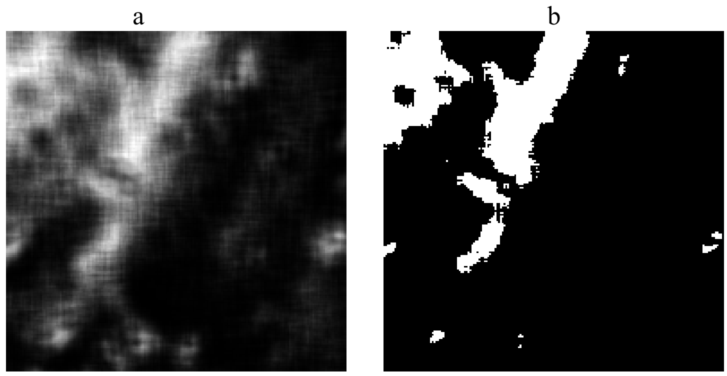
$$QED(k,p)=\sum_{G}\begin{cases}|i_1-i_2|+\left(\sqrt{2}-1\right)|j_1-j_2|, if\ \ |i_1-i_2|>|j_1-j_2|\\ \left(\sqrt{2}-1\right)|i_1-i_2|+|j_1-j_2|, otherwise\end{cases}. \tag{4}$$

where: $G=b(i_1,j_1)=1, b(i_2,j_2)=1, (i_1,j_1)\in MW, (i_2,j_2)\in MW$; MW – moving window; (k,p) - coordinates of central pixel; $b(i,j)$ - pixel value.

The result of image from Figure 3a SED transform is shown in Figure 4. The transform was calculated using Intel Visual Fortran 2013 and Intel Parallel Studio XE [32]. The choice of programming language and Parallel Studio is due to high speed of computation and the possibility of vector-matrix calculations. The fragment of Fortran vectorized code is shown below:

```
real*4 :: s
integer*4 i, j, i1, j1, ii1, jj1
integer*4, dimension(:), allocatable:: b1, b2, rkp2
allocate(b1(b))
allocate(b2(b))
allocate(rkp2(b))
rkp2 = 0
s=0.0
do i=1,a
    b1 = Bw(i,:)
    do i1 = 1,a
        b2 = Bw(i1,:)
        ii1 = (i - i1)**2
        forall(j = 1:b, j1 = 1:b, b1(j) .eq. 1 .and. b2(j1) .eq. 1) rkp2(j) = ii1 + (j - j1)**2
        s = s + sum(rkp2)
    end do
end do
make_E=0.5*s
```

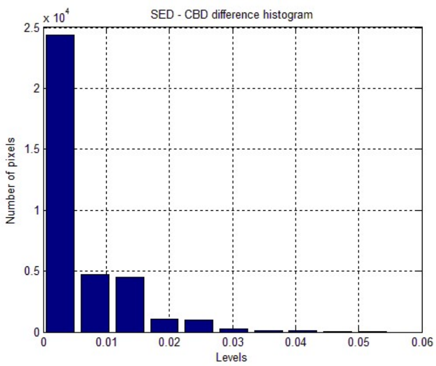
Figure 4. SED image transform: (a) normalized SED-image; (b) 0.5 level threshold binarization



Source: own preparation

Additional patterns could be seen when compared Figure 2, Figure 3 and Figure 4. It should be noted that results of other metrics are visually indistinguishable from SED. Let us consider the histogram of difference between SED and CBD images (Figure 5). It shows that difference of pixel values is very small: $|SED - CBD| < 0.01$. So, to get additional hidden patterns one should use algorithms of other kind.

Figure 5. Histogram of image difference between SED and CBD metrics



Source: own preparation

POTENTIAL TRANSFORM FOR PATTERN ANALYSIS

Let us assume that white points in binary image may be considered as particles. These particles create potential that may be calculated in the central pixel of MW. There are different kinds of particle interaction potential. There are distance dependent potentials (Coulomb potential) or distance independent potentials (Ising spin-spin interaction) [33] et al. In Ising model the spin-spin interaction is considered only between nearest spins [33]. In our study it corresponds to interaction only between particles in the limits of MW. In calculation we assume that white points have spin $S_i = 1$ and black points have spin $S_i = -1$. Also, we assume that total potential is the sum of two-particle interactions. So, we compute Ising potential as follows:

$$U_{I\sin g}(c) = -J \sum_{(t,q\in w)} S_t S_q = -\sum_{(t,q\in w)} (2b(t)-1)(2b(q)-1), \tag{5}$$

where: w – moving window; b – binary matrix of moving window; c – central point of moving window; t, q – white point numbers inside moving window; $S_t S_q \in \{1,-1\}$ – spin values of t-th and q-th white points; J – energy constant (in calculation used as $J = 1$).

In every position of MW, the total potential of spin interaction between particles is assigned to central pixel. The resulting gray-scale image we call Ising potential transform (IPT) of binary image.

In our study we also considered two algorithms with interaction of Coulomb type. The first algorithm (CPT1-algorithm) uses total potential of interaction only between white points (positive charge particles) $U(c) = \sum_{t<q} V(r_{t,q}) = \sum_{t<q} \frac{1}{r_{t,q}}$, where: $r_{t,q}$ - distance between two white points. We compute the total interaction between white particles as follows:

$$U(p,k) = \sum_G \frac{1}{\sqrt{(i_1 - i_2)^2 + (j_1 - j_2)^2}}, \tag{6}$$

where: w – moving window; (p,k) – central point of moving window; $G = \{t = (i_1, j_1) \neq q = (i_2, j_2) \neq c = (p,k) \in w, b(i_1, j_1) = 1, b(i_2, j_2) = 1\}$; b – binary image.

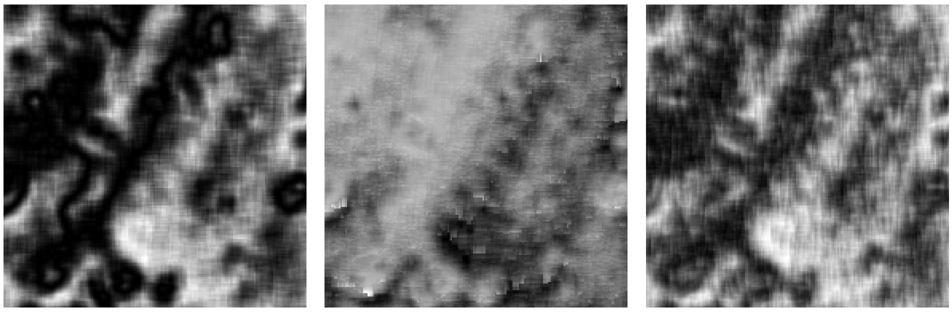
Second algorithm (CPT2-algorithm) uses for total potential calculation both white and black points (particles of any charge)

$$E(p,k) = \sum_G \frac{(2b(i_1, j_1) - 1)(2b(i_2, j_2) - 1)}{\sqrt{(i_1 - i_2)^2 + (j_1 - j_2)^2}}, \tag{7}$$

where $G = \{t = (i_1, j_1) \neq q(i_2, j_2) \neq c = (p,k) \in w\}$. The resulting gray-scale images we call Coulomb potential transform (CPT1 or CPT2) of binary image. We assume that using another kind of particle interaction, for example, the Lenard-Jones potential or Tersoff potential [34, 35], one may receive other patterns. Figure 6 shows resulting normalized gray-scale images of IPT, CPT1 and CPT2 for binary image in Figure 3a. Figure 7 shows their 0.5-threshold binarization and Figure 8 shows their histograms. From Figure 7a and Figure 7c it follows that resulting IPT and CPT2 show similar patterns.

Figure 6. Potential transforms of binary image from Figure 3a: (a) IPT; (b) CPT1-algorithm; (c) CPT2-algorithm

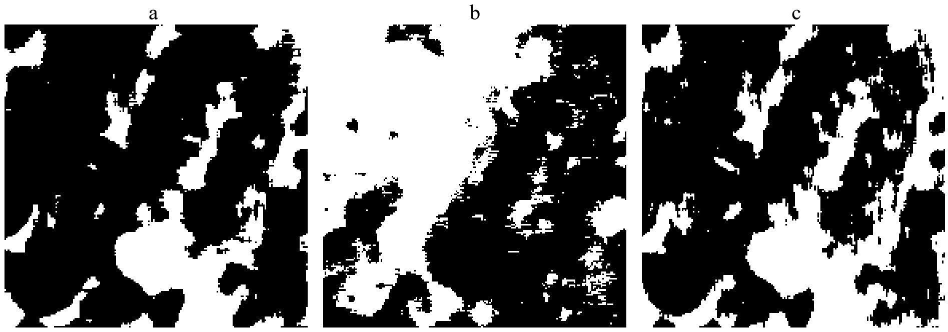
a b c



Source: own preparation

Figure 7. 0.5 level threshold binarization: (a) IPT; (b) CPT1-algorithm; (c) CPT2-algorithm

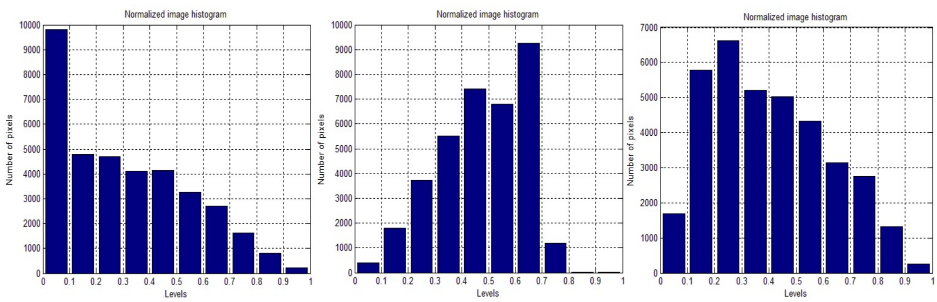
a b c



Source: own preparation

Figure 8. Histograms of potential transform gray-scale images: (a) IPT; (b) CPT1-algorithm; (c) CPT2-algorithm

a b c



Source: own preparation

Histograms show pixel distribution in MWDT images. Binary images show patterns. The patterns differ by statistical and geometric properties. To study pattern we used several statistical and geometric characteristics. In statistical analysis we used the following normalized statistical indices:

Normalizes average:

$$\bar{x}^{(n)} = \frac{\frac{1}{N}\sum_{i=1}^{N} x_i}{\max_i\left(|x_i|\right)}, \tag{8}$$

Normalized standard deviation:

$$\sigma^{(n)} = \frac{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}{\max_i\left(|x_i - \bar{x}|\right)}, \tag{9}$$

Normalized skewness:

$$Sk^{(n)} = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^3}{\max_i\left(|x_i - \bar{x}|\right)^3}, \tag{10}$$

Normalized kurtosis:

$$Ku^{(n)} = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^4}{\max_i\left(|x_i - \bar{x}|\right)^4}. \tag{11}$$

In geometric analysis we used five signatures: area, eccentricity, Euler number, orientation and perimeter.

Area – it is total number of pixels which form pattern objects in binary image [15,36]. Area is calculated as follows:

$$N_k = \sum_{(i,j)\in\Omega_k} 1, \tag{12}$$

where: (i, j) - pixel; Ω_k - set of all pixels forming *k*-object.

Eccentricity – it is the eccentricity of the ellipse that has the same second-moments as the object [15, 36]. Eccentricity is calculated as follows:

$$\varepsilon = \frac{\sqrt{I_{\max}^2 - I_{\min}^2}}{I_{\max}}, \tag{13}$$

where: $I_{\max}, I_{\min}$ - are the lengths of maximum and minimum axis of inertia; $I_{\max} = 2\sqrt{2}\sqrt{U_x + U_y + D}$; $I_{\min} = 2\sqrt{2}\sqrt{U_x + U_y - D}$; $U_x = \frac{1}{12} + \sum_{(i,j)\in\Omega} (i - i_c)^2 \big/ N_k$; $U_y = \frac{1}{12} + \sum_{(i,j)\in\Omega} (j - j_c)^2 \big/ N_k$; $D = \sqrt{(U_x + U_y)^2 + 4U_{xy}^2}$; $U_{xy} = \frac{1}{N_k} \sum_{(i,j)\in\Omega} (i - i_c)(j - j_c)$ $i_c = \frac{1}{N_k} \sum_{(i,j)\in\Omega} i$; $j_c = \frac{1}{N_k} \sum_{(i,j)\in\Omega} j$.

Euler Number – it is the number of objects in the region minus the number of holes in these objects [15, 36].

Orientation – it is angle (in degrees ranging from -90 to 90 degrees) between the x-axis and the major axis of the ellipse that has the same second-moments as the binary image object. Orientation is calculated as follows:

$$R = \begin{cases} \frac{180}{\pi} \arctan\left(\frac{U_y - U_x + D}{2U_{xy}} \right), U_y > U_x \\ \frac{180}{\pi} \arctan\left(\frac{2U_{xy}}{U_y - U_x + D} \right), \textit{otherwise} \end{cases} . \tag{14}$$

Perimeter – is computed by calculating the distance between each adjoining pair of pixels around the border of the region [15, 36].

The result of statistical and geometric analysis is presented in Table 1. Statistics over all pixels in binary image denotes calculation of above indices for all both white and black pixels in binary image and statistics of distances between white pixels denotes the same calculation for the whole array of distances between white pixels.

Table 1. Normalized signatures of binary images of Figure 7

	Average	Standard deviation	Skewness	Kurtosis
IPT (Figure 7a)				
Object signatures				
Area	0.0458	0.1602	0.0222	0.0219
Eccentricity	0.5789	0.7388	-0.2260	0.4217
Euler Number	0.7083	0.1769	-0.0244	0.0230
Orientation	0.1327	0.4262	-0.0174	0.1066
Perimeter	0.0832	0.2017	0.0295	0.0271
Statistics over all pixels of binary image				
	0.2375	0.5581	0.2145	0.2447
Statistics of distances between white pixels				
	0.1631	0.1918	0.0087	0.0034

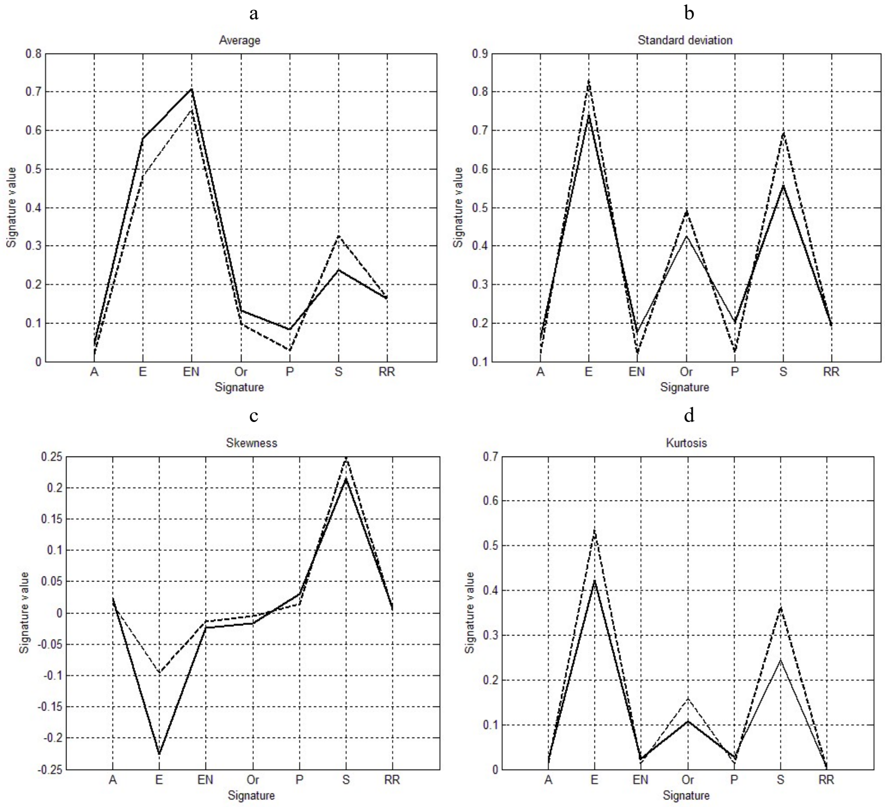
Table 1. continued

	Average	Standard deviation	Skewness	Kurtosis
CPT1 (Figure 7b)				
Object signatures				
Area	0.0089	0.0899	0.0080	0.0081
Eccentricity	0.5230	0.8025	-0.1820	0.5279
Euler Number	0.3810	0.0903	-0.0080	0.0081
Orientation	0.0928	0.3180	0.0343	0.0544
Perimeter	0.0149	0.0913	0.0080	0.0081
Statistics over all pixels of binary image				
	0.4784	0.9576	0.0761	0.8473
Statistics of distances between white pixels				
	0.1411	0.1603	0.0004	0.0007
CPT2 (Figure 7c)				
Object signatures				
Area	0.0210	0.1210	0.0142	0.0143
Eccentricity	0.4807	0.8281	-0.0964	0.5355
Euler Number	0.6528	0.1208	-0.0142	0.0143
Orientation	0.0969	0.4929	-0.0056	0.1581
Perimeter	0.0299	0.1243	0.0142	0.0143
Statistics over all pixels of binary image				
	0.3265	0.6962	0.2498	0.3636
Statistics of distances between white pixels				
	0.1648	0.1846	0.0047	0.0018

Source: own calculations

It follows from Figure 7a and Figure 7c that patterns in them are similar. So, we may use values in Table 1 as criteria for assessment of different statistical and geometric characteristics efficiency. The according graphs are shown in Figure 8.

Figure 9. Signature graphs: (a) – Average; (b) – Standard deviation; (c) – Skewness; (d) Kurtosis; symbols on x-axis denote: ‘A’ – Area, ‘E’ – Eccentricity, ‘EN’ – Euler Number, ‘Or’ – Orientation, ‘P’ – Perimeter, ‘S’ - statistics over all pixels of binary image, ‘RR’ - statistics of distances between white pixels; solid line – IPT; dotted line – CPT2



Source: own preparation

Figure 9 disclose the following satisfactory quantitative similarity between statistical - geometric combinations: (1) ‘Area - Average’; (2) ‘Distances between white pixels - Average’; (3) ‘Area – Standard deviation’; (4) ‘Distances between white pixels - Standard deviation’; (5) ‘Area - Skewness’; (6) ‘Euler Number - Skewness’; (7) ‘Orientation - Skewness’; (8) ‘Perimeter - Skewness’; (9) ‘Distances between white pixels - Skewness’; (10) – ‘Area - Kurtosis’; (11) ‘Euler Number - Kurtosis’; (12) ‘Perimeter - Kurtosis’; (13) ‘Distances between white pixels - Kurtosis’. Most often in combinations occur: ‘Area’ and ‘Distances statistics between white pixels’. So, these signatures may be proposed as satisfactory quantitative similarity criteria in comparing patterns in binary image.

CONCLUSION

At the present time image pattern analysis and recognition is of great practical interest for different applications. For example, the discovering of hidden patterns in image is of great importance for image biology, image medicine, material sciences et al. Many methods were worked out for pattern analysis including distance image transform. In this study we propose the potential transform as additional means for discovering hidden patterns in binary image. Three potentials were proposed:

- Ising model potential;
- Coulomb potential only from a system of positive charges (white foreground pixels) – CPT1-algorithm;
- Coulomb potential of a system both of positive (white foreground pixels) and negative charges (black background pixels) – CPT2-algorithm.

All calculations were made using moving window of a square shape with five pixels half-width. The resulting gray-scale images were transformed to binary images, using 0.5 thresholding.

Patterns in binary images were analysis using following statistical indices: average, standard deviation, skewness and kurtosis. Also we used the following geometric signatures: area, eccentricity, Euler number, orientation, perimeter. Statistical indices were also calculated for white - black pixel arrangement and for distance statistic between white pixels.

It was found that the most suitable geometric signature for pattern configuration analysis of IPT and CPT is area value. Similarly the most suitable statistics is distance statistics between white pixels.

REFERENCES

1. Cornelius T. Leondes (eds) (1998) Image Processing and Pattern Recognition. Academic Press.
2. Santosh Kumar Dash, Mrutyunjaya Panda (2016) Image Classification using Data Mining Techniques. Advances in Computer Science and Information Technology (ACSIT), 3(3) 157-162.
3. Arunadevi S., Daniel Madan Raja S. (2014) A Survey on Image Classification Algorithm Based on Per-pixel. International Journal of Engineering Research and General Science, 2(6), 387-392.
4. Fu K. S., Mui J. K. (1981) A Survey on Image Segmentation. Pattern Recognition, 13, 3-16.
5. Amanpreet Kaur (2014) A Review Paper on Material Analysis using Image Processing. International Journal of Science and Research (IJSR), 3(10), 624-625.
6. Reljin I. S., Reljin B. D. (2002) Fractal Geometry and Multifractals in Analyzing and Processing Medical Data and Images. Archive of Oncology, 10(4), 283-293.

7. Lopes R., Betrouni N. (2009) Fractal and Multifractal Analysis: a Review. Medical Image Analysis, 13(4) 634-649.
8. Uahabi K. L., Atounti M. (2015) Applications of Fractals in Medicine. Annals of the University of Craiova, Mathematics and Computer Science Series, 42(1), 167-174.
9. Image Processing for Remote Sensing (2006) CRC Press, Taylor & Francis Group.
10. Chen C.H. (2008) Image Processing for Remote Sensing. United States of America, Taylor & Francis Group, LLC.
11. James C. Bezdek, James Keller, Raghu Krisnapuram, Nikhil R. Pal (2005) Fuzzy Models and Algorithms for Pattern Recognition and Image Processing. Springer Science+Business Media, Inc.
12. Chee Sun Won, Robert M. Gray (2004) Stochastic Image Processing. Springer Science+Business Media New York.
13. Edward R. Dougherty, Roberto A. Lotufo (2003) Hands-on Morphological Image Processing. SPIE, Bellingham, Washington.
14. Wilhelm Burger, Mark J. Burge (2013) Principles of Digital Image Processing. Advanced Methods, Springer-Verlag London.
15. Rafael C. Gonzalez, Richard E. Woods (2008) Digital Image Processing. Pearson Education, Inc.
16. S. Allen Broughton, Kurt Bryan (2009) Discrete Fourier Analysis and Wavelets. John Wiley.
17. Stephane Marchand-Maillet, Yazid M. Sharaiha (2000) Binary Digital Image Processing. Academic Press.
18. Maria Petrou, Pedro Garcia Cevilla (2006) Image Processing. Dealing with Texture. John Wiley.
19. Stefano Cagnoni, Evelyne Lutton, and Gustavo Olague (eds) (2007) Genetic and Evolutionary Computation for Image Processing and Analysis. Hindawi Publishing Corporation.
20. P.-S. Liao, T.-S. Chen, and P.-C. Chung (2001) A Fast Algorithm for Multilevel Thresholding. Journal of Information Science and Engineering, 17, 713-727.
21. J. N. Kapur, P. K. Sahoo, and A. K. C. Wong (1985) A New Method for Gray-level Picture Thresholding using the Entropy of the Histogram. Computer Vision, Graphics, and Image Processing, 29, 273-285.
22. Frank Y. Shih (2009) Image Processing and Mathematical Morphology. Fundamentals and Applications. CRC Press, Taylor & Francis Group.
23. Pierre Soille, Peter Vogt (2009) Morphological Segmentation of Binary Patterns. Pattern Recognition Letters, 30, 456-459.
24. Borgefors G. (1986) Distance Transform in Digital Image. Computer Vision, Graphics and Image Processing, 34, 344-371.
25. Danielsson P-E. (1980) Euclidean Distance Mapping. Computer Graphics Image Processing, 14, 227-248.
26. Strand R. (2011) Sparse Object Representations by Digital Distance Functions. Discrete Geometry for Computer Imagery, 6607 in Lecture Notes in Computer Science. Springer Berlin Heidelberg, 211-222.
27. Abdul Ghafoor, Rao Naveed Iqbal, and Shoab Khan (2003) Image Matching using Distance Transform. Image Analysis, 13th Scandinavian Conference, SCIA Halmstad,

Sweden, June 29 - July 2, 2003, Proceedings. Springer-Verlag Berlin Heidelberg, 654-660.
28. Yves Lucet (2009) New Sequential Exact Euclidean Distance Transform Algorithms based on Convex Analysis. Image and Vision Computing, 27, 37-44.
29. Cuntz N., Kolb A. (2007) Fast Hierarchical 3D Distance Transforms on the GPU. Eurographics, 93-96.
30. Eşref Selvi, Merve Özdemir and M. Alper Selver (2013) Performance Analysis of Distance Transform based Inter-slice Similarity Information on Segmentation of Medical Image Series. Mathematical and Computational Applications, 18(3), 511-520.
31. CH. Nooka Raju, G. S. N. Raju, V. K. Varma Gottumukkala (2016) Studies on Watershed Segmentation for Blood Cell Images using Different Distance Transforms. IOSR Journal of VLSI and Signal Processing (IOSR-JVSP), 6(2), 79-85.
32. Intel Parrallel studio-xe, https://software.intel.com/en-us/parallel-studio-xe.
33. Binder K., Heermann D. (2010) Monte Carlo Simulation in Statistical Physics. 5th Ed. Springer.
34. Rapaport D. C. (1995) The Art of Molecular Dynamics Simulation. Cambridge, New York, Cambridge University Press.
35. Tersoff J. (1989) Modeling Solid-state Chemistry: Interatomic Potentials for Multi-component Systems. Physical Review B, 39, 5566-5568.
36. Matlab help system. https://www.mathworks.com/.

QUANTITATIVE METHODS IN ECONOMICS
Volume XIX, No. 1, 2018, pp. 28 – 37
Received: 01.09.2017
Accepted: 05.04.2018

SPATIAL ANALYSIS OF LOCAL HOUSING REAL ESTATE PRICE CHANGES (USING OLSZTYN AS AN EXAMPLE)

Radosław Cellmer (ORCID: 0000-0002-1096-8352)
Faculty of Geodesy, Spatial Engineering and Construction
University of Warmia and Mazury in Olsztyn, Poland
e-mail: rcellmer@uwm.edu.pl
Marcelina Zapotoczna (ORCID: 0000-0003-2542-7644)
Faculty of Economic Sciences
University of Warmia and Mazury in Olsztyn, Poland
e-mail: imz@uwm.edu.pl

Abstract: This paper analyses the spatial dynamism of price changes in the housing market in Olsztyn. A geographically-weighted regression was used to examine the relationship between price changes in time and features of properties that describe specific technical and functional parameters. Data from 2007-2015 obtained from RCiWN were used to construct one exponential GWR model as well as separate models for each year under analysis. The results are presented in background maps.

Keywords: housing market, spatial analysis, geographically-weighted regression

JEL classification: C21, R31

INTRODUCTION

The specific nature of the housing market has been noted by a number of researchers [e.g. Łaszek 2006, Belniak 2008, Kucharska-Stasiak 2016] who associated this market with the features of real estate which, on the one hand, distinguish it from other economic goods, but which restrict market operations, on the other. Such specific features include: the function that it plays in the life of each person, heterogeneity, durability, absence of substitutes and immobility. The latter feature makes location one of the key factors affecting the price of a property, because a buyer buys both an apartment and the place where it is situated. The

DOI: 10.22630/MIBE.2018.19.1.3

position itself can have various dimensions, e.g. it can concern the immediate neighbourhood of a property, (i.e. the building type, nature of the street, neighbours), but it can also describe, for example, the quality of other houses in the neighbourhood, the proximity of commercial and service facilities, transport connections with the place of one's work or natural and aesthetic value, or it can cover a broader area, e.g. a city, (i.e. economic activity and the attractiveness of labour markets or the prestige of the city). Therefore, a neighbourhood can have both positive and negative effects. The price is the key factor considered when making an apartment purchase decision. Therefore, since a property occupies a permanent place in space, prices are affected by variables that describe closer and more distant surroundings of a property, i.e. variables that are beyond the control of potential buyers. The effect of location on prices in the housing market has been dealt with, for example, by Rosen [1974], Li and Brown [1980], Clapp and Wang [2006], Kiel and Zabel [2008]. Models of hedonic regression are usually employed to describe the issue. On the other hand, according to Kulczycki and Ligas [2007], classic regression models do not include potential interactions which may occur between observation units (real-estate) and assume "stability" of the process related to price formation in the geographic space. Further, according to Cellmer [2013], the application of a geographically-weighted regression to a real-estate market analysis allows the effect of the particular attributes of a real-estate on its price to be determined depending on its location. Moreover, Cellmer [2010] further claims that this correlation is reinforced by the occurrence of submarkets within local markets. To determine the borders of submarkets which may form districts, Clapp and Wang [2006] used classification and regression trees (C&RT methods). The authors demonstrated that certain assessment parameters of a property location have a greater effect within sub-market borders. Lack of data describing a location (which was necessary for a detailed analysis) was an important issue faced by researchers. Appropriate methods of spatial analysis are still being sought to identify a pattern on local housing markets describing the relationship between the price and the location which takes into account the spatial dimension of the surroundings of a real estate property.

RESEARCH METHODOLOGY

This study analysed the spatial dynamism of price changes in the housing market in Olsztyn with the use of Geographically Weighted Regression (GWR). The analyses were based on transaction data for the sale of residential establishments in 2007 - 2015, obtained from the Real Estate Price and Value Register (Rejestr Cen i Wartości Nieruchomości – RCiWN). Transactions were selected for the study which contain information on: transaction date, location (address, geodesic precinct), transaction price, form of trade, floor area, position on a storey and year of construction. A total of 10,192 transactions were analysed.

The period taken for analysis was not a random one. The year 2007, recorded in the Polish economy as the last year of prosperity on the housing market, was the starting point. The next year, 2008, saw the beginning of an economic slowdown, which affected the housing market. Furthermore, a steady increase in the number of residential facilities entering the market was recorded, beginning with 2010, which translated into the number of flats put to use in the following years. A slow recovery was observed after 2011, which resulted from decreasing interest rates, unsatisfied housing demand and a growing supply of flats that met the conditions for a preferential credit[1]. Changes in the level of interest in flat purchases during the analysed period resulted in fluctuations of transaction prices both on the primary and secondary markets. Therefore, one of the important variables in constructing statistical models of transaction prices is the time specified as the transaction date expressed on the interval scale.

By estimation of parameters of multiple regression classic models, one can determine, for example, the effect of time on transaction prices, but those will be global parameters, not taking into account the spatial heterogeneity.

One of the methods by which the spatial structure can be taken into account in regression models is to assign weights to observations, which – due to their position in space – can have a theoretically greater effect on the issue than others. These weights are taken into account in GWR models. The equation of a typical GWR model will have the following form of results [Charlton, Fotheringham 2009]:

$$Y = \beta_0(x_i, y_i) + \sum_{i=1}^{k} \beta_j(x_i, y_i) \cdot X_i + \varepsilon_i \tag{1}$$

where Y denotes an explaining variable, β_0 model constant, β_j model parameters, X_j explaining variables, x_i and y_i coordinates of the point in which model parameters are estimated, whereas ε_i is a random element (residue) of a model. Detailed discussion of the principles of construction and evaluation on GWR model fitness GWR have been extensively presented in the literature [among others, Brunsdon et al. 2000; Charlton, Fotheringham 2009].

Parameters of a linear model with a dependent variable as a transaction price are interpreted as an amount per unit of change in an independent variable. Considering the fact that a number of market phenomena are not linear, and in view of the multiplicative nature of the effect of explaining variables on prices, it was assumed that the effect can be regarded as a multiplier (per cent). This allows for easier interpretation, especially with high spatial diversification of average prices. Therefore, a GWR exponential model in the following form was used to determine the relationship:

$$Y = \delta_0(x_i, y_i) \cdot \prod_{j=1}^{k} \left(\delta_j(x_i, y_i)\right)^{X_j} \cdot \xi_i \tag{2}$$

[1] Credits taken in government programmes: RnS and MdM, which are government subsidies.

where δ_0 is a model constant, δ_j model parameters interpreted as multipliers. Random element ξ can also, in this case, be treated as a multiplier. The remaining elements of the formula are the same as in formula (1)
If it is assumed that:

$$\delta_j(x_i, y_i) = r_j(x_i, y_i) + 1 \text{, and } r_j(x_i, y_i) = \delta_j(x_i, y_i) - 1 \qquad (3)$$

then r_j can be regarded as a direct relative effect of an attribute on transaction prices, at the point of known coordinates (x_i, y_i), expressed as a multiplier effect (compound interest).
When the model is brought to an additive form, estimation of the parameters can be performed by the least squares method, taking into account the location-dependent weights of observations [Charlton, Fotheringham 2009].
The direct percentage effect of the explained variable is then calculated as:

$$r_j = e^{\delta_j^{'}(x_i, y_i)} - 1 \text{, where } \delta_j^{'}(x_i, y_i) = \ln(\delta_j(x_i, y_i)) \qquad (4)$$

When the transaction date is regarded as one of explaining variables, determined with the resolution down to one month, information is obtained on the annual relative (expressed as percent) change of prices:

$$r_j \,/\, year = (1 + r_j \,/\, month)^{12} - 1 \qquad (5)$$

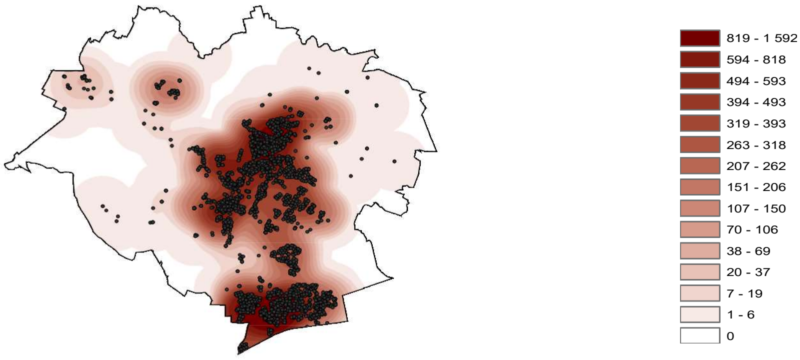
The average index of price changes for each geodesic precinct (as well as its error) was calculated as the arithmetic average of the values of raster cells (from interpolation) within its borders. This provided a distribution map of the average price change index for each year of the analysis. This helps to identify areas in which price changes can result not only from global, but mainly from local conditions. The calculations and visualisations were prepared with Statistica v. 12 and ArcGIS v. 10.2 software.

RESULTS AND DISCUSSION

The spatial activity of the housing market is naturally dependent on spatial distribution of existing housing resources and new investments. New investments are located on the outskirts of cities as well as in the middle and central zone, where the supply of land for housing investments is decreasing. Characteristic features of Olsztyn include natural spatial barriers such as a forest (in the north of the city) and numerous lakes in the city. Due to the presence of these barriers, the urban areas have mainly expanded to the south, where - according to the land use plan – the area of land for housing investment is the greatest.

The spatial diversity of the housing market in Olsztyn and the spatial distribution of transactions is shown in Figure 1.

Figure 1. The number of transactions per 1 km^2 on the housing market in Olsztyn in 2007 - 2015



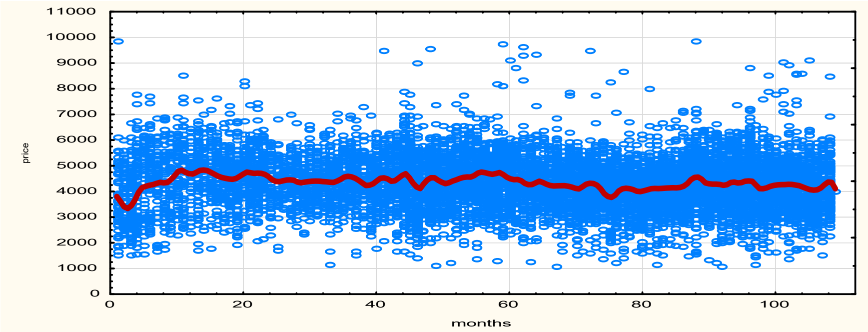
Source: the authors' research

The quartic kernel density estimation provided information on the average number of transactions per 1 km/m^2 [cf. Silverman 1986, Porta et al. 2009].

More than 100 transactions/km^2 annually were recorded in the city central area and in some of its residential quarters. The transactions in the centre and in the residential area (Zatorze) were usually conducted on the secondary market, which was the most active in the first two years of the analysis. Moreover, an increase in the supply on the primary market of flats that met the criteria for preferential loans attracted the interest of flat purchasers in this market segment. In effect, since 2010, the number of transactions increased in the south-eastern part of the city, i.e. in the area bordering on the communes of Purda and Stawiguda (areas where mainly multi-family residential buildings were constructed). This area is referred to in the land use plan as the reserve area for development of multi-family housing, on condition of conservation of the natural value of the land.

The difference between the supply and demand produced a slight increase in the flat prices in 2007-2008. A decrease in the amount of loans granted caused by an increase in the interest rate, and consecutive government programs (RnS and MdM), reversed the trend. Thus, the period of transaction price growth was followed by stabilisation at a medium level of ca. 4,500 PLN/m^2. Figure 2 presents the changes in transaction unit prices for residential premises (trend after inverse exponential smoothing).These fluctuations were not systematic and resulted from behavioural and random factors rather than from market conditions.

Figure 2. Trend of the changes in prices for the territory of the city of Olsztyn in 2007-2015



Source: the authors' research

Considering the fact that the greatest price change was observed in 2007 (38.5%), which considerably deviated from the other years of the analysis, data from this period were left out from the remaining part of the analysis and taken only for illustrative purposes. A detailed data analysis resulted in constructing an exponential GWR model for all of the 2008-2015 data and – independently – for the data from each of the years under analysis. The general results of GWR modelling are shown in Table 1.

Table 1. General GWR modelling results for 2008-2015 data

Model properties	Min	Max	Average
Observedvalue (log price)	6.967	9.194	8.349
Local R^2	0.031	0.880	0.226
Predictedvalue (log price)	7.147	8.663	8.349
Interceptδ_0	8.239	8.514	8.371
Std. error of δ_0	0.011	0.226	0.015
δ_1 (influence of time)	-0.004	0.003	-0.001
Std. error of δ_1	0.000	0.005	0.000
δ_2 (influence of flat area)	0.000	0.007	0.002
Std. error of δ_2	0.000	0.003	0.000
δ_3 (influence of floor)	-0.092	0.205	0.009
Std. error of δ_3	0.007	0.105	0.009
δ_4 (influence of year of building)	-0.009	0.010	0.002
Std. error of δ_4	0.000	0.004	0.001
Residual (ξ)	-1.301	1.346	0.000
Std. error of model	0.027	0.191	0.190
Prices dynamic [%/year]	-4.304	4.100	-1.760

Source: the author's calculations

Model parameters were estimated for each of the locations, where a transaction occurred. Thus, the min, max and average values refer to over 10,000 models. These values reflect the scale of spatial diversity of the parameters of the particular models.

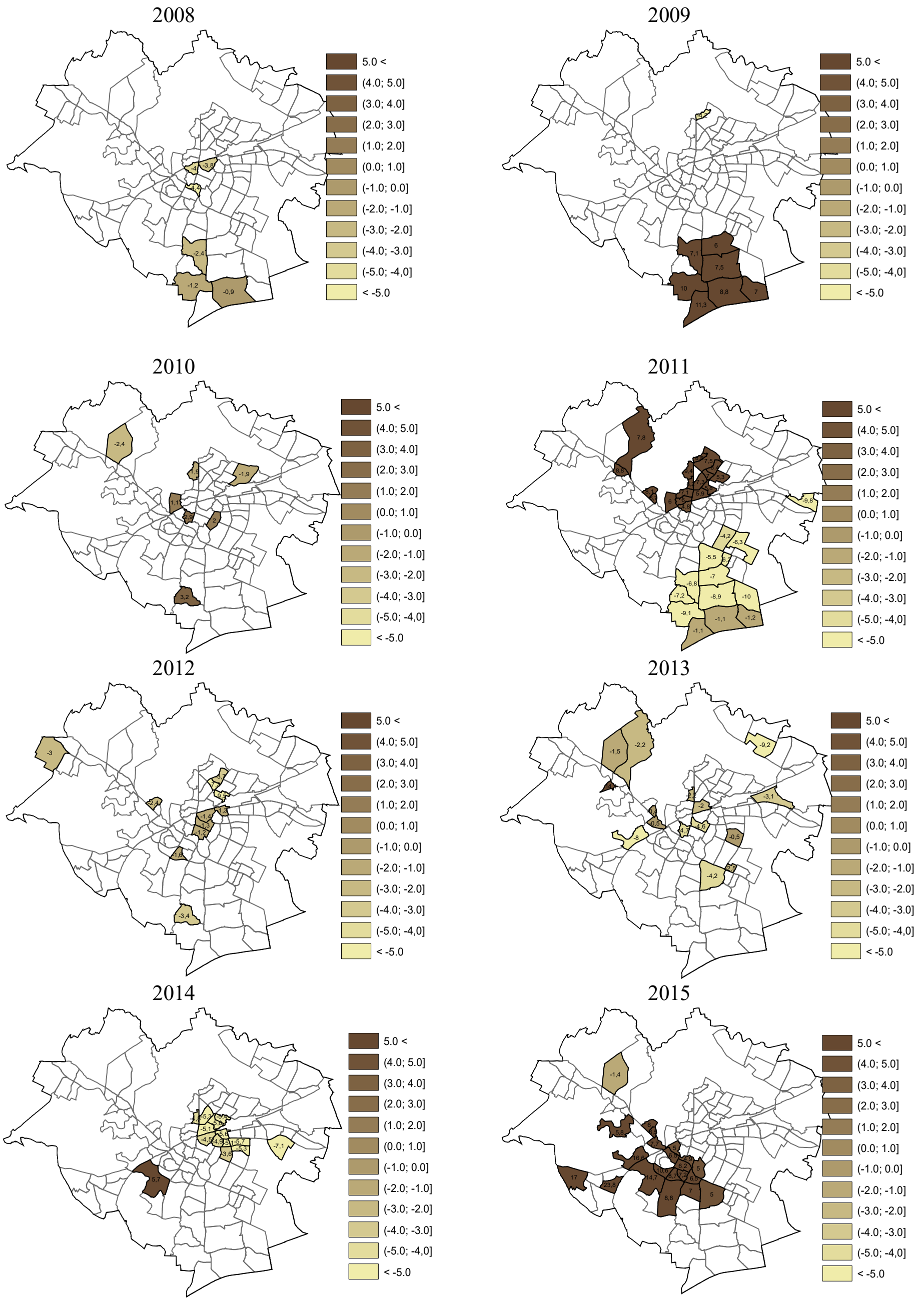
The trend parameters ranged from -0.004 to 0.003, which, according to formula (13), gives -4.3% to 4.1% for a year. The mean parameter for the impact of the area was 0.002 (which means that the 1 m^2 price decreased by 0.2%). The average impact of position on a storey was approx. 0.9% (per unit on the scale adopted). The mean parameter which shows an effect of the building age on the price indicates that each year affects the price by approx. 0.2%. The averaged relative error of the model was approx. 20%. It must be emphasised that these are averaged values; a detailed and precise analysis is possible independently for each of over 10,000 models created and they are not presented here to retain the paper's conciseness and clarity.

A great majority of the models indicate a negative trend, with nearly 1/4 of the observations ranging from -3% to -2.5% annually. The values of the local determination coefficient were mainly within the interval from 0.2 to 0.25 (for nearly 1/3 of cases). For a large majority of the points at which the GWR model was created, all the explaining variables (including the price change trend) proved to be significant at a level of significance not lower than 0.01. Considering the averaged results for the entire period, the unit prices decreased at 1.7% per year. In areas with the highest market activity, the price decrease was relatively small – usually approx. 1% annually. The smallest estimation errors were calculated for the city centre, whereas the highest (as expected) were calculated for the areas with relatively small numbers of transactions. No significant relationship was observed between the transaction density function and the annual price change index.

In the next stage of the study, GWR modelling was performed independently for each year of the analysis (beginning with 2008). For easier interpretation, a register precinct was taken as a reference unit. The average annual price change trend index was estimated independently for each precinct as an arithmetic average of the raster obtained by interpolation by the ordinary kriging method. Precincts with no housing were left out of the analysis. Fig. 3 shows the annual price change indexes in various city suburbs. Only precincts were taken into account in which the annual price change trend was statistically significant at the level of significance of $p < 0.05$.

No steady price change trends in various precincts of the city of Olsztyn were observed in the period under study. The price decrease observed was not permanent.

Figure 3. The spatial distribution of price change indices (%) in the city of Olsztyn in 2008-2015



Source: prepared by the authors

In 2009, the southern and south-eastern part of the city became the most attractive to flat buyers, which resulted in the price increase. This area of the city has become the most attractive to market participants due to an increase in the supply of land for construction caused by changes in the spatial structure, resulting from the transformation of agricultural land into land for residential building construction, and improvement of the living conditions by raising the technical and architectural value of existing buildings (the average yearly price increase coefficient was estimated at 4-5%).
In 2011, prices of housing property increased in the northern and north-western part of the city. The price increase in the north-western part resulted from the growing demand for detached and semi-detached single-family houses in a naturally attractive part of the city – Redykajny, which is surrounded by lakes, in the Łyna river valley and near the Municipal Forest complex. This trend reversed in 2013 due to the transport connection difficulties caused by a population increase in this part of city (an average estimated price decrease by 1-3%). Furthermore, a number of flats were put to use in the northern part in three completed multi-family buildings. The attractiveness of the location consisted in complementing the existing buildings or expansion or modernisation of the existing buildings while preserving the historic urban arrangement. On the other hand, the secondary market and scarce new investment projects were the most active in the eastern and south-eastern part of the city, whose location was perceived by flat purchasers as less attractive, which translated into decreasing transaction prices (the average annual price decrease index was estimated at 1-3%).
The greatest price increase in the last year, i.e. 2015, was recorded in the residential area of Podgrodzie, i.e. a suburb situated near the university campus.
The price change indexes were stable in the majority of precincts. Only in years 2009, 2011 and 2015 were the prices seen to increase (the average annual price increase index was estimated to be 4-5%), while a slight price decrease was observed in 2011, 2013 and 2014 (the average annual price decrease index was estimated at 1-3%).

SUMMARY

The spatial analysis of price changes employing geographically-weighted regression conducted for the housing market in Olsztyn, showed the trends and areas where the greatest price increases and decreases occurred during the analysed period. No steady trends in price changes were recorded in the years 2008-2015 in various parts of the city of Olsztyn, which may be a consequence of small fluctuations of the average transaction prices on the market under analysis. Extreme changes occurred in the south-western part of the city, where the greatest increases were observed in 2009, which were followed by the greatest decreases in 2011. Considering the increase (4-5%) and decrease index (1-3%), as well as the absence of changes in subsequent years, one should conclude that the prices stabilised at the average transaction price level for Olsztyn.

An analysis of prices on the housing market is a constant element of the market activity evaluation. The geographically weighted regression allowed for a detailed analysis of spatial prices diversity, which may be of particular use to market participants, especially entities making housing investments, institutions which provide market financing, real estate agents and urban planners.

REFERENCES

Belniak S. (2008) Makroekonomiczne uwarunkowania rynku mieszkaniowego w Polsce. Zeszyty Naukowe UE w Krakowie, 792, 6-14 (in Polish).

Brunsdon C., Fotheringham S., Charlton M. (2000) Geographically Weighted Regression as a Statistical Model. Spatial Analysis Research Group Department of Geography, University of Newcastle-upon-Tyne, UK.

Cellmer R. (2010) Analiza przestrzenna dynamiki zmian cen nieruchomości lokalowych z wykorzystaniem regresji ważonej geograficznie. Acta Sci. Pol., Administratio Locorum 9(3), 5-14 (in Polish).

Cellmer R. (2013) The Use of the Geographically Weighted Regression for the Real Estate Market Analysis. Folia Oeconomica Stetinensia, 17(2).

Charlton M. Fotheringam S. (2009) Geographically Weighted Regression. White Paper, National Centre for Geocomputation, National University of Ireland Maynooth.

Kiel K. A., Zabel J. E. (2008) Location, Location, Location: The 3L Approach to House Price Determination. Journal of Housing Economics 17, 175-190.

Kucharska-Stasiak E. (2016) Ekonomiczny wymiar nieruchomości. Wyd. Naukowe PWN, Warszawa (in Polish).

Kulczycki M., Ligas M. (2007) Regresja ważona geograficznie jako narzędzie analizy rynku nieruchomości. Geomatics and Environmental Engineering, University of Science and Technology Press, 1(2), 59-68 (in Polish).

Li M. M., Brown J. (1980) Micro-neighbourhood Externalities and Hedonic Housing Prices. Land Economics, 56, 125-141.

Łaszek J. (2006) Rynek nieruchomości mieszkaniowych i jego specyfika jako czynniki determinujące ryzyko kredytowania hipotecznego. Materiały i Studia NBP, 203 (in Polish).

Porta S., Latora V., Strano E., Cardillo A., Scellato S., Iacoviello V., Messora R. (2009) Street Centrality vs. Commerce and Service Locations in Cities: a Kernel Density Correlation Case Study in Bologna, Italy. Urban & Planning, B 36, 450-465.

Rosen S. (1974) Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition. Journal of Political Economy, 82, 34-55.

Silverman B. W. (1986) Density Estimation for Statistics and Data Analysis. New York, Chapman and Hall.

QUANTITATIVE METHODS IN ECONOMICS
Volume XIX, No. 1, 2018, pp. 38 – 47

Received: 25.01.2018
Accepted: 05.04.2018

SIMULATION RESEARCH IN THE PROCESS OF DETERMINATION OF STOCK PRICES USING A MODIFIED GORDON GROWTH MODEL

Marcin Halicki (ORCID: 0000-0002-5343-0093)
Faculty of Biology and Agriculture, University of Rzeszow, Poland
e-mail: mhalicki@ur.edu.pl
Tadeusz Kwater (ORCID: 0000-0001-5095-613X)
Faculty of Mathematics and Natural Sciences, University of Rzeszow, Poland
e-mail: tkwater@univ.rzeszow.pl

Abstract: In the latest literature there is a lack of broader discussion of the Gordon Model, with a desire to point out that its assumptions do not fit the expectations of modern investors and the nature of capital markets. The issue prompts us to ask whether this model can still be used by modern scientists and investors. The aim of the article is to present a classic Gordon model and show the direction of its modification. An attempt was also made to use simulation studies in a graphical interpretation of a selected share, which incorporates the Gordon's modified and classic models.

Keywords: risk, stock, investments, model, simulations

JEL classification: G11, G17

INTRODUCTION

The Gordon Growth Model is one of the most well known in the finance area, which, alongside Markowitz's theory, has undoubtedly contributed to the development of the field of investment management and portfolios of financial instruments. Nevertheless, one must not forget that capital markets, by their very nature, undergo constant transformation. They are facing new challenges, which are mainly attributable to the pressure of customers, other participants and regulators [PWC - Capital Markets 2020]. They form a tight corset that limits and encourages, among other things, the search for new or improved model solutions. And these seem to be desirable from the perspective of developing capital markets,

DOI: 10.22630/MIBE.2018.19.1.4

which are important mainly because they are an important part of any mature economy [Ouandlous 2010], positively influencing the functioning of its mechanisms. This corset of high demands does not allow for the fact that it will be loosened, so it has become an inspiration to consider the Gordon model. In addition, it was found on the basis of literature research that there was a lack of broader discussion of the model, and that brought the desire to point out that its assumptions did not fit the expectations of modern investors and the nature of capital markets. All this prompts us to ask whether this model can still be used by modern scientists and investors.

The purpose of this article is to present the Gordon model, characterizing its most important features and disadvantages, and showing the direction of its exemplary modification to fit the needs of both today's capital markets as well as investors and theorists. The study also attempted to use simulation studies in the process of graphical interpretation for theoretical stock based on selected models. For the purposes of research, the predicted values of the various parameters are assumed for specific probabilities for obtaining the price, which was calculated using the classical and modified Gordon model, taking the form different formulas. Literary research and scientific reflection were used in the discussion. It has been shown that the Gordon model continues to be an attractive research base that can be used to create a modified model that will be able to meet the expectations of modern economists.

GENERAL CHARACTERISTICS OF THE GORDON MODEL

The natural consequence of the development of capital markets and the development of capitalist states, was the development of numerous financial models in the 1950s and 1960s. One of the most important of these is the Gordon Growth Model [Gordon & Shapiro 1956], but the problem lies in the fact that the model seems to have been forgotten by researchers. This conclusion can be deduced from the literature. The starting point for the Gordon Growth Model characterisation is a approach of Williams [Williams 1938], who first introduced the classic stock valuation model, known as the dividend discount model (DDM). It assumed that the price of a company's shares depended on the sum of future dividends paid to shareholders whose value should be adjusted to determine their current value:

$$\mathrm{P_t} = \mathbb{E}_t\left[\sum_{\mathrm{n}=1}^{\infty} \frac{\mathrm{D_{t+n}}}{(1+\mathrm{R_{t+n}})^{\mathrm{n}}}\right], \tag{1}$$

where $\mathrm{P_t}$ is the stock price at period t, $\mathbb{E}_t[\,]$ - expected value, $\mathrm{D_t}$ is the value of future dividend paid at time t, and $\mathrm{R_t}$ is the rate used to discount cash flows or the rate of return required by investors. The Williams approach has contributed significantly to the development of fundamental analysis of listed companies and has become the basis for the development of a stock valuation model that ultimately adopts the following form:

$$P_0 = \frac{D_1}{(R-g)} = \frac{D_0(1+g)}{(R-g)}, \tag{2}$$

where P_0 is the price of the stock in the current period, D_1 is an expected dividends of one year from now, and g is the annual dividend growth rate, therefore $D_1 = D_0(1+g)$, but it should be assumed that $R > g$ and stock company will pay out dividends. Thus, the calculated share price in the current period takes into account continuous and uninterrupted growth, while the equation (2), which is presented above and which allows the stock to be valued in the current period, is known as the Gordon Growth Model and nowadays it seems to be losing popularity, so it is worth considering the way it evolved. In the first place, it is worth answering why the Gordon Growth Model is less and less used in research around the capital markets and around portfolio management. This is due to its main assumptions, which are considered unrealistic [Investopedia.com]. These assumptions are following:

- Sensitivity to the required rate of return and to the increase in the value of the dividend, which can consequently lead to very extreme stock valuations, making the analyzed model unsuitable,
- The rate of dividend growth must always be the same, irrespective of the company's financial situation and stock market conditions, and may not exceed the required rate of return.

Considering the above, one should also pay close attention to the clash of economic realities and assumptions on which the Gordon Growth Model is based. There is no need to further explain that they cannot be fulfilled in a real world that is variable and unpredictable. Moreover, the sensitivity of the stock price, calculated on the basis of this model to the variability of its variables, is extremely high, which is why economists have attempted to modify it to eliminate certain elements of the model or to add new, which are beneficial for equity investments.

MODIFICATION OF THE GORDON GROWTH MODEL

The Gordon model should be modified mainly for two reasons. First and foremost, the stock price calculated on its classical form is very sensitive to changes in the value of parameters that may change during the course of the investment, thereby reducing the theoretical and calculated price. Secondly, other and equally important parameters (such as book value of a company) are not included in such price calculations. The modification of the Gordon's model in literature [Chapados 2011] may be based on the classic RIM-Model (residual income valuation model), which takes into account the relationship between the book value and the net earnings per share and dividends per share:

$$B_t = B_{t-1} + E_t - D_t, \tag{3}$$

where B_t is the book value per share at time t, E_t is the earnings per share in the period t. Then, taking into account the Edwards' approach, Bella and Ohlson [Edwards & Bell 1961, Ohlson 1995], the relationship between shareholder payments and the so-called "abnormal" earnings can be determined, but including the interest rate in the form of their model, which is used to discount the cash flows. The model described here assumes the following form:

$$E_t^{\propto} \triangleq E_t - R * B_{t-1}, \tag{4}$$

where $E_t^{\propto}$ determines "abnormal" earnings, $B_{t\text{-}1}$ is the book value per share at time $t-1$, R is also a discount factor. By analyzing the aforementioned shots, one can build an equation allowing to calculate a dividend per share at time, but at a given book value per share and "abnormal" earnings, because:

$$D_t = E_t^{\propto} - B_t + (1 + R) * B_{t-1}, \tag{5}$$

This dividend calculation allows us to develop two equations that represent the modified Gordon model, which does not include dividends per share, or g , which most changes the price of shares in the classic model. Thus, the price of shares under the new approach can be calculated in two ways:

$$P_t = B_t + \mathbb{E}_t \left[\sum_{\tau=1}^{\infty} \frac{E_{t+\tau}^{\propto}}{(1+R)^{\tau}}\right], \tag{6}$$

and

$$P_t = B_t + \mathbb{E}_t \left[\sum_{\tau=1}^{\infty} \frac{E_{t+\tau} - R * B_{t+\tau-1}}{(1+R)^{\tau}}\right], \tag{7}$$

The modification of the Gordon's model is that it does not take into account the values g, so the price of the stock is not dependent on them. In contrast, it is sensitive to the change of the book value per share and the change of the "abnormal" earnings or of the earnings per share. The presented R is similar in role to the classical model, although the book value per share may lose its share of the stock price at low R values. Graphical interpretation of the presented formulas is unprecedented in the literature and may therefore be desirable for the use in the process of stock assessment and portfolio construction.

SIMULATION RESEARCH OF STOCK PRICE DETERMINATION USING THE CLASSIC AND MODIFIED GORDON MODEL

The MATLAB software was used for the simulation study and was analyzed for graphical analysis. They were divided into three parts to highlight the differences in the formulas analyzed. The first one was based on the classic Gordon model (2), the second on the modified model with the "abnormal" earnings (6), and the third model on the modified but without "abnormal" earnings (7). In-depth analysis of formulas indicates that, in order to obtain the desired dependency of the parameters, their graphical interpretation requires a series of values. The simulation study is therefore based on a theoretical value of the stock, with its parameters matched to the three variants corresponding to the models being analyzed. Table 1 presents the value of the parameters for calculating this share value for each of the analyzed models, and it is assumed that the purpose is to calculate stock prices in the current period (P_0).

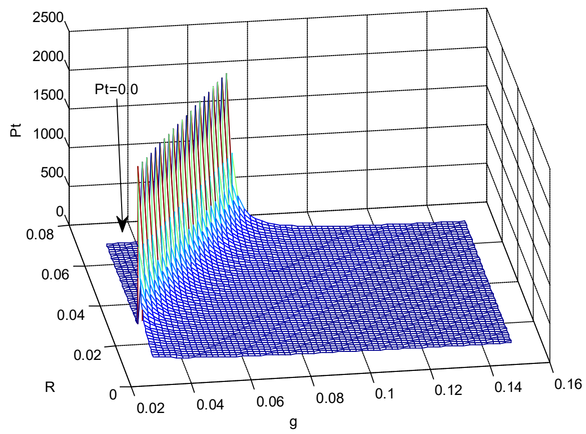
Table 1. Values of sample stock price simulations

Parameter	*Value (or changes)*
Classic model - equation (2)	
R (changes with step 0.002)	(0.03; 0.15)
g (changes with step 0.001)	(0.07; 0.01)
D_t $(t = 1)$	2.0
Figure number	1; 2
Modified model – equation (6)	
$E_t^{\propto}$ (changes with step 0.0048)	(3.0; 5.4)
R (changes with step 0.00024)	(0.03; 0.15)
B_t	3.0
τ	500
Probability (changes with step 0.002)	(0.8; 1)
Figure number	3; 4; 5
Modified model - equation (7)	
$E_{t+\tau}$ (changes with step 0.0048)	(3.0;5.4)
R (changes with step 0.00024)	(0.03;0.15)
B_t (changes with step 0.06)	(3.0;9.0)
τ	100 and 500
Probability (changes with step 0.002)	(0.8;1)
Figure number	6; 7

Source: own calculations

One can mention that for modified models, a value of $\tau = 500$ was chosen, because the smaller value of this indicator changes the stock valuation, and above this, the stock price is independent of τ. In addition, the most important distinguishing features of the presented formulas were expressed by two-dimensional and three-dimensional charts. Only the first numerical experiment for stocks, based on the classical model (2), is shown in Figures 1 and 2. The rest concerns the modified models.

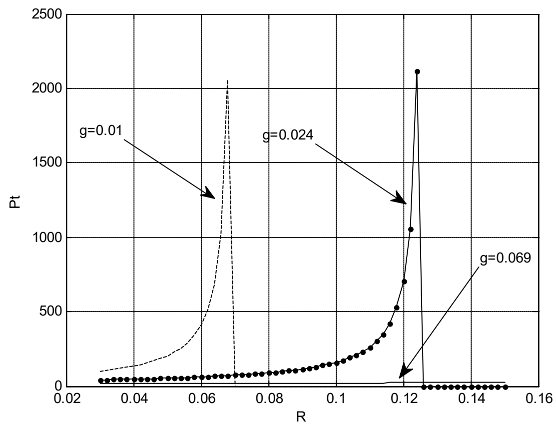
Figure 1. A three-dimensional graph of the relationship between the parameters R and g for the classical model (2)



Source: own study

Figure 1 provides that in certain cases it was assumed that for $R < g$ this price was "0". To enrich the presentation, this solution was highlighted not only on the presented three-dimensional graph, but also on the next two-dimensional graph (Figure 2). It presents the fragments of the solutions of Figure 1 for the selected values of the parameter g.

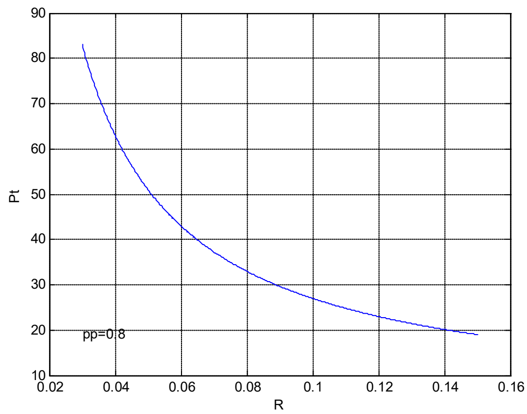
Figure 2. Two-dimensional graph of the relationship between the parameters R and P_t for the classical model (2) with the three values g



Source: own study

Under certain values g, the price of shares, is more sensitive and therefore change R can very quickly raise the price. It is easy enough to see that some g values (e.g. Figure 2 with g = 0.069) cause low sensitivity of stock prices to changes in R.

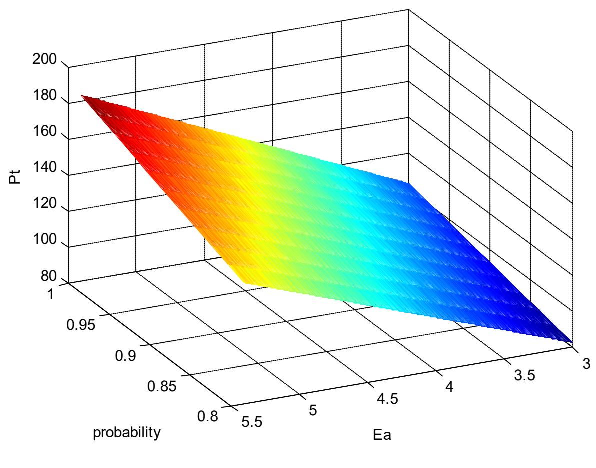
Figure 3. Relationships between R and P_t parameters for the modified model (6) at probability of 0.8 and variable abnormal earnings



Source: own study

Figure 3 shows that the increase in the required rate of return on stock, lowers its price. In Figure 4 the abnormal earnings is denoted as Ea.

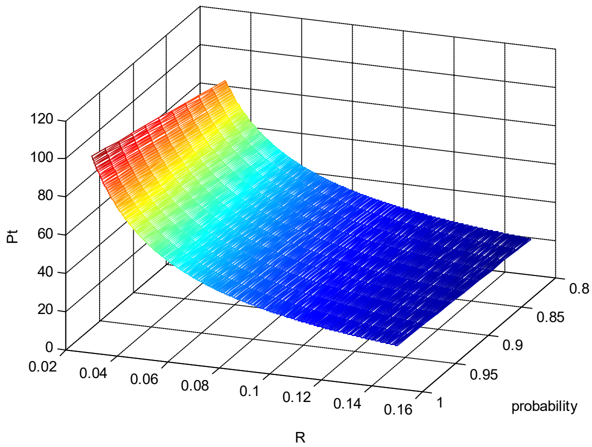
Figure 4. Three-dimensional graph of the relationship between the parameters P_t and Ea for the modified model (6) with increasing probability and R of 3%



Source: own study

By analyzing the above chart, it can be easily concluded that the probability of large abnormal earnings in companies significantly influences the stock price, but if the relatively high level of abnormal earnings is comparatively unlikely, then still the stock price is high.

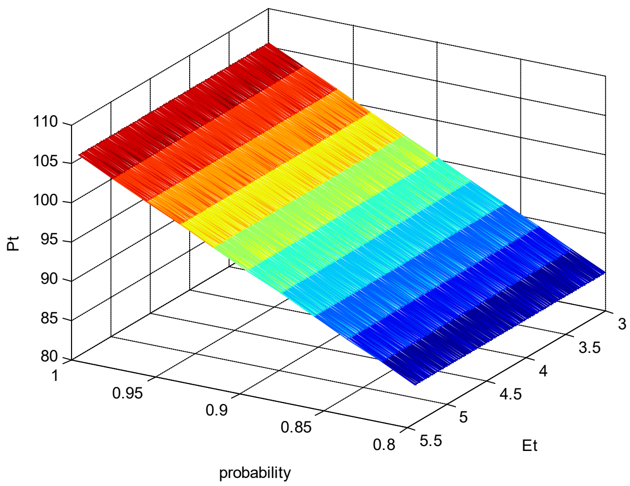
Figure 5. Relationship of P_t to R and probability for the modified model (6) at constant abnormal earnings of 3.0



Source: own study

The graph in Figure 5 confirms that the high required rate of return from stock, significantly decreases its price, even if the probability of abnormal earnings is high.

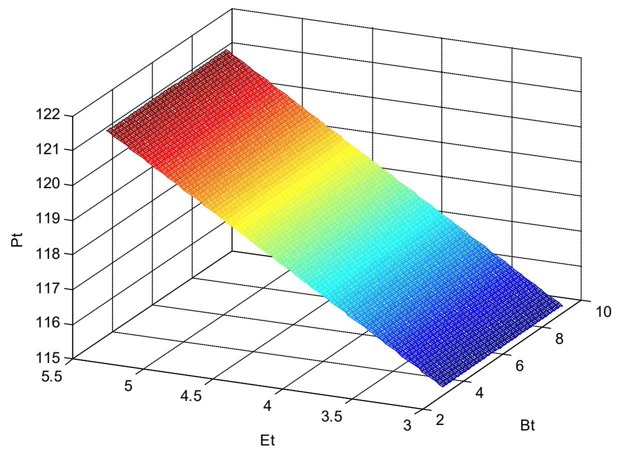
Figure 6. Price dependence of P_t on E_t and probability for the modified model (7) with a constant R of 3% and B_t of 3.0



Source: own study

The Figure 6 shows that the increase in earning per share at low probability, at constant R, does not raise the stock price to the extent that the probability increases. For any investor it means that, in the modified Gordon model, low returns but achievable are more important than high but unlikely.

Figure 7. Relationships between the parameters P_t, E_t and B_t for the modified model (7) with a constant R of 3% and a constant probability of 0.8



Source: own study

Figure 7 shows the effect of book value on stock price at different earning per share levels. Such appreciation can be considered cognitively interesting because the modified model (7) is the only one shown, which in its formula contains the book value multiplied in each period by the rate of return. Figure 7 reflects that with a given book value level, the stock price rises with the increase in earning per share. Conversely, the reverse relationship is different, because large book value changes, at constant earning per share, do not affect the price of the stock.

SUMMARY

To present the essence of the paper models, numerical experiments were conducted and the graphs were made. On the basis of the research, the following general conclusions were made. Firstly, the Gordon model in classical terms (2) should be treated as a theoretical tool that should be carefully used in practice, for at least the reason that, it allows analysis of shares only if R is higher than g and in the defined area the value of the g (g comparable to R) price of the stock price is very sensitive to the change in R. Secondly, the modified model (6) is an interesting analytical alternative to the classical solution, primarily because it does not take into account the g parameter but it is based on the abnormal earnings level, which most significantly influences the stock price, even with the probability change. Thirdly, the modified model in the second (7) spin also represents an alternative to the classical solution, since it is not based on the g parameter but primarily on the earning per share and book value. It has been shown that the share price is very sensitive to a given book value level with the simultaneous earning per share changes.

The final conclusions cannot be overlooked that the R parameter is also a factor influencing the price of shares in each of the models presented, but it is interesting that the modified model in each figure does not take into account either the parameter g or the level of dividends, making it an instrument of interest. for investors. But do not forget that recent empirical studies have shown [Khanal & Mishra 2017] that stock prices are rising in dividends announcements, so the classic Gordon model seems to be based on robust parameters.

REFERENCES

Chapados N. (2011) Portfolio Choice Problems - An Introductory Survey of Single and Multiperiod Models. Springer-Verlag New York, 27-29.

Edwards E. O., Bell P. W. (1961) The Theory and Measurement of Business Income. University of California Press, Berkeley, Los Angeles, London, 7 th edition.

Gordon M. J., Shapiro E. (1956) Capital Equipment Analysis: The Required Rate of Profit. Management Science, 3 (1), 102-110.

Investopedia.com (http://www.investopedia.com/ask/answers/032415/why-gordon-growth-model-not-more-widely-used.asp) [Access: 29 June 2017].

Khanal A. R., Mishra A. K. (2017) Stock Price Reactions to Stock Dividend Announcements: A Case from a Sluggish Economic Period. The North American Journal of Economics and Finance, 42, 338-345.

Ohlson J. A. (1995) Earnings, Book Values, and Dividends in Equity Valuation. Contemporary Accounting Research, 11(2), 661-687.

Ouandlous A. (2010) Capital Markets And Economic Development: A Framework For Newly Liberalized Economies. Journal of Business & Economics Research, 8(6), 9-16.

PWC-Capital Markets 2020, 5 (https://www.pwc.com/sg/en/banking-capital-markets/assets/capital-markets-2020.pdf) [Access: 29 June 2017].

Williams J. B. (1938) Theory of Investment Value. Harvard University Press, Cambridge, MA.

QUANTITATIVE METHODS IN ECONOMICS
Volume XIX, No. 1, 2018, pp. 48 – 59

Received: 23.01.2018
Accepted: 05.04.2018

IS PRODUCTIVITY PARADOX RELATED TO LOGISTICS? RESEARCH ON POLISH AGRI-FOOD INDUSTRY

Piotr Jałowiecki (ORCID: 0000-0002-3610-2151)
Faculty of Applied Informatics and Mathematics
Warsaw University of Life Sciences – SGGW, Poland
e-mail: piotr_jalowiecki@sggw.pl

Abstract: Paper presents research results of modern IT technologies and systems using impact for logistics activities in Polish food processing enterprises. Results indicate that a higher used IT solutions advancement level and, consequently, incurred expenditures on IT infrastructure do not directly translate to lower logistics costs. A clear relationships, according to which a higher IT solutions advancement level translates into a better company market position in the field of logistics and a higher level of knowledge about logistics solutions were found. Results confirmed productivity paradox existence in Polish agri-food processing companies identified earlier in relation to financial results.

Keywords: productivity paradox, information technologies, e-logistics, food processing, logistic costs

JEL classification: O33, Q19

INTRODUCTION

Agri-food producing industry is a significant sector of the Polish economy. According to data from the Central Statistical Office (GUS), in 2012, the value of its production sold accounted for 17.1% of production sold by the entire Polish industry and 20.4% of the value of sold production of industrial processing. In terms of food production, Poland ranks as the 6th place in Europe, and the food industry notes in recent years systematically increasing, a positive balance in foreign trade. In 2010-2012 trade balance increased in comparison to the previous year respectively by 9.3%, 21.3% and 59.6%. Polish sector of of food processing is responsible for producing approximately 6% of gross domestic product (GDP), its added value is approximately 4% of the total national economy value. Employment in the food

DOI: 10.22630/MIBE.2018.19.1.5

industry was represented 4.3% of total employment and 15.6% of total employment in the industry.

The sector of food processing in Poland is quite varied. According to the data in the database REGON, consists of 11 branches, of which by far the largest industry is bakery covering 44.3% of the companies. Significantly larger than other a meat industry is covering 20.2% of the companies. The share of other industries do not exceed 8.5%, and by far the smallest are oil-fat industry (0.8%) and tobacco (0.1%). It should also be noted that the Polish sector of of food processing is highly fragmented. The vast majority of 98.9% of the entities are companies belonging to the sector of small and medium-sized enterprises (SMEs), and as many as 69.7% are micro-enterprises employing up to 9 workers.

It is worth noting that the smaller companies, in particular, were classified as micro have far fewer opportunities in the acquisition, introduction and use of advanced IT technologies and systems. The reason is the need to incur significant costs the most, as well as relevant organizational preparation of company. But it is the ability to use in the practice of modern IT technologies is often mentioned as a key determinant of the market success possibility with regard to the SME sector. They just allow for more effective management of the enterprise, as well as being a kind of catalyst for innovation [Deep et al. 2004, Wong 2005, Wong and Aspinwall 2005, Terziovski 2010, Ząbkowski and Jałowiecki 2011]. Modern IT technologies and systems also play an important role in modern logistic systems to enable effective implementation and control of logistic processes, as well as the implementation of logistics services. Logistics implemented within the digital information systems or aided by them is defined as e-logistics nowadays. The obvious fact is so closely linking modern IT technologies and systems with modern logistics systems [Gnasekaran and Ngai 2003, Beheshti et al. 2007, Talbot et al. 2007].

A characteristic feature of the food industry in Poland is a large number of suppliers of agricultural raw materials and consumers of food products. The average food producing company acquires agricultural products from five categories of suppliers. Definitely the highest is the average number of farmers as suppliers (216.1), and in the dairy (492.2) and meat industries (371.4), it is still significantly higher. Subsequently, these are agricultural enterprises (14.0), purchasing companies (8.4), producer groups (6.9) and processing plants (6.0). Only in the case of fat-oil and other food products industries, you can talk about balance in terms of the average number of suppliers of agricultural raw materials belonging to each listed category [Jałowiecki and Jałowiecka, 2013].

A similar situation takes place with regard to recipients of food products of which there are 6 major categories. In this regard retail stores have a definite advantage (136.9). Subsequently, these are the warehouses (38.8), processing plants (27.3), institutional recipients (12.1), hotels and restaurants (10.3) and trading networks (6.1). The average number of retail stores as consumers of food products are significantly higher in the dairy (until 1809.1) and beverages industries (213.0).

It should also be noted that both the average number of farmers as suppliers of agricultural raw materials, as well as retail stores as consumers of food products is steadily increasing with the increase of the number of employees in the enterprise [Jałowiecki and Jałowiecka 2013]. Such large variations in both customers and market partners of Polish enterprises of food processing is one of the most important causative factors of enterprises functioning within the large and complex cooperative and logistics chains. In addition, most industries of agri-food sector, agri-food processing produces food for which a particularly important quality parameters of both agricultural raw materials, finished products as a freshness, and consequently delivery. In addition, most industries sectors of agri-food processing branch produces food for which a particularly important there are quality parameters of both agricultural raw materials, final food products as a freshness, and consequently delivery time. All this makes proper effectiveness of logistics chains is one of the most important factors in ensuring the competitiveness of enterprises on the market, although the same solutions in the field of IT technology and logistics are essentially the same as in other industrial sectors [Mangina and Vlachos 2005, Clements et al. 2008, Wicki and Jałowiecki 2010].

Known and frequently signaled in the literature phenomenon is called "Productivity paradox". In short, it consists in the fact that expenditure on informatization does not translate directly into financial results of companies. The paradox of productivity has been identified and formulated in the late 80s of the twentieth century by the famous American economists Robert Sollow and Norbert Strassman [Solow 1987, Strassmann 2010]. Increase in expenditure on information systems is usually caused by the implementation of more and more advanced, more complex and have greater possibilities of IT.

In the research conducted so far, in the food production sector enterprises, no correlation between the level of advancement of used IT technologies and financial situation was found, regardless of company size [Jałowiecki and Gostkowki 2013]. Still unpublished results of further studies indicate that such relationships exist in Poland only in the meat and dairy industries. In other sectors of agri-food processing branch, such relationships were not found. Since modern IT technologies are so strongly associated with modern logistics, theoretically, a higher level of their advancement should affect the growth effectiveness of logistics systems and, consequently, decrease the cost of logistics operations. That left investigate whether indeed there are dependencies between the level of advancement of used IT technologies and better market position the company in terms of logistics well as the level of logistics costs.

MATERIAL AND METHODS

n the research the results of a survey of 511 companies of the Polish sector of food processing performed in 2010-2011 were used. Surveyed companies were divided by the number of employees in accordance with the classification of GUS into 4 categories: micro (up to 9 employees), small enterprises (10-49 employees), medium (50-249 employees) and large (250 or more employees). Because of the small number of subjects, it was not isolated as a separate category of very large enterprises (1,000 or more employees). The studies were taken into account only 6 sectors: meat, fruit and vegetables, dairy, cereal and starchy, bread and other food products, which responded to the survey more than 20 companies.

The stage of advancement of used IT solutions was evaluated employing coefficient used in previous studies [6]. It takes into account factors such as: having the separated information system (yes, no) , computer aided each of the five areas of logistics (transportation, inventory, packaging and reverse logistics, warehouse management, order management and demand forecasting), transfer of information way in circulation within enterprises as well as between the company and the contractors and market partners (no specific, orally, on paper, by phone, fax , e-mail or Internet messengers, via the software), the class of used information system (no system, financial-accounting (FA), electronic data interchange (EDI), materiel resources planning (MRP), enterprise resources planning (ERP), business intelligence (BI) system) and the method The statistical significance of the correlation coefficients determined using the test was examined in accordance with the formula (2).of preparing forecasts of demand for manufactured products (no forecasts , production on the basis of received raw materials, production based on orders received, based on historical data from the company, based on market forecasts, based on data from the company and market). Coefficient could have values ranging from 0 to 6. In order to ensure the comparability with the rest of examined variables, its value categorized into 5 categories by assigning a value from 1 to 5, which marked level: very low, low, average, high, very high.

Their knowledge in the field of logistics evaluated on the basis of the respondents declaration also was categorized into 5 categories from 1 to 5: woefully inadequate; usually insufficient; as often enough as insufficient; usually sufficient and more than sufficient knowledge. On the basis of the declaration of the respondents also an assessment of the market position of the company against the industry in terms of each of the five areas of logistics activities (transportation, inventory, packaging and reverse logistics, warehouse management, order management and demand forecasting) were based. It was categorized into five categories: one of the worst in the against the sector, slightly weaker than the sector average, average, slightly better than the average in the sector, one of the better against the sector. Moreover, the total value of the coefficient of the market position of the company in terms of logistics was designated, and also categorized into 5 categories: very low, low, average, high, very high. The costs of logistics enterprises categorized into 5 values (less than 1%, from 1% to 4%, from 5% to 9%, from 10 to 14% and 15% or more of the total cost of the enterprise).

To assess the relationship between examined variables (advancement of used IT technologies, the assessment of the market position of the company in terms of individual areas of logistics activities, market position in terms of logistics and of logistics costs ratios) was used Spearman's rank correlation coefficient due to the categorization of all the studied variables. The statistical significance of determined correlation coefficients was examined with the test based on t-Student distribution. Correlations for all companies together, in 4 groups of employment size and for the six selected sectors separately were examined.

RESULTS

Among all surveyed companies the level of advancement of used IT solutions was at the level $\bar{x}$ = 2.51, which is in the middle between low and average. Its dispersion was ρ = 0.99, which accounted for 39.6% arithmetic average. This level definitely increased with the increase in the size category of employment of $\bar{x}$ = 2.07 for micro to $\bar{x}$ = 3.83 for large enterprises (see Table 1). Among the sectors by far the highest level of applied IT solutions was found among dairy enterprises $\bar{x}$ = 3.33, while by far the lowest among the baking $\bar{x}$ = 2.24 (see Table 2).

The average level of costs associated with logistics for all companies was as $\bar{x}$ = 2.92, which is on the border of categories from 1% to 4% and from 5% to 9% of the total costs of the company. Their dispersion was s = 1.10, which accounted for 37.6% of the arithmetic average. The average level of logistics costs incurred increased with increasing size class of enterprise from $\bar{x}$ = 2.57 in micro-enterprises to $\bar{x}$ = 3.07 in large enterprises, in which the dispersion was by far the lowest (see Table 1).

Table 1. Average levels of used IT solutions advancement, logistics solutions knowledge, market positions in term of five logistic activity areas, and in term of all logistic activities, share of logistic costs in total costs of company coefficients and its diversification in companies of Polish agri-food production sector according to employment size ($\bar{x}$ – average, s – standard deviation, V_x – diversification coefficient)

Employment size	Micro	Small	Middle	Large	All
Used IT solutions advancement level	$\bar{x}$ = 2.07 s = 0.71 V_x = 0.34	$\bar{x}$ = 2.30 s = 0.84 V_x = 0.37	$\bar{x}$ = 3.10 s = 1.04 V_x = 0.34	$\bar{x}$ = 3.83 s = 0.92 V_x = 0.24	$\bar{x}$ = 2.51 s = 0.99 V_x = 0.39
Knowledge of logistics solutions	$\bar{x}$ = 3.62 s = 1.16 V_x = 0.32	$\bar{x}$ = 3.46 s = 1.15 V_x = 0.33	$\bar{x}$ = 3.52 s = 1.03 V_x = 0.29	$\bar{x}$ = 3.55 s = 0.83 V_x = 0.23	$\bar{x}$ = 3.50 s = 1.11 V_x = 0.32
Market position in term of inventory control	$\bar{x}$ = 2.97 s = 1.15 V_x = 0.39	$\bar{x}$ = 3.40 s = 1.17 V_x = 0.34	$\bar{x}$ = 3.54 s = 1.13 V_x = 0.32	$\bar{x}$ = 3.69 s = 0.99 V_x = 0.27	$\bar{x}$ = 3.38 s = 1.18 V_x = 0.35
Market position in term of storage management	$\bar{x}$ = 2.93 s = 1.14 V_x = 0.39	$\bar{x}$ = 3.35 s = 1.19 V_x = 0.36	$\bar{x}$ = 3.51 s = 1.11 V_x = 0.32	$\bar{x}$ = 3.83 s = 0.99 V_x = 0.26	$\bar{x}$ = 3.35 s = 1.19 V_x = 0.36

Employment size	Micro	Small	Middle	Large	All
Market position in term of packaging management	$\bar{x}$ = 2.97 s = 1.17 V_x = 0.39	$\bar{x}$ = 3.09 s = 1.21 V_x = 0.39	$\bar{x}$ = 3.31 s = 1.14 V_x = 0.34	$\bar{x}$ = 3.45 s = 1.12 V_x = 0.32	$\bar{x}$ = 3.13 s = 1.20 V_x = 0.38
Market position in term of transport management	$\bar{x}$ = 3.15 s = 1.22 V_x = 0.39	$\bar{x}$ = 3.40 s = 1.23 V_x = 0.36	$\bar{x}$ = 3.70 s = 1.14 V_x = 0.31	$\bar{x}$ = 3.90 s = 0.99 V_x = 0.25	$\bar{x}$ = 3.45 s = 1.23 V_x = 0.36
Market position in term of information management	$\bar{x}$ = 2.87 s = 1.15 V_x = 0.40	$\bar{x}$ = 3.10 s = 1.18 V_x = 0.38	$\bar{x}$ = 3.28 s = 1.18 V_x = 0.36	$\bar{x}$ = 3.52 s = 1.04 V_x = 0.30	$\bar{x}$ = 3.13 s = 1.18 V_x = 0.38
Market position in term of all logistic activities	$\bar{x}$ = 3.16 s = 1.16 Vx = 0.37	$\bar{x}$ = 3.50 s = 1.16 V_x = 0.33	$\bar{x}$ = 3.74 s = 1.04 V_x = 0.28	$\bar{x}$ = 4.00 s = 1.02 V_x = 0.26	$\bar{x}$ = 3.53 s = 1.15 V_x = 0.33
Share of logistic costs in total costs of company	$\bar{x}$ = 2.57 s = 0.98 V_x = 0.38	$\bar{x}$ = 2.95 s = 1.12 V_x = 0.38	$\bar{x}$ = 2.99 s = 1.08 V_x = 0.36	$\bar{x}$ = 3.07 s = 0.88 V_x = 0.29	$\bar{x}$ = 2.92 s = 1.10 V_x = 0.38

Source: own preparation

In terms of logistics costs, studied sectors can be divided into 2 groups. In the first, including companies producing other food products ($\bar{x}$ = 3.18), dairy ($\bar{x}$ = 3.17) and fruit and vegetable processing ($\bar{x}$ = 3.09) the average level of logistics costs was much higher than in the second, covering the remaining branches (see Table 2). Assessing the relationship between the level of advancement of used IT solutions, and the level of costs connected with the logistics of all companies, there is a weak, but statistically significant correlation $r_S = 0.20$, according to which a higher level of IT solutions meant a higher level of logistics costs. Among the groups of companies similar relationship was found only in small enterprises $r_S = 0.22$, in the meat industry $r_S = 0.27$, bakery $r_S = 0.21$ and the strongest in the industry of other food products $r_S = 0.36$. Among the companies belonging to other groups of the number of employees and in other industries, there was no statistically significant relationships between the level of advancement of information technology used and the level of logistics costs. It is worth also be noted that although no statistical significant but negative correlations meaning depending on logistics costs decline with increasing severity of used IT technologies and systems found in large enterprises ($r_S = -0.14$), fruit and vegetable processing ($r_S = -0.15$) and milk ($r_S = -0.10$) (see Tables 3 and 4). The results of research conducted by the author indicate that they are a group of companies characterized by far the greatest complexity of the logistics structure [Jałowiecki et al. 2014]. The results obtained with regard to the level of logistics costs also confirmed the results of previous studies on the relationship between the level of used IT technology advancement, and the financial situation of companies in the branch of food production [Jałowiecki and Gostkowski 2013]. According to them only in small enterprises and in the meat and dairy industries statistically significant dependencies according to which a higher level of advancement of used IT technologies and systems translated into better financial results of companies. For all surveyed enterprises whereas there was no such statistically significant correlation.

Table 2. Average levels of logistics solutions knowledge, market positions in term of five logistic activity areas and its diversification in companies of Polish agri-food production sector according to sector of functioning ($\bar{x}$ – average, s – standard deviation, V_x – diversification coefficient)

Sector	Meat	Fruits and Vegetables	Milk	Cereal and Starch	Bakery	Other Grocery
Used IT solutions adv. level	$\bar{x}$ = 2.59 s = 1.01 V_x = 0.39	$\bar{x}$ = 2.97 s = 1.03 V_x = 0.35	$\bar{x}$ = 3.33 s = 0.91 V_x = 0.27	$\bar{x}$ = 2.47 s = 1.16 V_x = 0.47	$\bar{x}$ = 2.24 s = 0.82 V_x = 0.37	$\bar{x}$ = 2.76 s = 0.99 V_x = 0.36
Knowledge of logistics solutions	$\bar{x}$ = 3.40 s = 1.12 V_x = 0.33	$\bar{x}$ = 3.70 s = 0.74 V_x = 0.20	$\bar{x}$ = 3.79 s = 0.48 V_x = 0.13	$\bar{x}$ = 3.66 s = 1.06 V_x = 0.29	$\bar{x}$ = 3.45 s = 1.23 V_x = 0.36	$\bar{x}$ = 3.62 s = 0.86 V_x = 0.24
Market position in term of …						
inventory control	$\bar{x}$ = 3.36 s = 1.07 V_x = 0.32	$\bar{x}$ = 3.39 s = 1.14 V_x = 0.34	$\bar{x}$ = 3.75 s = 0.84 V_x = 0.22	$\bar{x}$ = 3.08 s = 1.21 V_x = 0.39	$\bar{x}$ = 3.40 s = 1.25 V_x = 0.37	$\bar{x}$ = 3.53 s = 0.91 V_x = 0.26
storage management	$\bar{x}$ = 3.31 s = 1.08 V_x = 0.33	$\bar{x}$ = 3.52 s = 1.10 V_x = 0.31	$\bar{x}$ = 3.50 s = 0.92 V_x = 0.26	$\bar{x}$ = 3.26 s = 1.30 V_x = 0.40	$\bar{x}$ = 3.31 s = 1.24 V_x = 0.37	$\bar{x}$ = 3.56 s = 0.94 V_x = 0.26
packaging management	$\bar{x}$ = 3.03 s = 1.12 V_x = 0.37	$\bar{x}$ = 3.18 s = 1.13 V_x = 0.36	$\bar{x}$ = 3.54 s = 0.83 V_x = 0.23	$\bar{x}$ = 3.26 s = 1.22 V_x = 0.37	$\bar{x}$ = 3.08 s = 1.25 V_x = 0.41	$\bar{x}$ = 3.51 s = 0.87 V_x = 0.25
transport management	$\bar{x}$ = 3.37 s = 1.11 V_x = 0.33	$\bar{x}$ = 3.45 s = 1.15 V_x = 0.33	$\bar{x}$ = 4.08 s = 0.83 V_x = 0.20	$\bar{x}$ = 3.50 s = 1.24 V_x = 0.35	$\bar{x}$ = 3.39 s = 1.29 V_x = 0.38	$\bar{x}$ = 3.62 s = 0.97 V_x = 0.27
information management	$\bar{x}$ = 3.03 s = 1.08 V_x = 0.25	$\bar{x}$ = 3.03 s = 1.08 V_x = 0.25	$\bar{x}$ = 3.46 s = 0.62 V_x = 0.24	$\bar{x}$ = 3.24 s = 0.83 V_x = 0.31	$\bar{x}$ = 3.12 s = 0.87 V_x = 0.33	$\bar{x}$ = 3.24 s = 0.72 V_x = 0.29
all logistic activities	$\bar{x}$ = 3.45 s = 1.10 V_x = 0.32	$\bar{x}$ = 3.64 s = 1.09 V_x = 0.30	$\bar{x}$ = 4.04 s = 0.81 V_x = 0.20	$\bar{x}$ = 3.53 s = 1.20 V_x = 0.34	$\bar{x}$ = 3.47 s = 1.19 V_x = 0.34	$\bar{x}$ = 3.76 s = 0.95 V_x = 0.25
Logistic costs share	$\bar{x}$ = 2.78 s = 1.09 V_x = 0.39	$\bar{x}$ = 3.09 s = 0.84 V_x = 0.35	$\bar{x}$ = 3.17 s = 0.82 V_x = 0.27	$\bar{x}$ = 2.87 s = 1.08 V_x = 0.47	$\bar{x}$ = 2.88 s = 1.12 V_x = 0.35	$\bar{x}$ = 3.18 s = 1.00 V_x = 0.36

Source: own preparation

In terms of market position in the 5 areas of logistics activity, as well as in terms of logistics in general, in all the surveyed companies stated regularity according to which the higher average employment group meant better market position. For example, the average summary assessment of logistics increased from $\bar{x}$ = 3.16, that is the average market position for micro to $\bar{x}$ = 4.00, which is slightly better market position than the average in the large companies. Top rated average market position of all surveyed enterprises was in the management of transport ($\bar{x}$ = 3.45), whereas the lowest were average market positions in terms of packaging and reverse logistics ($\bar{x}$ = 3.13) and information management ($\bar{x}$ = 3.13) (see Table 1). Taking into account sectors, definitely the highest average assessment of its market position in terms of logistics was found in dairy enterprises ($\bar{x}$ = 4.04), whereas the lowest in the meat ($\bar{x}$ = 3.45) and bakery industries ($\bar{x}$ = 3.47) (see Table 2).

Table 3. Spearman correlation between the level of used IT solutions advancement, logistics solutions knowledge, market positions in term of five logistic activity areas, and in term of all logistic activities, share of logistic costs in total costs of company coefficients depending on the employment size (r_S – correlation coefficient, t – empirical value of test statistic, tα – critical value of test statistic, α – significance level)

Employment size α = 0.05	Micro $t_\alpha = \pm 2.00$	Small $t_\alpha = \pm 1.97$	Middle $t_\alpha = \pm 1.99$	Large $t_\alpha = \pm 2.05$	All $t_\alpha = \pm 1.96$
Knowledge of logistics solutions	$r_S = 0.27$ $t = 2.12$ $p = 0.04$	$r_S = 0.34$ $t = 6.44$ $p < 0.01$	$r_S = 0.46$ $t = 5.11$ $p < 0.01$	$r_S = 0.64$ $t = 4.34$ $p < 0.01$	$r_S = 0.32$ $t = 7.00$ $p < 0.01$
Market position in term of …					
inventory control	$r_S = 0.16$ $t = 1.22$ $p = 0.23$	$r_S = 0.31$ $t = 5.79$ $p < 0.01$	$r_S = 0.46$ $t = 5.16$ $p < 0.01$	$r_S = 0.61$ $t = 3.96$ $p < 0.01$	$r_S = 0.35$ $t = 8.40$ $p < 0.01$
storage management	$r_S = 0.38$ $t = 3.11$ $p < 0.01$	$r_S = 0.40$ $t = 7.65$ $p < 0.01$	$r_S = 0.47$ $t = 5.27$ $p < 0.01$	$r_S = 0.55$ $t = 3.45$ $p < 0.01$	$r_S = 0.43$ $t = 10.72$ $p < 0.01$
packaging management	$r_S = 0.21$ $t = 1.63$ $p = 0.11$	$r_S = 0.32$ $t = 6.07$ $p < 0.01$	$r_S = 0.35$ $t = 3.70$ $p < 0.01$	$r_S = 0.54$ $t = 3.35$ $p < 0.01$	$r_S = 0.34$ $t = 8.19$ $p < 0.01$
transport management	$r_S = 0.27$ $t = 2.16$ $p = 0.04$	$r_S = 0.39$ $t = 7.44$ $p < 0.01$	$r_S = 0.45$ $t = 4.97$ $p < 0.01$	$r_S = 0.55$ $t = 3.41$ $p < 0.01$	$r_S = 0.41$ $t = 10.20$ $p < 0.01$
information management	$r_S = 0.22$ $t = 1.74$ $p = 0.09$	$r_S = 0.39$ $t = 7.56$ $p < 0.01$	$r_S = 0.57$ $t = 6.91$ $p < 0.01$	$r_S = 0.50$ $t = 2.98$ $p = 0.01$	$r_S = 0.42$ $t = 10.37$ $p < 0.01$
logistic activities	$r_S = 0.26$ $t = 2.03$ $p = 0.05$	$r_S = 0.37$ $t = 7.05$ $p < 0.01$	$r_S = 0.43$ $t = 4.53$ $p < 0.01$	$r_S = 0.57$ $t = 3.56$ $p < 0.01$	$r_S = 0.39$ $t = 9.48$ $p < 0.01$
Logistic costs share	$r_S = 0.11$ $t = 0.84$ $p = 0.41$	$r_S = 0.22$ $t = 3.90$ $p < 0.01$	$r_S = 0.15$ $t = 1.45$ $p = 0.15$	$r_S = -0.14$ $t = -0.70$ $p = 0.49$	$r_S = 0.20$ $t = 4.36$ $p < 0.01$

Source: own preparation

Assessing the relationship between the level of advancement of used IT solutions and market position of enterprises in terms of five areas of logistics activity, as well as in terms of logistics in general, it was found that the strength of this association increased with a higher and higher groups of the size of employment the enterprise. For example, the average aggregate assessment of market power in terms of logistics of $r_S = 0.26$ for micro to $r_S = 0.57$ for large enterprises. (see Table 3). Significantly stronger dependence was found in the areas of storage management ($r_S = 0.43$), transportation management ($r_S = 0.42$) and information management ($r_S = 0.41$) than in the areas of inventory management ($r_S = 0.35$) and the management of packaging and reverse logistics ($r_S = 0.34$). Taking into account individual, investigated sectors, conclusion is that there were significant differences between them in terms of the impact of used IT technologies and systems advancement level on the market position of enterprises in terms of five areas of logistics separately, as well as in terms of logistics in general. However, definitely strongest was the influence on fruit and vegetable, and dairy industries, whereas deciding the weakest in the cereal and starchy (see Table 4).

Table 4. Spearman correlation between the level of used IT solutions advancement, logistics solutions knowledge, market positions in term of five logistic activity areas, and in term of all logistic activities, share of logistic costs in total costs of company coefficients depending on the employment size (r_S – correlation coefficient, t – empirical value of test statistic, t_α – critical value of test statistic, α – significance level)

Sector α = 0.05	Meat tα = ±1.98	Fruits and Vegetables tα = ±2.04	Milk tα = ±2.07	Cereal and Starch tα = ±2.03	Bakery tα = ±1.97	Other Grocery tα = ±2.02
Knowledge of logistics solutions	$r_S = 0.42$ $t = 4.96$ $p < 0.01$	$r_S = 0.14$ $t = 0.78$ $p = 0.22$	$r_S = 0.39$ $t = 1.99$ $p = 0.06$	$r_S = 0.31$ $t = 1.99$ $p = 0.05$	$r_S = 0.32$ $t = 4.87$ $p < 0.01$	$r_S = 0.27$ $t = 1.84$ $p = 0.07$
Market position in term of …						
inventory control	$r_S = 0.27$ $t = 3.05$ $p = 0.12$	$r_S = 0.66$ $t = 4.92$ $p < 0.01$	$r_S = 0.50$ $t = 2.68$ $p = 0.01$	$r_S = 0.12$ $t = 0.73$ $p = 0.47$	$r_S = 0.35$ $t = 5.45$ $p < 0.01$	$r_S = 0.41$ $t = 2.99$ $p < 0.01$
storage management	$r_S = 0.37$ $t = 4.30$ $p < 0.01$	$r_S = 0.74$ $t = 6.14$ $p < 0.01$	$r_S = 0.41$ $t = 2.10$ $p = 0.05$	$r_S = 0.28$ $t = 1.72$ $p = 0.09$	$r_S = 0.43$ $t = 6.86$ $p < 0.01$	$r_S = 0.47$ $t = 3.47$ $p < 0.01$
packaging management	$r_S = 0.38$ $t = 4.38$ $p < 0.01$	$r_S = 0.36$ $t = 2.15$ $p = 0.04$	$r_S = 0.44$ $t = 2.32$ $p = 0.03$	$r_S = 0.23$ $t = 1.44$ $p = 0.16$	$r_S = 0.33$ $t = 5.10$ $p < 0.01$	$r_S = 0.40$ $t = 2.88$ $p = 0.01$
transport management	$r_S = 0.37$ $t = 4.19$ $p < 0.01$	$r_S = 0.44$ $t = 2.72$ $p = 0.01$	$r_S = 0.59$ $t = 3.46$ $p < 0.01$	$r_S = 0.36$ $t = 2.33$ $p = 0.03$	$r_S = 0.38$ $t = 5.97$ $p < 0.01$	r**S** = 0.37 $t = 2.58$ $p = 0.01$

Sector α = 0.05	Meat tα = ±1.98	Fruits and Vegetables tα = ±2.04	Milk tα = ±2.07	Cereal and Starch tα = ±2.03	Bakery tα = ±1.97	Other Grocery tα = ±2.02
information management	$r_S = 0.43$ $t = 5.03$ $p < 0.01$	$r_S = 0.61$ $t = 4.31$ $p < 0.01$	$r_S = 0.34$ $t = 1.70$ $p = 0.10$	$r_S = 0.38$ $t = 2.46$ $p = 0.02$	$r_S = 0.40$ $t = 6.42$ $p < 0.01$	$r_S = 0.40$ $t = 2.83$ $p = 0.01$
all logistic activities	$r_S = 0.36$ $t = 4.10$ $p < 0.01$	$r_S = 0.64$ $t = 4.59$ $p < 0.01$	$r_S = 0.55$ $t = 2.96$ $p = 0.01$	$r_S = 0.10$ $t = 0.62$ $p = 0.54$	$r_S = 0.36$ $t = 5.56$ $p < 0.01$	$r_S = 0.45$ $t = 3.26$ $p < 0.01$
Logistic costs share	$r_S = 0.27$ $t = 2.92$ $p < 0.01$	$r_S = -0.15$ $t = -0.77$ $p = 0.45$	$r_S = -0.10$ $t = -0.44$ $p = 0.67$	$r_S = 0.01$ $t = 0.07$ $p = 0.94$	$r_S = 0.21$ $t = 2.98$ $p < 0.01$	$r_S = 0.36$ $t = 2.36$ $p = 0.02$

Source: own preparation

Among all the surveyed companies declared level of knowledge in the field of logistics solutions reached the level of $\bar{x} = 3.50$, or between equally often enough that insufficient knowledge and knowledge usually sufficient. Dispersion was $\rho = 1.11$, which accounted for 31.9% of the arithmetic average. With the exception of micro-enterprises, which are found the highest level of knowledge logistics declared $\bar{x} = 3.62$, it increased slightly with the increase of the number of employees in the company since $\bar{x} = 3.43$ in enterprises small to $\bar{x} = 3.55$ in large enterprises. With increasing size of the company definitely decreased whereas dispersion of its values from $s = 1.16$ in the micro to $s = 0.83$ in large enterprises (see Table 1). The author believes declared the highest level of knowledge in the field of logistics, detected among micro-enterprises testifies rather to a lack of knowledge on modern logistics solutions than the real extensive knowledge in this field. An indirect confirmation of this interpretation are research results, according to which of micro-enterprises is by far the lowest level of complexity of the logistics and by far the lowest level of advancement of used logistic solutions [Jałowiecki et al. 2014]. Among the surveyed industries, by far the highest level of declared logistics expertise found among dairy enterprises $\bar{x} = 3.79$, by far the lowest among enterprises meat $\bar{x} = 3.40$ and bakeries $\bar{x} = 3.45$ (see Table 2).

As in the case other variables studied, an increase in strength of the relationship between the level of advancement used IT solutions, and the claimed level of knowledge in the field of logistics with the increase of the number of employees in the company showed since $r_S = 0.27$ for micro to $r_S = 0.64$ for large enterprises (see Table 3). Among the sectors such dependence was statistically significant only among enterprises of meat ($r_S = 0.42$) and baking enterprises ($r_S = 0.32$). On the border of statistical significance, it was also found such relationships in the dairy industry ($r_S = 0.39$) and cereal and starchy ($r_S = 0.31$). The average strength of this relationship for all the companies was not too strong and was $r_S = 0.32$.

CONCLUSIONS

The results presented in the paper confirm the existence of the productivity paradox also in relation to the cost of logistics enterprises in the sector of food processing. A higher level of advancement of used IT solutions and, consequently, greater investment in implementation of modern technologies and IT systems do not translate into lower costs for the logistics. In the case of small enterprises on employment from 10 to 49 employees and in the industries of meat, bakery and other food products branches, it was quite a positive correlation between higher levels of advancement of used IT solutions, and the increase in logistics costs. The only exceptions, but statistically insignificant, was found in large enterprises employing 250 or more employees, and fruit and vegetable and dairy branches. This does not change the fact that a higher level of advancement used IT solutions results in better market position of companies in terms of logistics. The results confirm the results of previous research on the relationship between the advancement of IT technologies and systems, and the financial situation of enterprises [Jałowiecki and Gostkowski 2013].

It seems, therefore, that expenditures on modernization of existing and implementation of new IT solutions in the Polish sector of food production primarily brings about an immeasurable benefits, such as just a better position in the market. In addition, it is very likely that the use of modern IT technologies and systems is primarily the need for a more complex structure of logistics and can also affect the severity of applied logistics solutions. That does not mean that it must directly translate into better financial results or lower costs. These benefits are more long-term and their direct impact on the financial results can take place in a much longer time horizon. Of course, those theses need to be confirmed by empirical studies, however partial, unpublished results of previous studies conducted by the author, seem to indicate the validity of just such conclusions.

Given the close relationship between modern IT technologies and logistics systems, significant implementation costs of both modern IT systems, as well as e-logistics, results obtained in terms of strength of the relationship between the level of advancement of used IT solutions and the level of knowledge of logistics prove two things. Firstly, in smaller enterprises, much lower level of logistic knowledge seems to be the result of just the lack or limitations on the possibility of access to modern IT solutions. Second, in smaller enterprises rather traditional, "non-IT" approach to logistics activities is dominating. On the one hand it may be very large due to the financial capabilities of the other, in turn, no need for the use of modern, expensive and complex IT systems and e-logistics. Especially this last point should be clarified in relation to the sector of food processing in the near future.

REFERENCES

Beheshti H. M., Hultman M., Jung M.L., Opoku R. A., Salehi-Sangari E. (2007) Electronic Supply Chain Management Applications by Swedish SMEs. Enterprise Information Systems, 1(2), 255-268.

Clements M., Lazo R., Martin S. (2008) Relationship Connectors in NZ Fresh Produce Supply Chains. British Food Journal, 110(4-5), 346-360.

Deep A., Guttridge P., Dani S., Burns R. (2004) Investigating factors affecting ERP selection in made-to-order SME sector. Journal of Manufacturing Technology Management, 19(4), 430-446.

Gunasekaran A., Ngai E. W .T. (2003) The successful management of a small logistics company. International Journal of Physical Distribution & Logistics Management, 33(9), 825-842.

Jałowiecki P., Gostkowski M. (2013) Productivity Paradox in Polish Food Processing Sector. Information Systems in Management, 4, 263-273.

Jałowiecki P., Jałowiecka E. (2013) Struktura i koszty logistyki w wybranych branżach sektora rolno-spożywczego. [w:] Lichota A., Majewska K. (red.) Wybrane zagadnienia logistyki stosowanej. Wydawnictwo AGH, Kraków, 155-166 (in Polish).

Jałowiecki P., Jałowiecka E., Olejniczak M. (2014) Ocena złożoności struktury logistyki w polskich przedsiębiorstwach przetwórstwa rolno-spożywczego. Zeszyty Naukowe Stowarzyszenia Ekonomistów Rolnictwa i Agrobiznesu, 16, 5, 67-72 (in Polish).

Mangina E., Vlachos I. (2005) The Changing Role of Information Technology in Food and Beverage Logistics Management: Beverage Network Optimization using Intelligent Agent Technology. Journal of Food Engineering, 70, 403–420.

Solow R. (1987) We’d Better Watch Out, review of S.S. Cohen and J. Zysman, Manufacturing Matters: The Myth of the Post-Industrial Economy. New York Times Book Review, 36.

Strassmann P.A. (1990) The Business Value of Computers. Information Economics Press, New Canaan, CT, USA.

Talbot S., Lefebvre E., Lefebvre L. A. (2007) Closed-loop Supply Chain Activities and Derived Benefits in Manufacturing SMEs. Journal of Manufacturing Technology Management, 18(6), 627-658.

Terziovski M. (2010) Innovation practice and its performance implications in small and medium enterprises (SMEs) in the manufacturing sector: a resource-based view. Strategic Management Journal, 31 (8), 892-902.

Wicki L., Jałowiecki P. (2010) Zróżnicowanie poziomu organizacji logistyki w wybranych branżach agrobiznesu. Logistyka, 3, 1-21 (in Polish).

Wong K.Y. (2005) Critical Success Factors for Implementing Knowledge Management in Small and Medium Enterprises. Industrial Management & Data Systems, 105 (3), 261-279.

Wong K. Y., Aspinwall E. (2005) An Empirical Study of the Important Factors for Knowledge-management Adoption in the SME Sector. Journal of Knowledge Management, 9(3), 64-82.

Ząbkowski T., Jałowiecki P. (2011) Rozwiązania informatyczne w logistyce małych i średnich przedsiębiorstw sektora rolno-spożywczego. Logistyka, 3, 62-65 (in Polish).

QUANTITATIVE METHODS IN ECONOMICS
Volume XIX, No. 1, 2018, pp. 60 – 70
Received: 14.02.2018
Accepted: 05.04.2018

EFFICIENCY OF HEALTHCARE SYSTEMS IN EUROPEAN COUNTRIES - THE DEA NETWORK APPROACH

Justyna Kujawska (ORCID: 0000-0002-0416-3048)
Faculty of Management and Economics
Gdansk University of Technology, Poland
e-mail: Justyna.Kujawska@zie.pg.gda.pl

Abstract: Healthcare systems in Europe are constantly undergoing reforms which adapt them to social, economic and political requirements. The aim of this article is to examine the efficiency of healthcare systems in 30 European countries in 2014. The Network Data Envelopment Analysis (NDEA) model was used. The efficiency of the countries' overall health systems and their two main components were examined: the public health system and the medical care system. The models include variables that are out of control of policy makers and the ones that can be controlled by them. The research results show that countries which reformed their healthcare systems achieved higher efficiency more often.

Keywords: healthcare system, public health, medical care, network Data Envelopment Analysis

JEL classification: C61, H51

INTRODUCTION

Ensuring healthcare to citizens is the goal of every government and the efficiency of the healthcare system is a recurrent and important topic of discussion as regards health policy. According to the Global Health Expenditure of the World Health Organization (WHO), health expenditure calculated as a percentage of GDP in recent years has increased significantly. Due to demographic changes, it can be assumed that this trend will continue in the coming decades.
Therefore, it is important to assess the efficiency of healthcare systems in different countries. The efficiency analysis may indicate a possible reduction of resources or an increase in health outcomes at a given level of expenses. For this reason, the

DOI: 10.22630/MIBE.2018.19.1.6

comparison of healthcare systems is important for identifying best practices and the most effective healthcare systems.

Global health challenges related to the ageing of the population, the imbalance between cost containment while maintaining access to and quality of healthcare, shifting from treatment of acute cases to management of chronic diseases that burden the resources of medical care more heavily, as well as fragmentation of treatment, prompted many European countries to introduce significant changes into their healthcare systems in the last decade [Yaya & Danhoundo 2015]. Some countries have completed reforms, while others are still implementing them, and yet others are on the planning stage.

The number of healthcare systems in Europe subjected to the long process of structural and organizational reforms is growing. Both tax-financed and contribution-based systems often change their organizational and institutional structures in order to ensure individual and populational public health. They also conduct initiatives aimed at encouraging entities operating in other sectors to implement health-oriented habits. A significant part of the political objectives implemented in the countries surveyed remains unchanged, while the strategies and mechanisms by which decision-makers want to achieve these goals undergo significant changes.

The wide-ranging debate on the reform of health systems has been ongoing since the end of the 1980s in Western Europe and the early 1990s in Central and Eastern Europe [Saltman & Figueras 1998].

There is no universal model according to which healthcare system should be reformed. Healthcare in each of the European countries is organized differently but all of them face similar problems, related to excessive demand for high-quality health services and insufficient resources of public health and medical care. Despite the diversity of healthcare systems, reforms in most countries have many common features. These include: expanding and strengthening primary healthcare, expanding environmental care, improving the availability of healthcare, changing the models of payment and reimbursement for health services, improving the quality of services or increasing the use of information technology in health [Yaya & Danhoundo 2015].

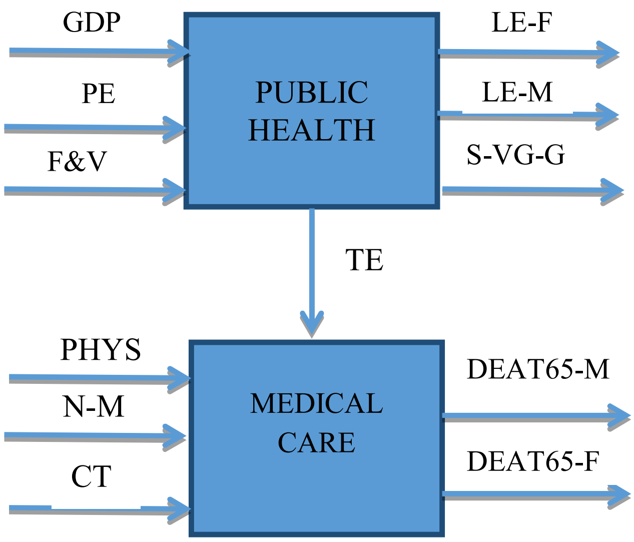
METHODOLOGY AND VARIABLES

The authors of many studies have attempted to investigate the efficiency of healthcare systems. Most of these studies are based on the traditional DEA (Data Envelopment Analysis) method, used to obtain results of the efficiency of healthcare systems in the compared countries [Afonso & St Aubyn 2006, Joumard et. al. 2010, Mirmirani et al. 2008, Retzlaff-Roberts et al. 2004]. In the traditional DEA model, the healthcare system is treated as one division in the calculation of efficiency based on a set of inputs and outputs. One of the drawbacks of these models is the neglect of intermediate products or linking activities. This makes it difficult to distinguish the efficiency of various elements of the healthcare system. According to a panel of experts from high-income countries, healthcare systems are based on the interaction of public health and medical care. It is recommended that both of these elements — public health and medical care — are examined when making comparisons between countries [Woolf & Aron 2013, Ozcan & Khushalani 2017].

The network-DEA approach makes it possible to describe the structure of processes in the healthcare systems in an unambiguous way while maintaining the advantages of the DEA method. The network-DEA model goes beyond the traditional DEA model, enabling the calculation of the efficiency of separate sub-processes, in addition to the efficiency of the entire healthcare system [Tone & Tsutsui 2009]. The DEA method is chosen mainly due to the difficulty of defining the production function, which would combine the inputs and outputs of the healthcare system through appropriate technology. In addition, the healthcare systems have multiple outputs.

The variables and combinations used in the NDEA model applied in this paper are shown in Figure 1.

Figure 1. Variables in network DEA model



Source: own elaboration

The inputs related to public health include non-medical determinants of health that are beyond the control of healthcare systems. These inputs are related to the wealth of the society and lifestyle and contribute significantly to individual health effects but their control and regulation is usually performed by the public health departments in each country. The most common variables include legal regulations and education aimed at promoting healthy lifestyles, the wealth of the society and the reduction of social inequalities. From among many variables, those that meet the isotonicity criterion have been selected, i.e. those for which there is a significant positive correlation between inputs and outputs. Therefore, annual gross domestic product (GDP) in EUR, converted according to the purchasing power standard (PPS) per inhabitant, preventive care expenditure (PE) per inhibitant in EUR and the percentage of the population declaring consumption of fruit and vegetables at least once a day were used as inputs to the public health subsystem. These inputs were used in previous studies evaluating the efficiency of healthcare systems [Ravangard et al. 2014, Hadad et al. 2013].

The inputs to the medical care subsystem represent capital, labour and technology and are widely used in assessing efficiency. They include the number of physicians (PHYS) and the number or nurses and midwives (N-M) per 100,000 inhabitants, and the number of computer tomographs (CT) per 100,000 inhabitants. The introduction of the number of computer tomographs variable is a response to the introduction of new technologies in treatment and the increasing frequency of diagnosing patients on the basis of imaging services.

These inputs were used, inter alia, in [Hadad et al. 2013, Mirmirani et al. 2008, Samut & Cafri 2016] studies. Annual per capita spending on healthcare taking into account the purchasing power standard of the currency in EUR (TE) is an intermediate variable connecting both subsystems and has been treated as an input to the subsystem of medical care and as an output of the public health subsystem.

It is a variable that is partly beyond the control of decision-makers — it is the result of the functioning of the entire healthcare system. Its value should be maximised in terms of public health and expenditure on preventive care and maintained at an appropriate level (minimised within reasonable limits) in the case of medical care.

The variables reflecting the outputs from the public health subsystem include life expectancy at birth for women (LE-F) and men (LE-M) as variables calculated for each of the countries and the percentage of inhabitants self-assessing their health as very good and good (S-VG-G) as a variable indicated in the social study. The outputs in the subsystem of medical care include reversed variables related to the standardised death rate of less than 65 years for women and men (DEAT65-F and DEAT65-M) per 100,000 inhabitants, describing the so-called premature death. It is believed that the vast majority of diseases before the age of 65 can be cured as long as the medical care is working properly.

All of these outputs were commonly used in calculating the efficiency of healthcare systems [de Cos & Moral-Benito 2014].

Statistical information from 2014 from the Eurostat and WHO databases was used.

Healthcare benefits belong to most important services performed in every country in the world. In general, ensuring health is effective if the healthcare providers (its producers) make the best use of available resources. Expenditure on health is a heavy burden for public finance and therefore a careful analysis of the efficiency of spending is required. An ineffective healthcare system would mean that outputs (or "performance") could be raised without spending extra money, or that care costs could be reduced without affecting the results, provided that greater efficiency is ensured. The research results indicate that there are cases in which the efficiency of health systems can be significantly improved without increasing financial resources [Grigoli 2012].

Particularly important is the fact that on average over 70% of spending on healthcare in EU countries is financed from public funds.

Table 1 presents the basic characteristics of the variables used.

Table 1. Descriptive statistics for all variables used in the Network DEA model

Statistics	GDP	PE	F&V	LE-F	LE-M	S-VG-G
Mean	28 476.7	65.2	51.8	82.5	76.7	67.2
Standard error	12 213.2	37.6	8.9	2.1	3.6	10.2
Median	24 650	63.2	53.1	83.3	78.3	69.4
Max	78 600	140.1	66.4	85.3	81.1	82.5
Min	12 900	6.3	29.2	77.8	67.9	44.9

Continued

Statistics	PHYS	N-M	CT	DEAT65-M	DEAT65-F	TE
Mean	349.9	899.2	2.2	316.2	145.6	2 465.6
Standard error	61.6	347.4	0.9	150.9	43.3	1 134.3
Median	336.4	825.4	2.1	259.5	131.6	2 208.4
Max	504.9	1786.3	3.8	670.2	234.8	4 709.8
Min	230.7	347.4	0.8	164.8	92.4	809.0

Source: own computation

The study covered healthcare systems of 30 European countries - 28 countries of the European Union, as well as Norway and Switzerland.

NETWORK-DEA MODEL

The healthcare system of each of the compared European countries consists of two sub-units — public health and medical care. Both sub-units were assigned the same weights, as both processes are equivalent components of the healthcare system [Woolf & Aron 2013]. The NDEA modification of inputs-oriented BCC model with variable return to scale (VRS) was applied. The input oriented of the model is a consequence of the fact that decision-makers cannot influence outputs. The size of medical personnel, the amount of equipment or health-promoting behaviours can be influenced but it is not possible to directly influence e.g. the level of mortality rate.

The calculations were made using the MaxDEA for Data Envelopment Analysis software provided by Beijing Realworld Software Company Ltd.

If every country is considered to be one DMU (decision making unit) within which a healthcare system operates, in this study we assume that each DMU_j (j = 1, 2, 3,..., n) has m^1 input column vector $X_{i^2 j}=(i^1=1,2,...m^1)$ and s^1 final output column vector $Y_{r^1 j}=(r^1=1,2,..,s^1)$ for the public health sub-process. In the medical care sub-process there is m^2 input column vector $X_{i^2 j}=(i^2=1,2,..,m^2)$ and s^2 final output column vector $Y_{r^2 j}=(r^2=1,2,..,s^2)$ for each country. There are p intermediate products $Z_{pj}=(p=1,2,..q)$ to connect the two sub-processes.

In order to take the scale effect into account, we introduce Models 1.1-1.3, based on the VRS assumption [Chen et al. 2009]. The model has no pre-assigned weights for the sub-processes [Guan & Zuo 2012].

$$
\begin{aligned}
\theta_k = \max\ & \sum_{r^1=1}^{s^1} u_{r^1} Y_{r^1 k} + \sum_{r^2=1}^{s^2} u_{r^2} Y_{r^2 k} + \sum_{p=1}^{q} w_p Z_{pk} - \mu_k^1 - \mu_k^2 \\
s.t. & \sum_{i^1=1}^{m1} \upsilon_{i^1} X_{i^1 k} + \sum_{i^2=1}^{m^2} \upsilon_{i^2} X_{i^2 k} + \sum_{p=1}^{q} w_p Z_{pk} = 1 \\
& \left(\sum_{r^1=1}^{s^1} u_{r^1} Y_{r^1 j} + \sum_{p=1}^{q} w_p Z_{pj}\right) - \sum_{i^1=1}^{m^1} \upsilon_{i^1} X_{i^1 j} - \mu_k^1 \le 0 \\
& \sum_{r^2=1}^{s^2} u_{r^2} Y_{r^2 j} - \left(\sum_{p=1}^{q} w_p Z_{pj} + \sum_{i^2=1}^{m^2} \upsilon_{i^2} X_{i^2 j}\right) - \mu_k^2 \le 0 \\
& u_{r^1}, u_{r^2}, \upsilon_{i^1}, \upsilon_{i^2}, w_p \ge \varepsilon,\ j = 1,2,...,\ n
\end{aligned}
\tag{1.1}
$$

$$
\begin{aligned}
& \theta_k^1 = \max \sum_{r^1=1}^{s^1} u_{r^1} Y_{r^1 k} + \sum_{p=1}^{q} w_p Z_{pj} - \mu_k^2 \\
& s.t. \sum_{i^1=1}^{m1} \upsilon_{i^1} X_{i^1 k} = 1 \\
& \left(\sum_{r^1=1}^{s^1} u_{r^1} Y_{r^1 k} + \sum_{r^2=1}^{s^2} u_{r^2} Y_{r^2 k} + \sum_{p=1}^{q} w_p Z_{pk}\right) \\
& - \theta_k \left(\sum_{i^1=1}^{m1} \upsilon_{i^1} X_{i^1 k} + \sum_{i^2=1}^{m^2} \upsilon_{i^2} X_{i^2 k} + \sum_{p=1}^{q} w_p Z_{pk}\right) - \mu_k^1 - \mu_k^2 = 0 \\
& \left(\sum_{r^1=1}^{s^1} u_{r^1} Y_{r^1 j} + \sum_{p=1}^{q} w_p Z_{pj}\right) - \sum_{i^1=1}^{m^1} \upsilon_{i^1} X_{i^1 j} - \mu_k^1 \le 0 \\
& \sum_{r^2=1}^{s^2} u_{r^2} Y_{r^2 j} - \left(\sum_{p=1}^{q} w_p Z_{pj} + \sum_{i^2=1}^{m^2} \upsilon_{i^2} X_{i^2 j}\right) - \mu_k^2 \le 0 \\
& u_{r^1}, u_{r^2}, \upsilon_{i^1}, \upsilon_{i^2}, w_p \ge \varepsilon,\ j = 1,2,...,\ n
\end{aligned}
\tag{1.2}
$$

$$
\begin{aligned}
&\theta_k^2 = \max \sum_{r^2=1}^{s^2} u_{r^2} Y_{r^2k} - \mu_k^2 \\
&s.t. \sum_{p=1}^{q} w_p Z_{pj} + \sum_{i^2=1}^{m^2} \upsilon_{i^2} X_{i^2k}) = 1 \\
&(\sum_{r^1=1}^{s^1} u_{r^1} Y_{r^1k} + \sum_{r^2=1}^{s^2} u_{r^2} Y_{r^2k} + \sum_{p=1}^{q} w_p Z_{pk}) \\
&- \theta_k (\sum_{i^1=1}^{m1} \upsilon_{i^1} X_{i^1k} + \sum_{i^2=1}^{m^2} \upsilon_{i^2} X_{i^2k} + \sum_{p=1}^{q} w_p Z_{pk}) - \mu_k^1 - \mu_k^2 = 0 \\
&(\sum_{r^1=1}^{s^1} u_{r^1} Y_{r^1j} + \sum_{p=1}^{q} w_p Z_{pj}) - \sum_{i^1=1}^{m^1} \upsilon_{i^1} X_{i^1j} - \mu_k^1 \le 0 \\
&\sum_{r^2=1}^{s^2} u_{r^2} Y_{r^2j} - (\sum_{p=1}^{q} w_p Z_{pj} + \sum_{i^2=1}^{m^2} \upsilon_{i^2} X_{i^2j}) - \mu_k^2 \le 0 \\
&u_{r^1}, u_{r^2}, \upsilon_{i^1}, \upsilon_{i^2}, w_p \ge \varepsilon,\ j = 1,2,\ldots,\ n
\end{aligned}
\tag{1.3}
$$

where μ^1_k and μ^2_k are unconstrained in sign.

The efficiency scores calculated using models (1.1) – (1.3) are more discriminative than those independently calculated from the traditional DEA models because models contain more constraints.

THE RESULTS AND THEIR INTERPRETATION

The results of the calculation of the efficiency of healthcare systems for 30 European countries are presented in Table 2. The "efficiency" column contains the result of efficiency and the countries are arranged according to the decreasing efficiency value.

The full efficiency of the network DEA method calculated for all public health and medical care variables was achieved by 3 countries: Cyprus, Greece and Luxembourg. All these countries have high life expectancy and low mortality rates among people up to the age of 65. Cyprus has a very high share of private spending in current spending on health, exceeding 50%, but the share of out of pocket spending amounts to only about 8%. However, this is not a decisive factor, as Cyprus is the country with the lowest mortality rate of women aged under 65. In the case of Greece, apart from good health results, the position in the ranking was also influenced by low inputs. Luxembourg has high level of expenditure but also high health outcomes.

Table 2. Efficiency Scores for the overall health system for the 30 European countries

Item	Country	Efficiency	Item	Country	Efficiency
1.	Cyprus	1.0000	16.	Italy	0.7735
2.	Greece	1.0000	17.	Bulgaria	0.7671
3.	Luxembourg	1.0000	18.	Czech Republic	0.7511
4.	Spain	0.9587	19.	Germany	0.7356
5.	Ireland	0.9457	20.	Denmark	0.7277
6.	Netherlands	0.9378	21.	Switzerland	0.7114
7.	United Kingdom	0.9305	22.	Norway	0.6975
8.	Portugal	0.9141	23.	Poland	0.6759
9.	Slovenia	0.9137	24.	Romania	0.6674
10.	Malta	0.8922	25.	Austria	0.6635
11.	Croatia	0.8434	26.	Slovakia	0.6214
12.	Finland	0.8237	27.	Latvia	0.6189
13.	France	0.7874	28.	Hungary	0.5954
14.	Sweden	0.7796	29.	Estonia	0.5921
15.	Belgium	0.7788	30.	Lithuania	0.4900

Source: own computation

Table 3 presents the results of the study of the efficiency and ranks of 30 countries separately for public health and medical care processes.

The TE P_H column shows the results of the efficiency of the public health subsystem and the TE M_C column the results for the medical care subsystem. In 2014, the full efficiency of the public health subsystem, equal to 1, was reached by 5 countries and the full efficiency of the medical subsystem was reached by 7 countries. In addition to the 3 countries mentioned above, Bulgaria and Romania have also achieved full efficiency as far as public health is concerned. The high position of these countries is due not to the implementation of profound reforms related to prevention and health promotion, but to a very low level of resources.

Four countries were selected for further analysis: two ranked 4th and 6th, i.e. directly behind the fully efficient ones, Romania, which has achieved full public health efficiency, and Lithuania, which has achieved the lowest public health efficiency among the countries surveyed.

Table 3. Efficiency scores and ranks of public health and medical care areas

Country	TE P_H	R	TE M_C	R	Country	TE P_H	R	TE M_C	R
Austria	0.6718	26	0.6552	21	Latvia	0.8730	11	0.3648	28
Belgium	0.5765	30	0.9811	8	Lithuania	0.7345	20	0.2455	30
Bulgaria	1.0000	1	0.5342	24	Luxembourg	1.0000	4	1.0000	4
Croatia	0.9595	7	0.7273	19	Malta	0.8511	13	0.9332	9
Cyprus	1.0000	2	1.0000	1	Netherlands	0.9733	6	0.9022	12
Czech Republic	0.8358	15	0.6664	20	Norway	0.6309	28	0.7641	18
Denmark	0.6890	24	0.7664	17	Poland	0.7500	18	0.6019	23
Estonia	0.6781	25	0.5061	26	Portugal	0.8979	9	0.9303	10
Finland	0.7401	19	0.9073	11	Romania	1.0000	5	0.3348	29
France	0.6966	23	0.8781	14	Slovakia	0.7283	22	0.5146	25
Germany	0.8372	14	0.6339	22	Slovenia	0.8274	16	1.0000	5
Greece	1.0000	3	1.0000	2	Spain	0.9174	8	1.0000	6
Hungary	0.7336	21	0.4572	27	Switzerland	0.7669	17	0.7923	16
Ireland	0.8914	10	1.0000	3	Sweden	0.6249	29	0.7979	15
Italy	0.6522	27	0.8948	13	United Kingdom	0.8610	12	1.0000	7

Source: own computation

On the basis of the model built, recommendations for inefficient countries will be formulated (actual and forecast values are presented in Table 4).

Table 4. Projection of changes in the inefficient countries

Country		Public health			Medical care		
		GDP	PE	F&V	PHYS	N-M	CT
Spain	data	24 900	43.62	55.65	380.08	514.9	1.75
	proj.	22 844	40.02	51.05	-	-	-
Netherland	data	35 800	140.07	36.15	335	1021	1.33
	proj.	30 518	102.62	35.19	302.23	921.14	0.95
Romania	data	15 200	6.28	29.2	269.82	633.36	1.07
	proj.	-	-	-	86.64	212.07	0.36
Lithuania	data	20 800	24.63	51.2	430.74	790.90	2.22
	proj.	15 277	6.31	29.35	100.48	194.17	0.54

Source: own computation

To be fully efficient as regards public health, Spain needs to reduce its expenditure, e.g. by reducing expenditure on preventive care to 40 EUR per capita, as the model shows that currently the part of GDP over 22,844 EUR per capita is not used efficiently to improve health outcomes. Moreover, fruit and vegetables could be consumed once a day by 51% of the population.

In the case of the Netherlands, expenditure should be reduced — the current health outcomes could also be achieved with: GDP reduced by 5 282 EUR per capita, PE reduced by 37.5 EUR per capita, social consumption of fruit and vegetables reduced by 1%, the number of doctors reduced by 33, the number of nurses and midwives reduced by 100 per 100,000 inhabitants, and the involvement of computer tomography reduced by 0.38 units per 100,000 inhabitants.

Romania, due to the lowest inputs, is achieving full public health efficiency, while the same mortality rates could be achieved with a lower expenditure: 183 fewer doctors and 421 fewer nurses and midwives per 100,000 inhabitants and 0.71 fewer computer tomographs per 100,000 inhabitants.

Lithuania, on the other hand, has the lowest life expectancy among men and the highest mortality rate among men up to the age of 65. Moreover, only 44% of the inhabitants assess their health as very good and good. These health outcomes could be achieved with much lower inputs — on the other hand, if the current resources were maintained, the mortality rates should be significantly reduced, particularly among men. Thus, for the outcomes achieved, GDP could be 27% lower, spending on preventive care 74% lower, the percentage of the population consuming fruit and vegetables 22 percentage points lower, the number of doctors 77% lower, the number of nurses 75% lower, and the number of computer tomographs 76% lower.

SUMMARY AND CONCLUSIONS

All analysed countries make changes in the functioning of their healthcare systems. The implemented reforms are often related to the financing system, which was not evaluated in this study.

The efficiency of the public health system plays an important role in the efficiency of the entire healthcare system and improving its effectiveness should be a priority for all countries.

The efficiency of healthcare systems is also affected by the relation of public and private out-of-pocket expenses. In countries where the private health insurance system is underdeveloped, excessive out-of-pocket expenses may result in resignation from the necessary medical services. This results in lower health results of the society.

However, it should be remembered that the inputs and outputs of public health and medical care systems are highly complex constructs. The variables selected to represent these structures serve only as a proxy to measure them. Although this study is based on variables commonly used in the literature, there are many other variables that could be selected as elements of the DEA model. The results from the DEA model may vary depending on the variables chosen to represent each of the constructs.

REFERENCES

Afonso A., St Aubyn M. (2006) Relative Efficiency of Health Provision: a DEA Approach with Non-discretionary Inputs. *ISEG-UTL* Economics Working Paper, No. 33/2006/DE/UECE, [access 20.04.2017].

Chen Y., Cook W. D., Li N., Zhu J. (2009) Additive Efficiency Decomposition in Two Stage DEA. European Journal of Operational Research, 196 (3), 1170-1176.

de Cos P. H., Moral-Benito E. (2014) Determinants of Health-system Efficiency: Evidence from OECD Countries. International Journal of Health Care Finance and Economics, 14(1), 69-93.

Grigoli F. (2012) Public Expenditure in the Slovak Republic: Composition and Technical Efficiency. IMF Working Paper, WP 173, Washington: International Monetary Fund.

Guan J., Zuo K. (2012) Modeling the Relative Efficiency of National Innovation Systems. Research Policy, 41, 102-115.

Hadad S., Hadad Y., Simon-Tuval T. (2013) Determinants of Healthcare System's Efficiency in OECD Countries. European Journal of Health Economics, 14(2), 253-265.

Joumard I., André C., Nicq C. (2010) Health Care Systems: Efficiency and Institutions. OECD Economics Department Working Papers, No. 769, OECD Publishing, Paris.

Mirmirani S., Li H. C., Ilacqua J. A. (2008) Health Care Efficiency in Transition Economies: An Application of Data Envelopment Analysis. International Business & Economics Research Journal, 7(2), 47-55.

Ozcan Y. A., Khushalani J. (2017) Assessing Efficiency of Public Health and Medical Care Provision in OECD Countries after a Decade of Reform. Central European Journal of Operations Research, 25(2), 325-343.

Ravangard R., Hatam N., Teimourizad A., Jafari A. (2014) Factors Affecting the Technical Efficiency of Health Systems: a Case Study of Economic Cooperation Organization (ECO) Countries (2004-2010). International Journal of Health Policy and Management, 3(2), 63-69.

Retzlaff-Roberts D., Chang C. F., Rubin R. M. (2004) Technical Efficiency in the Use of Health Care Resources: a Comparison of OECD Countries. Health Policy, 69(1), 55-72.

Saltman R. B., Figueras J. (1998) Analyzing the Evidence on European Health Care Reforms. Health Affairs, 17, 85-108.

Samut P. K., Cafrı R. (2016) Analysis of the Efficiency Determinants of Health Systems in OECD Countries by DEA and Panel Tobit. Social Indicators Research, 129(1), 113-132.

Tone K., Tsutsui M. (2009) Network DEA: a Slacks-based Measure Approach. European Journal of Operational Research, 197(1), 243-252.

Woolf S. H., Aron L. (2013) US Health in International Perspective: Shorter Lives, Poorer Health. The National Academies/Institute of Medicine, Washington.

Yaya S., Danhoundo G. (2015) Introduction: Special Issue on Innovations in Health Care System Reform in OECD Countries. The Innovation Journal, 20(1), 1-12.Samu

Quantitative Methods In Economics
Volume XIX, No. 1, 2018, pp. 71 – 77

Received: 17.03.2018
Accepted: 05.04.2018

SAMPLE ALLOCATION IN ESTIMATION OF PROPORTION IN A FINITE POPULATION DIVIDED INTO TWO STRATA: AN EXAMPLE OF APPLICATION

Dominik Sieradzki (ORCID: 0000-0001-5843-3413)
Wojciech Zieliński (ORCID: 0000-0003-0749-8764)
Faculty of Applied Informatics and Mathematics
Warsaw University of Life Sciences - SGGW, Poland
e-mail: dominik_sieradzki@sggw.pl, wojciech_zielinski@sggw.pl

Abstract: The problem of estimating a proportion of objects with particular attribute in a finite population is considered. This paper shows an example of the application of estimation fraction using new proposed sample allocation in a population divided into two strata. Variance of estimator of the proportion which uses proposed sample allocation is compared to variance of the standard one. In the paper an application of sample allocation described in Sieradzki & Zieliński [2017] is presented.

Keywords: survey sampling, sample allocation, stratification, estimation, election poll

JEL classification: C83, C99

INTRODUCTION

In the last years some of the election polls disappointed in their accuracy - the recent American Presidential Elections are the perfect example for that. Election polls are very important not only for the candidates, political party or media, but they can really make a serious impact on voters' decisions. Most of the voters use election polls to take one candidate's side. Moreover, some of them use election polls to decide whether even go to the elections or not! To prevent the feeling of guilt and the common view that the election polls do not mean a thing, it is very important, that they are the most accurate and precise as they can be and the quality of the standard way of the election polls are exquisite. Only this way we will be able to use the election polls as an Academic (scientific) tool.

Consider a problem of the estimation of the support for political parties or a particular candidate in the elections. We would like to know as accurately as possible

DOI: 10.22630/MIBE.2018.19.1.7

a real value of unknown support for a particular candidate in the elections. This magnitude would be known exactly if the society was subjected to exhaustive polling. In practice the easiest and the standard way is to take a sample of size n, count the "yes" answers and divide them number by a sample size. Therefore it could be dealt with sampling error and non-sampling error. The size of sampling error depends on the population variance and can be controlled by the sample size [Hansen et al. 1953]. Non-sampling error is associated with the non-response problem. We distinguish four types of non-response: non-coverage, not-at-homes, unable to answer and the "hard core" [Cochran 1977]. In the next part we are focused on sampling error only.

Consider a population $\mathcal{U} = \{u_1, u_2, \ldots, u_N\}$ which contains finite number of N people. In this population we could observe people (units, objects) which support for a one political option. So we can call this people as units or objects with a particular attribute. Let M denote an unknown number of units in population, which support a particular party in elections. We would like to estimate M, or equivalently, fraction $\theta = \frac{M}{N}$. Sample of size n is drawn due to simple random sampling without replacement scheme. In the sample number of objects which support a particular party in elections is observed. To estimate θ we count people in sample which support particular party and divide by size of sample. The number with certain attribute in the sample is a random variable. To be formal, let ξ be random variable describing number of units having a certain characteristic in the sample. The random variable ξ has hypergeometric distribution [Zieliński 2010] and its statistical model is

$$\left(\{0,1,\ldots,n\}, \{H(N,\theta N,n), \theta \in \langle 0,1\rangle\}\right),$$

with probability distribution function

$$P_{\theta,N,n}\{\xi = x\} = \frac{\binom{\theta N}{x}\binom{(1-\theta)N}{n-x}}{\binom{N}{n}},$$

for integer x from interval $\langle \max\{0, n-(1-\theta)N\}, \min\{n, \theta N\}\rangle$. Unbiased estimator with minimal variance of the parameter θ is $\hat{\theta}_c = \frac{\xi}{n}$ [Bracha 1998, Cochran 1977, Steczkowski 1995, Wywiał 2010]. It is the standard way to estimate unknown value of θ. Variance of that estimator equals

$$D^2_\theta \hat{\theta}_c = \frac{1}{n^2} D^2_\theta \xi = \frac{\theta(1-\theta)}{n}\frac{N-n}{N-1} \quad \text{for all } \theta.$$

It is easy to check that variance $D^2_\theta \hat{\theta}_c$ takes on its maximal value at $\theta = \frac{1}{2}$.

STRATIFICATION

The sample is drawn due to simple random sampling without replacement scheme, so when the support for a party is strongly variable and depends on region, gender of voters etc, it is possible that a part of population would be represented too often, while another part too rarely: the sample may contain only people which support the party or only people which do not support the party. To avoid this, let's divide our population into two disjoint strata $\mathcal{U}_1$ and $\mathcal{U}_2$, $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$ of N_1 and

N_2 units, respectively. For example, support in elections may depend on a gender or on dominant political option at the time. In each strata proportions of distinguished objects are θ_1 and θ_2, respectively. We are still interested in estimation the overall proportion θ, not θ_1 and θ_2. The question is, does the information of this division of the population into two strata improve estimation of the unknown proportion θ? We could answer the question, if we consider stratified estimator of the proportion θ.

Let contribution of the first strata be w_1, i.e $w_1 = N_1/N$. Hence, the overall proportion θ equals

$$\theta = w_1\theta_1 + w_2\theta_2,$$

where $w_2 = 1 - w_1$. Let n_1 and n_2 denote sample sizes from the first and the second strata, respectively. The whole sample size equals $n = n_1 + n_2$. Now we have two random variables describing number of units with property in samples drawn from each strata:

$$\xi_1 \sim H\left(N_1, \theta_1 N_1, n_1\right), \quad \xi_2 \sim H\left(N_2, \theta_2 N_2, n_2\right).$$

Values θ_1, θ_2 and θ are unknown. Since $\theta \in \langle 0, 1\rangle$, hence

$$\theta_1 \in \left\langle \max\left\{0, \frac{\theta - w_2}{w_1}\right\}, \min\left\{1, \frac{\theta}{w_1}\right\}\right\rangle$$

[Zieliński 2016]. Note that θ_1 is a rationale of type M_1/N_1.

Denote left end of the above interval by a_θ and its right end by b_θ, i.e.

$$a_\theta = \max\left\{0, \frac{\theta - w_2}{w_1}\right\}, \quad b_\theta = \min\left\{1, \frac{\theta}{w_1}\right\}$$

and let $L_\theta = b_\theta - a_\theta + 1$. Consider the estimator

$$\hat{\theta}_w = w_1\frac{\xi_1}{n_1} + w_2\frac{\xi_2}{n_2}.$$

The estimator $\hat{\theta}_w$ is unbiased estimator of unknown parameter θ [Sieradzki & Zieliński 2017]. Hence it is necessary to compare variances of estimators $\hat{\theta}_w$ and $\hat{\theta}_c$. The estimator with smaller variance would be more efficient. For given θ there are many θ_1 and θ_2 such that $\theta = w_1\theta_1 + w_2\theta_2$. We are not interested in estimating θ_1 and θ_2, hence we apply averaging with respect to θ_1 (parameter θ_1 is considered as a nuisance one). In such approach variance of estimator $\hat{\theta}_w$ equals:

$$\begin{aligned}
D^2_\theta\hat{\theta}_w =& D^2_\theta\left(w_1\frac{\xi_1}{n_1} + w_2\frac{\xi_2}{n_2}\right)\\
=&\frac{1}{L_\theta}\sum_{\theta_1=a_\theta}^{b_\theta}\left(\left(\frac{w_1}{n_1}\right)^2 D^2_{\theta_1}\xi_1 + \left(\frac{w_2}{n_2}\right)^2 D^2_{\frac{\theta - w_1\theta_1}{w_2}}\xi_2\right)\\
=&\frac{1}{L_\theta}\sum_{\theta_1=a_\theta}^{b_\theta}\left[\frac{w_1^2}{n_1}\theta_1(1-\theta_1)\frac{N_1 - n_1}{N_1 - 1} + \right.\\
&\left. + \frac{w_2^2}{n_2}\frac{\theta - w_1\theta_1}{w_2}\left(1 - \frac{\theta - w_1\theta_1}{w_2}\right)\frac{N_2 - n_2}{N_2 - 1}\right].
\end{aligned}$$

Let $f=\frac{n_1}{n}$ denote the contribution of first strata in the sample. For $0<\theta<w_1$ variance of $\hat{\theta}_w$ equals ($a_\theta=0$ and $b_\theta=\frac{\theta}{w_1}$):

$$\begin{aligned}\frac{h(f)}{-6(N_1-1)(N_2-1)Nf(1-f)n}\theta\\ +\frac{(N_2-1)N_1-(N(n+1)-2(N_1+n))f+(N-2)nf^2}{3(N_1-1)(N_2-1)f(1-f)n}\theta^2,\end{aligned}\tag{$*$}$$

where

$$\begin{aligned}h(f)&=N_1(N_2-3N_1(N_2-1)-1)\\&+\big(3N_1^2(N_2-1)+3N_2^2+2n+N_1\left(6N_2n-3N_2^2-4n+1\right)-N_2(4n+1)\big)f\\&+2\left(N_1(2-3N_2)+2N_2-1\right)nf^2.\end{aligned}$$

For $w_1\leq\theta\leq 1-w_1$ variance of $\hat{\theta}_w$ equals ($a_\theta=0$ and $b_\theta=1$):

$$\begin{aligned}&\frac{(N_2-(1-f)n)}{(N_2-1)(1-f)n}\theta(1-\theta)+\\&\quad-\frac{N_1\left(2(N+1)f^2+(3NN_2+N_2-N_1-2n(N+1))f-N_1(N_2-1)\right)}{6N^2(N_2-1)nf(1-f)}.\end{aligned}$$

To obtain explicit formula for variance of $\hat{\theta}_w$ for $1-w_1<\theta<1$ it is enough to replace θ by $1-\theta$ in $(*)$.

Detailed analysis of variance of estimator $\hat{\theta}_w$ could be found in Sieradzki & Zieliński [2017]. We would like to find "the worst" situation, i.e. the value of θ for which variance $D^2_\theta\hat{\theta}_w$ takes on its maximal value and then find optimal f which minimizes this maximal variance. General formula for the optimal f is unobtainable, because of complexity of symbolic computation. Nevertheless numerical solution is easy to obtain. In the next section we will considered an example of application.

EXAMPLE

Suppose we want to estimate support for a political party (it will be referred to as a party "A") in Poland. In Poland there is more than 30000000 people who may vote (due to official statistics, in 2011 there were $N=30762931$ voters[1]). The standard way of estimation is to take a sample of size $n=1000$ due to the scheme of simple sampling without replacement. Let ξ denote the number of "yes, I will vote on party A" answers. The standard estimator of the support is $\frac{\xi}{n}$.

In 2011 the party "A" won in 27 out of 41 regions. In those regions there were 20222414 people who may vote, while in the rest of regions there were 10540517 voters. To improve estimation of the support for party "A" we divide Poland into two strata: the first one of the weight $w_1=10540517/30762931=0.342636955$ and the second one of the weight $w_2=20222414/30762931=0.657363045$. The optimal f for

[1] http://wybory2011.pkw.gov.pl/wyn/pl/000000.html#tabs-1

Table 1. Possible results for $\xi = 200$, $\hat{v}_c(200) = 0.0001599948$

ξ_1	ξ_2	*variance*	*reduction*
25	175	0.000109763	5.23%
50	150	0.000158481	0.94%
75	125	0.000159807	0.11%
100	100	0.000155599	2.74%
125	75	0.000145855	8.83%
150	50	0.000130577	18.38%
175	25	0.000109763	31.39%

Source: own calculations

Table 2. Possible results for $\xi = 300$, $\hat{v}_c(300) = 0.0002099932$

ξ_1	ξ_2	*variance*	*reduction*
25	275	0.000183173	12.77%
50	250	0.000197636	5.88%
75	225	0.000206565	1.63%
100	200	0.000209959	0.01%
125	175	0.000207817	1.03%
150	150	0.000200141	4.69%
175	125	0.000186929	10.98%
200	100	0.000168183	19.91%
225	75	0.000143901	31.47%
250	50	0.000114085	45.67%
275	25	0.000078733	62.50%

Source: own calculations

this numerical case could be find. Finding the optimal f is equivalent to finding the optimal division (n_1, n_2) of the sample. After some calculations (in a mathematical software, for example Mathematica) we obtain optimal $f = 0.343$, hence $n_1 = 343$ and $n_2 = 657$.

Suppose that in the whole sample 200 "yes" answers were obtained. The point estimate of the support equals $\hat{\theta}_c = 0.2$ and its variance may be estimated as $\hat{v}_c(200) = 0.000159995$. If in the sample of size n_1 from the first stratum there were 25 "yes" answers and in the sample of the size n_2 from the second stratum there were 175 "yes" answers, then the point estimate of the support is $\hat{\theta}_w = 0.2$ and its variance may be estimated as $\hat{v}_w(25, 175) = 0.000109763$. Note that the stratified estimator has smaller variance than the one based on the non stratified sample. The relative reduction of variance equals

$$\mathit{reduction} = \left(1 - \frac{\hat{v}_w(25, 175)}{\hat{v}_c(200)}\right) \cdot 100\% = 5.23\%.$$

Table 1 shows other possible results of the pool, assuming that the overall "yes" answers equal to 200.

In the first and in the second column possible results of the pool are given. Values of estimated variances are given in the third column. The last column shows the relative reduction of variance, i.e. of how many percent estimator $\hat{\theta}_w$ is better than

Table 3. Possible results for $\xi = 400$, $\hat{v}_c(400) = 0.0002399922$

ξ_1	ξ_2	*variance*	*reduction*
25	375	0.0001842856	23.21%
50	350	0.0002063514	14.01%
75	325	0.0002228821	7.12%
100	300	0.0002338778	2.54%
125	275	0.0002393385	0.27%
150	250	0.0002392641	0.30%
175	225	0.0002336546	2.64%
200	200	0.0002225102	7.28%
225	175	0.0002058306	14.23%
250	150	0.0001836161	23.49%
275	125	0.0001558665	35.05%

Source: own calculations

estimator $\hat{\theta}_c$. In Tables 2 and 3 there are given possible results assuming, that the overall positive answers is 300 and 400 respectively. It is seen, that whatever results of pool are in strata the stratified estimator is better than the standard one.

CONCLUSIONS

In the paper an example of application of estimation of unknown fraction in population divided into two strata was presented. Estimators $\hat{\theta}_c$ and $\hat{\theta}_w$ were compared with respect to their variances. In that example for optimal allocation between strata it was shown that variance of stratified estimator is always smaller than variance of classical estimator. Hence in practice, it is recommended to use the information of the division of the population into two strata, because quality of stratified estimator is better than the quality of the classical one.

REFERENCES

Bracha Cz. (1998) Metoda reprezentacyjna w badaniach opinii publicznej i marketingu. PWN, Warszawa (in Polish).

Cochran W. G. (1977) Sampling Techniques (3rd ed.), New York: John Wiley.

Hansen M. H., Hurwitz W. N., Madow W. G. (1953) Sample Survey Methods and Theory. New York: John Wiley.

Sieradzki D. (2016) Estimation of proportion in finite population divided into two strata. master thesis, WZIiM SGGW Warszawa (in Polish).

Sieradzki D., Zieliński W. (2017) Sample allocation in estimation of proportion in a finite population divided into two strata. Statistics in Transition new series, 18(3), 541-548, 10.21307.

Steczkowski J. (1995) Metoda reprezentacyjna w badaniach zjawisk ekonomiczno-społecznych. PWN, Warszawa (in Polish).

Wywiał J. (2010), Wprowadzenie do metody reprezentacyjnej. Wydawnictwo Akademii Ekonomicznej w Katowicach, Katowice (in Polish).

Zieliński W. (2010) Estymacja wskaźnika struktury. Wydawnictwo SGGW, Warszawa (in Polish).

Zieliński W. (2016) A remark on estimating defectiveness in sampling acceptance inspection. Colloquium Biometricum, 46, 9-14.

QUANTITATIVE METHODS IN ECONOMICS
Volume XIX, No. 1, 2018, pp. 78 – 91
Received: 07.03.2018
Accepted: 05.04.2018

TESSELLATION AS AN ALTERNATIVE AGGREGATION METHOD

Kateryna Zabarina (ORCID: 0000-0003-0241-6977)
Faculty of Economic Sciences
University of Warsaw, Poland
e-mail: kzabarina@wne.uw.edu.pl

Abstract: The sensitivity of statistical results to the choice of a particular zoning system is known as the Modifiable Areal Unit Problem. Level of aggregation is a significant factor determining results. Moving from point data to areal data, one should take into account aggregation and existence of spatial autocorrelation in the data. Though usually Voronoi tessellation is used in different study fields, this paper suggests it can be an alternative aggregation method to connect point and areal data in economics. Paper shows the possibility to calculate spatial average from point data and vice versa.

Keywords: Voronoi tessellation, point data, regional data, random sample, spatial analysis, aggregation, MAUP, spatial average

JEL classification: C43, C63, C65, R12

INTRODUCTION

The sensitivity of statistical results to the choice of a particular zoning system is known as the Modifiable Areal Unit Problem (MAUP) [Briant et al. 2010]. Because of the MAUP, level of aggregation is a significant factor determining results [Pietrzak and Ziemkiewicz 2016], as it cuts spatial information and introduces errors [Jeffery et al. 2014]. Though spatial data are available either in point form (with geographical coordinates) or aggregated by administrative region (e.g. zip code) [Jeffery et al. 2014], aggregated data are more often used by researchers due to computational constraints [Stępniak and Jacobs-Crisioni, 2017]. To mitigate the MAUP, some advocate the use of methods that do not depend on spatial aggregation [Kwan 1998, Kwan and Weber 2008, Tobler 1989], but these

DOI: 10.22630/MIBE.2018.19.1.8

methods typically require substantial additional data, which are often unavailable [Stępniak and Jacobs-Crisioni 2017].

Aggregation and delimitation of space are not so obvious and fully understandable research issues. Talking about aggregation, one can apply different aggregation methods: based on fuzzy logics and networks [Lee W. C. 1995, Lee H. S. 1999, Liu P. 2011, Sharifzadeh and Shahabi 2004], preferences [Wang et al. 2005], ranks [Liu Y. T. et al. 2007, Kolde et al. 2012], constraints [Poon and Martins 2007] or weighting schemes (ordered [Xu 2008], dynamic [Jin et al. 2001], adaptive [Jagyasi et al. 2007, Hamada et al. 2010]). Spatial delimitation is usually described in researches connected with urbanism [Duque et al. 2012], resources management [Centeno 1997, Douvere et al. 2007], land-use [Xavier et al. 2018], tourism [Aubert et al. 2010, Bel et al. 2015], linguistics [Gvozdanović 1992], biology [Sabesan et al. 2008]. Both aggregation and delimitation are useful for spatial studies, however, are not widely applied in economics, especially in firm location studies.

Firms' locations are usually represented by a single point, and one can observe that they are correlated, i.e. dependent on each other. Tobler's first law of geography says: "everything is related to everything else, but nearby things are more related than distant things" [Tobler 1979]. This fact implies spatial autocorrelation for the observations in a geographic space. It means that there is a relation between values monitored at the neighboring locations [Sharifzadeh and Shahabi 2004].

Movement from point data to areal data should take into account also aggregation method. Point data cannot be treated as simple regional data aggregation - aggregation hides the variance of point data and 'cuts-off' the spatial information[1]. Most empirical work in economic geography relies on scattered geo-coded data that are aggregated into discrete spatial units, such as cities or regions. However, the aggregation of spatial dots into boxes of different size and shape is not benign regarding statistical inference [Briant et al. 2010].

Point measures are usually obtained from continuous surface, that's why one need to estimate the whole surface in order to get them. Aggregation from points to regions is needed, when value of some regional index has to be calculated. When spatial units do not have the same shape, averaging is less sensitive to changes in size than summation [Briant et al. 2010]. It can nonetheless be argued that administrative boundaries do not capture the essence of economic phenomena that often spill over boundaries [Briant et al. 2010].

Term 'spatial average' firstly appeared in the work of Weaver[2]. Later, the spatial averaging theorem was presented independently in 1967 by Anderson and

[1] Regional aggregates do not reflect spatial location, spatial distributions etc.

[2] Weaver J. C. (1956). The county as a spatial average in agricultural geography. Geographical Review, 46(4), 536-565.

Jackson [Anderson and Jackson, 1967], Slattery [Slattery 1976] and Whitaker [Whitaker, 1976] [all authors were mentioned in Hower, Whitaker 1985]. Problem of spatial average was widely discussed (among others) by Goodchild [Goodchild 1979][3] and Hower and Whitaker [Hower and Whitaker 1985][4]. It became a popular tool in geography (especially hydrology[5]) and sensor network[Sharifzadeh and Shahabi 2004] studies. In both cases, area of Voronoi tessellation cell was used as weighting coefficient and it was proved that this approach performs better than arythmetical mean and captures stochasticity of data.

To obtain more precise results, it is better to used weighting coefficients showing relative area of smaller administrative unit to bigger one (which includes that smaller unit). But taking into account fact, that even in the smallest administrative unit there could be more than one firm, calculation of some aggregated index could be biased. As Voronoi tessellation covers all the plane without no overlaps and no gaps, and there is only one point in each Voronoi cell, it can be considered as a division of space, which captures relation between values observed in the neighbourhood[6]. Despite the fact that usually Voronoi tessellation is used in cellular biology, image compression or resources distribution studies, it can be an alternative method which connects point data and areal data in economics.

Current research suggests, that in order to obtain proper results, area of Voronoi cell is divided by the sum of all Voronoi cells in the certain region[7]. Such weighting coefficient can be used when calculating spatial average from point data or vice versa – having spatial average, it is possible to set separate values for points, using weighting coefficient. Using firm location data from Slaskie voivodeship, two abovementioned cases are described. Method will be used for calculating economic indexes determining firm location. All calculations are made using R software[8] using randomly generated values[9].

---

[3] Goodchild M. F. (1979). The aggregation problem in location-allocation. Geographical Analysis, 11(3), 240-255.

[4] Howes F. A. & Whitaker S. (1985). The spatial averaging theorem revisited. Chemical engineering science, 40(8), 1387-1392.

[5] Precipitation spatial average. On-line access: https://earth.boisestate.edu/drycreek/education/hypsometric/

[6] As it comes from Voronoi tessellation properties, each cell contains exactly one point and every other location in a given cell is closer to its corresponding point than to any other point (Sharifzadeh and Shahabi, 2004).

[7] Dirichlet tessellation is computed exactly by the Lee-Schachter algorithm. When switching from bigger unit to smaller (for example, from voivodeship to powiat), area of tessellation cell is calculated with respect to the bigger unit and not to smaller one, so it makes sense to use a proportion.

[8] Except from standard packages for spatial data and task view “Spatial”, packages deldir, spatstat, qlcVisualize were used to calculation and visualization.

TESSELLATIONS

Tessellations were known for people since ancient times. The term comes from Latin 'tessella' – small square, (or Greek 'τέσσερα' - four) and means small cubical piece of clay, stone or glass used to make mosaics. But nowadays some new forms (such as Penrose tiling) are widely used [Boots et al. 1999].

One can define tessellation of d[10]-dimensional Euclidean space, $\mathfrak{R}^d$, either as subdivision into d-dimensional, non-overlapping regions or a set of d-dimensional regions which cover space without overlaps and gaps. Formal definition of a tessellation is the following.

Let S be a closed subset of $\mathfrak{R}^d$, $\mathfrak{I} = \{s_1, \dots, s_n\}$, where s_i is the closed subset of S and s_i' is the interior of s_i. If the elements of $\mathfrak{I}$ satisfy

$$\begin{cases} s_i' \cap s_j' = \emptyset, \text{for } i \neq j \\ \bigcup_{i=1}^{n} s_i = S \end{cases}, \tag{1}$$

then the set $\mathfrak{I}$ is called a tessellation of S. First property means that the interiors of the elements of $\mathfrak{I}$ are disjoint, and second property means that collectively elements of $\mathfrak{I}$ fill the space [Boots et al. 1999]. In more simple way of explanation, spatial tessellation is a set of regions that are collectively exhaustive and mutually exclusive except for the boundaries.

Planar tessellations are composed of three elements of d ($d \leq 2$) dimensions: cells (2-d), edges (1-d), and vertices (0-d). In GIS these elements are usually referred to as polygons, lines (or arcs), and points respectively. All tessellations should be divided in three groups: uniform (or Archimedean) (consist of regular polygons[11], which are not isohedral[12]), regular (the three uniform tessellations which are also isohedral (i.e. those consisting of regular triangles, squares, or hexagons)) and irregular (normal[13] tessellations consisting of only convex cells) [Boots et al. 1999]. The very best example of irregular tessellation is Voronoi tessellation[14].

Generic definition of the Voronoi diagram is the following. Let S denote a set of n points (called sites) in the plane. Since the regions are coming from

---

[9] There was no possibility to obtain any real numeric data except number of employees. However, distribution of employees was discrete with value 5 in around 80% of observations.

[10] When d=2, tessellation is called a planar tessellation.

[11] A regular polygon is one with equal side length and equal internal angles.

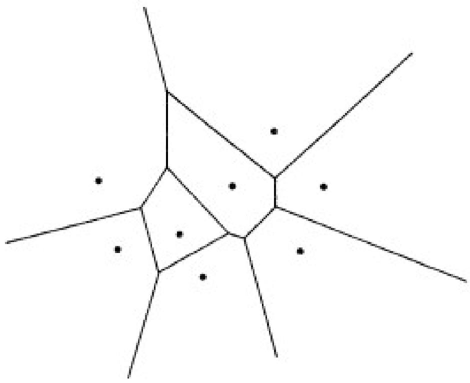
[12] If r_i denotes the number of edges meeting at the i^{th} corner of a cell in a monohedral tessellation, an isohedral tessellation is one in which the ordered sequence of values of r_i is the same for every cell.

[13] A d-dimensional tessellation in which every s-dimensional element lies in the boundaries of (d–s+1) cells ($0 \leq s \leq d-1$).

[14] The same as Voronoi diagram, Voronoi decomposition,Voronoi partition, Dirichlet tessellation, Thiessen polygons.

intersecting n – 1 half planes, they are convex polygons. Thus, the boundary of a region consists of at most n – 1 edges (maximal open straight-line segments) and vertices (their endpoints). Each point on an edge is equidistant from exactly two sites, and each vertex is equidistant from at least three. As a consequence, the regions are edge to edge and vertex to vertex, that is to say, they form a polygonal partition of the plane. This partition is called the Voronoi diagram, V(S), of the finite point-set S (Figure 1) [Aurenhammer 1991].

Figure 1. Voronoi diagram for eight sites in the plane



Source: Aurenhammer F. (1991) Voronoi Diagrams – A Survey of a Fundamental Geometric Data Structure

Current study bases on Voronoi tessellation, because it allows to cover all plane instead of regular tessellation. Thiessen polygons have wide usage: in natural sciences (cellular biology [Bock et al. 2009, Saribudak et al. 2016], hydrodynamics [Springel 2010], physics [Kasim 2017]), medicine [Sanchez-Gutierrez et al. 2016], image compression [Du et al. 2006], resource distribution studies [Liu et al. 2009]. In geographical analysis, it may represent administrative units, census tracts, postal zones, or electoral and school districts [Sadahiro 2010]. In this study Voronoi tessellation is suggested to be an instrument measuring different economic indexes.

DATASET AND PRELIMINARY STAGE OF PROPOSED APPROACH

Usually spatial data are presented as areal (one value for administrative unit) or point data (in case of firms, for example). Using weighting coefficients, one can split value of areal index between each single point or vice versa – calculate spatial average using data from each single one.

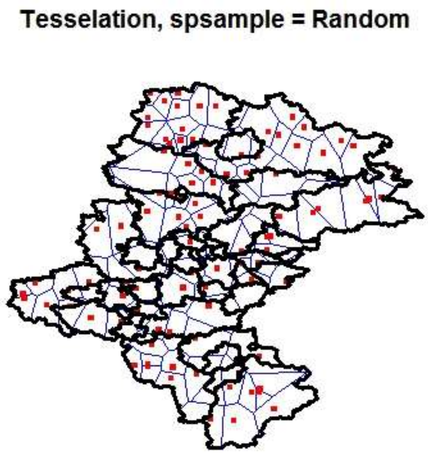
To conduct the study data from Slaskie voivodeship[15] were used. Data present 504917 firms, with information about their location (address, geographical

[15] NUTS2 unit.

coordinates, administrative unit), economic sector, number of employees. However, for the purpose of study we generate normally distributed data[16].

In case of Poland, sometimes data are presented on powiat[17] level. But when plotting tessellation and powiat structure together, one can observe difficulty in understanding which tessellation cell belongs to which powiat – administrative boundaries do not capture the essence of economic phenomena that often spill over boundaries [Briant et al. 2010]. Moreover, administrative units have a tendency to change their boundaries (in contrast to tessellation cells for the same sample). Figure 2 presents randomly selected 100 firms[18] from Slaskie voivodeship (square dots), with Voronoi tessellation based on that point (thin lines) and powiats (thick lines).

Figure 2. Tessellation and powiat structure of Slaskie voivodeship, plotted together



Source: own calculations

Proposed approach is the following. We first choose random sample of 1000 points. After it, we do a tessellation (Figure 3) in order to obtain Voronoi cell area[19], which will be used in further calculations.

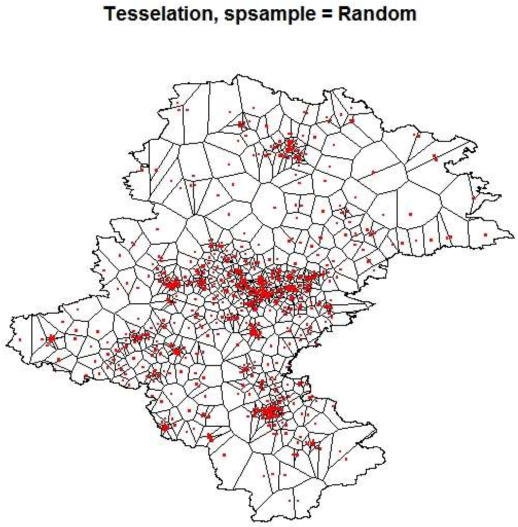
[16] Data were generated using normal distribution with mean=50 and standard deviation=15

[17] Smaller unit of voivodeship; there are 380 powiats in Poland.

[18] 100 firms from abovementioned dataset; hereafter ‘points’.

[19] Also known as Dirichet weight or Dirichlet area; in R ‘dirichletWeights()’and tile.areas()’ give the same results.

Figure 3. Tessellation of randomly chosen 1000 points



Source: own calculations

As it was stated in the introduction, this study presents two different methods of aggregation – calculating spatial average from point data and calculating point values having spatial average. Detailed description is in section below.

PROPOSED APPROACH

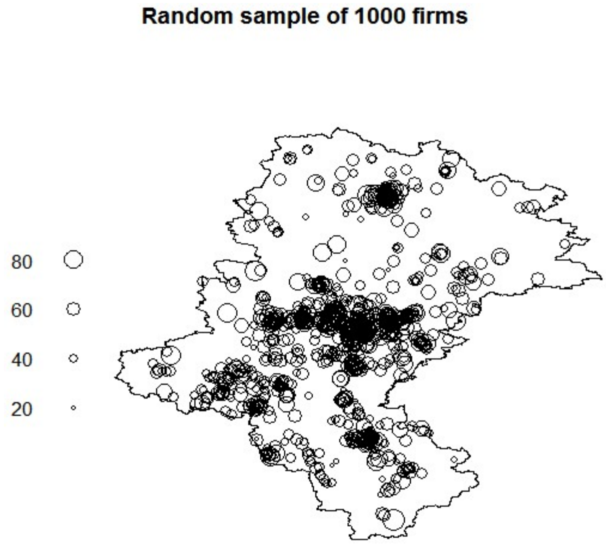
Proposed approach is described on the Slaskie voivodeship in Poland. Using this method, it is possible to make calculations for administrative units of different size and shape. Two different aspects of the proposed method are described below.

Calculating spatial average from point data

R software allows user to create a point pattern inside specified window and set values to each point[20]. Figure 4 shows initial state for this part of approach – point pattern with single value for each single point, where bigger circle size means bigger value in a certain point.

[20] Known in R as 'marks'. It is possible to create multiple marks, however, in this study for simplicity only one mark dimension was used.

Figure 4. Point pattern for calculating spatial average from point data (first aspect of approach) – initial state



Source: own calculations

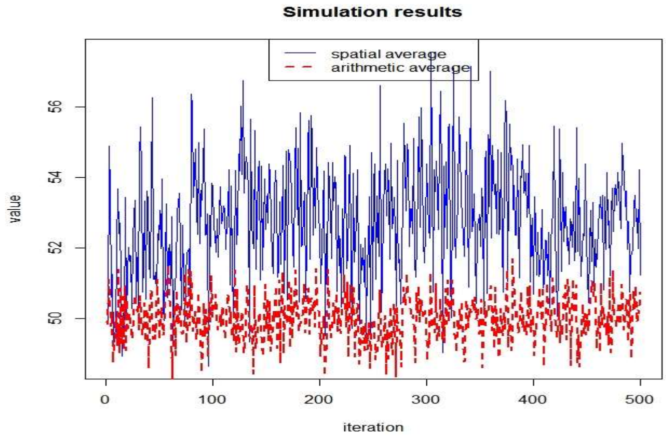
The next step is very easy – using weighting coefficient (calculated as area of certain Voronoi cell over sum of all Voronoi cells' areas), we can calculate spatial average as a sumproduct of weights and values (code in R is given below):

```
d_weights <- dirichletWeights(X.rand, exact=TRUE) #X.rand – is a point
        pattern for which we calculate Voronoi cells areas
suma <- 0 # initial value of spatial average
for (i in 1:length(X.rand$marks)){
  suma[i] <- X.rand$marks[i]*(d_weights[i]/sum(d_weights))
} #sumproduct of value and weighting coefficient
sum(suma) # summing up all sumproducts
```

After one iteration on 1000 randomly chosen points, spatial average (52.35) is bigger than arithmetic mean (50.76). However, arithmetic mean doesn't take into account spatial nature of data. Also, it produces more 'flat' value, especially in samples with huge outliers.

After simulation of 500 iterations (sampling 1000 points in each, and then calculating arithmetic mean and spatial average) we clearly see that in all cases spatial average exceeds arithmetical mean (see figure 5). Comparison of obtained values shows, that arithmetical mean is 'flatter', it doesn't characterize the real situation and doesn't take into account spatial nature of data.

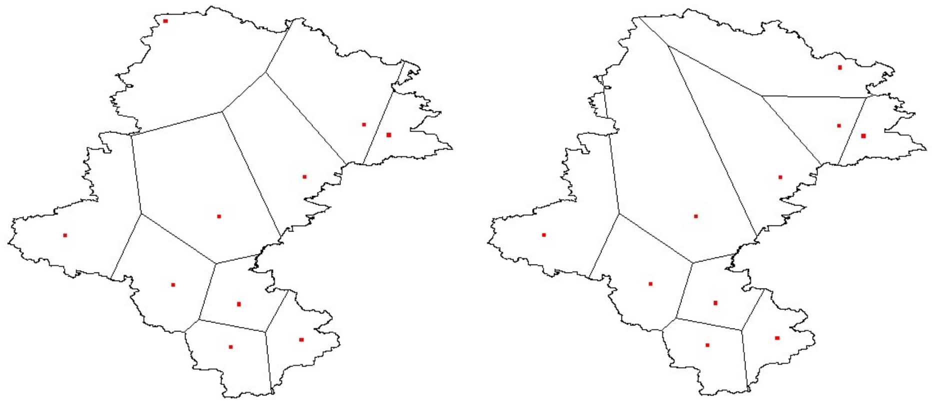
Figure 5. Simulation results: comparison of aggregated index and simple mean



Source: own calculations

Let's show on the simple example why location matters. Consider a sample of 10 points, for which we calculate both arithmetic and spatial average. On the next step, we change location of one of the points to a different random (only location, so value of parameter in an old point is set to the new point), and it can be observed, that location matters, so spatial average changed, but arithmetic didn't. Figure 6a shows the values of both indexes in the first step, 6b – values of them after changing location of 1 point[21].

Figure 6. Comparison of measures in case when location of one data point is different



Spatial average = 54,989	Spatial average = 46,13369
Arithmetic average = 52,4336	Arithmetic average = 52,4336
(a)	(b)

Source: own calculations

[21] For the purpose of experiment, only location and then tessellation is shown. Plotting points' value results in a mess on the graph.

As it is possible to calculate spatial average for big unit (as voivodeship), it is possible to do the same for smaller units (for example, powiats).

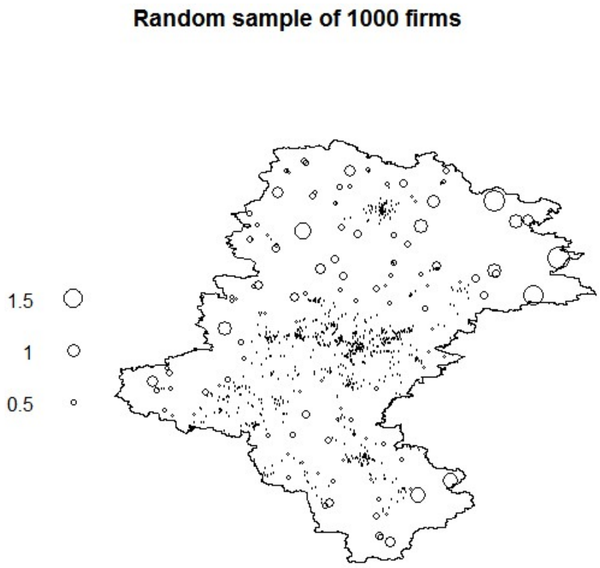
Calculating point values having areal index

This subsection describes how to calculate values for point data having spatial average. We still consider the same point pattern, however, without marks. Assume that there is some spatial average given. Having Voronoi cells areas, in the loop we set the new values for each point (code in R is given below):

```
marks(X.rand) <- 0 # setting values to zero
some_index <- 95 # assumed value of spatial average
new_marks <- matrix(0,1000,1)
d_weights <- dirichletWeights(X.rand, exact=TRUE)
for (i in 1:length(X.rand$marks)){
  new_marks[i] <- (d_weights[i]/sum(d_weights))*some_index
} # setting new values to the same points
marks(X.rand) <- new_marks
```

Arithmetical mean approach in this case resulted in value of 0.095 for each single point, which doesn't reflect spatial features of sample – points' location and neighbourhood structure, in contrary to spatial average approach. Figure 7 shows the result of above described procedure.

Figure 7. Point pattern for calculating point values from spatial average (second aspect of approach) – results



Source: own calculations

As it is possible to calculate point values from spatial average for one big unit, it is possible to do the same for other big unit as well, as for small ones.

CONCLUSIONS

This paper studies Voronoi tessellation from a different perspective – as an alternative aggregation method which can connect point data and areal data. It is shown that having Voronoi cell area, it is possible to calculate a weighting coefficient (as ratio of one Voronoi cell area to sum of all areas) and use it for computing spatial average from point data or vice versa.

Study results show, that proposed method performs better in terms of capturing spatial features of the data. Arithmetic mean flats the value of sample average, however spatial approach allows to set proper values for points, taking into account their location.

The results of the study will be helpful in economic studies for calculating different economic indexes. Furthermore, proposed approach could be useful in firm location studies – having some local economic index as a feature of region, it could be splitted between firms and used as an explanatory feature.

REFERENCES

Amini N., Rashidi T., Gardner L., & Waller S. T. (2016) Spatial Aggregation Method for Anonymous Surveys: Case Study for Associations between Urban Environment and Obesity. Transportation Research Record: Journal of the Transportation Research Board, 2598, 27-36.

Aubert A., Csapó J., Pirkhoffer E., Puczkó L., Szabó G. (2010) A Method for Complex Spatial Delimitation of Tourism Destinations in South Transdanubia. Hungarian Geographical Bulletin, 59(3), 271-287.

Aurenhammer F. (1991) Voronoi Diagrams – A Survey of a Fundamental Geometric Data Structure. ACM Computing Surveys, 23(3), 345-405.

Bel F., Lacroix A., Lyser S., Rambonilaza T., Turpin N. (2015) Domestic Demand for Tourism in Rural Areas: Insights from Summer Stays in Three French Regions. Tourism Management, 46, 562-570.

Bock M., Tyagi A. K., Kreft J.-U., Alt W. (2009) Generalized Voronoi Tessellation as a Model of Two-dimensional Cell Tissue Dynamics. Bulletin of Mathematical Biology, 72(7), 1696-1731.

Boots B., Okabe A., Sugihara K. (1999) Spatial Tessellations. Geographical Information Systems, 1, 503-526.

Briant A., Combes P. P., Lafourcade M. (2010) Dots to Boxes: Do the Size and Shape of Spatial Units Jeopardize Economic Geography Estimations? Journal of Urban Economics, 67(3), 287-302.

Centeno J. S. (1997, August) Segmentation of Thematic Maps using Colour and Spatial Attributes. International Workshop on Graphics Recognition, Springer, Berlin, Heidelberg, 221-230.

Douvere F., Maes F., Vanhulle A., Schrijvers J. (2007) The Role of Marine Spatial Planning in Sea Use Management: the Belgian Case. Marine Policy, 31(2), 182-191.

Dray S., Legendre P., Peres-Neto P. R. (2006) Spatial Modelling: a Comprehensive Framework for Principal Coordinate Analysis of Neighbour Matrices (PCNM). Ecological Modelling, 196(3), 483-493.

Du Q., Gunzburger M., Ju L., & Wang X. (2006) Centroidal Voronoi Tessellation Algorithms for Image Compression, Segmentation, and Multichannel Restoration. Journal of Mathematical Imaging and Vision, 24(2), 177-194.

Duque J. C., Royuela V. & Noreña M. (2012) A Stepwise Procedure to Determinate a Suitable Scale for the Spatial Delimitation of Urban Slums. Defining the Spatial Scale in Modern Regional Analysis. Springer Berlin Heidelberg, 237-254.

Fiene J. G., Sword G. A., Van Laerhoven S. L., Tarone A. M. (2014) The Role of Spatial Aggregation in Forensic Entomology. Journal of Medical Entomology, 51(1), 1-9, 2014.

Goodchild M. F. (1979) The Aggregation Problem in Location-allocation. Geographical Analysis, 11(3), 240-255.

Gvozdanović J. (1992). The Verbal Prefixes Po-and Pro-in Russian: Their Meanings and Uses. Studies in Slavic and General Linguistics, 17, 111-123.

Hamada N., Nagata Y., Kobayashi S., Ono I. (2010, July) Adaptive Weighted Aggregation: A Multiobjective Function Optimization Framework taking Account of Spread and Evenness of Approximate Solutions. Evolutionary Computation (CEC), 2010 IEEE Congress, 1-8.

Howes F. A., Whitaker S. (1985) The Spatial Averaging Theorem Revisited. Chemical Engineering Science, 40(8), 1387-1392.

Jacobs-Crisioni C., Stępniak M. (2017) Reducing the Uncertainty Induced by Spatial Aggregation in Accessibility and Spatial Interaction Applications. Journal of Transport Geography, 61, 17-29.

Jagyasi B. G., Merchant S. N., Chander D., Desai U. B., Dey B. K. (2007, August) AdWAS: Adaptive Weighted Aggregation Scheme for Single-hop and Multi-hop Wireless Sensor Network. Communications and Networking in China, 2007. CHINACOM'07. Second International Conference, 835-839.

Jeffery C., Ozonoff A., Pagano M. (2014) The Effect of Spatial Aggregation on Performance when Mapping a Risk of Disease. International Journal of Health Geographics, 13(1), 9.

Jin Y., Olhofer M., Sendhoff B. (2001) Dynamic Weighted Aggregation for Evolutionary Multi-objective Optimization: Why Does it Work and How? Proceedings of the 3rd Annual Conference on Genetic and Evolutionary Computation, Morgan Kaufmann Publishers Inc., 1042-1049.

Kasim M. F. (2017) Quantitative Shadowgraphy and Proton Radiography for Large Intensity Modulations. Physical Review E. 95(2).

Kolde R., Laur S., Adler P., Vilo J. (2012) Robust Rank Aggregation for Gene List Integration and Meta-analysis. Bioinformatics, 28(4), 573-580.

Kwan M.-P. (1998) Space-time and Integral Measures of Individual Accessibility: a Comparative Analysis using a Point-based Framework. Geogr. Anal., 30, 191–216.

Kwan M.-P., Weber J. (2008) Scale and Accessibility: Implications for the Analysis of Land Use–travel Interaction. Appl. Geogr., 28, 110-123.

Lee H. S. (1999) An Optimal Aggregation Method for Fuzzy Opinions of Group Decision. Systems, Man, and Cybernetics, 1999. IEEE SMC'99 Conference Proceedings. 1999 IEEE International Conference, 3, 314-319.

Lee W. C. (1995, April) Spanning Tree Method for Link State Aggregation in Large Communication Networks. INFOCOM'95. Fourteenth Annual Joint Conference of the IEEE Computer and Communications Societies. Bringing Information to People. Proceedings. IEEE, 1, 297-302.

Liu P. (2011) A Weighted Aggregation Operators Multi-attribute Group Decision-making Method based on Interval-valued Trapezoidal Fuzzy Numbers. Expert Systems with Applications, 38(1), 1053-1060.

Liu X. T., Zheng X. Q., & Li D. B. (2009) Voronoi Diagram-Based Research on Spatial Distribution Characteristics of Rural Settlements and Its Affecting Factors - A Case Study of Changping District, Beijing [J]. Journal of Ecology and Rural Environment, 2, 007.

Liu Y. T., Liu T. Y., Qin T., Ma Z. M., & Li H. (2007) Supervised Rank Aggregation. Proceedings of the 16th International Conference on World Wide Web, 481-490.

Marra F., Destro E., Nikolopoulos E. I., Zoccatelli D., Creutin J. D., Guzzetti F., Borga M. (2017) Impact of Rainfall Spatial Aggregation on the Identification of Debris Flow Occurrence Thresholds. Hydrology and Earth System Sciences, 21, 4525-4532.

Openshaw S., Taylor P. J. (1979) A Million or so Correlation Coefficients: Three Experiments on the MAUP. In Wrigley N (ed), Statistical Applications in the Spatial Sciences, London Pion, 127-144.

Pietrzak M. B., Ziemkiewicz B. (2016) Considering the Use of Random Fields in Modifiable Area Unit Problem. Institute of Economic Research Working Papers, No.40.

Plotkin J. B., Potts M. D., Leslie N., Manokaran N., LaFrankie J., Ashton P. S. (2000) Species-area Curves, Spatial Aggregation, and Habitat Specialization in Tropical Forests. Journal of Theoretical Biology, 207(1), 81-99.

Poon N. M., Martins J. R. (2007) An Adaptive Approach to Constraint Aggregation using Adjoint Sensitivity Analysis. Structural and Multidisciplinary Optimization, 34(1), 61-73.

Sabesan S., Konuganti H. K. R., Perumal V. (2008) Spatial Delimitation, Forecasting and Control of Japanese Encephalitis: India-a Case Study. Open Parasitol J, 2, 59-63.

Sadahiro Y. (2011) Analysis of the Relations among Spatial Tessellations. Journal of Geographical Systems, 13(4), 373-391.

Sanchez-Gutierrez D., Tozluoglu M., Barry J. D., Pascual A., Mao Y., Escudero L. M. (2016) Fundamental Physical Cellular Constraints Drive Self-organization of Tissues. The European Molecular Biology Organization Journal, 35(1), 77-88.

Saribudak A., Kucharavy H., Hubbard K., Uyar M.U. (2016) Spatial Heterogeneity Analysis in Evaluation of Cell Viability and Apoptosis for Colorectal Cancer Cells. IEEE journal of translational engineering in health and medicine, 4, 1-9.

Sharifzadeh M., Shahabi C. (2004, November) Supporting Spatial Aggregation in Sensor Network Databases. Proceedings of the 12th Annual Association for Computing Machinery International Workshop on Geographic Information Systems, 166-175.

Springel V. (2010) E pur si muove: Galilean-invariant Cosmological Hydrodynamical Simulations on a Moving Mesh. Monthly Notices of the Royal Astronomical Society, 401(2), 791-851.

Tobler W. R. (1979) Cellular Geography. Philosophy in Geography, Springer Netherlands, 379-386.

Tobler W. R. (1989) Frame independent spatial analysis. [in:] Goodchild, M., Gopal, S. (eds.), The Accuracy of Spatial Databases. Taylor & Francis, London, 115-122.

Tzirita Zacharatou E., Doraiswamy H., Ailamaki A., Silva C. T., Freire J. (2017) GPU Rasterization for Real-Time Spatial Aggregation over Arbitrary Polygons. Proceedings of the Very Large Database Endowment, 11, 3.

Wang Y. M., Yang J. B., Xu D. L. (2005) A Preference Aggregation Method through the Estimation of Utility Intervals. Computers & Operations Research, 32(8), 2027-2049.

Weaver J. C. (1956) The County as a Spatial Average in Agricultural Geography. Geographical Review, 46(4), 536-565.

Xie W., Ouyang Y. (2015) Optimal Layout of Transshipment Facility Locations on an Infinite Homogeneous Plane. Transportation Research Part B: Methodological, 75, 74-88.

Xu Z. (2008) Dependent Uncertain Ordered Weighted Aggregation Operators. Information Fusion, 9(2), 310-316.

https://earth.boisestate.edu/drycreek/education/hypsometric/

https://www.slideshare.net/jluchette/gis-spatially-weighted-averages

http://www.mcs.anl.gov/research/projects/mct/mct_APIs/node15.html

QUANTITATIVE METHODS IN ECONOMICS
Volume XIX, No. 1, 2018, pp. 92 – 103
Received: 08.01.2018
Accepted: 05.04.2018

LONG-TERM DEPENDENCE OF HOUSING PRICES AND CONSTRUCTION COSTS IN EASTERN POLAND

Rafał Zbyrowski (ORCID: 0000-0001-8499-8899)
Faculty of Management
University of Warsaw, Poland
e-mail: rzbyrowski@wz.uw.edu.pl

Abstract: The main purpose of the study is to verify the hypothesis on the occurrence of a long-term relationship between the average prices of 1m^2 UFA (i.e. Usable Floor Area) and the construction costs of residential real estate per 1m^2 for selected 5 cities of Eastern Poland (Białystok, Kielce, Lublin, Olsztyn & Rzeszów). The panel model for cointegrated variables and panel cointegration tests are the tools of the analysis. On the basis of the econometric model constructed, the long-term elasticity coefficient, which shows how average prices of flats change (i.e. the price of 1m^2 UFA) in selected cities of Eastern Poland as a result of changes in their production costs, was estimated.

Keywords: Panel cointegration, housing prices, cost of housing construction, long-term relationship, Eastern Poland

JEL classification: C23, R31

INTRODUCTION

According to the "Social and economic development strategy for Eastern Poland until 2020" adopted by the Council of Ministers on 30 December 2008, Eastern Poland covers the following voivodeships: Lubelskie, Podlaskie, Podkarpackie, Świętokrzyskie and Warmińsko-Mazurskie. The criterion for defining the macroregion is related not only to its geographical location, but also to the economic situation. The above-listed voivodeships in 2005 had the lowest GDP per capita in the extended European Union and for this reason they were covered by the special aid from the European Union Funds ("Development of Eastern Poland 2007-2013" programme). Thus, the whole region of Eastern Poland is the area that has common economic features, although particular voivodeships have

DOI: 10.22630/MIBE.2018.19.1.9

their distinctive features. The aim of this article is to examine the hypothesis on the impact of the construction costs on pricing of residential real estate in the area with a relatively low indicator of economic growth per inhabitant. The reports of the National Bank of Poland [2016] show that there is a significant difference between the construction costs and prices of residential real estate for the largest cities in Poland. This study is an attempt to narrow down the analysed area only to 5 selected cities particularly relevant to the areas which were provided with the special support of the European Union funds. General conclusions of the study will be drawn not only on the basis of quantitative data, but also on the basis of the econometric model for panel data.

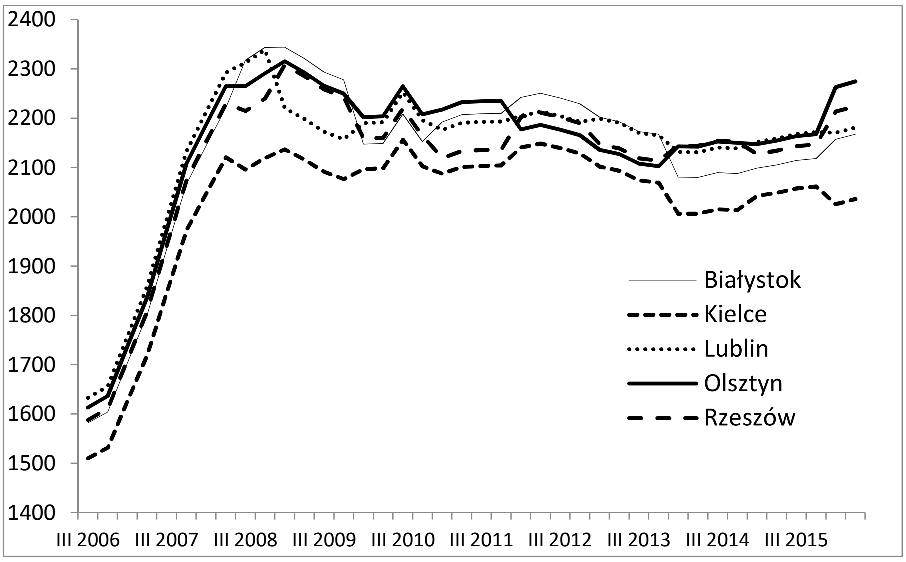
The literature review in the area of quantitative research on the real estate market leads to the conclusion that modeling in this case is not an easy task. This is mainly due to irregular economy cycles and the huge scale of the impact of local or qualitative factors. Reports on the housing market are regularly published by the National Bank of Poland [Łaszek 2016]. Usually econometric analyzes of long-term relationships may be found for banking, finance and macroeconomy [Majsterek 2014, Syczewska 1999]. On the other hand the long-term econometric analyzes are quite rare in the housing market [Zbyrowski 2017]. Incidentally in the area of operational applications, the regression models or core regression algorithms may be found for the initial or approximate valuation of residential real estate [Harney 2007]. In this context, the study of the cointegration relationship for panel data should be considered a completely new approach to analyze the real estate market.

The article contains the characteristics of the numerical data used and the analysis of data panel stationarity. Then a panel cointegration test was carried out and a long-term relationship model was built. Then the study contains conclusions regarding the long-term connections of prices with construction costs on the residential real estate market. Conclusions were formulated on the basis of the estimated panel cointegration model.

FIGURES USED IN THE STUDY

The empirical study was carried out on the basis of information made available by the National Bank of Poland (NBP) and the company Sekocenbud. Time series represent average transaction prices on the primary market of flats and construction costs of 1 square metre of UFA (i.e. Usable Floor Area). The abovementioned costs consist of the costs of building materials, work and the involvement of equipment for a building considered typical when estimating costs of the project. Time series cover the period from 3rd quarter 2006 to 2nd quarter 2016 for five selected cities of Eastern Poland (i.e. Białystok, Kielce, Lublin, Olsztyn and Rzeszów).

Figure 1. Construction costs of $1m^2$ UFA for 5 selected cities of Eastern Poland

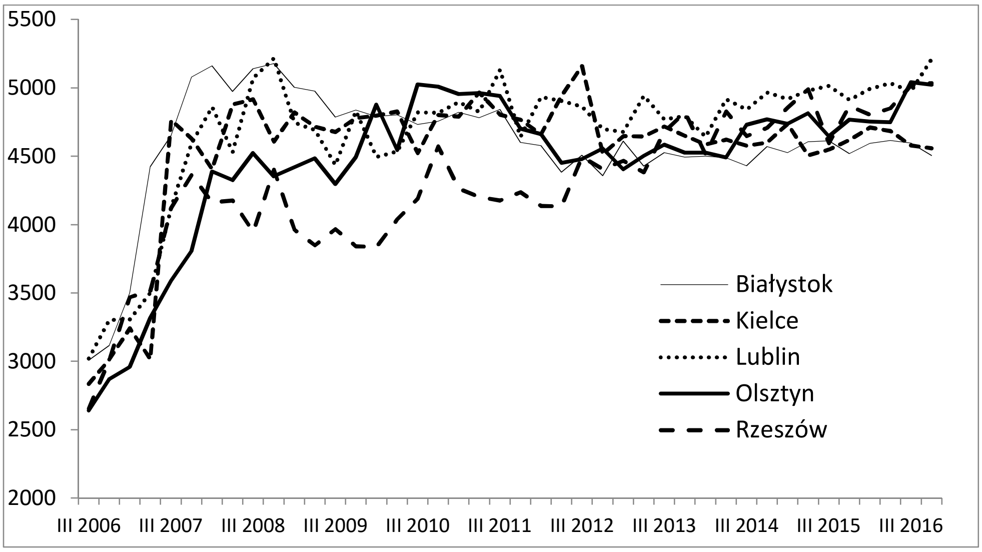


Source: own study on the basis of the NBP data and Sekocenbud report

On the basis of Figure 1 it can be observed that the $1m^2$ UFA construction costs had a similar course in the analysed period for all five cities in Eastern Poland. In the first years of the sample, the costs of construction grew dynamically, especially in 2007 and 2008. After 2008, the production cost of $1m^2$ UFA was over PLN 2,000, excluding expenses related to the purchase of a plot, paying taxes, preparing designs, land development, making connections and other expenses related to management and marketing. The construction cost covered in the study mainly includes the costs of building materials, equipment and work. The purchase of a plot by a developer actually increases costs by several hundred to over one thousand zlotys per 1 square metre. Thus, the land value is an important additional component of the price of each flat. However, development companies invest in land for development even several years before the construction starts, and the price of the plot depends, of course, on the specifics of the investment and location. In particularly attractive locations the prices of plots are extremely high, but in this case it is to be expected that the price per $1m^2$ UFA will be at least commensurate.

A graph of empirical data over time suggests that the lowest construction costs are in Kielce. In other analysed cities the $1m^2$ UFA production costs are very similar (Figure 1).

Figure 2. Prices of 1m² UFA for 5 selected cities of Eastern Poland



Source: own study on the basis of the NBP data and Sekocenbud report

Prices of 1m² UFA during the first few years of the analysed period grew rapidly, as did their production costs (Figures 1 and 2). In the short term, significant price fluctuations are visible, and in fact only Rzeszów had slightly lower prices in the period from the beginning of 2008 to the end of 2012. From the beginning of 2013, the average prices per 1m² UFA for all examined cities fluctuated from PLN 4,500 to around PLN 5,000. Since the beginning of 2011, Lublin is the only city that had slightly more expensive flats per square metre (Figure 2).

Testing the stationarity of data-generating processes:

Levin and Lin developed a whole group of panel stationary tests based on the Dickey-Fuller test [Dickey & Fuller 1979]. Panel data-generating process can be written in the following form [Strzała 2008]:

$$y_{it} = \propto_i + \delta_i t + \varphi_i y_{i,t-1} + \theta_t + \varepsilon_{it}, \quad i = 1, 2, \dots, N; \; t = 1, 2, \dots, T \tag{1}$$

where: $\varepsilon_{it} \sim \text{i.i.d.}(0, \sigma_\varepsilon^2)$ (2)

The model of form (1) and (2) contains varied individual effects $(\propto_i)$, differentiated linear trend (δ_i), heterogeneous autoregressive parameter (φ_i), and time effects (θ_t). The study of stationarity is related to the hypotheses concerning parameter (φ_i). Hypotheses of the stationarity test for panel data are generally written as follows [Levin & Lin 1992]:

$$H_0: \; (\varphi_i - 1) = \rho_i = 0; \; y_{it} \sim I(1) \tag{3}$$

$$H_A: \; (\varphi_i - 1) = \rho_i < 0; \; y_{it} \sim I(0) \tag{4}$$

Both variables subject to modeling in the further part of the article are integrated of order two. The non-stationarity of the series on the levels and the stationarity of their first differences are confirmed by the test results in Tables 1 to 4. Panel stationarity tests were performed for logarised data.

Table 1. Stationarity test of quarterly natural logarithm series of 1m^2 UFA transaction prices

Variable: ln (Price 1m^2 UFA)				
Testing method	Test statistic	p-value	number of cross-sectional units	number of observations
Levin, Lin & Chu t	1.798	0.964	5	182
Null hypothesis: Widespread occurrence of the unit root (for panel data)				
Testing method	Test statistic	p-value	number of cross-sectional units	number of observations
ADF - Fisher Chi2	1.472	0.999	5	182
PP - Fisher Chi2	0.611	1.000	5	195
Null hypothesis: Occurrence of the unit root for individual processes				

Source: own study in the Eviews programme

Testing of logarised time series of 1m^2 UFA transaction prices results in obtaining relatively low test values of the Levin, Lin & Chu and ADF (Augmented Dickey-Fuller) and PP (Phillips Perron) tests. All tests carried out verify the null hypothesis predicting the occurrence of a unit root in the data-generating process (Table 1).

Table 2. Stationarity test of quarterly increments of natural logarithm series of 1m^2 UFA transaction prices

Variable: D [ln (Price 1m2 UFA)]				
Testing method	Test statistic	p-value	number of cross-sectional units	number of observations
Levin, Lin & Chu t	-12.048	0.000	5	181
Null hypothesis: Widespread occurrence of the unit root (for panel data)				
Testing method	Test statistic	p-value	number of cross-sectional units	number of observations
ADF - Fisher Chi2	150.949	0.000	5	181
PP - Fisher Chi2	185.617	0.000	5	190
Null hypothesis: Occurrence of the unit root for individual processes				

Source: own study in the Eviews programme

Table 2 contains the results of Levin, Lin & Chu and ADF (Augmented Dickey-Fuller) and PP (Phillips Perron) tests estimated for the first logarithm differences of the natural series of 1m^2 UFA transaction prices. This time, tests checks take high absolute values, on the basis of which the null hypotheses should be rejected with a low probability of making the first type error (where “p-value” means “empirical level of significance” [Gajda 2004]).

Table 3. Stationarity test of quarterly natural logarithm series of 1m^2 UFA construction costs

Variable: ln (Cost 1m^2 UFA)				
Testing method	Test statistic	p-value	number of cross-sectional units	number of observations
Levin, Lin & Chu t	2.133	0.983	5	184
Null hypothesis: Widespread occurrence of the unit root (for panel data)				
Testing method	Test statistic	p-value	number of cross-sectional units	number of observations
ADF - Fisher Chi2	1.478	0.999	5	184
PP - Fisher Chi2	0.722	1.000	5	195
Null hypothesis: Occurrence of the unit root for individual processes				

Source: own study in the Eviews programme

Table 3 confirms the non-stationarity on the levels of natural logarithm time series of 1m^2 UFA construction costs. The null hypothesis on the occurrence of a unit root cannot be rejected due to the low values of the Levin, Lin & Chu and ADF and PP tests (Table 3).

Table 4. Stationarity test of quarterly increments of natural logarithm series of 1m^2 UFA construction costs

Variable: D [ln(Cost 1m^2 UFA)]				
Testing method	Test statistic	p-value	number of cross-sectional units	number of observations
Levin, Lin & Chu t	-6.752	0.000	5	187
Null hypothesis: Widespread occurrence of the unit root (for panel data)				
Testing method	Test statistic	p-value	number of cross-sectional units	number of observations
ADF - Fisher Chi2	57.303	0.000	5	187
PP - Fisher Chi2	47.451	0.000	5	190
Null hypothesis: Occurrence of the unit root for individual processes				

Source: own study in the Eviews programme

Table 4 confirms the stationarity of the increments of natural logarithm time series of 1m^2 UFA construction costs. The null hypotheses about the occurrence of the unit root should be rejected due to the very high values obtained in the Levin, Lin & Chu, ADF and PP tests.

The tests carried out in a panel and individual approach confirm the integration of natural logarithms of the first order time series analysed.

Testing the long-term dependence of 1m^2 UFA prices and construction costs.

In order to verify the hypothesis about the occurrence of a long-term dependence between the studied variables, the Pedroni panel cointegration tests were used [Strzała 2012]. These tests are modeled on the Engle-Granger procedure and have different variants (Pedroni developed 11 tests of the cointegrated panel test). The null hypothesis for Pedroni tests provides for a lack of cointegration between variables. Whereas the alternative hypothesis of panel cointegration tests predicts the existence of cointegration at the group level (between-dimension, group statistics test) or for the whole panel (within-dimension, panel statistics test).

The procedure of applying Pedroni cointegration tests is divided into two stages [Strzała 2012]:

1. In the first stage, the parameters of the general panel model are estimated in the following form:
$$y_{it} = \propto_i + \partial_i t + \beta_{1i}x_{1i,t} + \beta_{2i}x_{2i,t} + \cdots + \beta_{Ki}x_{Ki,t} + e_{it} \tag{5}$$
for $t = 1, 2, \ldots, N; t = 1, 2, \ldots, T; k = 1, 2, \ldots, K$ and $(y_{it}, x_{it}) \sim I$
The above equation (5) contains varied individual effects $\propto_i$, individual values of trend slope factors $\partial_i t$ and parameters β_{1i}.

2. In the second stage, the stationarity of the residuals should be investigated using auxiliary regressions for individual panel units. The auxiliary equations should be estimated as joint regressions in the following form:
$$\hat{e}_{it} = \rho_i \hat{e}_{it-1} + v_{it} \tag{6}$$
or in the extended form:
$$\hat{e}_{it} = \tilde{\rho}_i \hat{e}_{it-1} + \sum_{j=1}^{p_i} \varphi_{ij} \Delta \hat{e}_{it-j} + v_{it} \tag{7}$$
where: $\hat{e}_{it}$ – is a series of model residuals (5); $v_{it} \sim i.i.d.(0, \sigma_v^2)$ – is a random component of the auxiliary equation (joint regression) of the form (6) or (7).

The null hypothesis of the test assumes that the parameters in the auxiliary equations for individual units are equal to unity:
$$H_0: \rho_1 = \rho_2 = \cdots = \rho_N = 1 \tag{8}$$
In the Pedroni tests, two types of alternative hypotheses are found.

1. For a panel test, i.e. within-dimension, panel statistics test, when we assume parameter homogeneity ρ_i:
$$H_A: \rho_1 = \rho_2 = \cdots = \rho_N < 1 \tag{9}$$
2. For the between-dimension, group statistics test, when we assume the parameter heterogeneity ρ_i:
$$H_A: \rho_i < 1 \text{dla i=1,2,…,N} \tag{10}$$
with the assumption that $\lim_{N \to \infty} (m/N) = \delta, \quad 0 < \delta \leq 1$

On the basis of the auxiliary regression (6) or (7) residuals, Pedroni panel statistics $Z_{N,T}$ is determined, which in a standardised form is asymptotically convergent to the normal distribution [Pedroni 2004]:
$$\frac{Z_{N,T} - \mu\sqrt{N}}{\sqrt{\omega}} \to N(0.1) \tag{11}$$

The values of μ and ω were determined as a result of the Monte Carlo simulation. The variant of a homogeneous alternative hypothesis requires separate determination of average values of numerators and denominators of DF and ADF statistics for panel cross-sectional units. Then, the quotient of the values thus obtained should be standardised in accordance with the formula (11) on the basis of adjusted coefficients set out in the 1999 Pedroni article. In the case of a heterogeneous alternative hypothesis (Pedroni group-statistics test), DF or ADF statistics for individual panel units are averaged, which should then be standardised in accordance with the formula (11) [Strzała 2012].

Table 5. Cointegration test of quarterly price and cost series of $1m^2$ UFA for the region of Eastern Poland

Variables: (Price $1m^2$ UFA) and (Cost $1m^2$ UFA)		
Null hypothesis: no cointegration		
Alternative hypothesis: Panel cointegration (within-dimension)		
Testing method	Test statistic	p-value
Panel v-Statistic	2.697	0.049
Panel rho-Statistic	-4.792	0.000
Panel PP-Statistic	-4.091	0.000
Panel ADF-Statistic	-2.525	0.002
Alternative hypothesis: Panel cointegration (between-dimension)		
Testing method	Test statistic	p-value
Group rho-Statistic	-3.701	0.000
Group PP-Statistic	-5.428	0.000
Group ADF-Statistic	-3.295	0.001

Source: own study in the Eviews programme

Table 5 contains the high absolute values of the Pedroni cointegration tests. Hence, regardless of the type of test and the type of alternative hypothesis, the null hypothesis that there is no cointegration between the analysed variables should be rejected. Therefore, the long-term dependence between prices and construction costs of $1m^2$ UFA is in fact confirmed in the examined cities of Eastern Poland. In the studied region, developers are willing to take into account higher construction costs at transaction prices, transferring production costs to the final purchaser. In the further part of the study, a model of panel cointegration which shows the scale of transmission of construction costs to transaction prices using the long-term elasticity coefficient was estimated. The long-term equilibrium model was estimated using the FMOLS method (Fully Modified Least Squares, Phillips and Moon, 1999). It is characterised by moderate adjustment to empirical data. The adjusted coefficient of determination of 0.66 allows concluding that approximately 66% of the volatility of the $1m^2$ UFA price can be explained by the variability of the $1m^2$ UFA construction cost for the five cities studied located in the region of Eastern Poland (Enders, 2003). The model's fit coefficient is not high. However, the estimation of long-term elasticity coefficient of 1.315 with an error of 0.126 results in a high value of t-Student statistics of 10.43, which clearly confirms the statistical significance of the estimation.

Naturally, the 1m^2 UFA prices depend in practice on many factors. The purpose of this study is solely to analyse the relationship between the price and the construction costs of 1m^2 UFA. Although the tested relationship does not seem to be controversial in economic terms, the specificity of the real estate market, which is very difficult to model and is often determined by qualitative factors that effectively disrupt its regularity, should be taken into account.

The standard deviation of model disruptions is low (0.06661), since it represents less than 1% of the arithmetic mean value of the natural logarithm of the dependent variable of 8.413 (Borkowski, Dudek, Szczesny, 2007).

Dependent variable: $Price_{it}$ – transaction price of 1m^2 UFA on the primary market for the city and at the time t;
Independent variable: $Cost_{it}$ – the construction cost of 1m^2 UFA (without the cost of land purchase) for the city and at the time t.

Table 6. Model of 1m^2 UFA price and construction costs dependence for five selected cities of Eastern Poland

Dependent variable: Ln(CENA)				
Method: Panel Fully Modified Least Squares (FMOLS)				
Sample (adjusted): 2006Q4 2016Q2				
Periods included: 39				
Cross-sections included: 5				
Total panel (balanced) observations: 195				
Cointegration equation deterministics: C				
Variable	Coefficient	Std. Error	t-Statistic	p-value
Ln(KOSZT)	1.315132	0.126088	10.43031	0.0000
R-squared	0.668932	Mean dependent var		8.4131
Adjusted R-squared	0.660173	S.D. dependent var		0.1143
S.E. of regression	0.066611	Sum squared resid		0.8286
Long-run variance	0.015290			

Source: own study in the Eviews programme

On the basis of Table 6, the long-term dependence model can be written as follows:

$$\ln(\widehat{\mathrm{CENA}}_{\mathrm{it}}) = 1.315 * \ln(\mathrm{COST}_{\mathrm{it}}) + c_i \quad (12)$$

The model shows that if the construction costs of 1m^2 UFA increase by 1%, the transaction prices of 1m^2 UFA on the primary market will also increase on average by 1.315% over a long period of time with the ceteris paribus assumption [Gajda 2004]. The model (12) contains in the equation a constant value different for the five examined cities c_i (a deterministic element of the cointegration vector) which allows taking into account a fundamental part of the local heterogeneity [Maddala 2006]. The estimation of the parameter of 1.315 suggests that in cities of Eastern Poland there may be a highly flexible reaction of 1m^2 UFA price changes to changes in their production costs. It should be noted that the prices of residential real estate in the cities of Eastern Poland are quite low compared to other agglomerations of the country. On the one hand, it seems justified if GDP per capita of Eastern Poland is also record low. On the other hand, the study shows that in the case of an increase in production costs, transaction prices increase even faster in the east of our country. This may result from the fact that developers want to achieve additional profit as they are aware of higher flat prices on the national market. Another explanation of a high elasticity may be a long investment process. The construction of a complex of many flats takes years – if construction costs increase, it is expected that they will increase further during the investment period, which is reflected in the cost estimates. Developers therefore assume a greater margin of security for their investments so as not to incur losses, and this may stimulate further price increases.

SUMMARY

1. The analysed figures indicate very similar fluctuations in the 1m^2 UFA construction costs in all examined cities, with noticeably lower costs in Kielce compared to the remaining four. The 1m^2 UFA prices are more diversified in the short-run than the construction costs. Rzeszów definitely had the lowest UFA price per 1 square metre in 2008-2012, however, in the long run it joined the average for the entire examined group of cities in Eastern Poland. It can be said that Białystok, Kielce, Lublin, Olsztyn and Rzeszów form a homogeneous group in terms of the analysis of the residential real estate market.
2. On the basis of the panel cointegration tests performed, the existence of a long-term relationship between construction costs and the 1m^2 UFA price in all examined cities is confirmed. As already mentioned, although the dependence of the price on costs does not seem to be controversial in economic terms, in the case of the housing market, the differences between these two categories are counted in thousands of zlotys per square metre. Whereas, the housing market is subject to speculation and disruptions that may in practice hinder the impact of fundamental factors.
3. Estimation of the long-term elasticity coefficient equal to 1.315 suggests that in the cities of the eastern region, 1m^2 UFA prices on the primary market may on average increase faster than construction costs. Therefore, relatively low prices for 1m^2 in this region in comparison to other parts of the country may show a tendency to reduce the undervaluation of residential real estate in cities such as Białystok, Kielce, Lublin, Olsztyn and Rzeszów.

The area of future research will include farther econometric analysis of residential property prices in Poland. In particular, the author plans to carry out studies to estimate the scale of impact on the price level of many other exogenous factors.

REFERENCES

Borkowski B., Dudek H., Szczesny W. (2007) Ekonometria, wybrane zagadnienia. PWN, Warszawa (in Polish).

Dickey D. A., Fuller W. A. (1979) Distribution of the Estimators for Autoregressive Time Series With a Unit Root. Journal of the American Statistical Association, 74, 427-431.

Enders W. (2003) Applied Econometric Time Series. John Wiley & Sons, New York.

Gajda J. (2004) Ekonometria. C.H. BECK, Warszawa (in Polish).

Harney K. (2007) Reprisals on Appraisals. Washington Post.

Levin A., Lin Ch-F. (1992) Unit Root Test in Panel Data: Asymptotic and Finite Sample Properties. Working Paper, no. 92-23.

Levin A., Lin Ch-F. (1993) Unit Root Tests in Panel Data: New Results. Working Paper, no. 93-56.

Levin A., Lin Ch-F., Chu Ch-Sh. J. (2002) Unit root tests in panel data: asymptotic and finite sample properties. Journal of Econometrics, 108, 1-24.

Łaszek J. (2006) Rynek nieruchomości mieszkaniowych i jego specyfika. [w:] Kucharska-Stasiak E. (red.) Ryzyka banku w zakresie określania wartości nieruchomości dla celów kredytowych w Polsce na tle trendów w Unii Europejskiej, Fundacja na Rzecz Kredytu Hipotecznego, Warszawa (in Polish).

Łaszek J. (2016) Raport AMRON-SARFiN 4/2015. Ogólnopolski raport o kredytach mieszkaniowych i cenach transakcyjnych nieruchomości - komentarz do raportu. Warszawa (in Polish).

Maddala G. S. (2006) Ekonometria. PWN, Warszawa (in Polish).

Majsterek M. (2014) Modelowanie systemów skointegrowanych. Aspekty teoretyczne. Bank i Kredyt, 45(5), 433-466 (in Polish).

Narodowy Bank Polski (2015) Informacja o cenach i sytuacji na rynku nieruchomości mieszkaniowych i komercyjnych w Polsce w III kwartale 2014 roku. Warszawa (in Polish).

Narodowy Bank Polski (2016) Raport o sytuacji na rynkach nieruchomości mieszkaniowych i komercyjnych w Polsce w 2015 roku. Warszawa (in Polish).

Pedroni P. (1999) Critical Values for Cointegration Tests in Heterogeneous Panels with Multiple Regressors. Oxford Bulletin of Economics and Statistics, 61, 653-670.

Pedroni P. (2004) Panel Cointegration: Asymptotic and Finite Sample Properties of Pooled Time Series Tests with an Application to the PPP Hypothesis. Econometric Theory, 20, 597-625.

Phillips P. C. B., Moon H. R. (1999) Linear Regression Limit Theory for Nonstationary Panel Data. Econometrica, 67, 1057–1111.

Strzała K. (2009) Panelowe testy stacjonarności - możliwości i ograniczenia. Przegląd Statystyczny, 56(1), 56-73 (in Polish).

Strzała K. (2012) Panelowe testy kointegracji - teoria i zastosowania. Roczniki Kolegium Analiz Ekonomicznych, 27, 41-54 (in Polish).

Syczewska E. (1999) Analiza relacji długookresowych: estymacja i weryfikacja. Monografie i opracowania, Szkoła Główna Handlowa, Warszawa (in Polish).

Zbyrowski R. (2017) The Long Term Modeling of Residential Property Prices in Poland. Quantitative Methods in Economics, 18(1), 143-156.

Documents

„Strategia rozwoju społeczno-gospodarczego Polski Wschodniej do roku 2020", dokument przyjęty przez Radę Ministrów w dniu 30 grudnia 2008 r., uchwałą nr 278/08 (in Polish).

„Program Operacyjny Rozwój Polski Wschodniej. Narodowe Strategiczne Ramy Odniesienia 2007-2013". Wersja dokumentu po zmianach zaakceptowanych uchwałą nr 94/2011 Rady Ministrów z dnia 6 czerwca 2011 (in Polish).