METODY ILOŚCIOWE
W BADANIACH EKONOMICZNYCH

QUANTITATIVE METHODS
IN ECONOMICS

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GLOBALISATION AND WORLD ECONOMIC POVERTY: THE SIGNIFICANCE OF HIDDEN DIMENSIONS

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Abstract: The aim of our research is to examine how individual dimensions of globalization affect economic poverty in the World. For this, regression models are estimated with FGT_0 or FGT_1 poverty measures as dependent variables and KOF indices of globalization as dependent variables. The poverty indices are estimated for 119 countries' income distributions assuming log-normality and using Gini estimates from the WID2 database and GDP/capita from The World Bank database for the years 1990-2005. It has turned out that the "partial" impact of selected dimension of globalization on poverty is either linear or nonlinear, ceteris paribus. The nonlinear impact is of the U-shaped form or the inverted U-shaped form. Our results contradict some typical 'linear' findings when poverty measures are regressed only on one dimension of globalization. In other words, when some crucial dimensions of globalization are neglected in regression analysis the conclusions about impact of globalization on world poverty could be misleading.

Keywords: globalisation, poverty, world distribution of incomes

INTRODUCTION

The aim of the study is to obtain an answer to the question: Does globalisation and its components (dimensions) affect economic poverty in the world, and if so, in what manner?

Assuming that poverty is identified on the basis of an absolute poverty line, the above issue is explained by the following two hypotheses:

Hypothesis 1. The higher the general level of globalisation, the smaller the scale and depth of world economic poverty.

Hypothesis 2. The multi-dimensional approach reveals that the growth of globalisation in a single dimension initially reduces, and subsequently increases both the scale and depth of poverty, *ceteris paribus*.

Assuming that poverty is identified on the basis of a relative poverty line, the above-stated questions may be answered as follows:

Hypothesis 3: As globalisation grows, the scale and depth of world poverty progress along a bell-shaped curve.

Hypothesis 4. The multi-dimensional approach reveals that the influence of globalisation in single dimensions on the scale and depth of poverty is either linear or non-linear (U-shaped or bell-shaped), *ceteris paribus*.

The studies are encouraged by a number of circumstances. In political disputes between the supporters and opponents of globalisation, poverty assessments are important arguments. Moreover, it turns out that the evaluations of the influence of globalisation on world poverty tend to be radically different.

Ravallion (2010) points to three major causes of these discrepancies. The first one is the ambiguity of poverty identification, resulting from the use of different poverty lines, and the second one is the incomparability of the used statistical data in the analysis of poverty. The third cause, according to Ravallion, is the diversity of methodologies in studying poverty and globalisation.

The analysis presented in this paper reveals an additional cause consisting in the omission of important dimensions of globalisation. If a researcher is interested in the influence of just one selected dimension of globalisation on poverty, they may arrive at completely different conclusions than the ones they would have reached by taking into account more dimensions. The problem is well known to econometrics [Maddala, 2008, pp. 199-201].

The further structure of the paper is as follows. The second part provides a description of the methods used and the sources of data. The third part presents the results of empirical research on the relationship between economic poverty and dimensions of globalisation. Part four of the paper contains conclusions.

RESEARCH METHODOLOGY AND SOURCES OF STATISTICAL DATA

The fundamental difficulty in studying the influence of globalisation on economic poverty is the absence of data concerning national revenue distribution, allowing the estimation of poverty measures. The World Bank does publish evaluations of these indexes, but they are hardly credible, as they are calculated using incomparable statistical data. For this reason, the presented study employs the author's own approach, in order to obtain credible assessments of poverty in the global distribution of revenues.

It may be assumed that revenues in individual countries are subject to two-parameter log-normal distribution $\Lambda(\mu,\sigma)$ [Sala-i-Martin, 2006]. The unknown parameters μ and σ may be estimated on the basis of identity:

$$\bar{x} = \exp\left\{\mu + \frac{\sigma^2}{2}\right\} \tag{1}$$

$$G = 2\Phi\left(\frac{\sigma}{\sqrt{2}}\right) - 1 \tag{2}$$

[Kleiber, Kotz, 2003, p. 117], where G refers to Gini index, $\Phi(\cdot)$ refers to the standardised normal distribution, and mean $\bar{x}$ refers to GDP per person.

As poverty measures, two indexes FGT_0 and FGT_1 will be employed, from the FGT_α family, defined as follows:

$$FGT_\alpha = \frac{1}{n}\sum_{j=1}^{n}\left(\frac{z - x_j}{z}\right)^\alpha I(x_j < z) \tag{3}$$

with z referring to the poverty line, $x_1,\dots,x_n$ to revenues in a society composed of n persons, and the indicator function $I(x_j<z)$ amounts to 1 if the condition $x_j < z$ is met, and zero, if the condition is not met [Foster, Greer, Thornbecke, 1984].

If $\alpha = 0, FGT_0$ equals the fraction of the poor, and measures the scale of poverty. On the other hand, if $\alpha = 1, FGT_1$ measures the depth of poverty or the degree of impoverishment of the society as a whole.

It can be easily show that in a log-normal distribution, the measures of poverty FGT_0 and FGT_1 respectively amount to:

$$FGT_0 = \Phi\left(\frac{\ln z - \mu}{\sigma}\right) \tag{4}$$

$$FGT_1 = \Phi\left(\frac{\ln z - \mu}{\sigma}\right) - \frac{\bar{x}}{z}\cdot\Phi\left(\frac{\ln z - \mu - \sigma}{\sigma}\right) \tag{5}$$

After estimating the poverty measures FGT_0 (4) and FGT_1 (5) with respect to each country separately, the scale and depth of poverty in the global revenue distribution will be calculated as weighted means, weights being the given country's share in world population.

Gini index evaluations were mostly obtained from the WIID2 database. In order to achieve the maximum comparability, the analysis was limited to countries where the Gini index was estimated on the basis of revenue data obtained from surveys covering the entire territory and population of a given country. In the case of countries where time series of Gini indexes contained gaps, interpolations were performed using polynomials of the adequate-order.

Eventually, 119 were selected for the analysis[1]. The research period covered the years 1990-2005. The total number of people living in the selected countries under analysis was a very large part of the world population as a whole, in the order of 88-93%.

The data on GDP per capita were taken from World Development Indicators reports 1990-2005. They are expressed in international USD (purchasing power parity included) in fixed prices for 2005.

Two versions of poverty lines were adopted as bases for calculating poverty indexes. In version *a* it was the absolute poverty line z = 2$per day per person (730$ per day per person). In version *b* it was the relative poverty line amounting to half of the world's average revenue: z = GDP per capita/2. Correspondingly, poverty indexes FGT_{0a}, FGT_{0b} were calculated, measuring the scale of poverty as a percentage of the poor, and indexes FGT_{1a}, FGT_{1b}, measuring the depth of poverty, or impoverishment of society as a whole.

Globalisation in world countries was measured using KOF indexes presented in reports for 1990-2005 [Dreher et al. 2008]. The following symbols were adopted:

OGI - Overall Globalisation Index

Economic globalisation indices:
- *AFL- Actual Flows,*
- *RES- Capital Account Restrictions,*

Social globalisation indices:
- *PER -Personal Contacts,*
- *INF -Information Flows,*
- *CUL-Cultural Proximity,*

Political globalisation index:
- *POL*

Globalisation indices concerning the world as a whole were calculated as unweighted arithmetic means.

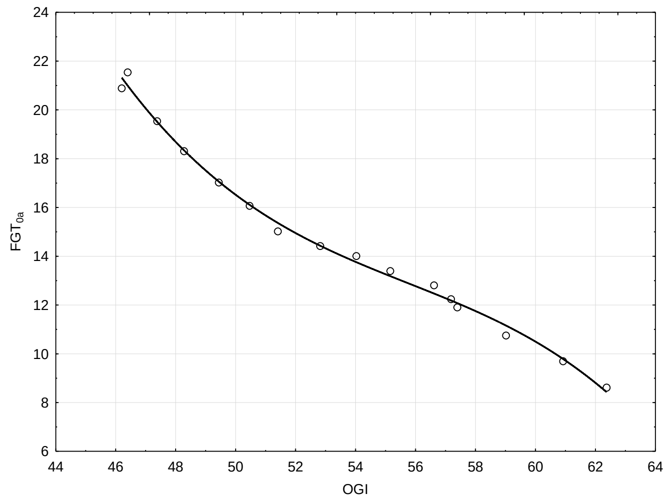
RESULTS

The choice of the poverty line (absolute or relative) may determine the assessment of the influence of globalisation on the scale of world economic poverty. Using the United Nations' absolute poverty standard, designed mainly for the analysis of developing countries, shows that as globalisation progresses, the scale of world economic poverty decreases (Fig. 1) A totally different conclusion

[1] A detailed description of the data interpolation process and the produced results were presented in a study by Kot and Adamkiewicz-Drwiłło (2013).

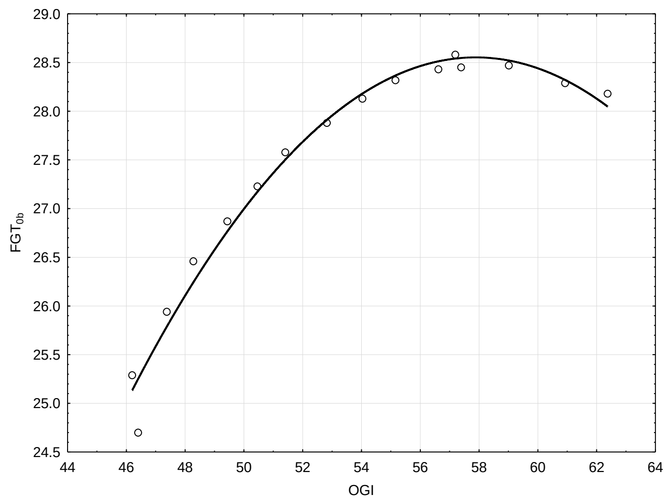
may be reached when using the relative poverty line applied to developed countries (Fig. 2).

Figure 1. Globalisation and the scale of world poverty (absolute poverty line)



Source: own study

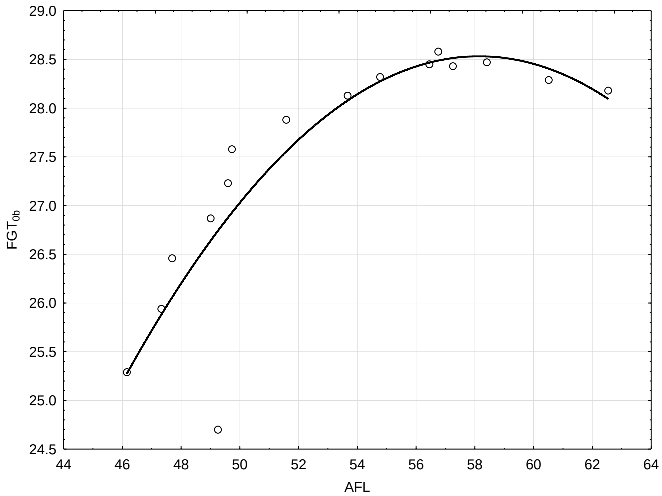
Figure 2. Globalisation and the scale of world poverty (relative poverty line)



Source: own study

Figure 3 illustrates the influence of actual financial leverage (AFL) on the scale of world poverty (relative poverty line).

Figure 3. Influence of a single dimension of globalization on the scale of poverty



Source: own study

It is evident that facilitating cash flows initially leads to the increase of the scale of poverty, and next to its drop when the AFL index rises above 58, *ceteris paribus*.

In order to obtain the correct image of the discussed relationships, the parameters of the poverty measure regression function were estimated against specific dimensions of globalisation. The results obtained using the method of backward stepwise regression are shown in Table 1.

Table 1. FGT_{0a} as RES function

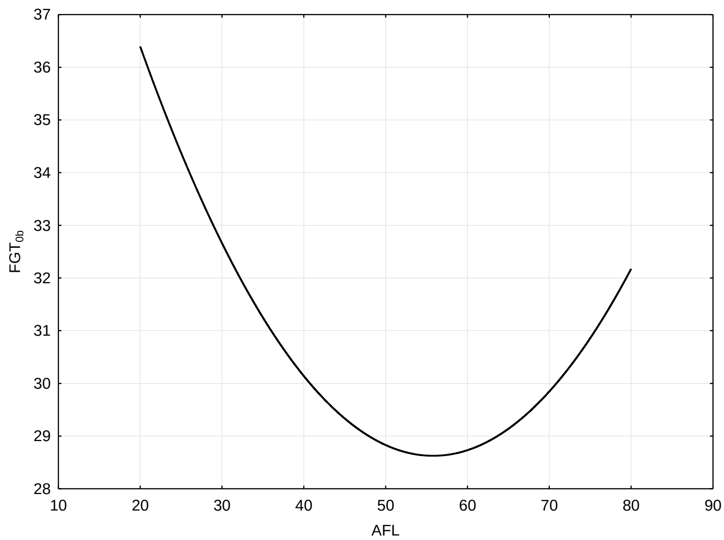
Variable	FGT_{0a}	FGT_{1a}	FGT_{0b}	FGT_{1b}
const	102.8290 (7.006647)	29.51880 (3.131659)	-9.08878 (2.470877)	-8.64788 (1.820636)
AFL			-0.67639 (0.069983)	-0.41988 (0.051566)
AFL^2			0.00606 (0.000678)	0.00391 (0.000499)
RES	-2.2278 (0.235223)	-0.49059 (0.105134)		
RES^2	0.0174 (0.002534)	0.00361 (0.001132)		
PER			-0.22257 (0.023819)	-0.14108 (0.017550)
INF	0.1093 (0.028410)	0.03984 (0.012698)	0.64179 (0.040387)	0.40960 (0.029758)

Variable	FGT_{0a}	FGT_{1a}	FGT_{0b}	FGT_{1b}
INF^2			-0.00540	-0.00357
			(0.000348)	(0.000257)
CUL	-0.3356	-0.13642	-0.24359	-0.24400
	(0.064151)	(0.028672)	(0.068256)	(0.050294)
CUL^2			0.00486	0.00431
			(0.001018)	(0.000750)
POL			1.41230	0.80691
			(0.084820)	(0.062498)
POL^2	-0.0029	-0.00122	-0.00999	-0.00585
	(0.000383)	(0.000171)	(0.000666)	(0.000491)
R^2	0.9982	0.9976	0.9999	0.9998

Source: own study

The presented results show that in the model for FGT_{Ob}, the AFL index appears in the form of a second-degree polynomial next to four other indexes that are also in the form of second-degree polynomials, except for the PER index reflecting international personal contacts. When these additional variables have average values, fractional influence of AFL on the scale of poverty will be illustrated by a parabola described by the following equation: $FGT_{0b} = 0.00606AFL^2 -0.67639\,AFL+47.5$. The function is presented in Fig.4.

Figure 4. Fractional influence of a single dimension of globalization on the scale of poverty



Source: own study

The above figure reveals the relationship between AFL and poverty that is completely different that in Fig. 3. Facilitation of cash flows between countries is initially accompanied by the decrease of world economic poverty, and subsequently by its rise, after AFL exceeds the value of 58.

FINAL CONCLUSIONS

The study produced a few important findings:

- Individual dimensions of globalisation have a varied influence on the scale and depth of world economic poverty. Results given in Table 1 show that for each poverty line, there is a separate set of factors. Nevertheless, within a given poverty line, the same sets of factors determine both the scale, and the depth of poverty.
- The influence of globalisation – in its individual dimensions – on the scale and depth of poverty is non-linear.
- Disregarding important dimensions of globalisation may lead to incorrect conclusions about its influence on world economic poverty.
- All four hypotheses presented in the paper have been confirmed.

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HUMAN RESOURCES ANALYSIS USING A DECISION SUPPORT SYSTEM

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Abstract: This paper presents the results of the analysis of labour resources in the sample manufacturing company. The process consisted of: scheduling, grouping, and assessment of personnel taking into account the criteria and preferences of executives. Solutions were obtained using the decision support system DSS 2.0.

Keywords: Analysis of human resources, ranking, clustering, decision support system.

INTRODUCTION

In the modern managed organization, managing people plays an important role. The management of such capital requires the proper use of knowledge, skills and talent and motivation for a better job, change and progress. Properly trained and competent staff is dynamic and active part of the resources of each company. Resource management work should be continuously improved since their quality is depreciated. [Penc 2007]

This article aims to analyse manual workers (fitters) from a sample production company. The scope of research is subject to DSS 2.0 system functions and is designed to highlight its relevance for supporting personnel and payroll decisions. The experiment takes into account the criteria and preferences of executives. The analysis includes: ranking of employees and the distribution of bonus fund, evaluation of the structure of the crew and the individual evaluation of the results compared to the level of wages. In addition, through simulation, a marginal increase in the value of the criteria for the crewmembers who would like to be included among the best is sought for.

MANAGEMENT OF HUMAN RESOURCES IN ORGANIZATION

The issues related to the acquisition of the right people and the effective use of their knowledge and skills to fulfil the objectives of the organization have long been the subject of research and practical solutions. Today, discussion of the place and role of the people in the organization defines the approach defined as the management of human resources. [Pocztowski 2008]

According to A. Pocztowski, people are the most important resource of the organization, and effective management is the key to success [Pocztowski 2000]. The concept of human resource management is understood strategically and a coherent approach to the management of organization's most valuable assets, which are employees, who individually and collectively contribute to the achievement of the objectives of the organization [Armstrong 2005]. On the other hand, personnel management is a philosophy that defines the way in which staff should be treated that such measures are consistent with the interests of the organization. Therefore, the approach can be divided into the „hard" and „soft" ones in human resource management. „Hard" approach emphasizes quantitative, strategic, business and spread sheet aspects. The staff is treated like other resources at the disposal of the organization and the organization invests in it as in the new technology [Armstrong 2005], [Król, Ludwiczyński 2006]. However, in terms of „soft" approach, the management is communication, leadership and motivation. The approach of the staff is more subjective than the subject. Great importance is attached to the culture and the employees who are members of one team, strive to achieve the goals set by the company. Most organizations, however, use the combination of „hard" and „soft" human resource management policies [Armstrong 2005].

Human resource function is the most important function of any organization. It contains all the activities of the personnel working in the company. The management of personnel consists of HR processes whose distribution depends inter alia on the size of the organization, its organizational structure, location, management philosophy and business. Among the key HR processes running in the company the following must be distinguished: beginning of work, work, and end of work [Pocztowski 2008].

EVALUATION OF WORK IN THE ORGANIZATION

An important function of personnel management is to evaluate employees, which consists of an expression in the form of oral or written evaluative view of their personal characteristics, behaviour and effects of work. Assessment of work can be done on a regular basis by supervisors or co-workers for example, and using a pre-prepared often formalized system of periodic evaluations of employees[Pocztowski 2008]. The main modules of the system evaluation of employees include: assessment purposes, methods of application, subject and object, the criteria, methods and techniques [Encyclopaedia of ... 1981].

One can distinguish the following evaluation of objectives:

- administration, the results are used to shape the personnel policy relating to: the hiring of workers, movement of employees within the company and compensation of employees;
- Information, data on employees' work are transmitted to managers, and information about the weaknesses and strengths to employees;
- motivation, the intention is to provide employees with information to motivate them to develop their personal skills and improve the efficiency of their work. [Anderson, 1993]

The technique of assessment, which is defined as the intentional way of making employee evaluations in the scope of the adopted criteria, should be mentioned. Among the most popular techniques are distinguished: ranking, pair wise comparison, the normal distribution method, normal recording, comparison with standards, techniques of critical events, check-lists, point scale, behavioural scales, management by objectives and assessment questionnaires [Szałkowski 2000].

Evaluating employees and, consequently, positive reinforcement and suppression of undesirable behaviour shapes relationships and increases organizational culture within the company. The managers and employees themselves and the persons responsible for the company's personnel and international institutions (trade unions) and external (National Labour Inspectorate), are interested in the opinion.

RESULTS OF DECISION ANALYSIS OF WORKERS

The study used a hypothetical „PRYM” company information, which specializes in the design, manufacturing and installation of steel structures, particularly steel buildings. It manufactures steel structures including: chimneys, silos, roof trusses, steel or composite construction, beams, floors, mezzanines and poles. It prepares own designs, or uses the documentation provided by the developer. The company has a dedicated and experienced staff and engineering and managerial personnel. The 50-people company is managed by the director. He is the immediate superior of a proxy for the quality, production director, quality controller and chief accountant, who in turn manages the accounting department and human resources (employee for the payroll and inspector for the Industrial Health and Safety). Production Manager manages production department and the immediate superior technical staff and production manager, who is head of supply staff, a warehouse and master. The latter is responsible for the work of a 40-people team performing physical work.

The study was conducted using the DSS 2.0 system (*Decision Support System*) developed in Szczecin universities (author: Budziński R., Becker J., version 2.0.098, 2013).The system is used to solve complex decision problems, which are dealt with in different homogeneous collections of objects. The concept has a dimension of abstract objects, in fact, they take on a specific form to represent, for example, objects, entities (people, organizations), factual events, objective scenarios (general – options for decision-making). The system enables the optimal choice, ordering and grouping objects according to a fixed structure criteria and preferences. Its functionality includes the analysis and econometric evaluation of objects, and found among them linguistic accuracies, using rough set theory.

Analysis of the staff was subordinated to system functions as a way to highlight the usefulness of the system for decision support personnel and payroll. The test procedure consisted of:

A. determining the structure of the evaluation criteria and give them preference
B. ranking employees and the distribution of bonus fund,
C. simulating the competitiveness of the workforce – the search for the adjustment of the minimum criteria for assessing the employee with a lower score with the top-rated crew members,
D. grouping of employees and assessment of the quality of the structure of the crew,
E. econometric assessment of the level of individual salaries.

A. The criteria and preferences

Persons performing work are evaluated. Therefore, there are four groups of criteria used during the operation, namely:

- efficiency criteria – relating to performance,
- eligibility criteria – depending on the job,
- behavioural criteria – regarding behaviour,
- personality criteria – including traits of intellect and character.

In this example, the evaluation was performed on workers, who were fitters, which is one of the main occupations in the metal and engineering industry. A fitter deals with manual or manual-machine metal processing. Fitter, in addition to the skills of mechanics, such as the removal and installation of equipment, cutting, sawing, drilling, riveting and soldering must master basic skills in the field of electrical engineering and hydraulics and metallurgy.

Among these groups of criteria for evaluating the first three were selected. Detailed information on each criterion are presented in Table 1 The analysis began by evaluating the performance of employees (d_1), which measured the amount of pieces of product made in one month ($d_{1.1}$) and the number of products with defects ($d_{1.2}$). The criterion of productivity achieved the highest priority (w_1= 0,45), as the data on the productivity of workers have the greatest impact on the system of rewards and punishments, the organization of the necessary training, which translates into increased productivity to the desired level, and awarded promotions and decisions on exemptions. The second group formed the eligibility criteria (d_2), which, due to the importance of the assessment system gained importance w_2= 0,35 Used in the study: work experience ($d_{2.1}$), familiarity with the devices ($d_{2.2}$) and physical condition ($d_{2.3}$). The next step concerned in the behavioural assessment (on behaviour) – d_3, with the weight w_3 = 0.15. In this group we find the following professional attitudes: accountability ($d_{3.1}$) initiative ($d_{3.2}$) and discipline ($d_{3.3}$). Acceptance of these evaluation criteria required by the employees that they are clearly defined and focused on behaviour, not personality traits [Assignment 2011]. This leaves the last group that formed personality traits, including intelligence traits, such as: memory, intelligence, capacity for analysis and synthesis, and the following character traits: activity level, emotionality, self-confidence, and sensitivity to criticism. Additionally, the skills assessment criterion was system introduced in to the system (d_4) with the weight of w_4= 0,05, for example: driver's license, certificates, courses, and foreign language skills. All the weighting factors at the level of the main criteria and sub-criteria were determined by the Saaty's method [Saaty 1980]. The degree of consistency of and assessments expressed in spot-verbal ratings measured by the convergence of coefficient CR for the main criteria was 0,068. It should be noted that the values of sub-criterias: $d_{1.1}$, $d_{1.2}$and $d_{2.1}$were introduced into the system at different units measurement and were transformed to the scale of 0-10.

Table 1. The structure of the evaluation criteria of fitters

Criteria and sub-criteria	Weighting factors
d_1 - Performance of employees [points]	w_1 = 0,45
$d_{1.1}$ - Measured the amount of pieces of product made in one month [pcs/month]	$w_{1.1}$ = 0,50
$d_{1.2}$ - The number of products with defects, which measured in month [pcs/month]	$w_{1.2}$ = 0,50
d_2 - Professional qualifications [points]	w_2 = 0,35
$d_{2.1}$ - Work experience [number of years]	$w_{2.1}$ = 0,27
$d_{2.2}$ - Familiarity with the devices [points]	$w_{2.2}$ = 0,54
$d_{2.3}$ - Physical condition [points]	$w_{2.3}$ = 0,19
d_3 - Professional attitudes [points]	w_3 = 0,15
$d_{3.1}$ - Accountability [points]	$w_{3.1}$ = 0,25
$d_{3.2}$ - Initiative [points]	$w_{3.2}$ = 0,50
$d_{3.3}$ - Discipline [points]	$w_{3.3}$ = 0,25
d_4 - Additional skills [points]	w_4 = 0,05

Source: own research

B. Allocation of bonus fund was based on the ranking of employees

Evaluation of fitters employed by the „PRYM" company was carried out in connection with the division of the bonus fund of PLN 20,000. By a simple weight additive method, available in DSS 2.0 system, a ranking of personnel, along with three examples of variations of the allocation of the premium was received – Table 2.

Table 2. Ranking of fitters with allocation of bonus variants

Obiekt	Nazwa potoczna	OZM(100%)		OZM (50%)		OZM (37%)		S	P	Sumy EMP	*Oceny PRF	[R]
AA0006	Pracownik 6 - filia 1 ...	3,450	-	1,725	-	1,277	-	-	-	27,400	31,191	1
AA0002	Pracownik 2 - filia 1 ...	2,900	-	1,450	-	1,073	-	-	⊗	32,500	30,763	2
AA0011	Pracownik 1 - filia 3 ...	3,990	-	1,995	-	1,476	-	-	-	28,400	30,404	3
AA0005	Pracownik 5 - filia 1 ...	3,560	-	1,780	-	1,317	-	-	⊗	30,200	29,784	4
AA0001	Pracownik 1 - filia 1 ...	2,980	-	1,490	-	1,103	-	-	⊗	29,300	28,234	5
AA0009	Pracownik 3 - filia 2 ...	3,330	⊗	1,665	-	1,232	-	-	-	23,300	27,902	6
AA0007	Pracownik 1 - filia 2 ...	3,950	⊗	1,975	-	1,462	-	-	-	26,500	27,807	7
AA0008	Pracownik 2 - filia 2 ...	3,680	⊗	1,840	-	1,362	-	-	-	23,900	27,398	8
AA0013	Pracownik 3 - filia 3 ...	3,060	⊗	1,530	-	1,132	-	-	-	21,600	26,455	9
AA0010	Pracownik 4 - filia 2 ...	3,060	⊗	1,530	-	1,132	-	⊗	-	19,500	24,746	10
AA0012	Pracownik 2 - filia 3 ...	3,020	⊗	1,510	-	1,117	-	⊗	-	19,200	24,237	11
AA0003	Pracownik 3 - filia 1 ...	2,670	⊗	1,335	-	0,988	-	-	⊗	23,100	22,105	12
AA0014	Pracownik 4 - filia 3 ...	2,860	⊗	1,430	⊗	1,058	-	⊗	-	18,400	21,427	13
AA0004	Pracownik 4 - filia 1 ...	2,710	⊗	1,355	⊗	1,003	-	⊗	-	16,800	20,851	14
AA0016	Pracownik 6 - filia 3 ...	2,670	⊗	1,335	⊗	0,988	-	⊗	-	14,400	18,389	15
AA0015	Pracownik 5 - filia 3 ...	2,670	⊗	1,335	⊗	0,988	-	⊗	-	12,500	15,370	16
AA0017	Pracownik 7 - filia 3 ...	2,630	⊗	1,315	⊗	0,973	-	⊗	⊗	14,800	13,693	17

Source: own using AHP module in the DSS 2.0

Each column headings are:

- [R] – ranking position,
- PRF assessments (pl *oceny PRF*) –weighted sum of points received by the employee on criteria,
- EMP sums (pl *sumyEMP*) – sum of points earned by the employee on the criteria (excluding preference, or with equal weights),
- P – information on a comparison of the values in column „PRF assessments" of „EMP sums" sign „⊗„ indicates that, for a given *t* employee (*t* = AA0001, AA0002, ..., AA0017) $prf^{(t)}$ note is less than $emp^{(t)}$, indicating greater participation in the evaluation of $prf^{(t)}$ points obtained in criteria less favoured by decision-makers,
- S – the results of comparing the $emp^{(t)}$ with the minimum level, the threshold $emp_{min} := emp_{max} \times S_{min}$ determined by the decision maker, sign '⊗' means that the employee did not receive a score determined by the decision makers S_{min} = 50% of the maximum points emp_{max}= 40, i.e. emp_{min} = 20 points,

- OZM – the bonus amount, acting in subsequent columns of interest: 100%, 50% and 37% of base salary („⊗„–no bonus awarded to the employee),
- ‘Nazwa potoczna’ – name of object, ‘Objekt’ – id of object.

As a result of simulation, it was observed that if the bonus was 100% of basic salary, it was granted to 5 best employees. Reducing the amount of award by 50%, caused five employees with the weakest did not receive a bonus. On the other hand, when the bonus was 37% of the basic salary, the extra money could be given to all employed fitters.

C. Simulation studies of competitiveness of employees

In the decision support common way of obtaining the best solutions from the point of view of the trustee is to use optimization methods. The system's DSS 2.0 exemplary structure of a WPL mathematical model has a large, theoretically unlimited potential for information. It allows describing the decision-making situation (including variants of decision) by the ability to define a number of decision variables, balances and limitations of resources and a lot of partial functions representing the selection criteria and preferences given to them. [Becker 2008]

In DSS 2.0 a fitter was represented by a simple mathematical model of partial linear programming, constructed on the basis of predefined template (Fig. 1). Automatic connection of specific partial models into one multimodel of linear programming allowed, through optimization of solving various scenarios of selecting employees. The template assumes that every fitter t = AA0001, AA0002, ..., AA0017 is represented by a binary variable of type 'X01_Wybór' and the vector of technical and economic parameters [c_2, c_3, d_1, d_2, d_3, d_4].

It was assumed that in the „PRYM” company at the end of the quarter, five best-rated employees would receive the bonus. At the beginning of the period the value cc1 = 5 was set in the system and potential leaders were selected. Employees who have not been admitted to the preferred leaders were able to improve their ratings using *subsystem simulation of appeal* included DSS 2.0. Its task is to find the minimum value of which the main criteria need to be corrected, so the employee has to compete with the leaders and earned a chance to win the next outcome (session optimization). Simulation algorithm provides three strategies to seek new values that vary in the order for amendments to the vector of criteria.

1) *Strategy A* – Annotating amendments starts from the least important criterion and can last until the threshold value, then modified in the same way more and more important criteria until exhausted.
2) *Strategy B* – Annotating amendments starts from the most important criterion and can last until the threshold value, then modified in the same way less and less important criteria until exhausted.
3) *Strategy C* (collectively, whole vectors) – Single corrections are applied to all of the criteria, the process can be repeated until the achievement of limit values around the vector.

Figure 1. Preview of the template of a mathematical model to choose a fitter

SZABLON [DSS033_000000]

ID	BILANSE	JM	X01_Wybór	F01_...	F02_...	F03_...	F04_...	R	WO
B01	¤Pracownik	Osób	1,000					<=	bb1
C01	¤Liczba pracowników	Osób	1,000					<=	cc1
C02	¤Stawka za roboczogodzinę ...	Osób	c2					<=	cc2
C03	¤Pensja	Tys.z	c3					<=	cc3
D01	Wydajność pracy	Pkt	d1	-n1				==	0
D02	Kwalifikacje zawodowe	Pkt	d2		-n2			==	0
D03	Postawa zawodowa	Pkt	d3			-n3		==	0
D04	Dodatkowe umiejętności	Pkt	d4				-n4	==	0
CEL	FUNKCJA WPL			a01	a02	a03	a04	=>	MAX

Source: the module of generator of WPL decision-making models in the DSS 2.0

It was assumed, for example, that the appeal system was used by three fitters, had the following places in the ranking: 6th, 12th and 17th place. The system juxtaposed the model describing each employee who was out of the leas with the leaders' models. Through optimization proposals for improving the criteria to guarantee entry to the group of winners were achieved. For example, analysing the situation of the worker, who took 6th place, it was advantageous to use scenario A. Disclosure of additional skills allowed him to move to the leaders with the least difficulties. For fitters, who were in the further positions, more advantageous was to select scenario B and C and their subsequent variants with reduced upper range of criteria.

Table 3. Simulation results of fitters' competitiveness – correction of notes

Id	Name	Position in the ranking	Main criteria	Values of the criteria [point]			
				Real	Simulated by algorithm		
					A	B	C
AA0009	Employee 3 - department 2	6	d1 - Performance of employees	8,0	8,0	8,2	8,1
			d2 - Professional qualifications	6,5	6,5	6,5	6,6
			d3 - Professional attitudes	6,8	6,8	6,8	6,9
			d4 - Additional skills	2,0	3,6	2,0	2,1
AA0003	Employee 3 - department 1	12	d1 - Performance of employees	6,7	6,7	10,0	8,3
			d2 - Professional qualifications	3,8	5,9	4,0	5,4
			d3 - Professional attitudes	5,6	10,0	5,6	7,2
			d4 - Additional skills	7,0	10,0	7,0	8,6
AA0017	Employee 7 - department 3	17	d1 - Performance of employees	3,6	3,6	10,0	7,3
			d2 - Professional qualifications	3,1	9,8	5,3	6,8
			d3 - Professional attitudes	3,1	10,0	3,1	6,8
			d4 - Additional skills	5,0	10,0	5,0	8,7

Source: own using WPL decision simulation module in the DSS 2.0 system

D. An analysis of the quality of staff assessed

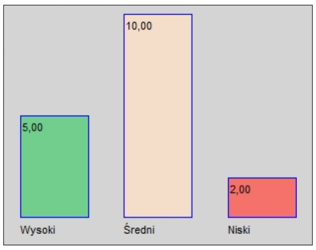
The next stage of the research was to allot fitters to three defined preferences classes: high, average and low. For this purpose, the ELECTRE TRI method was used [Roy, Słowiński 2008]. This procedure was designed to answer the following questions: what is the structure of the distribution of the crew, and is it consistent with the order obtained in the rankings, what category are the leaders of the ranking, whether they are high, average or low preferred fitters?

The system grouped employees in terms of preferences adopted to three quality grades: AB –low (pl *niski*), BC – medium (pl *średni*), CD – high (pl *wysoki*). Cut-off factor (level of evidence) was λ=0.76. According to the ELECTRE TRI method the allocation of options to designated classes was made using optimistic and pessimistic procedures (Table 4).

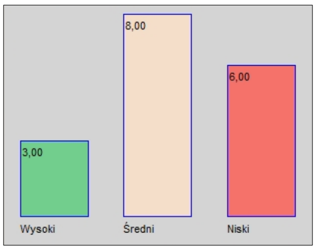
Table 4. Result of analysis of the quality structure of the crew – the division into three groups

Obiekt	Nazwa potoczna	O	OPTYMIZM	PESYMIZM	P
AA0006	Pracownik 6 - filia 1 ...	DE	Wysoki	Wysoki	DE
AA0002	Pracownik 2 - filia 1 ...	DE	Wysoki	Wysoki	DE
AA0011	Pracownik 1 - filia 3 ...	DE	Wysoki	Wysoki	DE
AA0005	Pracownik 5 - filia 1 ...	DE	Wysoki	Średni	CD
AA0007	Pracownik 1 - filia 2 ...	DE	Wysoki	Średni	CD
AA0001	Pracownik 1 - filia 1 ...	CD	Średni	Średni	CD
AA0009	Pracownik 3 - filia 2 ...	CD	Średni	Średni	CD
AA0008	Pracownik 2 - filia 2 ...	CD	Średni	Średni	CD
AA0013	Pracownik 3 - filia 3 ...	CD	Średni	Średni	CD
AA0010	Pracownik 4 - filia 2 ...	CD	Średni	Średni	CD
AA0012	Pracownik 2 - filia 3 ...	CD	Średni	Średni	CD
AA0003	Pracownik 3 - filia 1 ...	CD	Średni	Niski	BC
AA0014	Pracownik 4 - filia 3 ...	CD	Średni	Niski	BC
AA0004	Pracownik 4 - filia 1 ...	CD	Średni	Niski	BC
AA0016	Pracownik 6 - filia 3 ...	CD	Średni	Niski	BC
AA0015	Pracownik 5 - filia 3 ...	BC	Niski	Niski	BC
AA0017	Pracownik 7 - filia 3 ...	BC	Niski	Niski	BC

PRZYDZIAŁ OPTYMISTYCZNY



PRZYDZIAŁ PESYMISTYCZNY



Source: own using Electre Tri module in the DSS 2.0 system

Two applied approaches gave a slightly different division of employees. In the optimistic approach (Fig. in Tab. 4, pl *'PRZYDZIAŁ OPTYMISTYCZNY'*), there were 5 people highly preferred, 10 medium and 2 low. However, in the pessimistic procedure (fig. in tab. 4, pl *'PRZYDZIAŁ PESYMISTYCZNY'*) 3 people were highly preferred, 8 average, and the remaining low. The differences in allocations signalled the existence of certain ambiguities. Practically, in such situation, the final allocation is done by the decision-maker. Taking into account these two allocations, it was proposed to group the employees into five classes. Among the most favoured people were 3 employees, in the next class 2, the average group consisted of 6, while in the 4th class there 4 fitters, the last, lowest group contained 2 people. The resulting divisions were similar to the order obtained during the distribution of bonus fund (Stage B). Three out of five workers belonging to the leaders were definitely classified as a highly preferred group, one was on the border between the high and average class, and one was included in the group of averagely preferred.

E. Econometric analysis of the level of remuneration

Econometric analysis of the level of remuneration in the „PRYM" company included an examination of the effectiveness, which consisted of designing properly fitted econometric model describing the impact of the evaluation criteria of *t* employees at the level of their hourly rate of pay.Regarding the actual values of individual wages y_t('OZM' in Table 5) for each *t* employee to the dependent variable $\hat{y}_t$, one can measure economic efficiency.

Table 5. The results of the econometric analysis of wage levels (rate per hour)

Obiekt	Nazwa potoczna	D01...	D02...	D03_...	D04_...	Ocena	OZM	OS	R
AA0014	Pracownik 4 - filia 3 ...	6,900	4,200	4,300	3,000	WW	9,400	9,000	1
AA0011	Pracownik 1 - filia 3 ...	6,800	9,000	7,600	5,000	Wn	10,200	10,000	2
AA0003	Pracownik 3 - filia 1 ...	6,700	3,800	5,600	7,000	Wn	9,200	9,000	3
AA0006	Pracownik 6 - filia 1 ...	8,500	7,300	8,600	3,000	Pw	10,100	10,000	4
AA0004	Pracownik 4 - filia 1 ...	5,500	6,200	3,100	2,000	Pw	9,100	9,000	5
AA0008	Pracownik 2 - filia 2 ...	7,600	6,500	6,800	3,000	Pw	10,100	10,000	6
AA0005	Pracownik 5 - filia 1 ...	6,900	7,300	10,000	6,000	Pn	9,900	10,000	7
AA0009	Pracownik 3 - filia 2 ...	8,000	6,500	6,800	2,000	Pn	9,900	10,000	8
AA0007	Pracownik 1 - filia 2 ...	5,600	8,500	8,400	4,000	Pn	9,900	10,000	9
AA0015	Pracownik 5 - filia 3 ...	3,900	4,500	3,100	1,000	Nw	8,800	9,000	10
AA0016	Pracownik 6 - filia 3 ...	4,100	6,500	2,800	1,000	Nw	8,800	9,000	11
AA0013	Pracownik 3 - filia 3 ...	7,900	6,100	5,600	2,000	Nw	9,700	10,000	12
AA0012	Pracownik 2 - filia 3 ...	8,400	4,400	4,400	2,000	Nw	9,800	10,000	13
AA0010	Pracownik 4 - filia 2 ...	8,400	4,800	4,300	2,000	Nw	9,800	10,000	14
AA0017	Pracownik 7 - filia 3 ...	3,600	3,100	3,100	5,000	Nw	8,700	9,000	15
AA0002	Pracownik 2 - filia 1 ...	8,600	6,200	7,700	10,000	Nw	9,700	10,000	16
AA0001	Pracownik 1 - filia 1 ...	7,500	6,100	7,700	8,000	NN	9,600	10,000	17

Source: own using the econometric analysis module in the DSS 2.0 system

The estimated linear regression model, obtained in the study, for evaluable fitter (with R^2 = 91%) had form:

$$\hat{y}_t = 0{,}166\, d_1^{(t)} + 0{,}073\, d_2^{(t)} + 0{,}094\, d_3^{(t)} - 0{,}043\, d_4^{(t)} + 7{,}634 \qquad (1)$$

where $\hat{y}_t$ – the reference value ('OS' in Table 5), which *t* worker could achieve with reasonable work. Decision-makers thus receive an answer to the question: what is the current hourly rate of the employee („OZM") and what it should be ('OS')?

Based on the analysis of residuals, that is the difference between the actual value of the dependent variable and the value resulting from the theoretical model, taking into account the standard deviation of the residuals and the critical value read from tables t-student, five main salary groups were obtained: WW –*highest,* Wn–*high*, Pw, P, Pn –*average*, Nn –*low* and NN –*the lowest* [Budziński 2001]. Among the results two extreme positions deserve further attention. In the case of an employee AA0001, who was ranked in the top leaders, it can be concluded that the rate per hour. was too low at a high level of the adopted evaluation criteria. However, AA0014 employee received an exorbitant rate per hour of work in relation to the results of analysis.

CONCLUSION

Methods integrated in the decision support system DSS 2.0 function on a common set of data, under which it is possible to carry out a coherent, logical and comprehensive analysis of the decision-making process in the selected range. In the example, the information and decision-making process involved sphere of HR and payroll. The study used two different approaches, the first related to the achievements of American school, based on a functional model (Saaty's method, simple additive weight method and WPL optimization), the second stemming from the European school, based on a relational model (ELECTRE TRI method). Results of the methods well complement each other. The resulting order of employees referred to the defined classes of preferences. In this way a picture of the structure of the ranked crew was obtained. However, the use of models and WPL optimization expands the vector criteria for inclusion in the list of required specific constraints and balances, allowing from the point of view of the employee to seek solutions that offer assurance to receive the bonus (to be in the top five highest-rated employees). Complementary to the research is to identify the level of individual wages based on an econometric model for the assessment of employees.

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PRODUCTIVITY CHANGES OVER TIME – THEORETICAL AND METHODOLOGICAL FRAMEWORK

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Abstract: The TFPC Malmquist index is one of the tools that does not require knowledge of the price level and gives information on factors affecting productivity changes over time. The DEA-based approach allows decomposing of the TFPC indices into: technical change (ΔT), technical efficiency change (ΔTE) and scale efficiency change (ΔSE). A panel data from the companies of a key food processing sector, namely the meat processing, was used in the paper. The sample consisted of above 200 objects. The results indicated which of the decomposed indices (ΔT, ΔTE, ΔSE) had the greatest impact on productivity changes in the analyzed sector.

Keywords: productivity growth, the Malmquist index, food processing sector

INTRODUCTION

The purpose of the paper was to assess the productivity as a measure of efficiency-based relations in a selected sector of food processing. The accomplishment of this goal funds a strong economic background. The theoretical background of productivity and efficiency was presented within the framework of the paper. The assessment of productivity was conducted by using advanced computing techniques, namely the total factor productivity index – the TFPC[1] Malmquist index which bases on deterministic approach. A decomposition of the resulting index was conducted in the paper. Its components are a subject of evaluation, namely the change in manufacturing techniques (technical progress), the change in technical efficiency and scale efficiency change. The decomposition

[1] TFPC, *total factor productivity change*.

of the index extends the interpretation of the sources of the changes in efficiency-based relations over time.

The measuring of TFP (*total factor productivity*) indices is very commonly used in literature. Two main approaches, referred to as the axiomatic and the economic approach, can be distinguished in the literature of TFP indices [Diewert 1992]. The axiomatic approach postulates a number of properties that any index number should satisfied. By contrast, the economic approach is based on the economic theory and its behavioral assumptions. An example of the economic approach is the Malmquist index [Caves et al. 1982b]. One advantage of economic approach to TFP measurement is the availability of decompositions that gives information on the underlying sources of productivity growth [Kuosmanen and Sipilainen 2009]. In the paper the economic approach that bases on the application of the decomposed TFPC Malmquist index was applied.

ECONOMIC BACKGROUND

A firm operating on a competitive market seeks possibilities for maximization of expected profit by increasing production, especially by non-decreasing returns to scale[2]. Researchers confirm that the output growth in the agri-food sector is determined by a growth of demand for agri-food products occurring at a specific time[3]. The low growth rate of demand for agri-food products can limit the growth in the agri-food sector, and consequently, the production and processing growth inducing technical change. Therefore, the low growth rate of demand for agri-food products must determine the change of efficiency-based relations treated as a main growth factor in the sector. Thus, the authors believe that not the increase of input factors but the efficiency of its use is the main factor of firms' ability for long-term and effective growth and performance.

In the paper, it was assumed – based on W. Rembisz and A. Bezat-Jarzębowska – that changes in production efficiency are the endogenous condition of a company [Rembisz and Bezat-Jarzębowska 2013]. It is uncontested that the efficiency is a base source of profitability improving [Rembisz 2008]. Thus, it is important to understand which factors lead to improving of efficiency. The issue refers to both the allocation and distribution. Firstly, the aspect of allocation is described by the changes in production techniques (technical relations) – thus the productivity of each production factors. Secondly, the aspect of distribution refers to remuneration of each production factor. This should result – based on

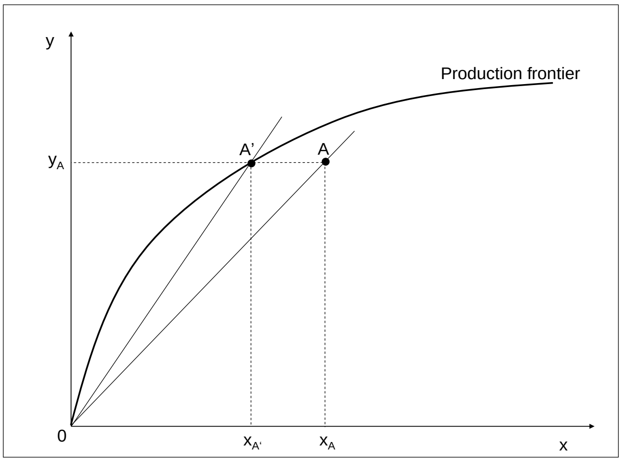
[2] Autors assume that in competitive market equilibrium the price is fixed for processor in agri-food sector.

[3] For more details see: Figiel Sz., Rembisz W. (2009) Przesłanki wzrostu produkcji w sektorze rolno-spożywczym – ujęcie analityczne i empiryczne, Multi-annual Programme 2005-2009, No. 169, Wyd. IERiGŻ, Warszawa.

neoclassical assumption – from the productivity of each factor, in particular from its marginal productivity (which is reflected by the scale of economic activity).

The concepts of productivity and efficiency are often used interchangeably; however, they do not mean exactly the same things [Coelli et al. 2005]. Some theoretical considerations on it were presented in the paper, nevertheless to illustrate the difference between them, a simple manufacturing process was shown, in which a single input (*x*) is used for obtaining a single output (*y*). Figure 1 illustrates the production frontier with one input and one output. Productivity of object A is calculated as output/input quotient (y_A/x_A), hence it is the value for the tangent of the angle between the x-axis and the 0A curve. The highest productivity to be achieved by object A (after reducing number of inputs for a given level of outputs) was determined by point A' ($y_A/x_{A'}$) [Coelli et al. 2005]. Further, it can be stated that the highest value of productivity is achieved by an object lying at the point of tangency of the production frontier and a curve drawn from the origin of the coordinate system [Jarzębowski 2011].

Figure 1. Efficiency and productivity - a comparison



Source: own work based on Coelli et al. 2005, p. 5

The efficiency of A is calculated as the ratio of minimum input ($x_{A'}$) needed to achieve a given level of output and actual input used to achieve this output (x_A). The highest efficiency (equal to 1) is obtained by objects lying on the production frontier. Therefore, efficient objects not necessarily achieve the highest level of productivity, since not each of them lies on the tangent to the production frontier. The result of the above evidence is the fact that productivity is not a synonymous term with efficiency.

There are various measures of productivity, depending on criterion (quantitative or valuable), used in order to express inputs and outputs. The decision on the choice of one from all the forms depends on the subject of the study, the purpose of its analysis and practical reasons (such as the availability of data) [Coelli et al. 2005].

For analysis of productivity change, indices of value are used. They are applied when changes in prices and volumes are measured over time; they allow also comparing productivity during the given time in a number of companies, industries, regions or countries. A price index may include consumer prices, prices of inputs and outputs, prices of import and export, etc., while a value index can measure changes in the volume of the outputs produced or inputs used in companies and industries taking into account changes over time or comparing companies among themselves.

Indices of value play an important role in economic sciences. The indices of Laspeyres and Paasche are one of the most important contributions, dated for the late XIX century. These indices are still used by statistical offices around the world. Irving Fisher's book "The Making of Index Numbers", published in 1922, showed application of many statistical formulas for determining value indices. And the Tornquist's index (1936) is formula which plays the main role in measuring of efficiency.

Traditional productivity measures can be applied in the analysis of companies' performance in one period. In contrary to the classical measures, the Malmquist index allows to analyze productivity change of companies or industries over time. The second problem connected with the productivity measurement is the fact that, one needs to know price levels. The most common examples of measurement using prices of production factors are the indices of: Laspeyres, Paasche and Fischer. Aggregation of the values expressed in other measurement units than prices is difficult. However, it is possible to express changes in analyzed inputs and outputs in a given period compared to the previous one.

The DEA-based Malmquist index is one of the tools that does not require knowledge of the price level and gives information on factors affecting productivity changes over time [Coelli et al. 2005].

METHODOLOGICAL BACKGROUND

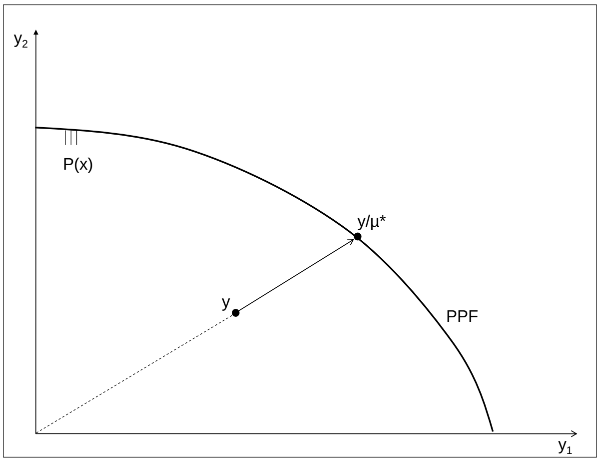
The productivity is almost always defined as the output-input ratio. However, it is quite common among researchers to analyse aggregated multi-outputs and multi-inputs cases. This is formally defined as *total factor productivity* (TFP) of a firm. In order to measure the changes in productivity a productivity change index has to be built. The *total factor productivity change index*is simply calculated as a ratio of the TFP of firm (i) in period (t) relative to the TFP of this firm (h) in period ($t+1$). This simple concept is used in case of the Mamlquist TFPC index which application was made in the paper.

The Malmquist productivity index was introduced as a theoretical index by D.W. Caves, L.R. Christensen and W.E. Diewert. [Caves at al. 1982a] and popularized as an empirical index by R. Färe, S. Grosskopf, B. Lindgren, P. Roos

[Färe et al. 1994]. The big advantage of the Malmquist TFPC index is that the interpreting of its value is relative simple and it is not necessary to know the factor prices. Further advantage is that the Malmquist TFPC index allows indicating the factors that affect productivity and its changes. The index can be decomposed into three different factors, namely: technical change (ΔT), technical efficiency change (ΔTE) and scale efficiency change (ΔSE). It is important not to miss any of these factors because the results could be interpreted incorrectly.

When calculating the TFPC Malmquist index one bases on the distance function. A distance function (defined as $D_O(\boldsymbol{x}, \boldsymbol{y}) = min\{\mu : \boldsymbol{x}/\mu \epsilon P(\boldsymbol{x})\}$, where $\boldsymbol{x}$ – input vector, $\boldsymbol{y}$ – output vector, P – output set) takes an output-expanding approach to the measurement of the distance from a producer to the production possibility frontier (PPF, see Figure 1) [Krumbhakar and Lovell 2004, p. 28]. This function gives the minimum value μ by which an output vector can be deflated and still remain producible with a given input vector. As it is shown in Figure 2, the output vector $\boldsymbol{y}$ is producible with input $\boldsymbol{x}$, but so is the radially expanded output vector (y/μ^*), and so $D_O(\boldsymbol{x}, \boldsymbol{y}) = \mu^* < 1$. An object for which $D_O(\boldsymbol{x}, \boldsymbol{y}) = 1$[4], is an efficient one, and if the value is smaller than one, an object is inefficient [Bezat 2012].

Figure 2. An output distance function



Source: own work based on Krumbhakar and Lovell 2004, p. 31

Application of distance functions (which are incorporated in the DEA method)allows measuring of the total productivity over time and allows making comparisons across firms and for a given firm over time.

[4] The relation is true for objects lying on the production possibility frontier.

The output-oriented Malmquist productivity index[5] using the period (t) benchmark technology is defined as [Färe et al. 1994]:

$$M_O^t(y_t, y_{t+1}, x_t, x_{t+1}) = \frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^t(x^t, y^t)} \quad (1)$$

where (x^τ, y^τ) ($\tau = t, t+1$) is an observed τ –period input-output vector, the "O" denoted the output-orientation.

The period ($t+1$) Malmquist index using the output orientation is defined as:

$$M_O^{t+1}(y_t, y_{t+1}, x_t, x_{t+1}) = \frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^t, y^t)} \quad (2)$$

The TFPC Malmquist index can be estimated by using the output-oriented Malmquist index between period (t) and period ($t+1$). It is defined as the geometric mean of two Malmquist indices between two time periods ($M_O^{t,t+1}$) [Cantner et al. 2007]. Thus, it's defined as[6]:

$$M_O^{t,t+1} = [M_O^t \times M_O^{t+1}]^{1/2} = \left[\frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^t(x^t, y^t)} \times \frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^t, y^t)}\right]^{1/2} \quad (3)$$

It is possible that observed productivity improvement (change) reflected in the Malmquist TFPC index could be the result of improvement of the underlying production technology (technical change) and/or of technical efficiency change (technical efficiency change). In this case the above Malmquist index (3) can be decomposed into ΔT[7] and ΔOTE[8] as follows [Cantner et al. 2007]:

$$M_O^{t,t+1} = \left[\frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^{t+1}, y^{t+1})} \times \frac{D_O^t(x^t, y^t)}{D_O^{t+1}(x^t, y^t)}\right]^{1/2} \times \frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^t(x^t, y^t)} = \Delta T \times \Delta OTE \quad (4)$$

Under the CRS (*Constant Returns to Scale*) assumption, in case of index (4), there will be no scale efficiency change. However the productivity of some firms can increase by changing the scale of its operation what leads to operating an optimal scale of production by the company. Following T.J. Coelli, P. Rao, Ch.J. O`Donnell and G.E. Battese (2005) we have the decomposition of the ΔOTE [Coelli et al. 2005]:

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[5] This section could be written using input distance functions to define an input-oriented Malmquist productivity index, and nothing in the way of decomposing would change.

[6] We can have $M_O^{t,t+1} > 1$, $M_O^{t,t+1} = 1$ or $M_O^{t,t+1} < 1$ according as productivity growth, stagnation or decline occurs between period (t) and ($t+1$).

[7] The ΔT is a geometric mean of the shift in production frontier in time (t) and ($t+1$) at inputs levels x^t and x^{t+1}(graphically – shift of production possibilities frontier).

[8] The ΔOTE measures the change in the technical efficiency between periods (t) and ($t+1$) (graphically – shift towards or away from production possibilities frontier).

$$\Delta OTE = \frac{D_{O^*}^{t+1}(x^{t+1}, y^{t+1})}{D_{O^*}^{t}(x^{t}, y^{t})} \times \left[\frac{\frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^{t+1}, y^{t+1})}}{\frac{D_O^{t}(x^{t}, y^{t})}{D_O^{t}(x^{t}, y^{t})}} \right] =$$

$$= \Delta TE \times \left[\frac{SE^{t+1}(x^{t+1}, y^{t+1})}{SE^{t}\,(x^{t}, y^{t})} \right] = \Delta TE \times \Delta SE \tag{5}$$

where the ΔTE measures technical efficiency change on the best practice technologies (production frontier), denoted as "O^*", and the ΔSE measures the change in scale efficiency from period (*t*) to period (*t+1*)[9]. The ΔSE is measured by using the output distance of observed input-output vectors from the CRS frontier relative to the VRS (*Variable Returns to Scale*) frontier. The product of this tree changes is equal to the TFPC Malmquist index in time period (*t*) to (*t+1*). Thus, the TFPC Malmquist index can be written as:

$$\text{TFPC} = \Delta T \times \Delta TE \times \Delta SE \tag{6}$$

If a data on a cross-section of companies in periods (t) and (t+1)is available then it is possible to use for example the DEA to calculate the TFPC index [Coelli et al. 2005]. The application of the DEA-based TFPC Mamlquist index was conducted in the next part of the paper.

APPLICATION OF THE DEA-BASED TFPC MALMQUIST INDEX

The productivity change assessment was carried out on the basis of data from the *Monitor Polski B*. The sample covered 223 companies from meat processing sector from across Poland from period 2006-2011 (the balanced panel). The selection of a specific sector was made because of different production's technologies in different food processing sectors. The data was reported as revenue/expenditure denominated in PLN in constant prices. The TFPC Malmquist index was calculated for a single output and two inputs. The inputs and the output are identified in table 1. The variables were selected to reflect the cost sources and production possibilities on the input side and the revenue sources on the output side.

Table 1. Inputs and outputs used to assess the efficiency scores

Inputs	Outputs
X1 – operational costs	Y –revenue on sales
X2 – value of assets	

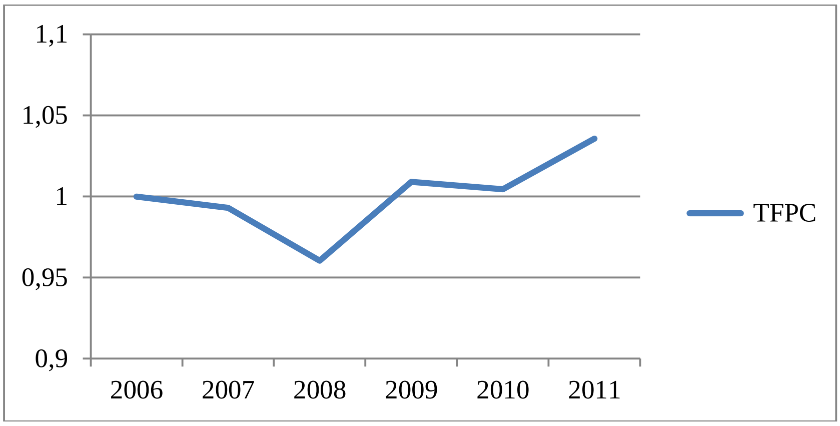
Source: own work based on Coelli et al. 2005, p. 5

Figure 3 shows the changes in productivity growth in the period 2006–2011. The significant progress (increase in productivity) can be observed in the period

[9]Graphically movements around the frontier surface to capture economies of scale.

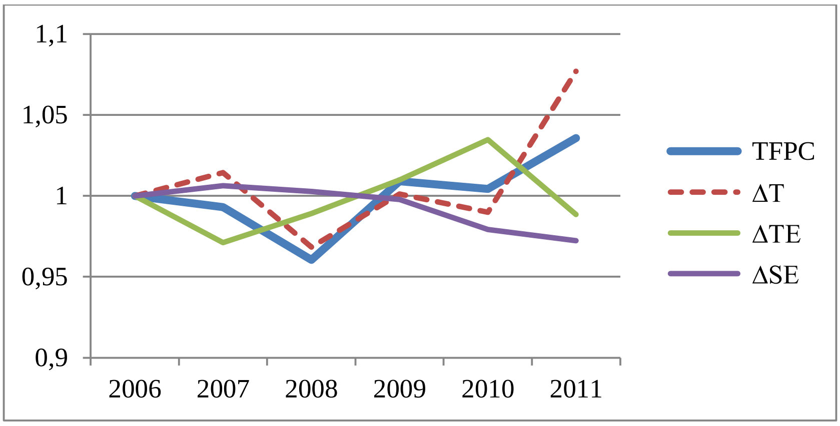
2008–2011. If the decomposition was not be made in the paper, the Figure 3 would be the only information regarding changes in the productivity in the Polish meet sector. The decomposition of the index allows researchers to analyze different sources of the productivity change (see Figure 3).

Figure 3. The changes of the mean value of the TFPC in the meat processing sector in period 2006–2011 (previous year = 100)



Source: own calculations

Figure 4. The changes of the mean value of the TFPC, the mean value of the ΔT, the ΔTE and the ΔSE in the meat processing sector in period 2006–2011 (previous year = 100)



Source: own calculations

Figure 4 reports the cumulative indices of each component of the proposed decomposition, namely technical change (ΔT), technical efficiency change (ΔTE) and scale efficiency change (ΔSE). The results suggest that the technical change was the main driver of TFPC change. An increase in the ΔT was observed in the period 2008–2011. It indicates the shift upwards of production frontier. The scale efficiency has slightly decreased in the sample. Other growth factor was the technical change which has not changed much over the analyzed years.

CONCLUSIONS

Within the framework of the paper the theoretical background for relationships between productivity and efficiency was presented. Some theoretical and analytical implications for modeling of productivity changes are detected in the study as well.

The analysis of productivity changes is of main interest of economic researcher. Among tools which can be used to measure productivity change one can find the TFPC Malmquist index. The use of the DEA-based Malmquist index allows including a number of inputs which might influence the productivity. The decomposition of TFPC index allows detecting some sources of productivity changes which might be: technical change (ΔT), technical efficiency change (ΔTE) and scale efficiency change (ΔSE). The decomposing of the productivity growth provides valuable information to managers and planners.

The application of the TFPC Malmquist index in the analyzed sample of companies allows concluding that the technical change (ΔT) was the main driver of the productivity changes in the meat processing sector in the period 2006–2011.

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QUANTITATIVE METHODS IN ECONOMICS
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SYNTHETIC RADAR MEASURES THAT FACTOR IN WEIGHTS OF FEATURES

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Abstract: This work is a direct continuation of previous works of authors ([Binderman, Borkowski, Szczesny 2008, Binderman, Szczesny 2009]). The goal of this work is to generalize known methods of constructing measures of synthetic objects, methods that use radar charts of vectors. The methods listed in this work factor in weights assigned to features and make the values of indicators independent from the ordering of considered features. The listed methods have been applied in the analysis of regional differentiation of agriculture in Poland between 2001 and 2010.

Keywords: synthetic measure, classification, ordering, weights, radar measure.

INTRODUCTION

Multidimensional analysis of empirical material requires many specialist tools. It is necessary to not only present, in a form of a clear and understandable chart, values of many measures assessing fragmentary aspects but also produce a summary valuation of area in a form of a normalized numerical coefficient. In simple situation analysis can be aided by a spreadsheet, as it provides basic functionality to create charts and calculations. Some of smaller companies decide to adopt such solutions. However, in practice that solution proves insufficient, because normally, one needs to carry out multi-criteria analysis that require processing gathered data in multiple ways. Many applications on the market have been designed for that purpose, as they offer management of data (protecting it from loss of integrity and allowing easy access to historical data arranged by present organizational structure) and standardization of measures and their presentation across the whole company. It is especially important when one is dealing with multilevel structures of measures and multilevel organizational structures. There are many provides of such tools and applications. Large corporations choose tools of multinational providers. The global market shares of particular global providers are available in annual IDC reports [see Vesset, McDonough, Wardley, Schubmehl 2012]. In Poland some of that also have a large market share as indicated by, e.g. **[**Odrzygóźdź, Karwański, Szczesny 2009**]**. While smaller companies utilize dedicated solution prepared by local provides. An interesting one is, for example, the ProRATE application offered by PRONOST ltd.

However, the key element to success when building such information systems is the right choice of means to present the results of observed fragmentary measures as well as normalized synthetic coefficient (or a set thereof). In subsequent analysis we will concentrate only on functionality available in a spreadsheet. With such constraints in place, we believe that the best way to present a set of fragmentary measures in a given moment in time is to utilize radar charts, as they allow to present in a clear way the values of desired measures (e.g. average, the positive and negative sample, etc.). Many analysts consider the area of a polygon defined by the values of fragmentary measures as a natural summary value (a synthetic measure) when the fragmentary measures are to be treated equally. In majority of cases the value of such a synthetic measures is dependent on the ordering of fragmentary measures (see **[**Binderman, Borkowski, Szczesny 2008, Binderman, Szczesny 2009**]**).

The goal of this work is to generalized known methods of constructing measures of synthetic objects, methods that utilize radar charts. Methods listed in this work take into consideration weights assigned to features as well as make values of synthetic coefficients independent from the order of features. In general, when constructing a synthetic coefficient one is more often in a situation when fragmentary measures are not uniformly important. Most often that is the case when e.g. constructing bonus indexes in corporations with many branches, because the evaluated activities are not always equally important. This means that the user should not only be presented on the chart (in this case a radar chart) with the values of specific fragmentary measures but also their weights in the summary synthetic value. In the next section we shall present how to achieve that.

DATA VISUALISATION WITH WEIGHTS OF VARIABLES

Let us consider a typical problem of ordering m∈*N* objects: **Q**$_1$, **Q**$_2$,...,**Q**$_m$ defined by a set of $n \in$ N variables (features): X_1, X_2,...,X_n. Let vector $\mathbf{x_i} = (x_{i1}, x_{i2}, \ldots, x_{in}) \in \Re^n$, $n \in \mathbb{N}$, *i=1,2,...,m*, define an i-th object **Q**$_i$.

An important phase in a comparative analysis is the calculation of synthetic coefficients which take into account non-uniform weights of variables (fragmentary coefficients). This phase is called the weighting of variables and its aim is to assign different importance to different variables. It is used because in the considered problems, variables often have different roles. This problem is yet to be fully analyzed, as literature presents different approaches and opinions as to should variables be weighted at all and if yes, how it should be done. Defining the weights of variables is tightly coupled with the ranking of information value of features, which in turn is considered when choosing diagnostic variables. That means that when choosing and weighting variables one should base their actions on knowledge of the analyzed process while statistical tools, which use information about distributions of variables, should only provide aid when necessary. Let us reiterate that one of the simplest ways of determining values of weights when aided by statistical tools is based on the coefficient of volatility of features after they have been normalized, when the assumption that they are all positive hold true:

$$w_j = \frac{V_j}{\sum_{k=1}^{n} V_k}, \quad j = 1,2...,n; \quad V_j = \frac{S_j}{\bar{x}_j},$$

where $\bar{x}_j$ – arithmetic average, S_j – standard deviation of j-th variable

Another way of determining weights utilizes a matrix of coefficients of linear correlation of diagnostic variables. The system of weights is created on the basis of measures of explicitness of variables. The explicitness of features means the certainty of classification of variables into sets of 'stimulator' and 'de-stimulator'. Weighting factors of features (assuming the number of features after reduction is equal to n) are calculated as follows [Grabiński, Wydymus, Zeliaś 1989, p.26, Zeliaś 2000]:

$$w_j = \frac{\left|\sum_{i=1}^{n} r_{ij}\right|}{\sum_{l=1}^{n}\left|\sum_{i=1}^{n} r_{il}\right|} \quad (j=1,2...,n),$$

where r_{ij} – coefficient of linear correlation between i-th and j-th variables.
If the identification is based on factor analysis then weighting factors are determined by:

$$w_j = \frac{|q_{1j}|}{\sum_{j=1}^{n}|q_{1j}|},\ j=1,...,n;$$

where q_{1j} – coefficient of linear correlation between j-th diagnostic variable and the first main factor.

Factors $q_{1j} = \sqrt{\lambda_1 a_{1j}}$, where λ_1 – maximal eigenvalue of correlation matrix $\mathbf{R} = [r_{ij}]_{n\times n}$, a_{1j} – subsequent elements of eigenvector, corresponding to value λ_1, $j = 1,2...,n$. The described above ways of determining differentiated weights fulfil the required criteria, they are non-negative: $w_j \geq 0$ and they sum to one: $\sum_{j=1}^{n} w_j = 1$

As mentioned above, the choice of a system of weights is for the researcher to make. However, when researching economic phenomenon, which can be very intricate, a demand raises to, for the purpose of analyzing statistic features of variables, consider only extra-statistic information about the process that comes from "field experts". More information on choosing a weight system can be found in the following works [Kukuła 2000, Nowak 1990, Pociecha 1996, Grabiński 1984, Borys 1984, Rusnak, Siedlecka, Siedlecki 1982, Stobiecka 1998, Młodak 2006, Malina 2004, Zeliaś 2000].

Let us introduce basic concepts and designations necessary in this work.

Definition 1. Set

$$\Omega := \left\{ w = (w_1, w_2, \ldots, w_n) \in \Re^n : \sum_{i=1}^{n} w_i = 1,\ w_i > 0 \right\},$$

will be called a set of weights and its elements Ω - vectors of weights.

Naturally, each weight is a structure, while the reverse do not always holds.

Definition 2. Let vector $\mathbf{w} = (w_1, w_2, \ldots, w_n) \in \Omega$,. Operator $\mathsf{B}_{\mathbf{w}} : \Re^n \to \Re^n$, is defined by:

$$\mathsf{B}_{\mathbf{w}}\mathbf{x} = \left(\frac{w_1 x_1}{w_{max}}, \frac{w_2 x_2}{w_{max}}, \ldots, \frac{w_n x_n}{w_{max}} \right), \tag{1}$$

Where $\mathbf{x} = (x_1, x_2, ..., x_n) \in \Re^n$, $w_{max} = \max(w_1, w_2, ..., w_n)$, will be called a weight operator induced by weight vector **w.**

Without loss of generality let us assume that features of object $\mathbf{x} = (x_1, x_2, ..., x_n) \in \Re^n$ are of stimulator variety and their values belong to the interval <0,1>, i.e. $\mathbf{0} \leq \mathbf{x} \leq \mathbf{1} \equiv 0 \leq x_i \leq 1$, $i=1,2,...n$,), where vectors $\mathbf{0}$:=(0,0,...,0),

1:=(1,1,...,1) will be called model vectors. This follows from the fact that variables can be normalized using the operation of zero notarization or a quotient mapping using the maximal value. Values of a set of features assigned to the objects can be then represented on a radar chart. In order to do this let us denote by x_i those points on axis O_i that lie on the intersection of axis O_i and a circle with the centre located at the origin of the coordinate system and a radius of x_i , $i=1,2,...,n$. By joining subsequent points x_1 with x_2, x_2 with x_3, ..., x_n with x_1 we create an n-polygon, which area $S_\mathbf{x}$ defined by:

$$S_\mathbf{x} = \sum_{i=1}^{n} \frac{1}{2} x_i x_{i+1} \sin\frac{2\pi}{n} = \frac{1}{2}\sin\frac{2\pi}{n}\sum_{i=1}^{n} x_i x_{i+1}, \quad \text{where } x_{n+1} := x_1. \tag{2}$$

The area of an inscribed, regular n-polygon induced by a model vector $\mathbf{1} = (1,1,\ldots,1)$ is defined by

$$S_\mathbf{1} = \sum_{i=1}^{n} \frac{1}{2}\cdot 1\cdot 1\cdot \sin\frac{2\pi}{n} = \frac{1}{2} n \sin\frac{2\pi}{n},$$

And the ration of those areas $S_\mathbf{x}/S_\mathbf{1}$ is defined as

$$S_{\mathbf{x}/\mathbf{1}} := S_\mathbf{x} / S_\mathbf{1} = \frac{1}{n}\sum_{i=1}^{n} x_i x_{i+1} \quad \text{where } x_{n+1} := x_1. \tag{3}$$

In further analysis let us assume that for equivalent features we have designated weight coefficients: w_1, w_2, ...,w_n, that make up a weight vector:

$$\mathbf{w} := (w_1, w_2, \ldots, w_n) \in \Omega,$$

i.e. $\mathbf{w} > \mathbf{0}, \sum_{i=1}^{n} w_i = 1.$

Let

$$w_{max} = \max_{1\le i\le n}(w_1, w_2, \ldots, w_n).$$

In a similar manner as above we can create radar charts for vectors $\mathbf{B_w}(\mathbf{x}), \mathbf{B_w}(\mathbf{1})$, calculate the areas of n-polygons that are generated by them: $S_{\mathbf{B_w}(\mathbf{x})}, S_{\mathbf{B_w}(\mathbf{1})}$:

$$S_{B_w(x)} = \sum_{i=1}^{n} \frac{1}{2}\frac{w_i}{w_{\max}} x_i \frac{w_{i+1}}{w_{\max}} x_{i+1} \sin\frac{2\pi}{n} = \frac{1}{2}\sin\frac{2\pi}{n}(w_{\max})^{-2}\sum_{i=1}^{n} w_i w_{i+1} x_i x_{i+1},$$

$$S_{B_w(\mathbf{1})} = \sum_{i=1}^{n} \frac{1}{2}\cdot\frac{w_i}{w_{max}}\cdot 1\cdot\frac{w_{i+1}}{w_{max}}\cdot 1\cdot\sin\frac{2\pi}{n} = \frac{1}{2}\sin\frac{2\pi}{n}(w_{max})^{-2}\sum_{i=1}^{n} w_i w_{i+1} \quad ,$$

where $x_{n+1} := x_1, w_{n+1} := w_1$, operator $\mathbf{B_w}$ is defined by equation (1).From that is follows that the ratio $S_{\mathbf{B_w}(\mathbf{x})} / S_{\mathbf{B_w}(\mathbf{1})}$ of areas of above polygons is defined by:

$$S_{\mathbf{x}/\mathbf{1}}^{\mathbf{w}} := S_{\mathbf{B_w}(\mathbf{x})} / S_{\mathbf{B_w}(\mathbf{1})} = \frac{\sum_{i=1}^{n} w_i w_{i+1} x_i x_{i+1}}{\sum_{i=1}^{n} w_i w_{i+1}} \tag{4}$$

Let us consider an object **Q** which features allow it to be described by a vector **x**= $(x_1,x_2,...,x_n) \in \Re_+^n$. If **x** has only one coordinate not equal to 0, that is $p\in$N: $1\le p\le n$, $x_p\neq 0$ and $x_i=0$ for $i\neq p$ then we shall call this vector a **singular** vector. If **x** is a singular vector then vector $\mathbf{B_w x}$, as defined by (1) is also a singular vector. A radar chard induced by a singular vector is only line segment with an area of 0. Considering the above, for definitiveness, let us assume that analyzed vectors are not singular. This can be achieved in many ways, for example by changing the coordinate system, of which a normalization of variables induced by those vectors is a special case. Another example of a change of a coordinate system is an affine transformation of the space $\Re^n$ into itself by [Stark 1970]

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b},$$

where the matrix:

$\mathbf{A} = \left[a_{ij}\right]_{n\times n}$, determinant $|\mathbf{A}| \neq 0$, vector $\mathbf{b}^T = \left[b_1, b_2, ..., b_n\right]$.

This approach allows for an intuitive visualization of individual objects, considering the importance of individual components (values of features). Let us illustrate this by a simple example posted in Table 1.

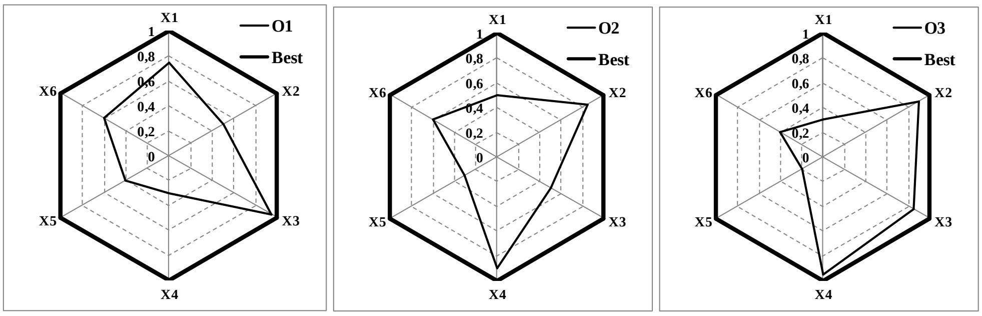
Table 1. The example's values of six variables of four objects in their original form (left hand side) and values of variables after they have been transformed by a weight operator defined by (1), induced by weight **w**=(0.225, 0.1, 0.1, 0.1, 0.25, 0.225) (right hand side).

	X1	X2	X3	X4	X5	X6		X1	X2	X3	X4	X5	X6
O1	0,75	0,5	0,95	0,3	0,4	0,6	**O1***	0,68	0,2	0,38	0,12	0,4	0,5
O2	0,5	0,85	0,5	0,9	0,3	0,6	**O2***	0,45	0,34	0,2	0,36	0,3	0,5
O3	0,3	0,9	0,85	0,95	0,2	0,4	**O3***	0,27	0,36	0,34	0,38	0,2	0,4
Best	1	1	1	1	1	1	**Best***	0,9	0,4	0,4	0,4	1	0,9

Source: own research

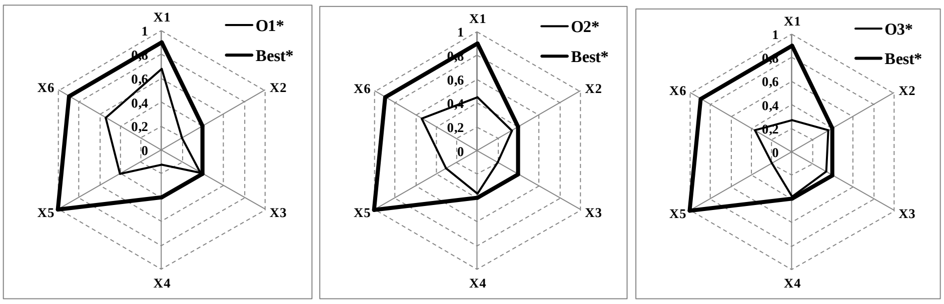
EXAMPLE 1.In the below pictures – Fig. 1, Fig. 2, we have presented visualizations of three objects defined by Table 1., with the forth object **Best** serving as the background. Object **Best** is evaluated as the best object considering each and every of its features. Those features can be interpreted as stimulators. It can be clearly seen that areas of objects **O1**, **O2** and **O3** in Figure 1 create an increasing sequence. However, when we consider the values of individual variables expressed by the weight vector **w**=(0.225, 0.1, 0.1, 0.1, 0.25, 0.225), then as shown in Figure 2, areas of objects **O1***, **O2*** and **O3*** creates a decreasing sequence. Moreover, let us note that while evaluating the analyzed phenomenon the radar chart of the dominant object **Best*** in Figure 2 clearly shows which variables are most important. If we were to use equations (3) and (4) to evaluate objects **O1**, **O2** and **O3**, then in a case when we consider all variables uniformly important we would obtain results of 0.32, 0.34, 0.37, respectively, for (3), and 0.32, 0.29, 0.21, respectively, for (4), when the evaluation was done using the weight vector **w**.

Figure 1. Radar charts for objects defined in Table 1 (left hand side)



Source: own research

Figure 2. Radar charts for objects defined in Table 1 (right hand side)



Source: own research

Let us note the formulas (3) and (4) use the intuition connected with visually representing objects as radar charts and even for a large number of features have the defect of depending on the ordering of those features. Which means that they are sensitive to changes in order of considered features which in many situation may result in a different ordering of evaluated objects.

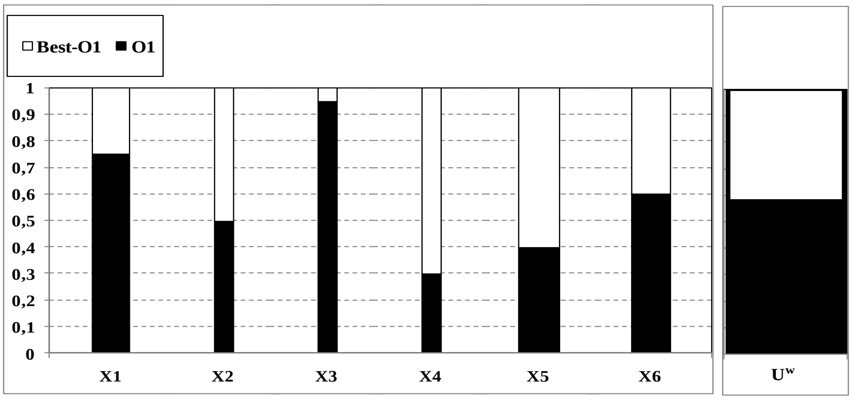
For example, if we were to order the variables form example 1 as: X1, X2, X5, X3,X6, X4, then both the graphical representation in Pic. 2, and the values obtained from (4) would result in a different ordering of objects. Indeed, the values obtained from (4) for objects **O1**, **O2** and **O3** would be 0.32, 0.35, 0.27, respectively, as opposed to previous values of 0.32, 0.29, 0.21.

In those cases when there are no instrumental limitations when creating reports one can imagine different ways of visualizing objects defined as sets of features. One can use, for example, an illustration such as one in Pic. 3. For the object **O1** from Table 1 and the vector of weights **w,** the illustration shows the levels of liquids in test tubes, as the information about both the importance of individual features and the value of the popular synthetic indicator **U,** defined by the formula [Cieślak 1974, Kukuła 2000]:

$$\mathbf{U}^{\mathrm{w}} = \sum_{i=1}^{n} w_i x_i \ , \tag{5}$$

where $\mathbf{w} = (w_1, w_2, ..., w_n), \mathbf{x} = (x_1, x_2, ..., x_n) \in \Re_{+}^{n} = \{\mathbf{y} : \mathbf{y} \ge 0\}, \sum_{i=1}^{n} w_i = 1.$

Figure 3. Graphical illustration of object **O1** from Table 1with object **Best** in the background



Source: own research

Unfortunately, this kind of visualization is not supported out of the box by the tooling available within a spreadsheet. Another way of representing data, especially useful when dealing with large numbers of variables and objects are overrepresentation maps [see Szczesny et al. 2012, Binderman, Borkowski, Szczesny, Shachmurove 2012]. In the following section we shall demonstrate how to obtain radar coefficients that are independent from the ordering of variables.

SYNTHETIC RADAR MEASURES

In order to construct radar weight measures that are independent from the ordering of features, let us consider an object **Q**, which features allow to describe it by a vector $\mathbf{x}=(x_1,x_2,...,x_n) \in \Re_{+}^{n}$ and a weight $\mathbf{w} := (w_1, w_2, \dots, w_n) \in \Omega$.

Let us denote the j-th permutation of the set of coordinates of the vector $\mathrm{B}_{\mathbf{w}}\mathbf{x} := (w_{max})^{-1}(w_1x_1, w_2x_2, \dots, w_nx_n)$.

By

$\mathbf{x}^{\mathbf{w}}{}_j := (x^w{}_{1j}, x^w{}_{2j}, \dots, x^w{}_{nj}) = (w_{max})^{-1}\left(w_{j1}x_{j1}, w_{j2}x_{j2}, \dots, w_{jn}x_{jn}\right)$, $(\mathbf{x}^{\mathbf{w}}{}_1 := \mathrm{B}_{\mathbf{w}}\mathbf{x})$,

where $j = 1,2,...,n!$.

Let us assume the following (see (4)):

$$S^{\mathbf{w}}\left(\mathbf{x}_j\right) := \sqrt{\frac{\sum_{i=1}^{n} w_{ji}w_{ji+1}x_{ji}x_{ji+1}}{\sum_{i=1}^{n} w_{ji}w_{ji+1}}} = \left(\sum_{i=1}^{n} w_{ji}w_{ji+1}\right)^{-\frac{1}{2}} \sqrt{\sum_{i=1}^{n} w_{ji}w_{ji+1}x_{ji}x_{ji+1}} \tag{6}$$

$$\mathrm{M}^{\mathbf{w}}(\mathbf{x}) := \max_{1\le j\le k} \mathrm{S}^{\mathbf{w}}(\mathbf{x}_j), \tag{7}$$

$$\mathrm{S}^{\mathbf{w}}(\mathbf{x}) := \frac{1}{k}\sum_{j=1}^{k} \mathrm{S}^{\mathbf{w}}(\mathbf{x}_j), \tag{8}$$

$$\mathrm{m}^{\mathbf{w}}(\mathrm{x}) := \min_{1\le j\le k} \mathrm{S}^{\mathbf{w}}(\mathbf{x}_j), \tag{9}$$

where k:=n!.

NOTE 1. If $\mathbf{x}=(x_1,x_2,...,x_n) \in \Re^n_+$ is a singular vector and j-th variable $x_j \neq 0$ then it's synthetic natural measure should be defined as $\frac{w_jx_j}{n}$.

NOTE 2. The defined above measures $\mathrm{M}^{\mathbf{w}}(\mathbf{x})$, $\mathrm{S}^{\mathbf{w}}(\mathbf{x})$ and $\mathrm{m}^{\mathbf{w}}(\mathbf{x})$ are **independent** from the ordering of coordinates of vector **x.** In case of object **O1** which was considered in Example 1, we have

$\mathrm{S}^{\mathrm{w}}(\mathbf{x})=0{,}56$; $\mathrm{M}^{\mathrm{w}}(\mathbf{x}) = 0{,}62$; $\mathrm{S}^{\mathrm{w}}(\mathbf{x})=0{,}57$; $\mathrm{m}^{\mathrm{w}}(\mathbf{x})=0{,}53$.

The methods of calculating synthetic measures that were defined above fulfil the basic postulate of stability of the used method [see Jackson 1970] – they are independent from the ordering of features that describe the given object. Radar methods may seem as computationally complex, but in the computer age this is no longer an issue, as they can be easily calculated in a spreadsheet – naturally, for a limited set of variables.

Let $\mathbf{w} := (w_1, w_2, \dots, w_n) \in \Omega$, $\mathbf{x} \in \Re^n_+, \alpha \in \Re_+$. Functions that were defined above have the following properties (with consideration of Note 1) [see Binderman, Borkowski, Szczesny 2008, Binderman, Szczesny 2009]:

a) $\mathrm{M}^{\mathrm{w}}(\mathbf{0})=\mathrm{S}^{\mathrm{w}}(\mathbf{0})=\mathrm{m}^{\mathrm{w}}(\mathbf{0})=0$,

b) $M^w(\mathbf{1})=S^w(\mathbf{1})=m^w(\mathbf{1})=1$,

c) $M^w(\mathbf{1/2})=S^w(\mathbf{1/2})=m^w(\mathbf{1/2})=1/2$,

d) $\mathrm{W}^{w}(\alpha\mathbf{x})=\alpha\mathrm{W}^{w}(\mathbf{x}),\ \mathrm{S}^{w}(\alpha\mathbf{x})=\alpha\mathrm{S}^{w}(\mathbf{x}),\ \mathrm{m}^{w}(\alpha\mathbf{x})=\alpha\mathrm{m}^{w}(\mathbf{x}),$

$m^w(\mathbf{x})\leq S^w(\mathbf{x})\leq M^w(\mathbf{x})$.

The authors have been researching into the subject of measuring similarity or dissimilarity of objects, with special attention given to the area of structural changes in agriculture for multiple years (see [Binderman et al. 2012, Binderman, Borkowski, Szczesny 2008, 2009, 2010a,b,c, 2011, 2012a Binderman, Szczesny 2009, 2011, Borkowski, Szczesny 2002]). Literature on this subject provides a wide array of instruments to compare and contrast objects represented by vectors. The authors in their other works use mainly measures based on visual representations via radar charts.

RESULTS OF EMPIRICAL RESEARCH

In the analysis of regional dissimilarity of agriculture in Poland, synthetic measures prove to be a fundamental tool. For they allow to describe analyzed objects (voivodeships) characterized by vectors from a multidimensional space of features by means of a single synthetic measure. By using this method one can perform comparative analysis of voivodeships with special consideration given to ordering of objects according to a predefine criterion and defining the place at which a given voivodeship ranks in comparison to other analyzed objects. Moreover, when analyzing a given time period one can define both the direction of change as well as the change in the level of dissimilarity in a group of objects. To illustrate the ideas and mechanisms described in previous sections the authors have used empirical data of Polish agriculture in an area formal (divided into voivodeships) for the period of 2001 – 2010. In order to eliminate or at least limit small seasonal deviations the period has been divided into three sub periods: 2001 – 2003, 2005 – 2007 and 2008 – 2010. Afterwards the data for those sub periods was averaged using CPI coefficient to realize the values given in PLN. 2004 has been eliminated from the analysis as in this year Poland joined the UE.

The following six features were used in the analysis:

1. GNP (X1) – Gross National Product (current prices) per capita given in PLN,
2. APG (X2) – Agricultural Production of Goods (PLN/ha),
3. AAH (X3) – Average Area of a single agricultural Household in hectares,
4. YIELD (X4) – yield of four crops in tones per hectare
5. UMF (X5) – Usage of Mineral Fertilizers in kilogram's per hectare
6. EMP (X6) – the number of people employed in agriculture, hunting and forestry, fishing for 100 hectares of general area.

All the features apart from X6 have had the character of a stimulator, X6 was assumed as a de-stimulator. The chosen features do not exhaust all aspects of agriculture, but on a general level, allow to show the differences which appear between voivodeships.

Based on the values of diagnostic variables in individual sub periods, we have created two constant (static) hypothetical voivodeships: minimal $\mathbf{Q}_0$ and maximal $\mathbf{Q}_{49}$, described by the least and most advantageous set of feature values, respectively. Those voivodeships, as models, will be denoted by vectors $\mathbf{x}_0$ and $\mathbf{x}_{49}$, each having 6 dimensions.

Thus we have 50 objects $\mathbf{Q}_0$, $\mathbf{Q}_1$, $\mathbf{Q}_2$,..., $\mathbf{Q}_{49}$, each described by n = 6 features $X_1, X_2,..., X_6$. As the chosen diagnostic variables had different units and orders of magnitude, they were all normalized. In order to make the variables comparable, from a set of approaches, the method of zero notarization was chosen [Kukula, 2000], converting the variables according to the following:

$$z_{ij} = \frac{x_{ij} - x_{mj}}{x_{Mj} - x_{mj}} \text{ – for stimulators, } j = 1,2,3,4,5;$$

$$z_{ij} = \frac{x_{Mj} - x_{ij}}{x_{Mj} - x_{mj}} \text{ – for de – stimulators, } j = 6, \tag{10}$$

where $x_{mj} = \min_{1 \le i \le 48} x_{ij}$, $x_{Mj} = \max_{1 \le i \le 48} x_{ij}$, $0 \le i \le 49,\ 1 \le j \le 6;$

The applied transformation can be represented symbolically as:

$$\mathbf{Z} = \varphi(\mathbf{X}),$$

where $\mathbf{X}$ – observation matrix, $\mathbf{Z} = [z_{ij}]_{48\times 6}$.

After the transformation of variables, the static model vectors are as follows:

$$\mathbf{z}_0 = \mathbf{0} = [0,0,...,0], \quad \mathbf{z}_{49} = \mathbf{1} = [1,1,...,1].$$

Additionally, one should note that it is non-trivial to show that they are equally important. Especially, GNP describes all economic activities, not just agricultural one, which input into GNP is at a level of a few percent. Yields of crops are largely dependent on the quality of soil and general environment conditions. Thus, is a comparative analysis, this fact should be noted by assigning different weights to features. So, to illustrate the problem, for the purposes of this work the following set of weights was assumed:

$$\mathbf{w} = (0{,}05;\ 0{,}225;\ 0{,}275;\ 0{,}05;\ 0{,}05;\ 0.35). \tag{11}$$

Empirical data, after normalization is shown in the following Table 1.

Table 1. Input data after normalization via zero unitarization (Formula 10)

	2001-2003						2005-2007						2008-2010					
	X1	X2	X3	X4	X5	X6	X1	X2	X3	X4	X5	X6	X1	X2	X3	X4	X5	X6
DOLNOŚLĄSKIE	0,20	0,16	0,45	0,58	0,23	0,87	0,38	0,21	0,44	0,55	0,57	0,89	0,54	0,29	0,49	0,72	0,68	0,88
KUJAWSKO-POMORSKIE	0,13	0,45	0,59	0,33	0,54	0,70	0,23	0,49	0,61	0,32	0,88	0,76	0,31	0,58	0,67	0,44	0,78	0,80
LUBELSKIE	0,00	0,16	0,22	0,15	0,25	0,44	0,08	0,24	0,22	0,07	0,45	0,51	0,16	0,37	0,24	0,27	0,35	0,59
LUBUSKIE	0,11	0,02	0,48	0,10	0,33	0,91	0,25	0,12	0,55	0,13	0,50	0,86	0,31	0,25	0,66	0,32	0,38	0,92
ŁÓDZKIE	0,13	0,49	0,23	0,03	0,26	0,58	0,27	0,74	0,24	0,08	0,62	0,61	0,38	0,75	0,25	0,21	0,48	0,64
MAŁOPOLSKIE	0,09	0,22	0,00	0,18	0,15	0,00	0,22	0,39	0,00	0,23	0,12	0,16	0,32	0,37	0,02	0,29	0,04	0,39
MAZOWIECKIE	0,55	0,38	0,27	0,05	0,14	0,59	0,81	0,61	0,28	0,00	0,39	0,75	1,00	0,73	0,30	0,12	0,28	0,74
OPOLSKIE	0,06	0,33	0,45	0,73	0,59	0,77	0,19	0,40	0,46	0,76	0,82	0,79	0,28	0,59	0,53	1,00	1,00	0,86
PODKARPACKIE	0,00	0,00	0,02	0,17	0,02	0,34	0,08	0,03	0,03	0,15	0,04	0,41	0,16	0,00	0,04	0,28	0,00	0,40
PODLASKIE	0,04	0,16	0,53	0,01	0,17	0,67	0,13	0,32	0,55	0,00	0,24	0,78	0,20	0,41	0,57	0,11	0,29	0,81
POMORSKIE	0,19	0,08	0,59	0,25	0,52	0,88	0,32	0,25	0,64	0,29	0,52	0,87	0,41	0,45	0,69	0,39	0,52	0,91
ŚLĄSKIE	0,24	0,28	0,06	0,34	0,25	0,77	0,39	0,43	0,09	0,28	0,40	0,75	0,52	0,54	0,11	0,39	0,36	0,83
ŚWIĘTOKRZYSKIE	0,04	0,27	0,13	0,04	0,10	0,44	0,14	0,40	0,11	0,02	0,35	0,26	0,25	0,45	0,12	0,15	0,19	0,44
WARMIŃSKO-MAZURSKIE	0,04	0,18	0,87	0,23	0,18	0,93	0,14	0,27	0,93	0,18	0,48	0,92	0,21	0,36	1,00	0,47	0,41	0,92
WIELKOPOLSKIE	0,22	0,64	0,53	0,31	0,36	0,68	0,38	0,86	0,51	0,32	0,76	0,79	0,49	1,00	0,53	0,43	0,72	0,78
ZACHODNIOPOMORSKIE	0,16	0,04	0,71	0,26	0,39	0,98	0,26	0,13	0,94	0,30	0,45	0,98	0,34	0,23	1,00	0,54	0,47	1,00

Source: own calculation based on data published by GUS

Based on the above data, the synthetic measures were calculated using the weight vector as defined in (11) and their results placed in Table 2. Table 3 shows the ranking of voivodeships according to those measures. Moreover, a division into four groups was performed according to E. Nowak's method [Nowak 1990], and its results places in Table 4.

Table 2. Values of synthetic coefficients as defined by (5) – (9), calculated using the weight vector as defined by (11). U* denotes the popular coefficient defined by (5) with the assumption that all weight are identical

	2001-2003						2005-2007						2008-2010					
	U*	U	S	$\mathcal{M}$	$\mathcal{S}$	m	U*	U	S	$\mathcal{M}$	$\mathcal{S}$	m	U*	U	S	$\mathcal{M}$	$\mathcal{S}$	m
DOLNOŚLĄSKIE	0,42	0,52	0,35	0,57	0,44	0,33	0,51	0,55	0,45	0,58	0,50	0,42	0,60	0,61	0,54	0,64	0,58	0,50
KUJAWSKO-POMORSKIE	0,46	0,56	0,48	0,59	0,53	0,43	0,55	0,62	0,55	0,66	0,59	0,52	0,60	0,67	0,61	0,70	0,65	0,58
LUBELSKIE	0,20	0,27	0,19	0,28	0,24	0,18	0,26	0,32	0,26	0,33	0,29	0,22	0,33	0,40	0,32	0,42	0,37	0,31
LUBUSKIE	0,32	0,48	0,26	0,56	0,35	0,17	0,40	0,52	0,37	0,61	0,44	0,31	0,47	0,61	0,45	0,67	0,54	0,45
ŁÓDZKIE	0,29	0,40	0,31	0,44	0,35	0,23	0,43	0,49	0,43	0,58	0,46	0,36	0,45	0,52	0,45	0,60	0,49	0,40
MAŁOPOLSKIE	0,11	0,07	0,05	0,10	0,06	0,02	0,19	0,18	0,13	0,22	0,16	0,09	0,24	0,25	0,18	0,33	0,22	0,12
MAZOWIECKIE	0,33	0,40	0,36	0,44	0,37	0,29	0,47	0,54	0,51	0,62	0,50	0,40	0,53	0,58	0,56	0,67	0,55	0,44
OPOLSKIE	0,49	0,54	0,44	0,57	0,50	0,42	0,57	0,58	0,51	0,62	0,56	0,50	0,71	0,69	0,64	0,75	0,68	0,63
PODKARPACKIE	0,09	0,13	0,03	0,12	0,06	0,01	0,12	0,17	0,09	0,15	0,11	0,08	0,15	0,17	0,10	0,19	0,12	0,04
PODLASKIE	0,26	0,43	0,25	0,49	0,34	0,16	0,34	0,52	0,36	0,56	0,44	0,24	0,40	0,56	0,43	0,60	0,50	0,34
POMORSKIE	0,42	0,54	0,36	0,63	0,45	0,29	0,48	0,59	0,46	0,66	0,53	0,44	0,56	0,68	0,57	0,72	0,63	0,55
ŚLĄSKIE	0,32	0,39	0,27	0,41	0,31	0,23	0,39	0,44	0,35	0,50	0,38	0,28	0,46	0,50	0,40	0,58	0,45	0,33
ŚWIĘTOKRZYSKIE	0,17	0,26	0,17	0,28	0,22	0,12	0,21	0,24	0,21	0,28	0,22	0,17	0,27	0,32	0,26	0,37	0,29	0,23
WARMIŃSKO-MAZURSKIE	0,40	0,63	0,36	0,74	0,50	0,30	0,48	0,68	0,48	0,79	0,58	0,42	0,56	0,73	0,56	0,83	0,66	0,52
WIELKOPOLSKIE	0,46	0,57	0,51	0,60	0,54	0,42	0,60	0,68	0,63	0,74	0,66	0,58	0,66	0,73	0,69	0,79	0,71	0,64
ZACHODNIOPOMORSKIE	0,42	0,59	0,34	0,71	0,46	0,25	0,51	0,68	0,44	0,82	0,57	0,39	0,60	0,74	0,55	0,87	0,66	0,51
min	0,091	0,069	0,034	0,096	0,056	0,008	0,124	0,171	0,092	0,146	0,108	0,085	0,147	0,175	0,102	0,195	0,116	0,040
max	0,488	0,626	0,506	0,741	0,542	0,426	0,604	0,684	0,631	0,816	0,661	0,576	0,711	0,744	0,690	0,867	0,709	0,639
max-min	0,397	0,557	0,472	0,644	0,487	0,418	0,480	0,513	0,539	0,670	0,553	0,491	0,564	0,569	0,588	0,673	0,593	0,598
mi	0,322	0,423	0,295	0,470	0,358	0,239	0,407	0,487	0,390	0,544	0,438	0,337	0,474	0,548	0,457	0,608	0,505	0,413
sigma	0,123	0,160	0,131	0,187	0,147	0,125	0,140	0,167	0,146	0,192	0,159	0,143	0,156	0,172	0,162	0,184	0,171	0,169
V	0,381	0,378	0,445	0,398	0,410	0,521	0,344	0,343	0,374	0,354	0,362	0,424	0,330	0,313	0,356	0,303	0,338	0,410

Source: own research

In the above Table 2 mi, sigma, V denote average value, standard deviation, volatility, respectively. Values in columns U^w, S^w, M^w, S^w, m^w are values of synthetic measures calculated according to (5) – (9), using the weight vector as defined by (11), while the values in column U* are the values of a synthetic measures as defined by (5) but with the assumption that all weights are identical, that is the weight vector !! $\mathbf{w} = \mathbf{1/n} = (1/n, 1/n, \ldots, 1/n)$!!.

Table 3. Division of voivodeships into four groups according to the values of coefficients from Table 2

	2001-2003						2005-2007						2008-2010					
	U*	U	S	$\mathcal{M}$	$\mathcal{S}$	*m*	U*	U	S	$\mathcal{M}$	$\mathcal{S}$	*m*	U*	U	S	$\mathcal{M}$	$\mathcal{S}$	*m*
DOLNOŚLĄSKIE	6	7	7	7	7	4	5	7	7	9	8	6	3	8	8	9	7	7
KUJAWSKO-POMORSKIE	2	4	2	5	2	1	3	4	2	5	2	2	4	6	3	6	5	3
LUBELSKIE	13	13	13	13	13	11	13	13	13	13	13	13	13	13	13	13	13	13
LUBUSKIE	9	8	11	8	10	12	10	9	10	8	11	10	9	7	9	8	9	8
ŁÓDZKIE	11	11	9	10	9	9	9	11	9	10	9	9	11	11	10	10	11	10
MAŁOPOLSKIE	15	16	15	16	16	15	15	15	15	15	15	15	15	15	15	15	15	15
MAZOWIECKIE	8	10	6	11	8	6	8	8	4	7	7	7	8	9	6	7	8	9
OPOLSKIE	1	5	3	6	3	3	2	6	3	6	5	3	1	4	2	4	2	2
PODKARPACKIE	16	15	16	15	15	16	16	16	16	16	16	16	16	16	16	16	16	16
PODLASKIE	12	9	12	9	11	13	12	10	11	11	10	12	12	10	11	11	10	11
POMORSKIE	5	6	4	3	6	7	7	5	6	4	6	4	6	5	4	5	6	4
ŚLĄSKIE	10	12	10	12	12	10	11	12	12	12	12	11	10	12	12	12	12	12
ŚWIĘTOKRZYSKIE	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
WARMIŃSKO-MAZURSKIE	7	1	5	1	4	5	6	3	5	2	3	5	7	2	5	2	4	5
WIELKOPOLSKIE	3	3	1	4	1	2	1	1	1	3	1	1	2	3	1	3	1	1
ZACHODNIOPOMORSKIE	4	2	8	2	5	8	4	2	8	1	4	8	5	1	7	1	3	6

Source: own research

Table 4. Ranking of voivodeships according to coefficients from Table 2

	2001-2003						2005-2007						2008-2010					
	U*	U	S	$\mathcal{M}$	$\mathcal{S}$	*m*	U*	U	S	$\mathcal{M}$	$\mathcal{S}$	*m*	U*	U	S	$\mathcal{M}$	$\mathcal{S}$	*m*
DOLNOŚLĄSKIE	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
KUJAWSKO-POMORSKIE	1	2	1	2	1	1	1	2	1	2	2	1	2	2	2	2	2	2
LUBELSKIE	3	3	3	3	3	3	4	3	3	4	3	3	3	3	3	4	3	3
LUBUSKIE	2	2	3	2	3	3	3	2	3	2	2	3	3	2	3	2	2	2
ŁÓDZKIE	3	3	2	3	3	3	2	2	2	2	2	2	3	3	3	3	3	3
MAŁOPOLSKIE	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
MAZOWIECKIE	2	3	2	3	2	2	2	2	2	2	2	2	2	2	2	2	2	2
OPOLSKIE	1	2	1	2	2	1	1	2	2	2	2	1	1	2	1	2	1	1
PODKARPACKIE	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
PODLASKIE	3	2	3	2	3	3	3	2	3	2	2	3	3	2	3	3	3	3
POMORSKIE	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
ŚLĄSKIE	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ŚWIĘTOKRZYSKIE	4	4	3	4	3	3	4	4	4	4	4	4	4	4	4	4	4	4
WARMIŃSKO-MAZURSKIE	2	1	2	1	2	2	2	1	2	1	2	2	2	1	2	1	2	2
WIELKOPOLSKIE	1	2	1	2	1	1	1	1	1	1	1	1	1	1	1	2	1	1
ZACHODNIOPOMORSKIE	2	1	2	1	2	2	2	1	2	1	2	2	2	1	2	1	2	2

Source: own research

Analysis of results in Tables 2 – 4 has shown that the level of agriculture is spatially variable and the division of voivodeships into four groups according to analyzed features is in accordance with regional dispersion. The chosen example is a set of sixteen objects, which differ significantly between each other. In those cases, every method will result in a similar ordering of objects. That is why all coefficients indicate that the average level increases between subsequent periods. However, this cannot be stated when differentiation is considered (standard deviation has a different trend than volatility). The results show that even in this relatively stable example the measure of ordering S^w, defined by (6) return with one of the worst (random) orderings. Those are significant in cases of a few voivodeships and the magnitude of inconsistency between the ordering and other coefficients is comparable only to that of inconsistency represented by coefficient U*. However, that coefficient is using a completely different set of weights (so it is based on a different valuation criteria). Research has shown that the best ordering is given by a method based on coefficient defined by (7). The results are relatively close to those returned when using coefficient U, but, as mentioned before, when objects are so different between each other, most methods should lead to similar orderings.

SUMMARY

In reports generated by information system, graphical representations are desirable as they depict in an easy to understand and process way even very difficult ideas by using the recipients intuition and not requiring them to thoroughly prepare in terms of both knowledge and toolkit. However, improper use can lead to misunderstandings and incorrect conclusions and, as a result, decline of trust in the information system. In this work it has been shown that preparing radar charts requires special care. Charts that are a promising and easy-to-access tool available in a spreadsheet, which allow to present and evaluate objects described by many features. Analysis performed on data describing the level of agriculture between different voivodeships has shown that used methods of classification and ordering of objects with six features, even though the objects were very dissimilar, have given different results. Moreover, it has been noted that the very intuitive synthetic coefficient S^w that uses the area of an n-polygon is not constant in relation to the ordering of variables, which can, in many cases, be a large disadvantage. A similar behavior can be observed when considering the weighting process. Coefficient U* which considers all variables equally and coefficient U^w which takes weights into account returned with different results, even though the base material was very diverse.

Because of this it seems necessary to place in visualization reports results of individual objects (processes) in a multidimensional view (for example by using radar charts with an ideal and “desired” or “expected” objects in the background) of multiple values, rather than a single synthetic coefficient which evaluates the given object as the most significant one. In case when they will return with different evaluation the recipient will, at the very least, investigate the causes of the discrepancies. It is especially important when the report consists of a set of hierarchical visualizations and evaluations, because then unexpected situations on lower levels can go unnoticed.

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MODEL OF COMPETENCE OF EXPERTS IN THE COMPUTER DECISION SUPPORT SYSTEM

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Abstract: The paper presents the module of competence management of experts that has been developed for computer decision support system (DSS 2.0). At the top of the hierarchical structure of the model is the European standard of core competencies, the decomposition of which allows the mapping of different specializations. The flexible structure of information competence module allows defining any set (profile) of qualifications of experts and the strength of their impact in solving specific problems of decision-making.

Keywords: expert competence, competency management, decision support system

INTRODUCTION

Fundamental competency management issues revolve around the creation of models (profiles) of competences. The concept of such a model is understood as an ordered set of competencies specific to the job, the role of the organization, the profession, which is used as a multi-purpose tool in the management of human capital [Oleksyn 2006]. Appropriate models of competence clearly define the skills that are necessary and required to achieve that objective, the strategy adopted by the company and the applicable regulations and standards (in institutions, professional group). Creating models is an extremely difficult and complex task. They should accurately reflect the goals and at the same time be flexible in relation to changing environmental conditions and take into account patterns of human behaviour. A common problem in the literature is the lack of use of a universal list of competencies for all job post profiles [Borkowska 2006].

The paper presents the module of competency management of experts that has been developed for computer decision support system DSS 2.0 [Budziński, Becker, 2008-2013]. The advantage of this solution are flexible information structures that allow for modelling of expert competency profiles required in supporting specific decision problems. These issues are presented in the form of mathematical models (decision-making tasks), whose components are variables, parameters and constraints [Becker 2010]. The decision support information system assumes that the values of the decision-making tasks are: the result of a group of expert assessments (ratings linguistically expressed or numerically), defragmented into the components and assessed together (including preferences) and transposed to the desired output forms, such as the scope of binary common in multi-step tasks.

COMPETENCE MODELLING STANDARDS

The theory and practice of management has developed (in general and specific) many definitions that describe the concept of competence. One of them states that they usually include relatively stable characteristics of a man making a causal cause and result relationship with the high or above average work results, which have universal dimensions [Pocztowski 2003]. Today, in the literature there are two strands to define the notion of competence. The first is directly related to the person who has the power and it defines as the knowledge, skills, responsibilities and powers of action [Oleksyn 2006]. He identifies them with a set of behaviours that some people take over better than others, which makes them operate more efficiently in a specific situation [Sidor-Rządkowska 2006], as well as abilities, interests, personality traits, as examples of the parameters that differentiate between them [Levy- Lboyer 1997]. The second trend combines the skills of their work or duties related to his office. Therefore, this term shall include a set of characteristics of a person, which includes, among other things: motivation, personality traits, skills, self-esteem associated with the functioning of the group and the knowledge [Witaszek 2011].

Competencies can be grouped into the following categories:

- *core competencies* required from all employees (teamwork, integrity, customer orientation, communication skills),
- *competencies specific to the role*, which allows the employee to play a certain attitude in the organization (team management, planning long-term, strategic thinking),
- *competencies specific to the function*, required from employees, depending on the area of your business (eg, sales and negotiation skills, knowledge of the industry and the local market).

Competencies management, from the point of view of the organization, begins with the development kit (models, profile) competencies needed to perform the work [Witaszek 2011]. At the same time the management of professional competence consists in following a personnel policy, in which the concept of competence combines activities in different areas of human resource management, such as recruitment and selection, employee evaluation, training, developing, motivating and rewarding. Therefore, there are two methods of supporting the process:

- expert – work on the identification and description of competencies and developing implementation procedures performs a team of external and independent experts,
- participatory – creation of appropriate competency profiles required number of workshops, involving specific positions within the company and their supervisors, and in some cases customers.

The mission of competency management is to ensure that the adequate resources in the company, necessary to achieve its strategic objectives.

Build a model of competence required to determine the level of detail descriptions and competencies. Frequently this decision determines the time required to develop the model and the possibility of its application in various areas of the organization. In practice, a specific competence for one company may be a set of competencies as possible to further disaggregate into various components for another company. The more detailed the model is, the longer its construction lasts and the higher are its development costs. There may also be problems with the comparison of information from a variety of tasks and the people who perform them. Considerable detail to define the competence limits the use of creative, new ways of achieving the desired results. On the other hand, it allows a more accurate description of the expected results and better performance management. However, the most effective results are achieved using computer-integrated management systems. [Kopczewski, Szwarc 2009]

Competency model can be constructed in the form of clusters or arrays of competence. The structure must be clear, understandable and do not contain ambiguous wording or complicated descriptions. All elements of the model should be independent of each other and reflect the specifics of the organization and reflect the true nature of roles and positions.

Competence profiles can describe a particular employee, position, or role in the company, and a group of employees, group of positions (eg, management or employees of financial and accounting department) or the whole company. The primary objective of building a competency profiles it is possibility to assess them. From this point of view, there are two types of competency profiles:

- models of the desired competencies describing relevant properties for action or an ideal situation in the past,
- models of present competencies at our disposal at the time of the description [Kopczewski, Szwarc 2009].

A rule of thumb approach used to identify competence is the use of two or more of the methods and comparing the results obtained. Techniques to identify competencies include: examination of documents, surveys, interviews, direct observation and simulation [Kopczewski, Szwarc 2009]. In business practice, there is a shortage of system solutions, which in an objective manner, taking into account the time factor, would select such candidates to the teams and evaluate their decisions. This is a serious problem, because the skills are the basis for decisions on recruitment, selection, training, opportunities for development and evaluation of employees. Previous studies show that over 90% of the implemented software for competence management is based on simple and inefficient methods [Borkowska 2006], [Dale 2006], [Galen, Dean 2005].

INFORMATION STRUCTURE OF EXPERT COMPETENCE MODULE

Presented in Figure 1 model of expert competence management subsystem has a hierarchical structure, which can be divided into general level (phase 1) and detailed (phase 2 and 3). The highest level of the structure was constructed based on the European Standard of Core Competencies (a set of *K)*, i.e.: k_1 – Communication in the mother tongue, k_2 – Communication in foreign languages, k_3 – Mathematical competence, k_4 – Competences in science and technology, k_5 – Digital competence, k_6 – Learning to learn, k_7 – Social and civic competence, k_8 – Sense of initiative and entrepreneurship, k_9 – Cultural awareness and expression (cf. [Recommendation of the European Parliament ..., 2006]). Denote by *OC* set of evaluators (experts), and the oc_j person from this set (j = 1, 2, ..., m). In the first phase (Figure 2) for each *j*-expert key competences shall be made, using a point scale or the same as the scale of linguistic levels. They are a set of declared (for example, by self-assessment) of the vector values W^j expressing the levels of competence based on the scientific and proven performance. It was assumed that these indicators take a value between ⟨0; 1⟩:

$$W^j = \{w_1^j, w_2^j, \dots, w_9^j\},\ (j\ =\ 1, 2, \dots, m). \tag{1}$$

In the second phase of the procedure (Fig. 3) and the second level of the hierarchical structure of the model, the standard of competence for the specific problem of decision-making is defined $(z\ =\ 1, 2, \dots, v)$. For this purpose a proportional comparison matrix for selected key competencies is built. Then, using the method of Saaty [1980], using the technique of pair wise comparisons [Trzaskalik 2006] a vector scale is defined R^z. The procedure refers to the assignment of competence in the set

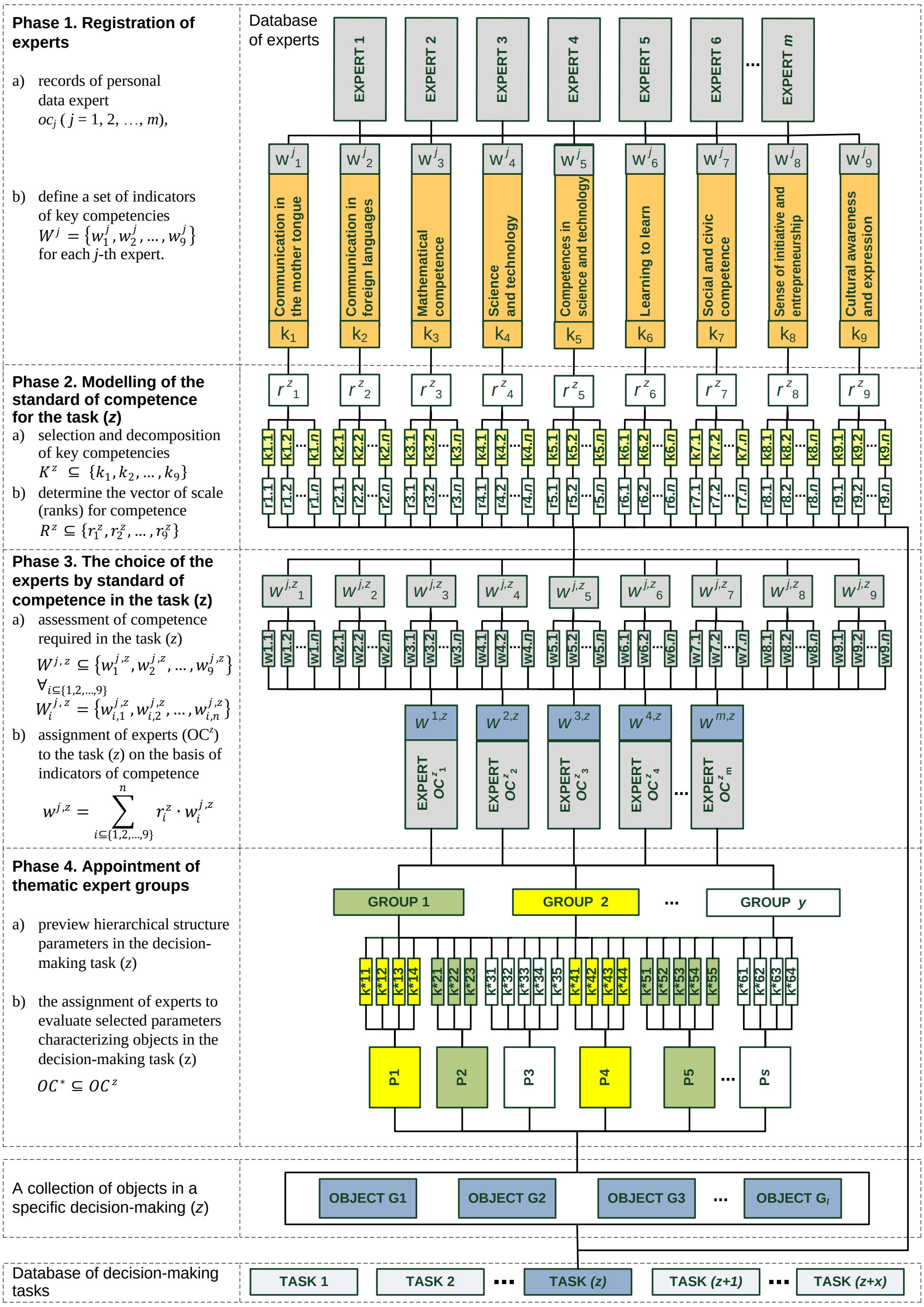
$$K^z\ \subseteq\ \{k_1, k_2, \dots, k_9\}, (z\ =\ 1, 2, \dots, v) \tag{2}$$

of standardized indicators preferences

$$R^z \subseteq \{r_1^z, r_2^z, \dots, r_9^z\}, \tag{3}$$

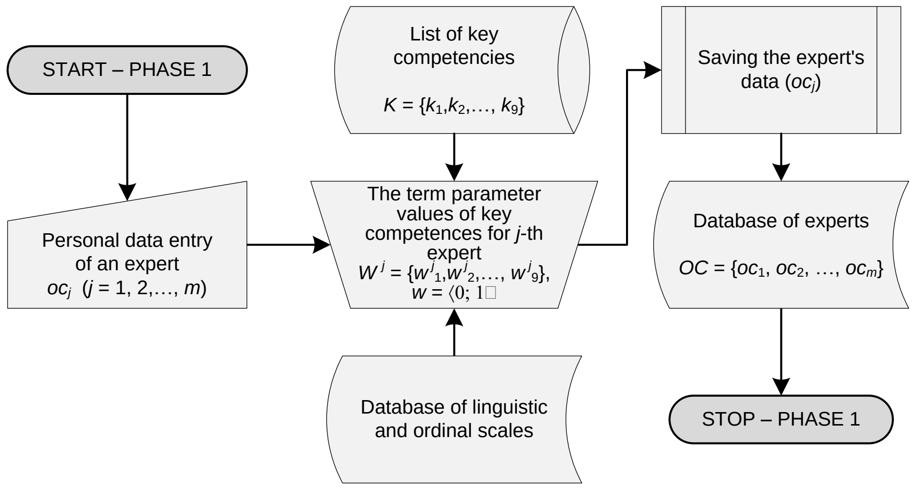
that in a given decision-making tasks (z) allow them to be ranked.

Figure 1. Model of subsystem for competence management in the DSS-class system



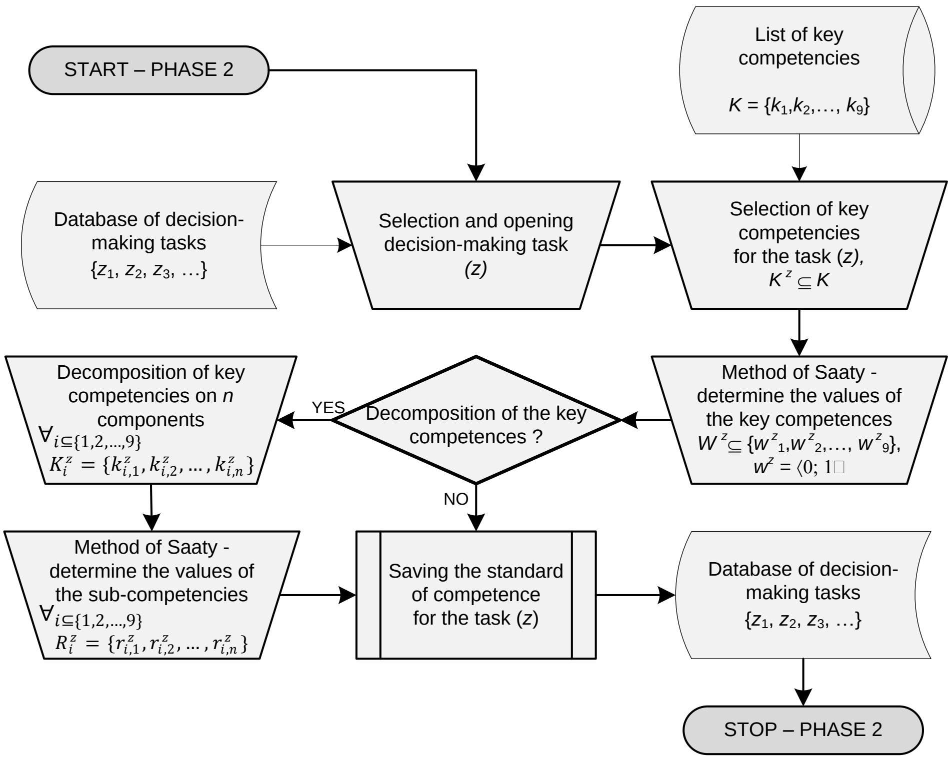
Source: own work

Figure 2. Registration of experts in the DSS –phase 1



Source: own research

Figure 3. Modelling the standard of competence for the task –phase 2



Source: own research

The resulting vector scale R^z is positive and normalized (the sum of its elements equals 1). Non-obligatorily, each core competency can be decomposed into n constituent notes

$$\forall_{i\subseteq\{1,2,\ldots,9\}} K_i^z = \{k_{i,1}^z, k_{i,2}^z, \ldots, k_{i,n}^z\} \quad (4)$$

and evaluated for their vector scale (rank) in the same way as for the key competences

$$\forall_{i\subseteq\{1,2,\ldots,9\}} R_i^z = \{r_{i,1}^z, r_{i,2}^z, \ldots, r_{i,n}^z\}. \quad (5)$$

The adopted two-level structure of competence allows for mapping a wide variety of expertise specialities required in the decision-making procedure (z).

The selection of experts for research included two stages:

1) the pre-selection of people on the basis of the generic competences W^j and evaluation (assessing) of the competence of the persons selected according to a certain pattern of competence $\{K^z; R^z\}$ for the task (Fig. 4, phase 3),
2) the formation of teams of experts to assess individual performance criterion characterizing objects (variants of decision) are reported to the system, for example in the form of applications (Fig. 5, phase 4).

The initial steps in phase 3 are to respect individual competence (K^j) to a defined pattern (K^z) for the task. This action takes the form of dialogue and is made by the analyst (DSS system user). As a result, the selection shall be given a set of people $OC^z \subseteq OC$ with the highest competence, in accordance with the task (z). The analyst has at his disposal the tools to prioritize and retrieval of experts (database records) according to any logical conditions. The query can be either coarse, consisting of ranking of the average values of standardized coefficients of competence $\overline{w}^j$, consistent with the K^z for each expert. The search can also be more precise, based on a comparison of the elements of the vector W^j in the database of given experts.

In the next step of phase 3 the degree of competence of the experts prequalified for the task (z) is assessed (estimated). For each element of the set K^z, a set within the scope of $\langle 0; 1\rangle$ expressing the degree of competence of *the main competencies,* should be determined using a point scale or identical linguistic scale

$$W^{j,z} \subseteq \{w_1^{j,z}, w_2^{j,z}, \ldots, w_9^{j,z}\}. \quad (6)$$

Exceptions to this rule are the competence (of the set K^z) which decomposed at n component notes as set out in (4). Competence particulars referred to as a model for the task are subject to the same process of evaluation, the result is a set of ratings

$$\forall_{i\subseteq\{1,2,\ldots,9\}} W_i^{j,z} = \{w_{i,1}^{j,z}, w_{i,2}^{j,z}, \ldots, w_{i,n}^{j,z}\}, \quad (7)$$

for each pre-qualified j-th expert. Individual indicators for general competence ratings are calculated as the sum of products of elements of the vector rank (5) and vector of partial marks (7):

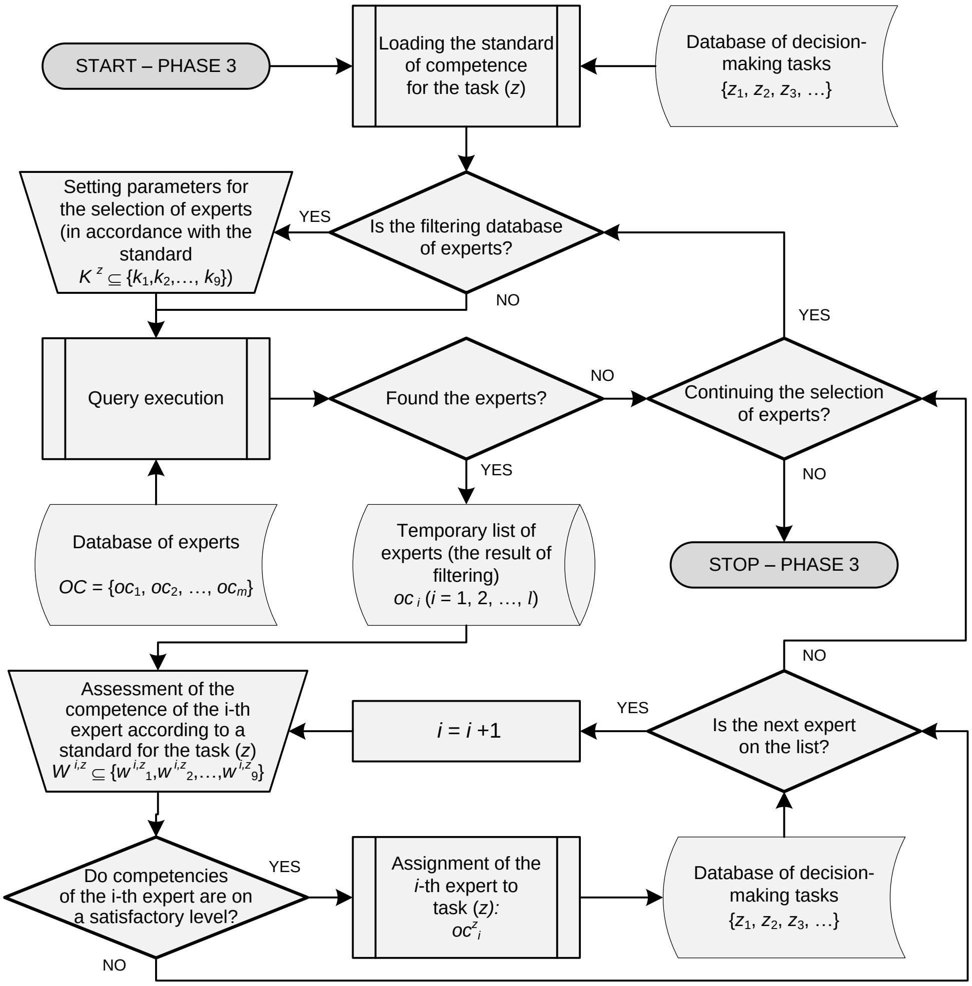
$$w_i^{j,z} = \sum_{s=1}^{n} r_{i,s}^z \cdot w_{i,s}^{j,z}. \quad (8)$$

Similarly, based on the vector ratings (8) and ranks (3) for key competencies system calculates the *global competence indicator of an expert (j) in the task (z)*

$$w^{j,z} = \sum_{i\subseteq\{1,2,\ldots,9\}}^{n} r_i^z \cdot w_i^{j,z}. \tag{9}$$

Rate $w^{j,z}$ is positive in the range ⟨0, 1⟩ and expresses the degree of competence (strength reviews) expert in the review of applications, specifically in the evaluation of the parameters characterizing the variations of decision-making in the task. In practice, it should not take too low values (e.g. less than 0.7) for the vast number of selected experts, as this will have a negative impact on the reliability of the results of the analysis of decision-making: choice, ranking and grouping options in the decision-making system.

Figure 4. The choice of the experts to the task –phase 3



Source: own research

In the second stage of the selection of experts (Figure 5, phase 4) it is assumed that each parameter (p^*) With decision-making tasks (z) can be assigned with the individual team of experts $OC^* \subseteq OC^z$. Denoted by oc_j^* an assessor parameter p^* where j = 1, 2, ..., m^*. Then m^* will determine the cardinality of the set OC^*, or expert opinion relating the parameter p^* in each object (variant decision) G_t(t = *1*, *2*, ..., *l*) filed into the DSS system in the form of application.

The analyst can determine the parameter p^* a set of sub-criteria k_i^* (i = 1, 2, ..., n^*) and give them the values rang validity r_i^* (preferences of the decision maker). It is assumed that the elements r_i^* single-column matrix of rank R^* express the main parameter of interest p^* and their sum is equal to unity:

$$R^* := (r_i^*)_{n^* \times 1}, \quad \sum_{i=1}^{n^*} r_i^* = 1. \tag{10}$$

No division into sub-criteria will undergo parameter p^* direct assessment ($r_i^* = 1$, $i = n^* = 1$).

Let τ is the number of linguistic values $a^{(\alpha)}$ (α = 1, 2, ..., τ), which form a system-defined simple scale in the DSS (e.g. $a^{(1)}$: small, $a^{(2)}$: medium, $a^{(3)}$: large). Considering a simplified technique of scaling, which is to divide the range evaluated parameter (p^*_{min}, p^*_{max}) on τ-1 equal sections, assignment $a^{(\alpha)} \mid p^{(\alpha)}$ is obtained by calculating:

$$p^{(\alpha)} = p^*_{\max} + \frac{(p^*_{\max} - p^*_{\min}) \cdot (\alpha - 1)}{\tau - 1}, \text{ for } \tau > 1. \tag{11}$$

If the parameter p^* was divided into n^* sub-criteria k_i^* (i= 1, 2, ..., n^*), which underwent a collective assessment by m^* people OC_j (j = 1, 2, ..., m^*) then we obtain an individual scoring matrix

$$\mathrm{A} := (a_{i,j}^{(\alpha)} \mid p_{i,j}^{(\alpha)})_{n^* \times m^*}. \tag{12}$$

For each sub-criterion k_i^*, a row in the matrix A, we calculate the arithmetic mean of the partial, creating a matrix column

$$\mathrm{P} := (\bar{p}_i)_{n^* \times 1}, \; where \quad \bar{p}_i = \frac{\sum_{j=1}^{m^*} p_{i,j}^{(\alpha)}}{m^*}. \tag{13}$$

Then, multiplying the vector of average grades of group P by the vector of ranks (preferences) of decision-makers R^*, we obtain the vector of values of sub-criteria K^*:

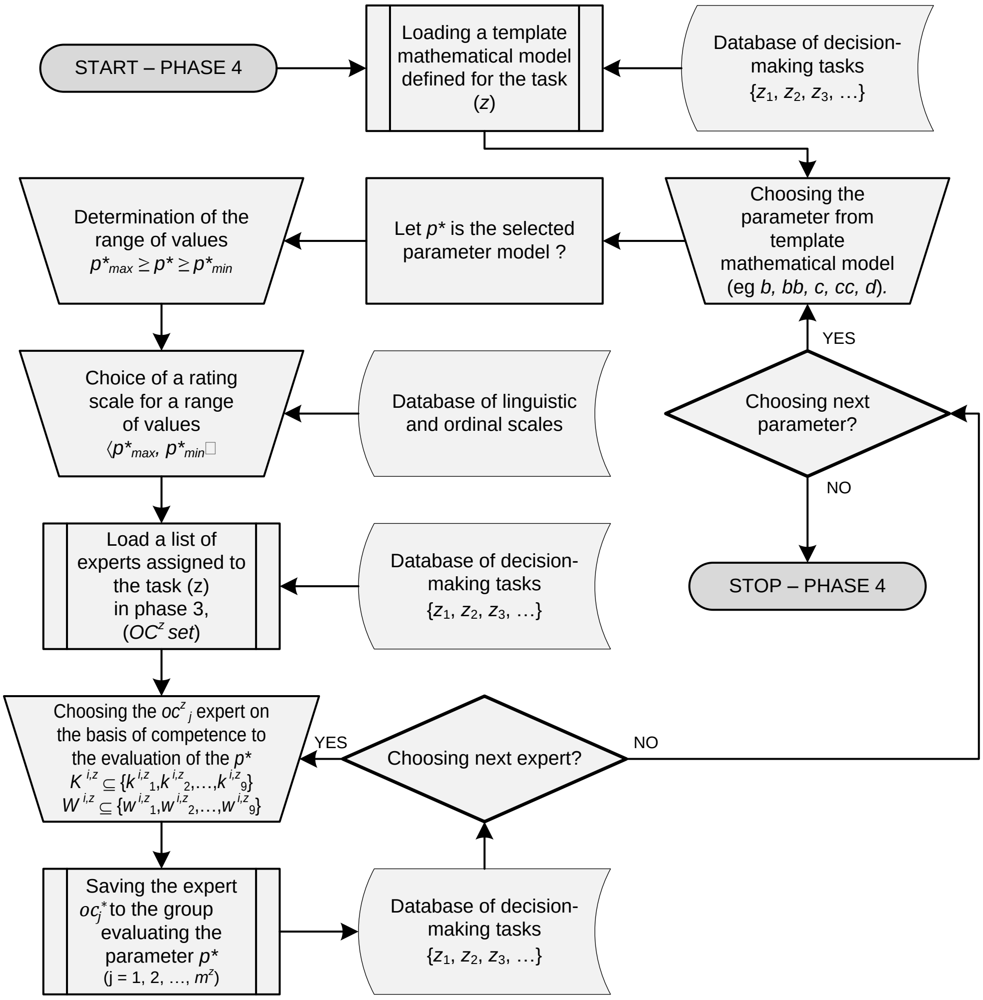
$$\mathrm{K}^* := (k_i^*)_{n^* \times 1}, \; \mathrm{K}^* = \mathrm{P} \times \mathrm{R}^*. \tag{14}$$

As a result of the vector sum of the individual values of K vector we obtain a synthetic value of group evaluation (assessment) parameter p^*:

$$p^* = \sum_{i=1}^{n^*} k_i^*. \tag{15}$$

Value p^* will be in the predetermined range (p^*_{min}, p^*_{max}). However, it does not take on the form of equation (9), i.e. *global competence indicator expert in the task*.

Figure 5. The assignment of experts to evaluate selected parameters characterizing objects in the decision-making task (z) – phase 4



Source: own research

The reference of group evaluation parameter p^* to set up in the task (z) level of competence of the experts (j) involves the determination of vector weighted average partial marks

$$\mathrm{P}_{ocn} := (\bar{p}_i)_{n^* \times 1}, \ \ where \ \ \bar{p}_i = \frac{\sum_{j=1}^{m^*} p_{i,j}^{(\alpha)} \cdot w^{j,z}}{\sum_{j=1}^{m^*} w^{j,z}}. \qquad (16)$$

Then, in accordance with the provisions of (14) and (15), a vector is calculated for sub-criteria

$$\mathrm{K}^{*}_{ocn} \coloneqq (k^{*}_{i(ocn)})_{n^{*}\times 1},\ \mathrm{K}^{*}_{ocn} = \mathrm{P}_{ocn} \times \mathrm{R}^{*} \tag{17}$$

and the value of the parameter group evaluation

$$p^{*}_{ocn} = \sum_{i=1}^{n^{*}} k^{*}_{i(ocn)}\ , \tag{18}$$

which takes into account the strength of competence of the experts on the team OC^{*}.

CONCLUSION

The inclusion of competence management module with the function of the group evaluation (assessment) parameters of objects into the structure of the DSS system, is justified in practice, because it allows for seeing and comparing the results of decision analysis (selection – WPL, ranking – AHP and grouping – Electre TRI) in two sections, including and excluding the impact of the competence of individual experts.

The following methods were used for the creation of a basic criterion which is the analysis of competence *ex-ante* of people taking strategic decisions. A deficiency in decision support systems is their assessment *ex-post*. This assessment is defined as a final score, which is run after the implementation of the various actions, decisions, interventions, programs, projects, etc. The main objective of this review is to determine the quality of teams as well as competence *ex-ante* with respect to the decisions taken. In this way, the utility evaluated the effects of implemented measures. This is due to answer a number of questions. Were people matched well to the teams? Did they have sufficient competencies? Did decisions taken really lived up to expectations? Were the effects of the measures taken sustainable? Evaluation *ex-post* also functions as a feedback on the quality of performed activities and advisory groups in decision-making.

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POLISH TECHNOLOGY PARKS AND REGIONAL DEVELOPMENT: ARE THEY EFFECYIVE?

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Abstract: The effective development of regions depends not only on the size of the national income, but also on the source of its origin. Differences in approach to the factors describing the competitiveness of regions and cities have evolved from a more general level in the direction of specialization, including both factors related to the quality of human capital and economic potential. Technology parks are an example of a high concentration of innovative human capital. In the past five years several new park initiatives were established in Poland. The activities of parks should be the development measured, for example, by employment growth in the regions, and innovativeness of companies. The main aim of the study is a multi-dimensional assessment of spatial differentiation in development of technology parks in Poland in terms of dynamics and identification the factors affecting the socio-economic development of cities and regions. The study used data from a project of PARP entitled "Benchmarking technology parks in Poland." The selected methods of multidimensional comparative analysis in terms of dynamics was selected to study the effectiveness of the parks.

Keywords: multidimensional comparative analysis, effectiveness, technology parks

INTRODUCTION

The main objective of the technology parks, which are one of the main instruments of innovation policy, is to increase the competitiveness of the Polish economy. Although the first technology parks were created in Poland in the mid-nineties, it was this type of co-financing projects from EU funds that led to their rapid development. With the development of parks, there was also a need to evaluate the effectiveness of their performance and to identify the main directions of development.

Technology parks in Poland are located especially within large cities, such as, Wrocław, Poznań and Gdańsk. The weaker regions with less developed urban centers, which are in transition results deviated from their industrial roots, technology parks do not exist or they are in the embryonic phase such as it is in Lubuskie region. Therefore, the level of development of the spatial differentiation parks is significant, and hence their efficiency of operation.

The main goal of this work is a multi-dimensional assessment of the level of spatial differentiation of the development of technology parks in a dynamic approach on the basis of the data from the years 2009-2011 in Poland and an indication of the factors affecting the efficiency of the system.

In order to verify this posed to the study, the following research hypotheses were taken:

1. The variation in the development of technology parks in Poland (including the different stage of development of parks) results in the need for a comprehensive approach to study the efficiency of their operations taking into account this diversity.
2. Analysis of the effectiveness of the parks requires evaluation from the perspective of many different factors that describe not only the potential infrastructure technology parks or their financial results using methods for making multi-dimensional assessment of the level of development of parks.

For the study of spatial differentiation in the development of technology parks in Poland a taxonomic measure of the development z_i in a dynamic approach was used which was based on the statistical information collected during the study „Benchmarking of technology parks in Poland –2010 and 2012 edition".

THE ECONOMIC IMPORTANCE OF THE TECHNOLOGY PARKS AND THEIR IMPACT ON THE COMPETITIVENESS OF THE REGIONS

In the literature, there are many institutions whose activities relate to innovation policy. These institutions can function independently or function such as technological parks, they are as follows: science parks, research parks, industrial parks, research and innovation centres. The definition can be significantly different depending on the country in which these institutions operate.

Polish legislation separately interprets the term technology park and industrial park[1]. In the case of the definition of a technology park the main

[1] Under the Act of 20 March 2002 on financial support for investment (Dz. U. Nr 41, pos. 363)

emphasis is put on the implementation of scientific knowledge in business practice and the development of enterprises that use modern technology. One of the major areas of technology parks is also the implementation of a function of incubation of the existing park incubators. However, the main purpose of industrial parks is to support restructuring processes, business development and local labor markets [Pelle et al. 2008].

The need for the creation of technology parks is associated with the possibility to create positive externalities as a result of research and development park (as an institution) and businesses operating within the park (park tenants).

The high concentration of knowledge and human capital can lead to significant changes to improve the competitiveness of the regions in which parks operate. It is very important if we take into account the changes in the approach to the factors describing the competitiveness of the regions, which have evolved from a more general level in the direction of specialization, including both factors related to the quality of human capital and economic potential.

The main changes concern the increasingly observed the duality of the labor market, associated with significant differences between the primary and secondary market [Gębski 2009]. From this point of view, among the new factors of competitiveness of regions and cities listed include [Sassen 2006, Parteka 2007]:

- instead of general human, the part of them that is capable of manufacturing and service permanently on the market;
- not so much the size of a scientific center in the vicinity, which is located in the technology park as its ability to generate innovation and absorb them;
- willingness to rapid changes in the economic profile and the variety and flexibility in the so-called specialization. Smart specialization, and not so far indicated the stability of the development of cutting-edge sectors and specializations fixed region.

In Poland, the potential for innovation due to the structure of companies is only for located mainly in small and medium-sized enterprises. Unfortunately the company of this size does not usually have the infrastructure or facilities or more research to implement new technology solutions more effectively. The solution in this regard may be the support of the business environment to facilitate access both to information as well as technical infrastructure, services and financial assistance for example, by the possibility of using seed capital [Kowalak 2010].

Technology parks are a good example of this type of support for small and medium-sized enterprises. In the past five years there have been several in Poland new initiatives parks created by the regional authorities, universities, and private owners. Each of the entities forming or co-participating in creating the park aims at different targets, including: the growth of entrepreneurship and employment in modern companies with high potential for innovation, for example in case of regional authorities and commercialization of knowledge and innovation, for example in the case of higher education . In this context there is a different way of approaching the effectiveness of the park.

The evaluation of the effectiveness of the technology parks in Poland is made as part of research commissioned by the Polish Agency for Enterprise Development, "Benchmarking of technology parks in Poland" [Hołub-Iwan et al. 2012].

Benchmarking is defined most commonly as a modern tool for managing the organization, the essence of which is to identify best practices in the business capable of achieving success in the industry and in the policy area. The benchmark indicator is one form of the benchmarking, under which compares similar to each organization based on a set of highlighted indicators.

The main aim of conducted research is a comparison of the rate and direction of development of parks. The conducted research involves two steps, which aim is to determine the phase of the life cycle of the park and stage appropriate essential benchmarking study. It should be noted that despite a full set analyzed in this study, the evaluation is done through the prism of a single cluster or in a set of two features within the map of strategic groups. The analysis does not take into account the time factor and conducted comparisons remain static. As a complement to the analyses, the use of taxonomic meter of development z_i was offered in a dynamic approach.

THE USE OF TAXONOMIC MEASURE TO ANALYSE THE DEVELOPMENT OF SPATIAL DIFFERENTIATION OF DEVELOPMENT LEVEL OF TECHNOLOGY PARKS IN POLAND

The statistical information was analyzed collected during the investigation „Benchmarking technology parks in Poland" - edition 2010 and 2012 for 13 selected technology parks in both editions of the survey.

In the first stage of research, the collected information has been subjected to a preliminary analysis. From the set of potential diagnostic features eliminated variables that do not meet the accepted criteria of formal and substantive. It is assumed that the final set of features should include the variables [Zeliaś et al. 2000]: with high spatial variability with low correlating and an asymmetric distribution. Finally, a set of 46 diagnostic features were selected for the final set of 15 variables.

This collection, which became the basis for further empirical research created the following features:

1. Value of funds raised from the European Union (or as grants from other international organisations)/ revenues (%).
2. Total revenue/park assets (%).
3. Number of cooperating companies/ number of tenants.
4. Number of collaborating independent experts/ number of tenants.

5. Number of projects executed by a technology park in partnership with other institutions.
6. Park building area (m^2).
7. Number of tenants.
8. Number of start-up companies/number of newly created companies.
9. Ratio of used park building area (%).
10. Number of services provided to tenants during the last 12 months/ number of tenants.
11. Expenditure on ICT of the park/total sales (%).
12. Innovative companies/number of park tenants (%).
13. Park tenants engaged in R&D activity/number of park tenants (%).
14. Employees with a scientific degree of a PhD at least or an academic title/total number of park employees (%).
15. Number of scientific-industrial teams realizing research initiatives.

The scope of the variables used to determine the level of development of technology parks surveyed contains features describing the one hand, the potential of parks infrastructure (building area, or the percentage level of its use), on the other hand is focused primarily on the indication of the potential of the parks in the possibility of developing co-operation with the environment (e. g. the number of research groups - industrial pursuing scientific initiatives that the number of cooperating companies in terms of the number of tenants), and in assessing the potential of companies operating in the park for innovation (e.g. number of legally protected patents and trademarks in terms of the number or percentage of tenants the share of innovative firms in the total number of tenants park).

The extracted both from the substantive criteria and formal statistical variables formed the basis of a comparison and classification of discrete spatial units (technology parks) into groups with similar levels of development.

For the study of the spatial differentiation of the development of technology parks, a taxonomic measure of the development was implemented, on the basis of the following formula [Nowak 1990]:

$$z_i = \frac{1}{K}\sum_{k=1}^{K} z_{ki} \,, \tag{1}$$

where:

z_i – value of a taxonomic measure of development for *i*-object,
z_{ki} – standardized value of *k*-feature in *i*-object,
K – number of features examined.

As a basis for standardization of individual characteristics, assumed average values determined on the basis of statistical information analyzed for 13 technology parks in 2009 year. In order to make comparative analysis, variable

z_i was transformed into synthetic variable determined on the basis of the following formula:

$$z'_i = \frac{z_i}{\max\{z_i\}} \qquad (i = 1, \ldots, m), \tag{2}$$

Facilities ordered by decreasing value of taxonomic measure development are divided into groups with similar levels of development of the phenomenon under study. The study examined the set of all technological parks divided into 4 groups, including values of the meter with the development of the following ranges [Zeliaś 2004]:

- the first group of parks, for which $z_i \geq \bar{z} + S_z$,
- the second group of parks, for which $\bar{z} + S_z > z_i \geq \bar{z}$,
- the third group of parks, for which $\bar{z} > z_i \geq \bar{z} - S_z$,
- the fourth group of parks, for which $z_i < \bar{z} - S_z$.

Results of grouping parks in 2009-2011 are presented in Table 1.

Table 1. Dividing technology parks according to the synthetic meter of the level of development in 2009-2011

Lp.	Group	2009		Group	2010		Group	2012	
		Park/phase	z_i		Park/phase	z_i		Park/phase	z_i
1	I	Park 5/EM	1,000	I	Park 6/M	1,000	I	Park 8/G	1,000
2		Park 16/M	0,924		Park 5/EM	0,986	II	Park 14/G	0,587
3	II	Park 14/G	0,624	II	Park 7/M	0,816		Park 6/M	0,485
4		Park 6/M	0,569		Park 14/G	0,802		Park 5/EM	0,412
5		Park 15/M	0,563		Park 16/M	0,797	III	Park 16/M	0,350
6		Park 7/M	0,527	III	Park 9/M	0,625		Park 9/M	0,336
7		Park 9/M	0,525		Park 12/EM	0,578		Park 15/M	0,326
8	III	Park 1/M	0,421		Park 8/G	0,568		Park 7/M	0,313
9		Park 13/EM	0,349		Park 1/M	0,538		Park 2/M	0,304
10		Park 2/M	0,325		Park 2/M	0,523		Park 1/M	0,289
11		Park 8/G	0,276		Park 15/M	0,510		Park 19/EM	0,274
12	IV	Park 12/EM	0,214		Park 19/EM	0,482	IV	Park 13/EM	0,200
13		Park 19/EM	0,186		Park 13/EM	0,357		Park 12/EM	0,161

Source: own analysis,
where G – growth phase, EM – early maturity phase, M – maturity phase

In the table, in addition to the development of the standardized measure also the information is included about the group which is assigned to the park and the life cycle phase of the park is indicated:

- a growth phase (G),
- an early maturity phase (EM)
- or a maturity phase (M), which was defined in the benchmarking study in 2012.

The analysis of the results shown in the table, demonstrate significant changes in the ordering of the units (parks) during the years of analysis. For some parks these changes are more favorable but for others less favorable. This is particularly noticeable in the case of two parks qualified for the growth phase (the park no 8 and the park no 14), whose position had improved significantly, which in 2011 topped the rankings.

In the case of dynamic analysis is an important issue despite the place occupied by various parks also measure the value obtained in the subsequent years of the analysis. It is clear that in spite of the improvement of the position of indicated parks, the measure in the following years is lower than in 2009. This means lower growth rate compared to 2009.

The study was conducted in the dynamic approach for all analyzed years. Including time in the spatial differentiation of the development of technology parks in Poland allowed the isolation of parks, where we see the growth rate of parkland on the same level of development and those for which the deterioration in the level of development takes place.

As a result of the transformation manner used, the analyzed variables are measured in the interval scale. The dynamic analysis was, therefore, conducted using the methods which can be used in the case of this type of scales.

The analysis of dynamics was conducted using the absolute chain increment on the basis of the formula: [Zeliaś et al. 2000]:

$$\Delta_{i(t+1,t)} = z_{it+1} - z_{qit} \, (i = 1, \ldots, 13;\ t = 1, 2, 3) \tag{3}$$

where:

$\Delta_{i(t+1,t)}$ – absolute chain increment of a Z_i synthetic variable for an i object calculated for t and t+1 time units.

Subsequently, the mean pace of change in time was determined on the basis of the formula:

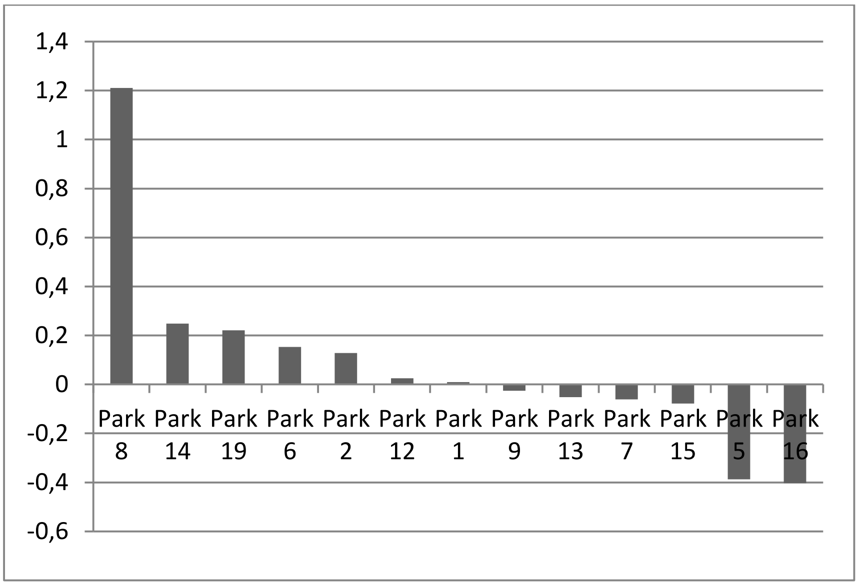
$$G_i = \frac{z_{i3} - z_{i1}}{2} \, (i = 1, \ldots, 13) \tag{4}$$

where:

G_i – mean pace of change in time of the Z synthetic variable for the i object.

In the Figure 1 the distribution of the average rate of change for each analyzed technology parks is presented.

Figure 1. Average rate of change in the development of taxonomic meter of technology parks



Source: own calculations

The mean pace of change in 2009-2011 of taxonomic meter development is characterized by a right-sided asymmetry. The median value (0,0089) lower than the arithmetic average (0,0758) means that a bigger number of parks (7) reached during the analyzed period higher change rate.

To the group of parks, for which the synthetic variable increase in 2011 compared to 2011 was the largest belongs park 8 (1.211). In the group with average values above the average rate are parks which were classified in 2011 to the third and fourth typological groups(park No. 2, No. 3 and park No. 13), with the values of taxonomic meter below the average. However, the negative rate of change was observed in the case of six parks, including park No. 5, which in 2011 was classified to typological Group II with the values of taxonomic meter above the average.

CONCLUSION

Parks increasingly recognize the need for both active acquisition of new tenants as well as their maintenance in the park. Due to the wider spectrum of business parks, the intermingling of different areas of the business, a more comprehensive approach to evaluating the effectiveness of the technology park and considering the level of development of the park, not only from the point of view of the individual indicators, but also on the basis of a number of studies classified as diagnostic variables.

The previous studies of the effectiveness of the technology parks in Poland, for example, in the study of PARP despite a full set of variables, have been based primarily on an assessment of individual indicators or sets of two features in the map of strategic groups. The analysis has not taken into consideration the time factor. A good solution in this case is to use the methods of multidimensional comparative analysis, including e.g. taxonomic meter development in a dynamic approach. The use in the studies of this type the taxonomic meter of development enabled, due to the simultaneous analysis of multiple variables collected, to determine ranking position occupied by parks participating in the study and the analysis of the rate of development of surveyed parks.

Measuring the effectiveness of technology parks is so important that it is the parks with a developed system of pre-incubators and incubators are one of the elements that describe the so-called, the modern metropolis of knowledge, or clusters of institutions, entrepreneurs and investors focused on functioning in the Knowledge-Based Economy [Maskell 1997, Parteka 2007].

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THE ATTEMPT TO CREATE AN INTERNAL CREDIT RISK RATING OF PRODUCTION COMPANIES WITH THE USE OF OPERATIONAL RESEARCH METHOD

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Abstract: In the article the ratings developed by Moody's Corporation, Standard & Poor's Ratings Services and financial data of Polish windows manufactures were analyzed. Ratings published by international agencies were compared with an independently developed rating. Authors made an attempt to verify the hypothesis whether the internal rating created by means of operational research method, significantly differs from the ratings prepared by international rating agencies. In the article mathematical possibilities of potential changes in credit rating were presented.

Keywords: rating, credit risk, operational research, DEA

CREDIT RISK RATINGS

Introduction

Credit risk is most simply defined as the potential for the loss due to borrower's failure to meet its contractual obligation to repay a debt in accordance with the agreed terms [Credit Risk Management, The GARP Risk Series]. A popular tool for credit risk assessment are ratings prepared by international ratings agencies. According to the most famous rating agencies: Moody's Corporation and Standard &Poor's Ratings Services the main purpose of ratings is to provide investors with a simple system of gradation by which future relative creditworthiness of securities may be gauged [Ratings Definitions] or to provide a simple, efficient way to communicate creditworthiness and credit quality [About Credit Ratings]. It is worth noting that efficient use of the credit risk ratings by the company can also help in its development, gaining new customers and increasing market share.

Moody's Corporation and Standard & Poor's Ratings Services practically created a duopoly in the global market for credit rating agencies with all its consequences for the prices and quality of service. Moody’s defines credit risk as the risk that entity may not meet its contractual, financial obligations as they come due and any estimated financial loss in the event of default [Moody’s Rating Symbols & Definitions June 2013]. S&P definition is very similar: S&P credit ratings express the agency’s opinion about the ability and willingness of an issuer, such as a corporation or state or city government, to meet its financial obligations entirely and on time [Credit Ratings Definitions & FAQs]. In other words Moody's and S&P’s credit ratings are opinions of the credit quality of individual obligations or of an issuer's general creditworthiness. But none of the agencies gives any warranty, expressed or implied, as to the accuracy, timelines, completeness, merchantability or fitness of any rating or other opinions or information given or made by them in any form or manner whatsoever [Moody’s Rating Symbols & Definitions June 2013]. Moody's and S&P ratings of credit risk are expressed as symbols grades that range from ‘Aaa’ to ‘C’ (Moody's) or from ‘AAA’ to ‘D’ (S&P). The general meaning of credit rating symbols is summarized in table 1.

Table 1. Moody's and S&P credit ratings symbols

Moody's rating	S&P rating	Opinions of the credit quality of obligations or of an issuer’s general creditworthiness
Aaa,	AAA	Highest quality and lowest credit risk.
Aa1, Aa2, Aa3	AA+, AA, AA-	High quality and very low credit risk.
A1, A2, A3	A+, A, A-	Low credit risk
Baa1, Baa2, Baa3	BBB+, BBB, BBB-	Moderate credit risk
Ba1,Ba2,Ba3	BB+,BB,BB-	Substantial credit risk
B1, B2, B3	B+, B, B-	Speculative, high credit risk
Caa/C	CCC/C	Lowest creditworthiness, very high credit risk
D	D	

Source: based on [Moody’s Rating Symbols] and [Guide to Credit Rating Essentials]

Authors made an attempt to verify the hypothesis whether the rating created with the use of one of the operational research methods: Data Envelopment Analysis (DEA), differs significantly from ratings prepared by credit rating agencies (CRA). Authors are aware that Moody's and S&P ratings belong to Through-The-Cycle credit indicators. Traditional DEA utilize available and pertinent information as of a given date so that provides a Point-In-Time credit risk measure. The assessment of changes in credit risk allows for instance the combination of DEA method and Malmquist index [Chodakowska 2013]. Nevertheless, limited by the availability of data, in the article, ratings published by international agencies for Polish windows manufactures were compared with an independently developed rating. Evaluation of similarities between classifications was done using Rand Index and correlation matrix. At the end, mathematical possibilities of potential changes in credit rating were presented.

Data

The analysis covered 35 largest Polish companies in terms of revenue in 2010 estimated by the Centre of Sectoral Analysis in the industry: Manufacture of builders' ware of plastic (22.2.3 by Statistical Classification of Economic Activities in the European Community). Financial data for analysis was collected from ISI Emerging Markets Database. Available reports included employment, simplified profit and loss account, simplified balance sheet, simplified cash flow statement and selected financial ratios, e.g. in terms of financial leverage — total debt ratio, in the area of short-term liquidity — the share of cash in current assets, in the area of long-term liquidity — the share of equity and non-current liabilities in fixed assets and in the area of profitability — return on gross assets (ROA). The analyzed companies were classified by Moody's and S&P from Baa1/BBB+ to D (table 3).

Data Envelopment Analysis

Data Envelopment Analysis (DEA) is more and more popular and widely used method for determining the effectiveness of both commercial and non-profit organizations. Determining the effectiveness of the units using DEA method is to find the optimal technology by solving the adequate linear programming task [Cooper at al. 2007]. The optimal technology of the unit minimizes its inputs to a level not higher than the authentic and allows to get results not worse than the authentic (or maximizes the results to a level not lower than the authentic inputs, not higher than authentic) [Guzik 2009]. Comparing the optimal technology and empirical efficiency ratio is obtained. The resulting ratio has a value in the range of <0,1> [Guzik 2009].

From a variety of DEA analytical capabilities in the article authors used: determining the effectiveness of the units, rankings of the units' effectiveness and determining structure of inputs and outputs that guarantee 100% effectiveness.

ANALYSIS

Variables selections

In this article Authors made an attempt to compare the rating created with the use of operational research method and ratings prepared by CRA. For this purpose nine variables were selected: one informing about the size of the company — employment (X1), and eight financial indicators: debt ratio (X2), cash and cash equivalents in current assets (X3), equity and long-term liabilities in fixed assets (X4), sales revenue (X5), fixed assets (X6), current assets (X7), short-term liabilities

(X8), ROA (X9). Variables X10 and X11 denote Moody's and S&P rating. Correlation matrix for all the variables shown in table 2.

Table 2. Correlation matrix

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
X1	1.000	-0.196	0.188	-0.218	**0.833**	**0.663**	**0.656**	**0.526**	0.124	-0.223	-0.194
X2		1.000	**-0.364**	-0.243	-0.215	-0.181	-0.285	0.158	**-0.625**	**0.775**	**0.734**
X3			1.000	0.161	0.123	-0.058	0.285	-0.100	0.324	**-0.640**	**-0.672**
X4				1.000	-0.158	-0.239	-0.184	-0.268	**0.413**	**-0.374**	-0.266
X5					1.000	**0.798**	**0.801**	**0.783**	0.183	-0.229	-0.213
X6						1.000	**0.787**	**0.831**	-0.043	-0.112	-0.094
X7							1.000	**0.759**	0.143	**-0.343**	**-0.340**
X8								1.000	-0.140	0.092	0.086
X9									1.000	**-0.786**	**-0.764**
X10										1.000	**0.974**

Source: own calculations using STATISTICA 10;
bold — correlations are significant $p < 0.05$,
cells filled with grey — correlations between variable selected and Mood’s and S&P rating

On the basis of correlation matrix Moody's and S&P ratings depend mainly on debt ratio (X2), cash and cash equivalents in current assets (X3) and ROA (X9). These variables as outputs and employment (X1) as the input indicator of the size of the company were chosen for further analysis with DEA BCC-O model for variable returns to scale [Cooper at al. 2007; Guzik 2009].

In order to fulfill the postulates of the DEA methodology i.e. unity of preference of all outputs and positive sign of the all the variables a differential transformation was applied for the variables X2 and X9:

for X2:

$$x_{i2} = a - bx_{i2}, \tag{1}$$

where:

$a = \max_i \{x_{i2}\}$,

$b = 0.9999$,

for X9:

$$x_{i9} = a - bx_{i9} \tag{2}$$

where:

$a = \min_i \{x_{i9}\}$,

$b = 0.9999$.

The authors are aware that every transformation of variables, except for scaling, affects the estimated using DEA efficiency, however, authors decided to apply transformations than exclude variable or units

Ratings

Analyzed polish manufactures of builders’ ware of plastic have the rating from range Baa1/BBB to D by Moody's and S&P. Agencies are very similar in credit risk assessment. Differences between classes in their ratings are usually by one position up/down, maximum by two. CRA are fully completed in opinions about companies with lowest creditworthiness (Table 4). The correlation coefficient between these ratings is 0.974. Generally, CRA estimate credit risk rather guardedly: substantial credit risk concerns 15 (16) enterprises. Only 5 (4) companies are judged higher.

Efficiency estimated using selected variables and DEA vary in the range from 18.8% to 100% (table 3). 11 out of the 35 enterprises are classified as 100% effective. This is due to the specificity of the linear programming problem solved in DEA, in which weights of the inputs are optimized to maximize the efficiency of each unit. Fully effective enterprises have the lowest credit risk in analyzed group, but not necessary in the whole sector. Nevertheless, the results of the DEA and the ratings issued by Moody's and S&P are similar. The correlation coefficient is 0.877 (Moody's) or 0.844 (S&P).

Because of the large number of classes they have been grouped for comparison. CRA' ratings for four categories according to the rating symbols' definitions as: moderate credit risk (1), substantial credit risk (2), speculative, high credit risk (3) and lowest creditworthiness, very high credit risk (4).

Similar, enterprises with the credit risk estimated using DEA were grouped. In this article for classification was used arithmetic mean $\bar{z}$ and standard deviation σ. 100% effective enterprises belong to first class of credit rating The other three groups include the companies with the values of effectiveness from the following ranges: second class $100\% > z_i \geq \bar{z}$, third class: $\bar{z} > z_i \geq \bar{z} - \sigma$, the fourth class: $\bar{z} - \sigma > z_i$. Categorized results are shown in table 4.

Table 3. Moody's, S&P and DEA credit ratings

Code	Decision Making Unit (DMU)	Moody's rating	S&P rating	DEA score	DEA rating
P1	Anwisa G Wiśniewscy sp.j.	Ba2	BB-	94.47	5
P2	Budvar Centrum SA	Ba1	BB	91.79	6
P3	Classen Pol SA	Caa/C	CCC/C	58.69	18
P4	Defor SA	D	D	38.17	23
P5	Drutex SA	Baa1	BBB+	100	1
P6	Eljako Al sp. z o.o.	Ba3	BB-	86.58	8
P7	Eurocolor sp. z o.o.	B3	B-	64.2	15
P8	Excellent Profile Grzybczyk Rogoda Szczepocki Ziębicki sp.j.	Ba1	BB	100	1
P9	Fakro Gp sp. z o.o.	Ba1	BB	100	1
P10	Firma Produkcyjno Handlowo Usługowa Wiśniowski	Ba3	BB-	100	1
P11	Hormann Legnica sp. z o.o.	Ba2	BB	88.02	7
P12	Invado sp. z o.o.	B3	B-	67.15	13
P13	Ispol sp. z o.o.	D	D	49.33	20
P14	Komandor SA	Ba1	BB	100	1
P15	Krispol sp. z o.o.	Ba2	BB-	79.3	11
P16	Markisol International Ltd sp. z o.o.	Ba2	BB-	100	1
P17	Mercor SA	Ba2	BB	66.55	14
P18	Nb Polska sp. z o.o.	B1	B	85.5	9

Code	Decision Making Unit (DMU)	Moody's rating	S&P rating	DEA score	DEA rating
P19	Okna Rąbień sp. z o.o.	D	D	43.16	22
P20	Oknoplast sp. z o.o.	Baa3	BBB-	100	1
P21	Opal Ryszard Szulc Wacław Olejniczak sp.j.	B3	B-	63.32	17
P22	Petecki sp. z o.o.	B3	B-	56.47	19
P23	Podlaska Fabryka Okien I Drzwi Witraż sp. z o.o.	B3	B+	63.86	16
P24	Pol Skone sp. z o.o.	Ba2	BB-	82.79	10
P25	Porta Kmi Poland SA	Ba1	BB	96.87	4
P26	Portos Renata Tomasz Szukalscy sp.j.	Baa2	BBB	100	1
P27	Pozbud T R SA	Baa2	BBB+	100	1
P28	Przedsiębiorstwo Produkcyjno Usługowo Handlowe Filplast Głogówek sp. z o.o.	B1	B+	97.4	3
P29	Roto Frank Okna Dachowe sp. z o.o.	Baa3	BB	100	1
P30	Seeger Dach sp. z o.o.	B2	B	100	1
P31	Sokółka Okna i Drzwi SA	D	D	23.55	24
P32	Sonarol sp.j. Najda	Ba3	BB-	78.04	12
P33	Stolbud Włoszczowa SA	D	D	18.82	25
P34	Stollar Systemy Okienne sp. z o.o.	D	D	47.86	21
P35	Velux Polska sp. z o.o.	Ba1	BB+	98.96	2

Source: ISI Emerging Markets Database and own calculations

Table 4.Moody's, S&P and DEA credit ratings

Moody's rating	S&P rating	DEA Class	Enterprises		
			Moody's classification	S&P classification	DEA classification
Baa1	BBB+	1	P5	P5, P27	P5, P8, P9, P10, P14, P16 ,P20, P26, P27, P29, P30
Baa2	BBB		P26, P27	P26	
Baa3	BBB-		P20, P29	P20	
Ba1	BB+	2	P2, P8, P9, P14, P25, P35	P35	P1, P2, P6, P11, P15, P18, P24, P25, P28, P35
Ba2	BB		P1, P11, P15, P16, P17, P24	P2, P8, P9, P11, P14, P17, P25, P29	
Ba3	BB-		P6, P10, P32	P1,P6, P10, P15, P16, P24, P32	
B1	B+	3	P18, P28	P23, P28	P3, P7, P12, P17, P21, P22, P23, P32
B2	B		P30	P18, P30	
B3	B-		P7, P12, P21, P22, P23	P7, P12, P21, P22,	
Caa/C	CCC/C	4	P3	P3	P4, P13, P19, P31, P33, P34,
D	D		P4, P13, P19, P31, P33, P34	P4, P13, P19, P31, P33, P34	

Source: ISI Emerging Markets Database and own calculations

In Table 5 were shown correlation coefficients between origin CRA' ratings, DEA rating, and grouped on the basis of DEA ratings (DEA class).

Table 5. Correlation

	DEA score	DEA ranking	Moody's rating	S&P rating	Moody's class	S&P class	DEA class
DEA score	**1.000**	**-0.980**	**-0.868**	**-0.835**	**-0.835**	**-0.834**	**-0.939**
DEA ranking		**1.000**	**0.877**	**0.844**	**0.838**	**0.832**	**0.964**
Moody's rating			**1.000**	**0.974**	**0.968**	**0.964**	**0.865**
S&P rating				**1.000**	**0.957**	**0.970**	**0.828**
Moody'sclass					**1.000**	**0.985**	**0.839**
S&P class						**1.000**	**0.830**

Source: own calculations using STATISTICA 10
bold — correlations are significant $p < 0.05$

For a formal comparison of ratings Rand index (R) and Adjusted Rand index (AR) were used (table 6). Rand index takes values from the range <0, 1>. The higher values indicate greater similarity with classification results.

Table 6. Rand Index and Adjusted Rand Index

	S&P classification		DEA classification	
Moody's classification	R=0.968	AR=0.922	R=0.739	AR=0.333
S&P classification			R=0.721	AR=0.290

Source: own calculations

High compatibility of ratings allows to induce that changes in levels in variables in DEA model should also change Moody's and S&P ratings. However it is worth notice that there is a large disparity between AR and R measures.

Potential changes

Potential improvements were identified through the analysis. Mathematical possibility to change the position of inefficient enterprises in the DEA ranking was presented. The optimum value, determined by the peer object was calculated. As a result, possibilities of rise enterprises efficiency by enhancing outputs and reducing inputs were showed. To show the possibility of changes in the ranking DEA for objects outside the first-class rating (inefficient) the target technology was calculated (technology that guarantees 100 percent efficiency). Optimal technology can be written as [Guzik 2009]:

$$T_i^* = \sum_{j=1}^{J} \lambda_{oj} T_j \qquad (3)$$

where λ_{oj} means non-negative optimal weight for $j = 1,..,J$ object. T_j means j-th object technology. Weights are calculated with assumption that the i-th object to obtain optimal results uses not more than the real inputs.

Table 7. Optimal weight for inefficiency objects

DMU	Score	λ_{oj}										
		P5	P8	P9	P10	P14	P16	P20	P26	P27	P29	P30
P1	94.47	0	0	0	0	0.87	0	0	0.13	0	0	0
P2	91.79	0	0	0	0	0.845	0	0	0.155	0	0	0
P3	58.69	0	0	0	0	0.615	0	0	0.385	0	0	0
P4	38.17	0	0	0	0	1	0	0	0	0	0	0
P6	86.58	0	0.26	0	0.95	0	0	0	0	0	0.645	0
P7	64.2	0	0	0.273	0	0.269	0	0	0.458	0	0	0
P11	88.02	0	0	0	0	0.849	0	0	0.151	0	0	0
P12	67.15	0	0	0	0	0.294	0	0	0.76	0	0	0
P13	49.33	0	0.488	0.344	0	0	0	0	0.139	0	0.29	0
P15	79.3	0	0	0.71	0	0.169	0	0	0.341	0	0.419	0
P17	66.55	0.163	0	0	0	0	0.138	0	0.7	0	0	0
P18	85.5	0	0	0	0	0.542	0	0	0.458	0	0	0
P19	43.16	0	0	0	0	1	0	0	0	0	0	0
P21	63.32	0	0.27	0	0	0	0.14	0	0.32	0	0.558	0
P22	56.47	0.71	0	0	0	0	0.89	0	0.839	0	0	0
P23	63.86	0	0	0.75	0	0.25	0	0	0	0	0	0
P24	82.79	0	0	0	0	0.354	0	0	0.646	0	0	0
P25	96.87	0	0	0	0	0.588	0	0	0.412	0	0	0
P28	97.4	0	0	0.217	0.75	0	0	0	0	0	0	0.33
P31	23.55	0	0	0	0	0	0	0.238	0	0.762	0	0
P32	78.04	0	0	0.39	0	0.29	0	0	0.663	0	0	0
P33	18.82	0.169	0	0	0	0	0.831	0	0	0	0	0
P34	47.86	0	0.969	0	0	0	0.31	0	0	0	0	0
P35	98.96	0	0.59	0	0	0	0.34	0	0.14	0	0.361	0
Numbers of references		3	5	6	2	12	6	1	16	1	5	1

Source: own calculations using Banxia Frontier Analyst

In the table 7 optimal weights for inefficient objects were presented. Peers for them are: P5, P8, P9, P10, P14, P16, P20, P26, P27, P29 and P30 (units with the 100% efficiency score). The enterprise P26 was peers most frequently (16 times). This object was classified by the analyzed ratings in 1 class (table 4).

An example of changes in the ratings and classification of the group will be discussed at P1 enterprise. This object received in the Moody's rating Ba2 grade, at S&P — BB and was on the 5th place in DEA ranking with 94.47% efficiency score (table 4). From table 7 and by the actual data, the optimal technology for the enterprise P1 was written as:

$$T_{P1}^{*} = \lambda_{P14}T_{14} + \lambda_{P26}T_{26} = 0{,}87\begin{vmatrix} x_{2,P14} \\ x_{3,P14} \\ x_{9,P14} \\ x_{1,P14} \end{vmatrix} + 0{,}13\begin{vmatrix} x_{2,P26} \\ x_{3,P26} \\ x_{9,P26} \\ x_{1,P26} \end{vmatrix} = \begin{vmatrix} 0.986 \\ 0.364 \\ 0.379 \\ 182 \end{vmatrix}$$

The actual values for the object P1 are: $x_2 = 0.5596$, $x_3 = 0.0783$, $x_9 = 0.3582$, $x_1 = 250$. Comparison of actual results with the optimal allows to determine the direction and magnitude of change. The company P1 to reach a value that will change places in the DEA ranking must: reduce the debt ratio of about 76%, increase cash and cash equivalents in current assets over three times (364%) and ROA about 6% even reduce employment of about 27%. These improvements would enable change the result of the efficiency, and thus change the classification of a group of 2 to 1. The same procedure can be used for other inefficient objects.

The example shows one of the possible changes in the ranking with using math calculations. In particular the target values for the analyzed variables was showed. In fact, the size of such modification (such as increase cash and cash equivalents in current assets over three) is not always possible. However this analysis may indicate direction of changes which leads to a reduction of credit risk. Computing capabilities are obviously larger (e.g., what kind of improvements enterprises can make to change only one level in the classification). This is not the purpose of this article to show all possibilities changes but only show that DEA can be used to determine them.

SUMMARY

Main issues of this article are relatively widely present in literature studies. The U.S. credit rating agencies after the spectacular scandals have been subjected to severe criticism. It is worth recalling the most famous: the Enron scandal revealed in 2001 — the giant U.S. energy company, which until its downfall had the highest credit rating AAA, or a fraud committed by Goldman Sachs, which mortgage securities awarded AAA in 2006 and 2007 were later downgraded to junk status [Faux 2011]. Rating agencies are criticized mainly for lack of transparency, conflicts of interest and ratings shopping, and lack of accountability for the ratings prepared [Deb 2011]. However, the ratings published by U.S. credit rating agencies still have great impact both on individual companies and global economy. Credit rating agencies play a significant opinion-forming role in global securities and banking services. Their ratings are often treated as 100% sure assessment, although agencies have borne no direct liability for errors in their judgments. The main reason is that ratings, although based on complex fundamental analysis and advanced mathematical models can be used mechanically by investors regardless of their education, knowledge or experience. The rating agencies, though undeservedly, still possess impeccable reputation and for many market participants are a sources of objective analysis [Brylak 2011].

This article shows how to create ratings and classified enterprises in groups using mathematical methods without the use of sophisticated analysis and based on the basic data (debt ratio, cash and cash equivalents in current assets, ROA and employment). The DEA ranking was almost 90% compatible with the rankings of the specialized agencies. In addition, the use of the DEA method allows to show the mathematical possibilities changes in the ranking. The results can be used by managers for determine directions of change that will allow the group to move to a lower credit risk. The study had cognitive and application character. It was shown that the Data Envelopment Analysis can also be used as a tool for the analysis of credit risk.

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PANEL MODELS AS A TOOL FOR INVESTIGATING THE EFFECTS OF CONSUMPTION VAT IN POLAND IN THE YEARS 1995-2011

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Abstract: Econometric models are employed to establish the redistributive properties of tax systems, but this area of their application is weakly covered in the literature. Used to investigate the burden of consumption taxes, the panel data models allow identifying differences in the burden of indirect taxes falling on households and establishing household types where it is the greatest. This article is an attempt at applying panel regression to explore the redistributive effects of VAT in Poland in the years 1995-2011.

Keywords: VAT, consumption, households, tax regression, panel models

THE REDISTRIBUTIVE CONSEQUENCES OF VAT IN POLAND

The redistributive effects of VAT in Poland have been studied since its introduction in 1993. The first study in this field was carried out in the years 1992-1993 by the *Integrated Tax and Transfer Research Group*[1]. Its purpose was to analyse the amount of VAT paid by households in relation to their disposable incomes, size and socio-economic status. This methodological approach was modified and improved by all subsequent investigations dealing with the redistributive effects of consumption taxes.

As shown by the studies conducted in Poland, the structure of the VAT burden carried by households is strongly related to their wealth. Because Polish VAT is regressive, its impact on the consumption of the lowest-income households is relatively stronger [Dobrowolska 2008]. It has also been found that between

[1] The research team consisted of scientists representing the University of Łódź, the Justus Liebig University in Gieβen, the University of Naples Federico II, the Institute of Finance and the Institute of Labour and Social Studies.

1995 and 2011 the average tax burden on households due to the standard VAT rate rose more than twofold, whereas the burden of reduced VAT rates decreased by 18% on average. The situation was caused by the increasing, from 1994, range of goods and services taxed with the standard VAT rate and the decreasing range of goods and services to which preferential VAT rates applied, but also by changes affecting the structure of household consumption [Dobrowolska et al. 2011].

From the social perspective, the main disadvantage of consumption taxes is that they are not fair. They are not directly related to taxpayer's incomes and property status and their shiftability causes that they are comparatively more burdensome for the low-income taxpayers whose consumption accounts for a large share of their expenditure than for taxpayers whose consumption constitutes a small proportion of high incomes. This regularity is perceived as breaching the ability-to-pay principle and is strongly determined by the regressive character of the taxes. Because indirect taxes are regressive, it is not trivial from the social policy standpoint what proportions of budget revenues are derived from direct and indirect taxes. „Because an indirect tax is shiftable, it is difficult to determine what tax burden is actually carried by different agents and to pursue a redistribution policy preferred by the state and accepted by the citizens" [Krajewska 2004].

An attempt at marrying social justice and the economic efficiency of indirect taxes is the application of practically uniform tax rates (this approach is recommended by the European Union and also imposed by the mechanism of the almost ubiquitous VAT tax) that do not affect the structure of consumption, however making some exemptions, out of respect for social justice, for essential goods and luxury goods, in the latter case mainly through the excise mechanism [Ostaszewski et al. 2004].

The above observations have led to a widespread opinion that the distribution of the VAT burden among particular types of households needs to be constantly monitored, not only for the purpose of controlling its social impacts, but also to gain very useful knowledge on how its rates can be respectively diversified or homogenised.

METHODS OF INVESTIGATING THE REDISTRIBUTIVE EFFECTS OF INDIRECT TAXES

There are two main methods in the literature that allow the burden of indirect taxes to be assessed. One method involves observations of the relative tax burden in successive income groups. The percentage share of the tax in the incomes, or expenditures, of individual income groups provides an indication of its character. If the share is decreasing while incomes (expenditures) are going up, than the tax is regressive; a constant share means that the tax is proportional (linear); and a share increasing with rising incomes shows that the tax is progressive. This classification refers to the mean tax rate. In some cases, the marginal tax rate is used to establish whether the tax is progressive or regressive. With this criterion, a tax is considered progressive when the marginal tax rate is increasing with growing incomes. As a matter of fact, the two approaches produce different results [Neneman 1997].

Another approach makes use of simple regression, for instance one of the form [Adams 1980]:

$$\ln(VAT)_i = \alpha + \beta \ln(Y)_i + \varepsilon_i \tag{1}$$

where:

- $(VAT)_i$ the amount of VAT paid by a household in the i-th income group,
- α a fixed effect,
- β a coefficient of tax burden elasticity with respect to household expenditures or disposable income,
- $(Y)_i$ expenditures or disposable income of a household in the i-th income group,
- ε_i normally distributed random term.

The estimate of β directly indicates the redistributive character of the tax system. If the estimate is greater than 1, then the system is progressive; if it is smaller than 1, then the system is regressive.

The redistributive effects of the tax system can also be determined with econometric models, but this area of their applications is still insufficiently covered in the literature [more on this subject in Dobrowolska 2008]. The relevant approaches use classical estimation methods and estimation methods with panel data models.

When the data are generated by economic processes that are very similar and the same econometric model can be applied to describe them, then the data characterising the investigated objects can be aggregated and estimated jointly. This type of estimation is more efficient than one using single models. A typical example of the panel data is household budget statistics. The panel models[2]can also generate information on fixed effect decomposition.

The method that this analysis proposes as a means of investigating the redistributive effects of VAT estimates the panel models that, *inter alia*, make it possible to specify how being a given type of a household contributes to differences in its burden. With the fixed effect estimates being known, the differences in the amounts of VAT paid by households in Poland can be identified and thereby household types where its burden is the greatest. A one-way model allows differences related exclusively to the household type (other factors being omitted) to be found. A two-way model enables a concurrent investigation into the household type and the time factor effect on the burden of VAT.

[2] For the purpose of this article, the term panel models will be used as equivalent to econometric panel data models.

The redistributive effect of VAT on households can be analysed with respect to their incomes, but also their size or socio-economic status. The distribution of tax burden determined by these characteristics provides grounds for indicating which household types are the most burdened by VAT.

APPLYING THE PANEL MODELS TO INVESTIGATE THE REDISTRIBUTIVE EFFECTS OF VAT IN POLAND IN THE YEARS 1995-2011

The redistributive effects of VAT in Poland in the years 1995-2011will be analysed with the panel regression models.

The research uses a sample of Polish households divided into decile income groups, which the GUS (Poland's Central Statistical Office) analysed during a household budget survey carried out in the years 1995-2011. Therefore, 170 observations (10 decile groups observed over 17 years) are used to verify the panel models empirically.

VAT amounts paid by particular types of household are estimated with detailed, unpublished GUS data on household expenditures. A full description of the assumptions for estimating VAT amounts paid by households can be found in the article by B. Dobrowolska [Dobrowolska 2008].

In the panel regression used in this analysis the explained variable is the PLN amount of VAT paid by a household and the explanatory variable is household net incomes, also in PLN. The net income was accepted as the most appropriate because it includes all monetary components that are really available to households, unlike the gross income that contains also amounts that households cannot use, such as income tax or social insurance premiums, and thereby not increasing their potential for consumption.

Some relationships between the selected variables were omitted from the analysis, as its main purpose is to determine how being a particular type of a household influence sits burden of VAT.

Testing for the most appropriate function showed a linear function and a power function to have the best statistical characteristics indicating the model's usefulness. A somewhat better fit between the empirical and theoretical values was obtained for the linear models. The stratification procedure was performed by income groups. Because the models with fixed effect decomposition turned out to yield better results for most estimated equations than those with the decomposition of the random term **(**as indicated, for instance, by $R^2_{adj.}$values[3], but mainly by the results of the Hausman test), only the estimates of:

- - the one-way model with fixed effect decomposition (variant A),
- - the two-way model with the decomposition of the random term (variant B)

[3] R^2_{adj} -adjusted determnination coefficient

were used in further analysis.

All variables' values are given in current prices, because the same approach is used in the financial models [Łapińska–Sobczak 1997] and because adjustment of the estimated models' variables to their real values deteriorated the properties of the specified equations. For these reasons, the nominal data were used.

The statistical quality of the estimated equations is very good, as shown by the high value of the determination coefficient R^2; it must be noted, however, that the two-way model time offered a better fit to the empirical data (see Table 1).

Table 1. Parameter estimates of the linear model of *VAT* paid, where the explanatory variable is net incomes (*incom*) of successive decile groups– models with fixed effect decomposition

Explanatory variable	Model 1 A One way model			Model 1 B Two way model		
	coefficient	t	p	coefficient	t	p
Incom Constant	0.0995	80.944	0.0000	0.084 -4.860	87.326 -4.851	0.0000 0.0000
R^2	0.988			0.998		
LRT	227.271		0.00000	296.770		0.00000
F	49.594		0.00000	42.274		0.00000

Fixed-effect estimates in model 1 A

Group	Coefficient	Standard Error	t-ratio
1	-0.09301	1.73947	-0.05347
2	-7.08765	1.75922	-4.02885
3	-7.75230	1.80379	-4.29778
4	-10.83693	1.85304	-5.84819
5	-12.46440	1.91223	-6.51827
6	-15.17600	1.98579	-7.64228
7	-18.86736	2.08166	-9.06361
8	-24.50245	2.22951	-10.99008
9	-30.59516	2.47176	-12.37791
10	-72.81382	3.64098	-19.99844

Fixed-effect estimates in model 1 B

Group	Coefficient	Standard Error	t-ratio
1	10.90776	2.05665	5.30366
2	4.70450	1.98468	2.37041
3	5.58290	1.85704	3.00635
4	3.96231	1.75419	2.25877
5	3.89126	1.66809	2.33276
6	2.91581	1.60489	1.81683
7	1.27186	1.58014	0.80490
8	-1.50764	1.63868	-0.92004
9	-3.38881	1.89671	-1.78668
10	-28.33995	3.90414	-7.25894

Fixed-effect estimates in model 1 B (cont'd).

Period	Coefficient	Standard Error	t-ratio
1995	-7.08015	2.49027	-2.84313
1996	-7.13962	2.36240	-3.02219
1997	-9.11738	2.26208	-4.03054
1998	-7.02281	2.19616	-3.19776
1999	-4.41018	2.16918	-2.03311
2000	-4.16867	2.13920	-1.94870
2001	-3.58284	2.12791	-1.68374
2002	-3.91593	2.12002	-1.84712
2003	-5.92313	2.11248	-2.80387
2004	0.99618	2.10724	0.47274
2005	-1.53606	2.10691	-0.72906
2006	-2.51144	2.12524	-1.18172
2007	-0.07633	2.18526	-0.03493
2008	5.20451	2.29346	2.26928
2009	11.53478	2.35859	4.89055
2010	13.43074	2.44982	5.48234
2011	25.31830	2.50885	10.09160

where: Y_i- net per capita incomes of households (PLN) in the decile group

Source: calculated by the author with the Limdep 7.0 software package based on unpublished GUS data derived from the Household budget surveys 1995-2011

The amount of household net incomes used as the explanatory variable was statistically significant in both equations, and the signs of the slope coefficients were as expected. Increasing household incomes usually cause the amounts of VAT paid to grow too.

The research has demonstrated that the amount of VAT a household will pay depends on its wealth. Both models point to the statistical significance of the group effects, which is also confirmed by the values of the *LRT* and *F* statistics. Therefore, the household's belonging to a particular decile group has a major effect on how much it will pay in VAT.

Let us remind that the two-way models, unlike the one-way ones, generate not only individual effects, but also an estimate of the fixed effect pertaining to all investigated groups of households. The fixed effects specific to particular decile groups of households should therefore be viewed as deviations from the fixed effect common to the entire sample.

An analysis of the group effects obtained from the two-factor model of VAT paid by particular decile groups of households (Table 1 B) has showed the deviations from the „general fixed effect" to have opposite signs for households in the first seven decile groups than for the households in decile groups VIII – X. While the tax burden carried by the less wealthy households (deciles I – VII) is heavier than average, in the wealthier households (above decile VII) it is below average. This exposes the regressive character of Polish VAT. This regressiveness is probably caused by households having different structures of consumption spending. A proven fact is that the poorer a household is, the larger share of consumption expenditures in its total spending.

Further, the fixed-effect values for the time effects (see Table 1 B) show that in the period under consideration the amount of VAT paid by Polish households increased the most in 2011.The main cause of the increase was rises in VAT rates that were introduced that year[4]. The most important changes consisted in the standard VAT rate and the reduced VAT rate being raised by 1 percentage point, respectively from 22% to 23% and from 7% to 8%.

An additional reduced VAT rate of 5% has been introduced permanently for some goods that previously benefitted from super reduced VAT rates of 3% and 0%. The 5% rate applies to basic foodstuffs, such as grain products (bread, flour, oats, pasta), dairy products, meat products, juices, as well as specialist books and magazines[5]. The reasons for VAT rates being changed from 2011 was the expiry of derogations negotiated with the European Commission that allowed the application of reduced VAT rates to some products, as well as the condition of public finances in Poland[6].

[4] Regulation of the Minister of Finance of 22 Dec. 2010 concerning the implementation of the provisions of the valued added tax act (Dz.U. Nr 246, poz. 1649)

[5] The full catalogue of goods taxed with 5% VAT rate can be found in Annex 10 to the valued added tax act of11 March 2004.

[6] The 8% and 23% VAT rates were introduced for a period to expire at the end of December 2013. Then the previous rates of 7% and 22%were to be restored, unless the ratio between public debt and

A notable fact is that accounting for the time effect in the decomposition of the fixed effect increased the value of R^2.

CONCLUSION

The panel regression models can serve as a tool for investigating the redistributive effects of indirect taxes. One advantage of using panel regression to study the redistributive consequences of consumption taxes is that it allows the differences in the tax load carried by particular types of households to be identified, and thereby the types of households where the load is the greatest. Unlike the one-way models that can expose differences related exclusively to the household type (other factors being unaccounted for), the two-way models are useful in investigating the impact of both the household type and the time factor on the burden of indirect taxes.

The conducted analysis used the panel regression models to study the economic consequences of VAT imposed on Polish households in the years 1995-2011. The analysis has demonstrated that Polish VAT is regressive. The regressiveness of indirect taxes is probably caused by the structure of households' consumption spending. It is generally known that the poorer a household is the larger share of consumption expenditures in its total spending.

Further, the application of two-way panel models showed, owing to the isolation of the time effect, that the amounts of VAT paid by particular decile groups of households in the investigated period were related to the social and tax policy of the state.

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BAYESIAN CONFIDENCE INTERVALS FOR THE NUMBER AND THE SIZE OF LOSSES IN THE OPTIMAL BONUS–MALUS SYSTEM

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Abstract: Most of the so far proposed Bonus–Malus Systems (BMSs) establish a premium only according to the number of accidents, without paying attention to the vehicle damage severity. [Frangos and Vrontos 2001] proposed the optimal BMS design based not only on the number of accidents of a policyholder, but also on the size of loss of each accident. In our work, we apply the approach presented by Frangos and Vrontos to construct the Bayesian confidence intervals for both the number of accidents and the amount of damage caused by these accidents. We also conduct some simulations in order to create tables of estimates for both the numbers and the sizes of losses and to compute the realizations of the corresponding Bayesian confidence intervals. We compare the results obtained by using our simulation studies with the appropriate results derived through an application of an asymmetric loss function and its certain modification.

Keywords: optimal BMS, number of claims, severity of claims, Bayesian analysis, Bayesian confidence intervals, asymmetric loss functions

INTRODUCTION

The Bonus-Malus Systems are commonly used in the calculation of insurance premiums in the area of vehicle insurance. They penalize the car owners who caused accidents by premium surcharges and reward the drivers with accident-free year(s) by discounts. The term "Bonus" means a discount in the premium of the policyholder, which is given on the renewal of the policy if no at-fault accident occurred to that client in the previous year. In turn, the term "Malus" denotes an increase in the premium of the insured driver who caused an accident in the last year. The main drawback of the BMSs is that they calculate a premium only according to the number of accidents, disregarding the size (severity) of loss that each accident incurred; in this way, a policyholder who had an accident with a small size of loss is unfairly penalized like a policyholder who had an accident with a big size of loss (thus, the policyholders with the same number of accidents pay the same malus, irrespective of the size of damage). [Frangos and Vrontos 2001] proposed the optimal BMS design based not only on the number of accidents, but also on the size of losses. It was a certain development of the method introduced by [Dionne and Vanasse 1989]. The works of the cited authors attracted our attention towards the subject of the optimal BMSs and encouraged us to undertake some research in this field.

The objectives of our studies are:

(i) application of the Frangos and Vrontos approach in the construction of the Bayesian confidence intervals for both the number of accidents and the size (severity) of damage that these accidents incurred,

(ii) application of the loss functions in the estimation of the expected number of losses and the expected size of losses in the year $t+1$, given the numbers and the sizes of losses in the previous years (i.e., in the years $1,...,t$),

(iii) conducting some simulation studies in order to create tables of estimates for both the number and the size of losses and to compute the realizations of the Bayesian confidence intervals for the expected number and size of claims,

(iv) calculation of the change in the frequency of the number of claims and the net premium in the optimal BMS,

(v) making comparisons of the results obtained by using our simulation studies with the appropriate results derived through an application of an asymmetric loss function and its certain modification.

BAYESIAN CONFIDENCE INTERVALS FOR THE EXPECTED NUMBER AND THE EXPECTED SIZE OF LOSSES

We denote by X the number of accidents (claims) and by λ the underlying risk of a policyholder to have an at-fault accident. We assume that the conditional distribution of X given the parameter $\lambda > 0$ is the $Poisson(\lambda)$ distribution, i.e.,

$$P(X = k \mid \lambda) = \frac{e^{-\lambda}\lambda^k}{k!}, \ \lambda > 0, k = 0,1,2,... \tag{1}$$

The Poisson distribution is often applied to describe random and independent events, such as vehicle accidents. It is a particularly useful distribution for modeling numbers of automobile accidents in the case when a random variable is introduced into the regression component in the formula for the risk λ. In this setting the regression component contains all significant information about individual characteristics of the insured driver, which may affect the policyholder's driving skills and habits. For more details on this issue see the paper of [Dionne and Vanasse 1989].

In addition, we assume that λ is a random variable with the prior distribution $Gamma(a,b)$, i.e., its density function is given by

$$g(\lambda) = \frac{\lambda^{a-1} b^a e^{-b\lambda}}{\Gamma(a)}, \;\; a > 0, \; b > 0. \tag{2}$$

Such a determination of λ gives a certain a priori knowledge about the *proneness* of a policyholder to cause an accident (parameters a, b specify our a priori knowledge about how *accident-prone* a policyholder is).

Denote by $X_1,...,X_t$ the numbers of accidents that a policyholder caused in the i th year, $i = 1,...,t$. We assume that $X_1,..., X_t$ are conditionally independent given unobserved variable λ. It may be verified that $X_1,..., X_t$ are unconditionally dependent and that, under the conditions in (1), (2), the unconditional distribution of the number of claims X is the *Negative Binomial* (a,b) distribution, with a probability function of the form

$$P(X = k) = \binom{k + a - 1}{k} \left(\frac{b}{1+b}\right)^a \left(\frac{1}{1+b}\right)^k, \;\; k = 0,1,2... \tag{3}$$

Then, we have that the posterior distribution $\lambda | X_1,..., X_t$ (i.e., the a posteriori structure function of λ for a policyholder with the historical claim numbers $X_1,..., X_t$) is the $Gamma(a + K, b + t)$ distribution, where $K = \sum_{i=1}^{t} X_i$. Consequently, the $(1-\alpha)\cdot 100\%$ Bayesian confidence interval for $\lambda_{t+1} = \lambda_{t+1}(X_1,..., X_t)$ – the expected number of losses in the year $t+1$ of a policyholder with the claim numbers history $X_1,..., X_t$ – may be derived from the relation $P(A_\alpha \le \lambda_{t+1} \le B_\alpha | X_1,...,X_t) = 1-\alpha$ by putting:

$$A_\alpha = \Gamma^{-1}(\alpha/2, a + K, b + t), \tag{4}$$

$$B_\alpha = \Gamma^{-1}(1-\alpha/2, a + K, b + t), \tag{5}$$

where $\Gamma^{-1}(\,\cdot\,, a + K, b + t)$ stands for the corresponding quantile of the $Gamma(a + K, b + t)$ distribution.

Now, let us refer to the issue concerning the size of claims of the insured drivers.We denote by Y the size of claims (losses) and by β the mean size of claims of each insured. We assume that the conditional distribution of Y given the parameter $\beta > 0$ is the $Exponential(\beta)$ distribution, i.e.,

$$P(Y \le y \mid \beta) = 1 - e^{-y/\beta}, \quad \beta > 0. \tag{6}$$

Furthermore, we also assume that β is a random variable having the prior distribution $Inverse\ Gamma(s, m)$, i.e., its density function is given by

$$g(\beta) = \frac{\frac{1}{m} e^{-m/\beta}}{\left(\frac{\beta}{m}\right)^{s+1} \Gamma(s)}, \quad s > 0, m > 0. \tag{7}$$

Denote by $Y_1,...,Y_t$ the sizes of losses incurred as a result of accidents that a policyholder caused in the i th year, $i = 1,...,t$. We assume that $Y_1,...,Y_t$ are conditionally independent given unobserved variable β. It can be checked that $Y_1,...,Y_t$ are unconditionally dependent and that, under the conditions in (6), (7), the unconditional distribution of the size of losses Y is the $Pareto(s, m)$ distribution, i.e., it has a density function of the form

$$f(y) = \frac{sm^s}{y^{s+1}} I(y \ge m), \quad s > 0,\ m > 0, \tag{8}$$

where I stands for the indicator function.
Then, the posterior distribution $\beta | Y_1,...,Y_t$ (i.e., the a posteriori structure function of β for a policyholder with the historical claim sizes $Y_1,...,Y_t$) is the $Inverse\ Gamma(s+t, m+L)$ distribution, where $L = \sum_{i=1}^{t} Y_i$.
Therefore, the $(1-\alpha)\cdot 100\%$ Bayesian confidence interval for $\beta_{t+1} = \beta_{t+1}(Y_1,...,Y_t)$ – the expected size of losses in the year $t+1$ for a policyholder with the claim sizes history $Y_1,...,Y_t$ – may be easily derived from the relation $P(C_\alpha \le \beta_{t+1} \le D_\alpha | Y_1,...,Y_t) = 1-\alpha$ by making the following substitutions:

$$C_\alpha = I\Gamma^{-1}(\alpha/2, s+t, m+L), \tag{9}$$

$$D_\alpha = I\Gamma^{-1}(1-\alpha/2, s+t, m+L), \tag{10}$$

where $I\Gamma^{-1}(\cdot, s+t, m+L)$ denotes the corresponding quantile of the $Inverse\ Gamma(s+t, m+L)$ distribution.

APPLICATION OF THE LOSS FUNCTIONS IN THE ESTIMATION OF THE EXPECTED NUMBER AND THE EXPECTED SIZE OF LOSSES

A map of the form $L(x) = e^{-cx} + cx - 1$ is called an asymmetric Linex Loss function. It is now a widely used function in the actuarial statistics (for its applications in the area of BMSs, we refer to [Bermudez et al. 2001]). In contrast to the quadratic loss function, this type of loss function avoids high penalties by breaking the symmetry between the overcharges and underchages. If $c < 0$, it gives a greater penalty for overestimation than for underestimation of losses. If $c > 0$, it gives a greater penalty for underestimation than for overestimation of losses.

It can be shown that in our model, the optimal Bayesian estimator of the parameter λ_{t+1} (interpreted as the expected number of losses in the year $t+1$), obtained by minimizing the expectation $E_{post}L\left(\hat{\lambda}_{t+1} - \lambda_{t+1}\right)$, where E_{post} is calculated with respect to the posterior distribution $\lambda | X_1, ..., X_t$, has the following form (see Appendix for the corresponding proof)

$$\hat{\lambda}_{t+1(Linex)} = \frac{1}{c} \ln\left(\frac{b+t}{b+t-c}\right)^{a+K}, \quad K = \sum_{i=1}^{t} X_i. \tag{11}$$

For comparison, the mean of the posterior distribution $\lambda | X_1, ..., X_t$ (i.e., an average of the $Gamma(a+K, b+t)$ distribution) is equal to

$$\hat{\lambda}_{t+1} = (a+K)/(b+t). \tag{12}$$

Let us now move on to the issue concerning the estimation of the parameter relating to the size of losses. Since there is no optimal Bayesian estimate of β_{t+1} for any $c > 0$ in our model, the Linex1 Loss function, instead of the Linex Loss function, is used in the estimation of β_{t+1}. [Basu and Ebrahimi 1991] proved that the optimal Bayesian estimator of β_{t+1}, obtained through an application of the Linex1 Loss function, has the following form

$$\hat{\beta}_{t+1(Linex1)} = \left(-\frac{L+m}{c}\right)\left(1 - \exp\left(\frac{c}{t+s+1}\right)\right), \quad \text{where } L = \sum_{i=1}^{t} Y_i. \tag{13}$$

In order to obtain (13), we minimize $E_{post} L\left(\hat{\beta}_{t+1} / \beta_{t+1} - 1\right)$, where L is the Linex function and E_{post} is calculated with respect to the posterior distribution $\beta | Y_1, ..., Y_t$. For comparison, the mean of the posterior distribution $\beta | Y_1, ..., Y_t$ (i.e., an average of the $Inverse\ Gamma(s+t, m+L)$ distribution) is equal to

$$\hat{\beta}_{t+1} = (m+L)/(s+t-1). \tag{14}$$

CALCULATION OF THE NET PREMIUM IN THE OPTIMAL BMS

In this part of our work, we give the formulas for the changes in the frequency of the number of claims and for the net premium in the optimal BMS. Namely, let us notice that:

(i) By using (12), we have the following formula for the change in the frequency of the number of losses

$$(\hat{\lambda}_{t+1} / EX) \cdot 100\%, \tag{15}$$

where $EX = a/b$ is an unconditional expected value of the number of claims (we recall that $X \sim Negative\ Binomial\left(a,b\right)$);

(ii) By using (11) (and the Linex Loss function in particular), we have the following formula for the change in the frequency of the number of claims

$$(\hat{\lambda}_{t+1(Linex)} / EX) \cdot 100\%; \tag{16}$$

(iii) By using (12), (14), we obtain the following formula for the optimal BMS net premium (interpreted as the total loss in the year $t+1$)

$$\hat{\lambda}_{t+1} \cdot \hat{\beta}_{t+1}; \tag{17}$$

(iv) By using (11), (13) (and the Linex Loss functions in particular), we get the following formula for the optimal BMS net premium (interpreted as the total loss in the year $t+1$)

$$\hat{\lambda}_{t+1(Linex)} \cdot \hat{\beta}_{t+1(Linex1)}. \tag{18}$$

By putting for the values of parameters a, b the values considered in [Frangos and Vrontos 2001, p. 16], we may compute the quantity in (15) for different t, K and give the interpretations of the obtained results as follows:

(1) Put: $a = 0.228, b = 2.825, t = 3, K = 0$.

Then: $X \sim Negative\ Binomial\left(0.228, 2.825\right)$, $EX = a/b = 0.08$, and $\left(\hat{\lambda}_4 / EX\right) \cdot 100\% = 48\%$. It means that provided a policyholder has not had any accident for the first 3 years of insurance duration, then the premium paid at the beginning of the 4th year of insurance duration should amount to 48% of the premium paid at the beginning of the 1st insurance year;

(2) Put: $a = 0.228, b = 2.825, t = 4, K = 1$.

Then: $X \sim Negative\ Binomial\left(0.228, 2.825\right)$, $EX = a/b = 0.08$, and $\left(\hat{\lambda}_5 / EX\right) \cdot 100\% = 223\%$. It means that provided a policyholder has had 1 accident for the first 4 years of insurance duration, then the premium paid at the beginning of the 5th year of insurance duration should amount to 223% of the premium paid at the beginning of the 1st insurance year.

SIMULATION STUDIES

I. Simulation of the numbers of losses (claims)

This subsection consists of two parts. In the first one, we construct (based on the previously introduced theoretical background) the procedure leading to the simulation of the numbers of losses; the simulated numbers of losses are collected in the appropriate table. In the second part, the realizations of the confidence intervals for the expected number of losses are computed by using the earlier derived formulas.

I.1. Table of the numbers of losses

We simulated the numbers of claims for a portfolio of 100 polyholders by applying the following procedure:

(i) We generated a sample of size 100 from the $Gamma(0.3, 1/3)$ distribution (i.e., we generated 100 values of the random variable $\lambda \sim Gamma(0.3, 1/3)$ (see (2))),

(ii) For the given $\lambda(s)$ ($s = 1, ..., 100$) from the generation in (i), we generated 100 independent 10–element samples $\mathbf{x}^{(s)} = \left(x_1^{(s)}, x_2^{(s)}, ..., x_{10}^{(s)}\right)$ from the $Poisson(\lambda(s))$ distribution, which represented the numbers of losses in a portfolio of 100 polycyholders in the period of 10 years.

Based on the generations in (i), (ii), we obtained the simulated numbers of losses in the course of 10 years. The corresponding results are collected in Table 1.

Table 1. The simulated numbers of losses (claims)

Years/Numbers of losses												
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*≥11*
1	67	16	7	6		1		1		2		
2	73	11	5	3	3	2	2			1		
3	78	9	4	3	2	1		1	2			
4	77	9	4	6	2	1	1					
5	71	14	6	3	2	2					1	1
6	69	12	8	5	2	1	2					1
7	70	15	8	2	1	2		1				1
8	76	9	4	5	2		1	2				1
9	74	13	6	2	1	1	2					1
10	78	8	5	4		1	1	1				2

Source: own calculations

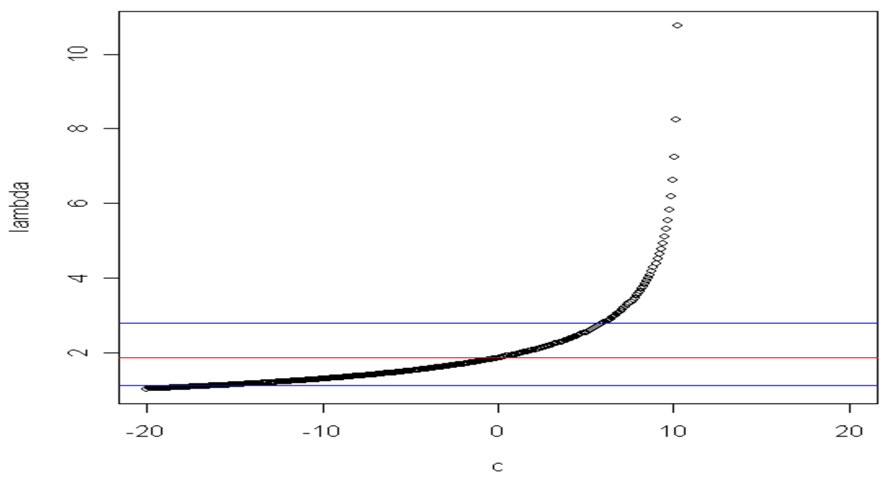
I.2. Realizations of the confidence intervals for the expected number of losses

Based on the first 10 observations $x_1^{(1)},...,x_{10}^{(1)}$ (relating to the simulated numbers of losses of the first polycyholder - see the generation $\mathbf{x}^{(1)}$ in (ii) above), we obtained the realization of the 95% confidence interval (CI) for $\lambda_{t+1} = \lambda_{11}$ by putting: $a = 0.3$, $b = 1/3$, $K = \sum_{i=1}^{10} x_i^{(1)} = 19$, $t = 10$ into the formulas in (4), (5).

The received realization of the 95% CI for λ_{11} - the expected number of losses in the 11th year - was $[1.13; 2.79]$.

Fig. 1 depicts the Bayesian estimates of λ_{11}, obtained through an application of the Linex Loss function $L(x) = e^{-cx} + cx - 1$ for c from -20 to 20 with the step 0.1. The outer horizontal lines correspond to the limits 1.13, 2.79, of the estimated 95% Bayesian CI for λ_{11}. In addition, by the inner horizontal line the expected value 1.87, of the posterior distribution $\lambda | X_1,..., X_{10}$, which is the $Gamma(0.3+19, 1/3+10)$ distribution, is presented (this expected value has been calculated by substituting the values: $a = 0.3$, $K = 19$, $b = 1/3$, $t = 10$ into (12)).

Figure 1. The Bayesian estimates of λ_{11} (based on the application of the Linex Loss function), the estimated 95% Bayesian CI for λ_{11} and the expectation of an appropriate posterior distribution



Source: own calculations

In practise, the values of a, b are unknown, but we may estimate them from a portfolio by the method of moments (MM). Since X - the number of losses - has the negative binomial distribution with the mean $EX = a/b$ and the variance $VarX = (a/b)(1+1/b)$, then: $\hat{a} = \overline{X}\hat{b}, \hat{b} = \overline{X}/(S^2 - \overline{X})$, where $\overline{X}$ and S^2 are the sample mean and the sample variance. The obtained values of the MM estimators (calculated for the previously generated samples relating to 100 policyholders): $\hat{a} = 0.25$, $\hat{b} = 0.35$ are very close to the true values of $a = 0.3$, $b = 1/3$. The realization of the approximated Bayesian CI for λ_{11}, computed by substituting the estimates $\hat{a}$, $\hat{b}$ for a, b into (4), (5), is $[1.13; 2.78]$.

It may be verified that the MM estimators $\hat{a}$, $\hat{b}$ are the consistent estimators of the parameters a, b, respectively.

II. Simulation of the sizes of losses (claims)

We simulated the sizes of claims by using the following scheme:

(i) We generated a 1-element sample from the *Inverse Gamma* $(2.5, 1/5000)$ distribution (i.e., we generated a value of the random variable $\beta \sim$ *Inverse Gamma*$(2.5, 1/5000)$ (see (7))),

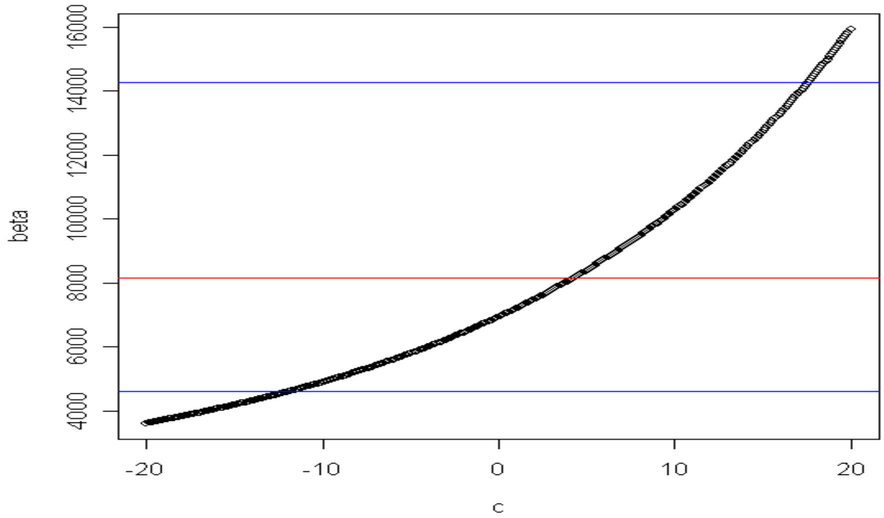
(ii) From the *Exponential(β)* distribution, where β was a value generated in (i), we generated a sample $y_1, y_2, ..., y_{10}$, of the sizes of losses in the period of 10 years.

Based on the first 10 observations $y_1, y_2, ..., y_{10}$, of the sample generated in (ii), we obtained the realization of the 95% CI for $\beta_{t+1} = \beta_{11}$ by substituting the values: $s = 2.5$, $m = 1/5000$, $t = 10$, $L = \sum_{i=1}^{10} y_i = 93608.96$ into (9), (10).

The received realization of the 95% CI for β_{11} – an average size of losses in the 11th year – was [4606.00; 14296.96]. Furthermore, we simulated the sizes of losses from the last 10 years and obtained the following values: 1351.418, 24872.665, 15063.568, 1083.870, 27508.688, 14729.373, 1839.331, 1332.476, 4024.451, 1803.115.

Figure 2 depicts the Bayesian estimates of β_{11}, obtained through an application of the Linex1 Loss function for c from -20 to 20, with the step 0.1. The outer horizontal lines correspond to the limits 4606.00, 14296.96, of the estimated 95% Bayesian CI for β_{11}. Furthermore, by the inner horizontal line the expected value 8139.91, of the posterior distribution $\beta | Y_1, ..., Y_{10}$, which is the *Inverse Gamma* $(2.5 + 10, 1/5000 + 93608.96)$ distribution, is presented (this expected value has been calculated by substituting the values: $m = 1/5000$, L = 93608.96, s = 2.5, t = 10 into (14)).

Figure 2. The Bayesian estimates of β_{11} (based on the application of the Linex1 Loss function), the estimated 95% Bayesian CI for β_{11} and the expectation of an appropriate posterior distribution



Source: own calculations

If the prior parameters s, m are unknown, we may estimate them by the method of moments (MM). We proceed as follows. By generating from the *Inverse Gamma*(2.5,1/5000) and the appropriate *Expotential(β)* distributions, we simulate 100 independent sizes of losses according to the steps (i), (ii) from the current subsection (with the difference that, we generate a sample of size 100 in (i) and 100 10–element samples in (ii)). Then, by using the formulas for the mean and the variance of the Pareto distribution and the mentioned method of moments, we have the following formulas for the MM estimators: $\hat{m} = (\hat{s}-1)\overline{Y}/\hat{s},\ \hat{s} = 1+\sqrt{1+c}$, where $c = \overline{Y}^2 / S^2$. Based on these formulas and the generated sample, we obtain the following values of the MM estimators: $\hat{m} = 0.8 \cdot 10^{-4}$, $\hat{s} = 2.13$.

The realization of the approximated Bayesian CI for β_{11}, computed by substituting the estimates $\hat{m}$, $\hat{s}$ for m, s into (9), (10), was $[4716.05;\ 14873.42]$.

It may be verified that the MM estimators $\hat{m}$, $\hat{s}$ are the consistent estimators of the parameters m, s, respectively.

III. Some simulations of the net BMS premium and the related quantities

In this subsection, the values of the derived estimators are presented. The calculations have been carried out for the following parameter values: $a = 0.3, b = 1/3, t = 10, K = 19, L = 93608.96, s = 2.5, m = 1/5000$. The remarks "*a, b, s, m* - estimated" mean that the values *a, b, s, m* are replaced by the MM estimates: $\hat{a} = 0.25,\ \hat{b} = 0.35,\ \hat{s} = 2.13,\ \hat{m} = 0.8 \cdot 10^{-4}$.

Table 2. The simulated net BMS premiums and the related estimates

Estimators	Computed values	Numbers of the applied formulas
$\frac{\hat{\lambda}_{11}}{EX} \cdot 100\%$	208	(12), (15)
$\frac{\hat{\lambda}_{11}}{EX} \cdot 100\%$ (*a*, *b* - estimated)	260	(12), (15)
$\frac{\hat{\lambda}_{11(Linex)}}{EX} \cdot 100\%$	169, c=-5 284, c=5	(11), (16)
$\frac{\hat{\lambda}_{11(Linex)}}{EX} \cdot 100\%$ (*a*, *b* - estimated)	212, c=-5 356, c=5	(11), (16)
95% Bayesian CI (in %)	$\lambda_{11} \in [126;$ $310]$	(4), (5)
95% Bayesian CI (in %) (*a*, *b*- estimated)	$\lambda_{11} \in [157;$ $389]$	(4), (5)
$\hat{\lambda}_{11} \cdot \hat{\beta}_{11}$	15203	(12), (14), (17)
$\hat{\lambda}_{11} \cdot \hat{\beta}_{11}$ (*a, b, s ,m* - estimated)	15643	(12), (14), (17)
$\hat{\lambda}_{11(Linex)} \cdot \hat{\beta}_{11(Linex1)}$	8828, c=-5 21426, c=5	(11), (13), (18)
$\hat{\lambda}_{11(Linex)} \cdot \hat{\beta}_{11(Linex1)}$ (*a, b, s, m* - estimated)	8997, c=-5 22044, c=5	(11), (13), (18)

Source: own calculations

COROLLARY

In our paper, the Bayesian confidence intervals for the expected number of losses and the expected size of losses in the optimal Bonus-Malus Systems have been established. Although both of the parameters of the prior distribution, relating to the number and the size of loss, respectively, are unknown, they may be estimated by the method of moments. The realizations of the obtained confidence intervals have been compared with the Bayesian estimates of the corresponding parameters, obtained with the help of the Linex Loss function and its modification, called the Linex1 Loss function. The proposed constructions of the Bayesian confidence intervals can be easily generalized to the models with additional explanatory, deterministic variables. Apart from the constructions of the confidence intervals, the procedures leading to the simulations of the numbers and the sizes of losses are also presented. Furthermore, the formulas for the net premiums and the related quantities have been established and applied.

APPENDIX

Our objective here is to prove the formula in (11).

Let $\Delta = \hat{\theta} - \theta$. For the Linex Loss function $L(x) = e^{-cx} + cx - 1$, we search $\hat{\theta}$ minimizing the posterior risk $E_{post}L(\hat{\theta} - \theta)$, where the expected value is computed with respect to the posterior distribution $\theta | X_1, \dots, X_t$. Since the function L is convex, the given expected value attains its minimum if it is finite.

Thus, it is sufficient to find the solution of the equation $\frac{\partial E_{post} L(\Delta)}{\partial \hat{\theta}} = 0$. By taking the derivative of the integral, we obtain that $-cE_{post}e^{c\hat{\theta} - c\theta} + c = 0$. Therefore, $\hat{\theta} = \frac{1}{c} ln\left(E_{post} e^{c\theta}\right)$.

In the case when $X_1, \dots, X_t$ have the marginal Poisson distribution with the parameter λ, the prior distribution of λ is the $Gamma(a, b)$ distribution and the corresponding posterior distribution is the $Gamma(a + K, b + t)$ one, where $K = \sum_{i=1}^{t} X_i$. Hence, $E_{post}e^{c\lambda} = \int_0^\infty e^{c\lambda} \frac{\lambda^{a+K-1}(b+t)^{a+K}}{\Gamma(a+K)} e^{-(b+t)\lambda} d\lambda = \left(\frac{b+t}{b+t-c}\right)^{a+K}$.

Consequently, we obtain $\hat{\lambda}_{(Linex)} = \frac{1}{c} \ln\left(\frac{b+t}{b+t-c}\right)^{a+K}$, which is a desired result (11).

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ANP APPLICATION: RISK MANAGEMENT IN PRODUCTION OPERATIONS

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Abstract: This academic paper presents results of research conducted in 2012 in a production business in south Poland where the Analytic Network Process (ANP) method was applied. The objective of the research was to establish the level of risk for selected factors resulting from higher production output of the business in focus. Moreover, selection of the decision-making variant burdened by the lowest risk priority for the achievement of goals set comprised an important research criterion. The application of ANP facilitated a comprehensive approach to the issue in focus. The overriding goal was to demonstrate the efficacy of the method in solving multi-criteria decision problems, especially those which require risk analysis.

Keywords: risk, company, decision-making, Analytic Network Process, ANP

INTRODUCTION

Risk management should comprise a component of an integrated company management system. As much as managing pure risk refers to operational risk and to marginal degree to tactical management level, managing speculative risk constitutes a domain of strategic management. This allocation may result from the fact that risk factors identified at the lowest level (i.e. operational level)

[1] Surname at birth: Gręda.

of management for the most part refer to pure risk (insured risk, which is a result of random events,) and to a minor degree, to speculative risk (risk in action.) The readiness to accept risk must result from the calculation of potential benefits and losses related to actions taken [Cymanow 2010].

An increased insecurity accompanying decision-making process and the corresponding increase in the level of risk for actions taken comes to be an important management feature in the post-industrial era. Market dynamics – which refers to both domestic and international markets – hence the growing number of variable factors in the environment, forces companies to identify underlying risk. Managerial staff should be aware and skilled enough to recognize dangers related to business operations and by the same to introduce procedures which facilitate effective actions, in turn leading to taking the right decisions. Identification of risk allows to eliminate or at least partially effectively limit its adverse implications, and by the same to increase the likelihood of achieving goals set for the organization [Jedynak, Szydło 1997].

A thorough study and assessment of risks areas identified should trigger actions aimed to ensure proper management of risks faced by the business. Hence, actions which may be taken to eliminate or neutralize risk and / or its implications should be defined. The following risk response methods are identified [Tarczyński, Mojsiewicz 2001]:

- Risk acceptance (risk monitoring only)
- Risk minimization (specific preventive actions are required)
- Risk avoidance (discontinuation of actions which may be causing risk)
- Risk transfer or diversification (reduction or elimination at source.)

Managers of the organization should focus on areas which facilitate maintaining or increasing existing competitive advantages at a specified acceptable risk level.

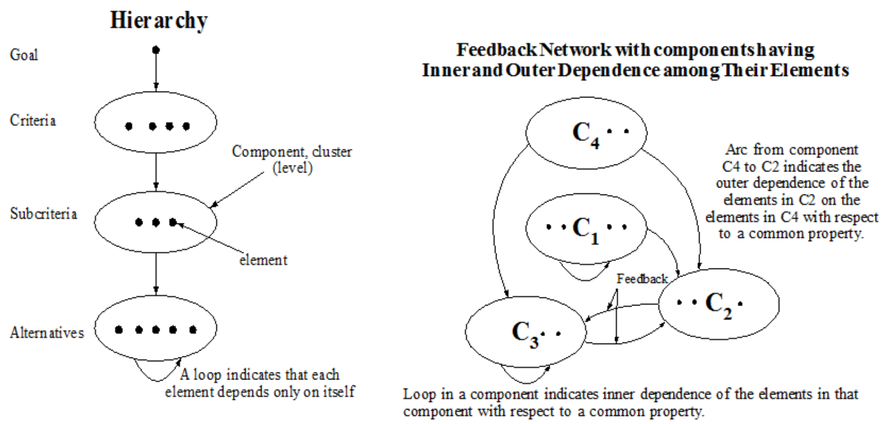
The primary objective of research conducted is to assess the level of risk while implementing specific decision-making variants related to increasing production output in the business discussed and to ensure that the optimum decision is taken in line with criteria assumed. The optimization is to consist in selecting an alternative which is marked by the lowest risk level from among factors determining enhanced scale of operations. The application of ANP facilitated a comprehensive presentation of the stage in the decision-making process related to risk management and by the same allowed to demonstrate a significant efficacy of the tool in focus while solving multi-criteria problems.

SUBJECT AND METHOD OF RESEARCH

Subject of Research and General Structure of Analytic Network Process

A comprehensive research of benefits, costs, opportunities, and risks related to the generation of “higher production output[2]” was conducted in 2012 based on a survey interview with managerial staff of the production business located in south Poland. This paper reviews a collection of survey interviews conducted about risk analysis for the problem in focus, developed in line with Analytic Network Process. The structure of the ANP risk model is a decision-making network marked by interdependencies and links between key elements (selected as key elements by the Authors) included in the decision-making process. The following structure is assumed under the model discussed (Figure 1): level 1 is the key goal, i.e. ‘higher production output,’ level 2 includes key organizational, production, economic, and technological criteria. As part of every criterion, sub-criteria are assumed, comprising level 3 of the ANP decision-making model[3] and facilitating a better understanding of the problem at hand. The next level of the model comprises sub-networks developed for key sub-criteria, with the value of their global priority higher or equivalent to 0.03 (3%). They have the biggest impact on the selection of the optimum decision-making alternative (variant,) which is interpreted as the decision marked by the lowest risk priority under the model discussed.

Figure 1. Comparison between general hierarchy structure and decision-making network



Source: Based on [Saaty 2004]

Three decision-making alternatives are posited under the model: (1) an upgrade of the existing production line, (2) an acquisition of a new technological

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[2] Key objective of ANP.
[3] Up to this level, the model follows the pattern of controlled hierarchy.

line as well as (3) a two-shift operation. An upgrade of the existing technological line refers to an increase of the output generated by the present drawing machine from the level of 2.5 tons per hour to 3.6 tons per hour. The cost of the refurbishment will come up to PLN150 thousand whereas the number of staff will increase by 4 people (up to 24.) If the decision to purchase a new line is made, the drawing machine with the output of 5 tons per hour will apply. An estimated cost of purchase will come up to PLN 350 thousand whereas due to an automation of the line, the number of staff will decrease by 4, i.e. down to 24 people. On the other hand, reengineering will lengthen the working time up to 16 hours per day (2 shifts.) An introduction of this solution will not generate costs related to the purchase or upgrades, however, the machine will be utilized more intensively. Moreover, the number of staff will increase to 30 people.

As part of the risk network model for 'higher production output' of the business in focus, sub-networks were developed so that elements could be grouped in general feedback system clusters as part of which connections in line with their external and internal dependencies and impacts were made. This is indicated by arrows which connect clusters marked by common links between elements.

The importance of decision-making elements in ANP and risks were identified by way of comparing pairs of elements as follows: key criteria, sub criteria, and clusters (in decision-making sub-networks developed) against their impact on every element in the subsequent cluster with which they are connected (the so-called external dependency) or in elements within the same cluster (the so-called internal dependency.)

When benchmarking, the criteria/sub-criteria against which items are benchmarked are taken into account. Items are benchmarked in line with how a certain element impacts on an element to a larger degree and how this extent is larger from that for a different element from the controlled hierarchy sub criterion. While conducting benchmarking in the risk model, the following question is posed: which element is marked by higher risk (is more risky)? The fundamental scale for pair-wise comparisons by Saaty was applied in the exercise (1-9). Opinions were presented in the form of the so-called unweighted supermatrix which was then recalculated and presented in the form of the weighted and limited supermatrix. Examples of such matrices can be found in the following studies: [Saaty 2001], [Saaty, Ozdemir 2005], [Saaty, Cillo 2008], and [Gręda 2009]. When seeking a solution to the problem posited, Super Decisions computer software was used. When calculating variants, the software automatically processes only these criteria and sub-criteria which are accompanied by networks or sub-networks.

General Framework of Research Method Applied: Analytic Network Process

In order to solve the problem posited concerning the selection of an optimum solution marked by the lowest level of risk to ensure a higher production output, the Analytic Network Process method was applied. ANP is an extension of the Analytic Hierarchy Process (AHP) and AHP has a particular place within ANP.

Hence, references frequently cite AHP/ANP methods. AHP/ANP is one of the most widely recognized methods globally as well as one of the fastest growing mathematical methods in recent years, applied to solve multicriteria decision problems. Both these theories have revolutionized the way complex mathematical problems are solved. Thomas L. Saaty from the University of Pittsburgh (USA) developed the methods.

The difference in the ANP consists in dependencies (interdependencies) occurring between groups of elements and within the groups, feedback, as well as in the presentation of the problem's structure not as hierarchy as is the case with AHP but as a network which comprises the system of interlinked components.
The ANP method allows to demonstrate the complexity of the problem at hand and facilitates comprehensive assessment of diverse links and interdependencies as well as assigning importance to quantitative and qualitative decision-making factors. The prioritization takes place through the pair-wise comparison against a given objective, criterion, or sub-criterion, in line with the 9-degree fundamental preference scale of Saaty [Gręda 2010]. The scale is presented in Table 1.

Table 1. Fundamental scale for pair-wise comparisons by T. L. Saaty

Intensity of importance	Definition	Explanation
1	Equal meaning	Equivalence of both of the elements benchmarked (two factors contribute equally to the objective)
3	Moderate importance	Weak (moderate) importance or preference of one factor over the other (one activity is slightly more important than the other)
5	Strong importance	Strong preference (importance) of one factor over the other
7	Very strong or demonstrated importance	Dominant importance or very strong preference of one factor over the other
9	Extreme importance	Absolutely higher importance (preference) of one factor over the other (advantage of one factor over the other is at the highest possible level of affirmation)
2, 4, 6, 8	For compromise between the above values	At times, numerical interpolation is required when a compromising opinions occur (central values from the above scale are applied in such instances)
1.1 – 1.9	For closely connected factors	If the meanings of factors are close and almost impossible to tell apart, the average equivalent to 1.3 is adopted while the extreme = 1.9
Reciprocals of above	Reciprocal values	If factor I has one of the above non-zero numbers assigned to it when compared with factor j, then j has the reciprocal value when compared with I When X and Y are compared and 'a' value is allocated, it must be automatically assumed that '1/a' must be the result of the Y and X comparison.

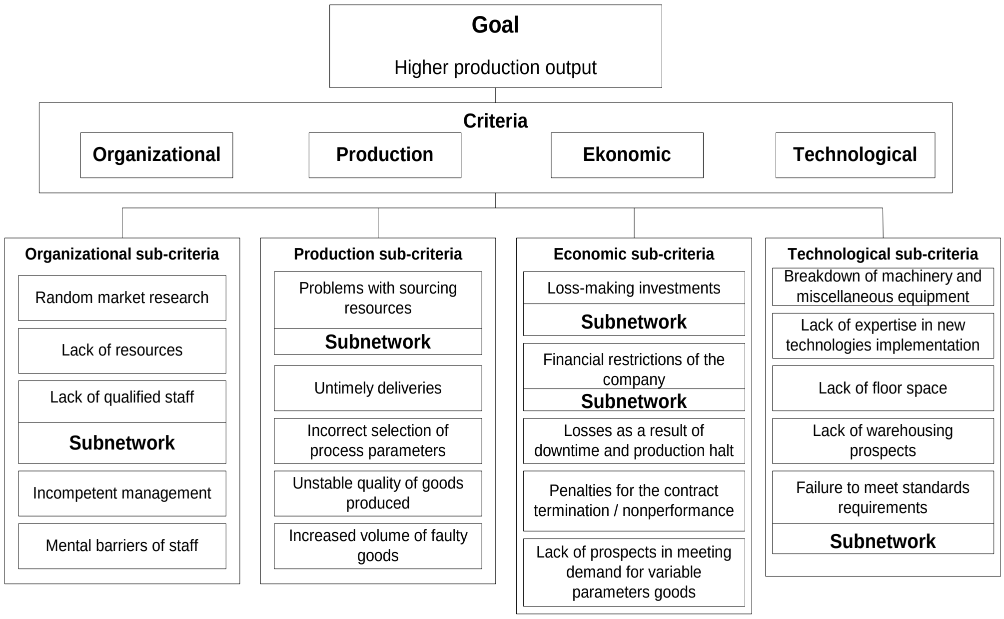
Source: [Saaty 2001]

Application of multicriteria decision techniques allows to answer the following question: which of the decision-making variants assumed (alternatives) will be burdened by the lowest risk if sensitive factors are introduced as well as which will allow to increase production output in the production business? The ANP method will also facilitate the sensitivity analysis for models developed, which will help answer the question of replacement alternatives. The AHP/ANP method facilitates the selection of the most beneficial decision from a range of alternatives.

RESULTS OF RESEARCH AND DISCUSSION

Application of ANP at work facilitates risk analysis and decision-making concerning the selection of a relevant decision-making variant for higher production output for the business in question. In comparison to the AHP method, results produced by way of ANP are more precise. They are based on interdependencies and feedback of elements in various random directions and at various levels of the network structure of the risk model analyzed. The AHP model compares pairs at every level of the hierarchy structure towards a decreasing priority (they are ordered.) In the ANP method, the direction of comparisons is not defined. It results from links between comparable elements and their interdependencies.

Figure 2. ANP risk model for 'higher production output' of the business



Source: own study

Figure 2 depicts the ANP 'higher production output' risk model. Selection of the variant which carries the lowest risk priority value was made by way of benchmarking all components of the decision model (criteria, sub-criteria, and items as part of sub-networks under development.) For each element of the risk model, local and global priorities were calculated. Global priorities indicate the importance of each and every network component in the process to ensure that the main goal is achieved whereas local components indicate the significance of those components within each sub-system cluster. Values of local and global priorities for specific components of the risk model are presented in Table 2.

Table 2. Significance of decision elements in risk model

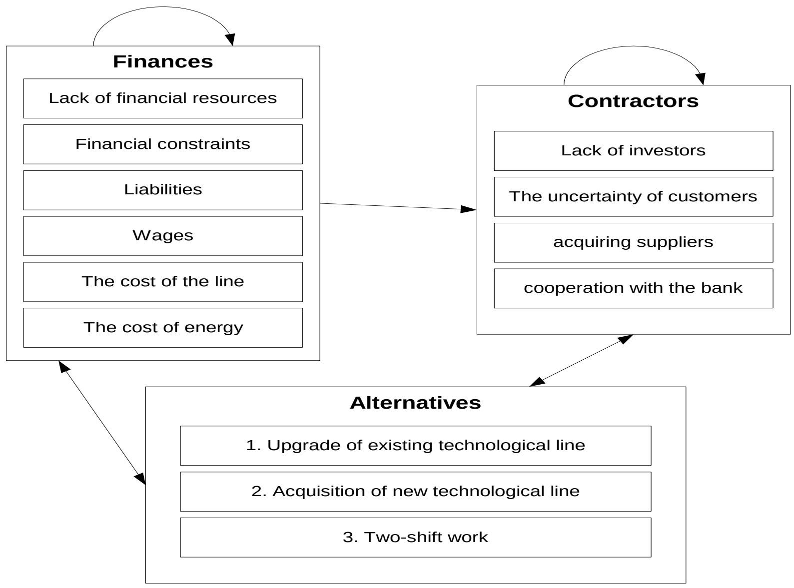
Criterion	Sub-criterion	Local Priorities	Global Priorities
Organizational criterion (0.1409)	Random market research	0.1080	0.0076
	Lack of resources	0.1753	0.0123
	Lack of qualified staff	0.5465	0.0385
	Incompetent management	0.1105	0.0078
	Mental barriers of staff	0.0597	0.0042
Production criterion (0.2628)	Problems with sourcing resources	0.5235	0.0688
	Untimely deliveries	0.1150	0.0151
	Incorrect selection of process parameters	0.1011	0.0133
	Unstable quality of goods produced	0.1202	0.0158
	Increased volume of faulty goods	0.1402	0.0184
Economic criterion (0.4554)	Loss-making investments	0.2901	0.0661
	Financial restrictions of the company	0.5213	0.1187
	Losses as a result of downtime and production halt	0.0418	0.0095
	Penalties for the contract termination / nonperformance	0.0758	0.0173
	Lack of prospects in meeting demand for variable parameters goods	0.0710	0.0162
Technological criterion (0.1409)	Breakdown of machinery and miscellaneous equipment	0.2446	0.0172
	Lack of expertise in new technologies implementation	0.1116	0.0079
	Lack of floor space	0.0856	0.0060
	Lack of warehousing prospects	0.0673	0.0047
	Failure to meet standards requirements	0.4909	0.0346

Source: own study

The risk network model stipulates 5 decision-making sub-networks for the following factors: (a) financial restrictions of the company (0.1187), (b) problems with ensuring resources (0.0688), (c) lack of return on investment (0.0661), (d) lack of qualified staff (0.0385), (e) failure to meet standards (quality) requirements (0.0345).

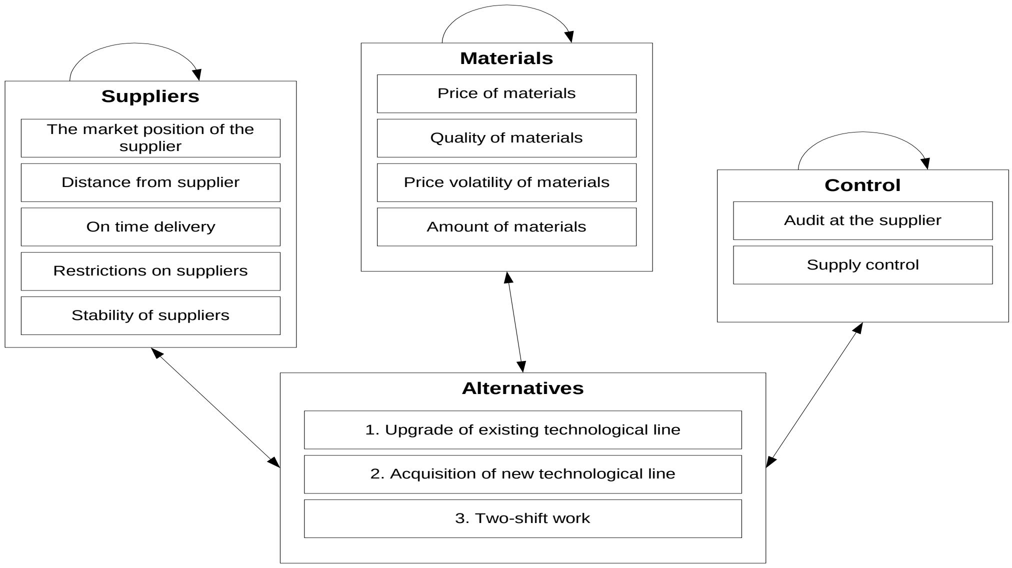
Due to editing restrictions, Figure 3 provides an example of a sub-network for the following sub-criterion: financial restriction of the business whereas Figure 4 provides an example of the following sub criterion: problems with ensuring resources.

Figure 3. Sub-network for the sub-criterion of 'financial restrictions of the business' in ANP risk model for 'higher production output'



Source: own study

Figure 4. Sub-network for the sub-criterion of ‘issues with ensuring resources’ in ANP risk model for ‘higher production output’



Source: own study

Values of priorities presented in Table 3 for specific variants in the risk model were determined as a result of a pair-wise comparison of their significance in the achievement of each sub-criterion in the organizational, production, technological, and economic sphere as part of sub-networks under development as well as related impact factors (this is indicated by incoming and outgoing arrows around the decision-making variants cluster.)

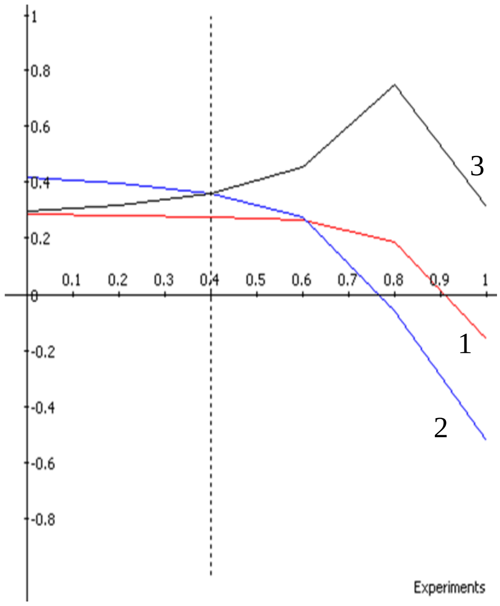
Table 3. End results for decision variants

Criterion	Organizational criterion (0.1409)	Produ-ction criterion (0.2628)	Economic criterion (0.4554)		Technologi-cal criterion (0.1409)	Total	Norma-lized value
Sub-criterion / Variant	Lack of qualified staff (0.5465)	Problems with ensuring resources (0.5235)	No return on invest-ment (0.2801)	Financial restrict-ions of the company (0.5213)	Failure to meet standards require-ments (0.4909)		
Variant 1	0.1333	0.0418	0.0671	0.0568	0,1126	0.0369	0.2072
Variant 2	0.1395	0.0931	0.1387	0.1470	0,1441	0.0867	0.4870
Variant 3	0.0804	0.0818	0.0738	0.0862	0,0985	0.0545	0.3058

Variant 1: Upgrade of existing technological line
Variant 2: Acquisition of new technological line
Variant 3: Two-shift work

In order to confirm the stability of solutions arrived at, the sensitivity analysis was conducted. Based on this, it can be verified how the solution suggested will change once the risk value has been modified (upwards or downwards) in the presented network model for the higher production output. Thanks to this analysis, the stability of solutions assumed may be verified if any criterion or sub-criterion assumed in the model is modified.

Figure 5. demonstrates the sensitivity analysis



1. Upgrade of existing technological line.
2. Acquisition of new technological line.
3. Two – shift work.

Source: own study based on Super Decisions software calculations

When conducting the sensitivity analysis for the risk model, it is noted that up to the risk priority value of 0.6, the best solution (burdened by minimum risk) consists in upgrading the existing technological line. The second-best solution consists in selecting two-shift work, whereas the solution to acquire a new technological line bears the highest risk (linked to high costs of investment.) From the risk priority value of 0.4 onwards, the risk level for the 'acquisition of a new technological line' option decreases and becomes the least risky solution above the value of 0.6.

Taking into account the risk analysis performed, it is concluded that the decision concerning an acquisition of the new technological line in a longer-term perspective seems the most beneficial solution from the point of view of the business discussed due to an increased stability of parameters of goods produced, smaller number of staff needed to service the new equipment, equipment's higher output and lower risk of breakdown, which is linked to timely deliveries as well as facilitating the prospect of new orders (which would not have been possible to ensure by the old machine.) In order to acquire the new technological line, the business can apply for Technology Loan and once granted, use EU subsidies to write off a percentage of the loan – as part of Technology Bonus from EU funds for entrepreneurs under 4.3. Innovative Economy Operational Program.

SUMMARY

The development of the risk model for higher production output results from multiple threats which may be noted in the course of business operations. These threats may result in the lack of desired business performance, and be accompanied by unintended losses or higher outlays against those anticipated. Hence, risk is involved[4]. The ANP risk network model demonstrates a dynamic approach to risk management[5]. It is focused on the future of the business and consists in anticipating dangers (and their neutralization and elimination,) lateral thinking, and prevention. To ensure this, relevant decision variants were assumed, and the sensitivity analysis was conducted to assess the stability of solutions suggested if internal or external factors of business operations changed.

The business should be managed so that any new foreseeable risks and accompanying responses can be accounted for[6]. It is recommended that the management and the staff are aware of the risks which may be encountered in various areas of business operations. Hence, the risk analysis for 'higher production output' considers the following four areas: organizational, production, technological, and economic – all of which were adopted to facilitate this study of the ANP model.

To recapitulate, the risk study concerning higher production output and the solution proposed in line the Analytic Network Process deems it a useful and practical tool, which may be applied to solve other multi-criteria decision problems as well[7].

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[4] Technical literature defines the concept of *risk* in a variety of ways. Economics and decision science references interpret risk as a nondeterministic concept whereby the probability of various scenarios – both positive and negative - are determined [Findeisen 1985].

[5] K. Lisiecka [2000] also distinguishes a passive approach to risk management. It is past-oriented and is focused on the discovery and identification of threats, as well as on the analysis and action. In line with this approach, risk is treated as a negative concept and the source of losses which should be taken into account when operating a business.

[6] Ibid.

[7] Examples of such problems are discussed in e.g. books by [Saaty, Ozdemir 2005], and [Saaty, Cillo 2008].

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A SYNTHETIC EVALUATION OF THE INNOVATIVENESS OF FOOD INDUSTRY BRANCHES IN POLAND

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Abstract: The aim of the presented study is the assessment of the innovativeness of particular food industry classes in Poland conducted on the basis of statistical methods for linear objects grouping. In the study there were used unpublished statistical data characterising the innovativeness of enterprises conducting economic activity in particular branches of the food industry.

Keywords: innovativeness, competitiveness, food industry, statistical methods for linear objects grouping

INTRODUCTION

Poland's integration with the European Union and globalisation processes that make the country increasingly open to contacts with the world economy increase competition among companies in both domestic and international markets. To maintain their market positions, companies must constantly develop and be innovative. This means that the expansion of the food industry in Poland depends on its innovativeness which, quite naturally, implies the need to analyse the aspects of the process.

This research assesses the level of innovativeness of the food industry branches in Poland with a linear ordering of objects used in statistics. It is based on the unpublished GUS (Polish Central Statistical Office) statistics on the innovativeness of companies operating in particular branches of the food industry in the years 2005-2011.

THE LINEAR ORDERING PROCEDURE

A linear ordering of objects generally follows a pattern made of six successive steps[1], i.e.:

1. Defining the purpose of the analysis and the preliminary hypotheses.
2. Specifying the substantive (objects, indicators) and temporal scope of research.
3. Setting up a database containing the values of acceptable diagnostic indicators.
4. Preliminary data analysis:
   - Descriptive analysis of the diagnostic indicators (measures of location, range and variance).
   - Analysis of correlation, reduction and selection of diagnostic indicators.
   - Determination of the character of the diagnostic indicators and applying a stimulation procedure if necessary.
   - Assigning weights to the diagnostic indicators.
5. Linear ordering of objects:
   - Normalization of the diagnostic indicators.
   - Selecting an aggregation formula for the diagnostic indicators (a model-based method, a non-model-based method, the orthogonal projection of objects onto a line).
   - Evaluation of the quality of the results and selection of the optimal solution.
6. Interpretation of the completed linear ordering of objects.

EVALUATION OF THE INNOVATIVENESS OF THE MANUFACTURING SECTOR' DIVISIONS – AN ATTEMPT AT CONSTRUCTING ITS SYNTHETIC MEASURE AND CARRYING OUT AN EMPIRICAL VERIFICATION

This part of the article describes the process of developing a synthetic measure of innovativeness of food industry branches based on the aforementioned linear ordering procedure.

Indicators of innovativeness in manufacturing companies

The set of diagnostic indicators that could show the innovativeness of food companies in Poland was compiled with the unpublished data from a GUS survey based on the PNT-02 form *„Sprawozdanie o innowacjach w przemyśle"*(Statement of Innovations in the Industry), which was carried out as part of the *CIS* programme. The means of the indicators were calculated for the years 2005-2011

[1] Developed by the author based on [Grabiński et al. 1989, pp. 87-89] and [Kolenda 2006, pp. 139-140].

and then were used as a starting point for analysing innovativeness in this sector of industry. The indicators were the following:

1. innovative companies as a share of all companies,
2. companies that introduced new or significantly improved products as a share of all companies,
3. companies that introduced products that are new to the market, or significantly improved, as a share of all companies,
4. companies that introduced new or significantly improved processes as a share of all companies,
5. companies involved in innovative activity that made outlays to innovate as a % of all companies,
6. total outlays on innovative activities (R+D, the purchase of technologies, software, investment outlays on capital assets, personnel training, marketing, other outlays) per company,
7. total R&D outlays as a share of total innovation outlays,
8. outlays on company's own R&D activity as a share of total innovation outlays,
9. purchase of ready-made technologies as a share of total innovation outlays,
10. software outlays as a share of total innovation outlays,
11. outlays on buildings, structures and land as a share of total innovation outlays,
12. outlays on machinery and technical equipment as a share of total innovation outlays,
13. outlays on imported machinery and technical equipment as a share of total innovation outlays,
14. outlays on personnel training as a share of total innovation outlays,
15. marketing outlays as a share of total innovation outlays,
16. sales of innovative products as a share of total sales,
17. sales of innovative products that are new to the market as a share of total sales,
18. sales of innovative products that are new only to the company as a share of total sales,
19. manufacturing companies with cooperation agreements on innovation activity concluded with other entities as a share of all companies,
20. manufacturing companies with cooperation agreements on innovation activity concluded with other entities as a share of all actively innovative companies,
21. the number of automated production lines per company,
22. new or significantly improved products sold as a share of the total sales of products.

Preliminary data analysis–selecting the diagnostic indicators

In selecting the diagnostic indicators, the following informational criteria should be applied [Ostasiewicz 1999, p. 110]: universality, variation, significance, correlation.

The variation of the potential diagnostic indicators was assessed with the classical coefficient of variation (V_j). Indicators with $V_j < 0.1$ were removed from the set.

Another measure of variation used in course of the analysis was the coefficient of the relative amplitude of fluctuations $A(X_j)$ [Kukuła 2000]:

$$A(X_j) = \frac{\max_i x_{ij}}{\min_i x_{ij}}, \ (i = 1,...,n; j = 1,...,m), \tag{1}$$

where $\min_i x_{ij} \neq 0$. Taking an additional condition $A(X_j) \geq c$, where $c = 1,2$ allows variables with a low amplitude of fluctuations to be eliminated.

In the first step of the preliminary analysis, variables 16, 17, 18 with data gaps caused mainly by changes in the PNT-02 methodology were taken out from the set of the potential diagnostic indicators which were selected to evaluate the innovativeness of food industry branches in Poland.

The remaining 19 indicators were found satisfactory regarding variation and amplitudes of fluctuations.

In the next step, in order to carry out the reduction procedure and to select the final set of indicators, the potential indicators were assessed for correlation. Diagnostic indicators (j rows) with the greatest sum of the absolute values of correlation coefficients in the row of the correlation matrix R were rejected. In adding up the coefficients in the j row of the correlation matrix R only the strongly correlated variables were taken into account $(r_{j\bullet} > 0,5)$, thereby diagnostic indicators showing the strongest (total) correlation with other indicators were eliminated.

Summing up, as a result of the correlation analysis indicators 2, 5, 19, 3, 8, 4, 12, 15, 20, 21, 22 and 14 were excluded from further processing. These indicators were removed because they had a small informative capacity; the high correlation with other indicators means a transfer of the same information about compared objects. The final set of diagnostic indicators that was used to rank Polish food companies by their innovativeness consisted of 7 indicators (Table 1) that in the evaluation of innovativeness were treated as stimulants.

Table 1. The diagnostic indicators of manufacturing companies' innovativeness

No.	Symbol [a]	Preferences [b]	SPECIFICATION
1	Z1	S	Innovative companies as a % of all companies
2	Z6	S	Innovation outlays per enterprise carrying on innovative activity (PLN thousands)
3	Z7	S	R&D outlays as a % of innovation outlays
4	Z9	S	Outlays on the purchase of ready-made technologies as % of innovation outlays
5	Z10	S	Software outlays as a % of innovation outlays
6	Z11	S	Investment outlays on buildings, structures and land as a % of innovation outlays
7	Z13	S	Investment outlays on imported machinery and technical equipment as a % of innovation outlays

[a] Corresponds to the indicator's number in section **Indicators of innovativeness …** ;
[b] S – stimulant

Source: developed by the author

In this research, individual variables were assumed to be equally important for the lack of non-trivial ways enabling the determination of their weights with additional information; hence:

$$\alpha_j = 1/m, (j = 1,...,m). \quad (2)$$

Linear ordering of objects

The most important requirement that a normalisation procedure is expected to meet is that the transformation does not affect the correlation between the characteristics as well as the basic indicators determining the shape of their distribution (skewness, kurtosis). This requirement is satisfied by transformations based on standardisation (3) and unitarisation (4) [Zeliaś 2000, p. 792]:

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j}, (j = 1,...,m), \quad (3)$$

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{\max\limits_i x_{ij} - \min\limits_i x_{ij}}, \ (j = 1,...,m). \quad (4)$$

Because the literature of the subject offers a range of normalisation methods[2], the theoretical properties of particular approaches [Kukuła 2000, pp. 77-

[2] As far as normalisation procedures are concerned, [Grabiński et al. 1989, pp. 27-28] indicate 3 transformations that are used the most frequently; [Domański et al. 1998, pp. 49-

100] must be assessed to establish their usefulness before a transformation with the best characteristics is selected and applied to perform a linear ordering of objects.

In this research two variants of calculations are presented; one is based on the unitarisation of diagnostic indicators (variant I) and the other uses the classical standardisation of diagnostic indicators (variant II) – see Table 2.

The synthetic measure M with values μ_i for the $i-$th object $(i=1,...,n)$ computed on the basis of normalised variables z_{ij} and weights α_j $(j=1,...,m)$ assigned to variables $\alpha_j \in (0;m)$, particularly those weights for which $\sum_{j=1}^{m}\alpha_j = 1$, can be expressed as the arithmetic, harmonic and geometric means of the diagnostic indicators. The comparative studies typically use a formula based on the mean of normalised diagnostic indicators [Gatnar, Walesiak 2004, p. 355]:

$$\mu_i = \frac{1}{m}\sum_{j=1}^{m}\left(z_{ij}\alpha_j\right), \tag{5}$$

the formula can be applied if all variables were originally measured on an interval scale or a quotient scale and the normalisation procedure was based on standardisation, unitarisation or zero unitarisation.

In the calculations, the diagnostic indicators were aggregated by adding up their normalised values. This approach yields exactly the same linear order of objects as aggregation making use of the arithmetic mean of the normalized values of the diagnostic indicators (see formula 5).

It was only recently, in 2006, that the need to evaluate ranking quality and to choose the optimal solution started to emerge in the literature as a precaution against inference based on rankings constructed with „*ad hoc*” selected partial procedures.

The statistical quality of the rankings can be assessed with the directional variance of the synthetic variable M^* given by the formula [Kolenda 2006, pp. 137-140]:

$$s^2\left(M^*\right) = \frac{\sum_{i=1}^{n}\left(\mu_i^* - \bar{\mu}^*\right)^2}{n}\ ,\ (i=1,...,n), \tag{6}$$

48] present 5 standardisation and 10 quotient transformations; [Kukuła 2000, pp. 106-110] takes a different division with 10 normalisation transformations; [Zeliaś 2002, pp. 792-794] discusses 2 standardisation methods, 4 unitarisation methods and 6 quotient transformations; [Walesiak 2006, pp. 16-22] analyses a total of 11 transformations; [Młodak 2006, pp. 39-42] presents 4 standardisation methods, 7 unitarisation methods and 8 quotient methods, including author’s own proposals based on location statistics.

where μ^* stands for the values of the synthetic variable M^*, n is the number of objects, and $\overline{\mu}^*$ denotes the arithmetic mean μ_i^* of the synthetic variable M^* $\left(\overline{\mu}^* = 0\right)$ determined from:

$$\mu_i^* = \sum_{j=1}^{m}\left(z_{ij} w_j\right),\ \left(i = 1,\ldots,n\right), \tag{7}$$

$$\sum_{j=1}^{m} w_j^2 = 1,\ w_j > 0, \tag{8}$$

where w_j are weights being the coordinates of the unit vector.

It is so, because any other result of the ordering of objects obtained with the values of, for instance, the synthetic measure M with any weights α_j adding up to one, is transformable into the result of the ordering of objects according to the values of M^* (orthogonal projection) with weights w_j meeting condition (8) derived from formula (9):

$$w_j = \frac{\alpha_j}{\sqrt{\sum_{j=1}^{m} \alpha_j^2}}, \tag{9}$$

If the sum of the values of μ_i^* calculated with the normalised values of individual variables z_{ij} tends to a maximum, then the mean sum of squares $s^2\left(M^*\right)$, i.e. the directional variance of the synthetic measure given by formula (6), also tends, under $\overline{\mu}^* = 0$, to a maximum and provides an unambiguous criterion for selecting the best ordering of objects[3].

The results of the linear ordering of food industry branches generated by variants I and II are presented in Table 2.

[3] The directional variance method applied to evaluate the correctness of the ordering of objects can be found in Kolenda 2006, pp. 137-140; Mikulec 2008, pp. 35.

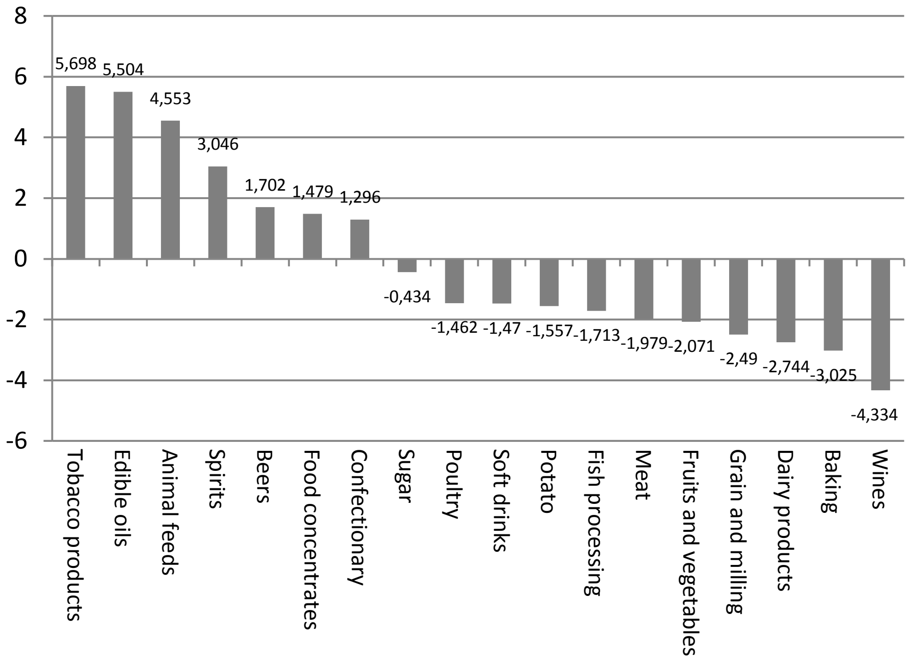
Table 2. The numerical characteristics of the linear ordering of food companies and the evaluation of ordering quality by variant: I – unitarisation of the diagnostic indicators, and II – classical standardization of the diagnostic indicators

Industry (branch)	Synthetic measure M 2005-2011		Values of $\left(\mu_i^* - \bar{\mu}^*\right)^2$ 2005-2011	
	Variant I - unitarisation	Variant II - standardisation	Variant I – unitarisation	Variant II - standardisation
Meat	1.323	-1.979	0.0008	0.0014
Poultry	1.437	-1.462	0.0005	0.0062
Fish processing	1.374	-1.713	0.0006	0.0086
Dairy products	1.142	-2.744	0.0014	0.0220
Potato	1.433	-1.557	0.0005	0.0071
Fruits and vegetables	1.288	-2.071	0.0009	0.0125
Edible oils	3.337	5.504	0.0065	0.0883
Grain and milling	1.224	-2.490	0.0011	0.0181
Sugar	1.731	-0.434	0	0.0006
Animal feeds	2.875	4.553	0.0031	0.0604
Baking	1.051	-3.025	0.0018	0.0267
Confectionary	2.152	1.296	0.0003	0.0049
Food concentrates	2.227	1.479	0.0004	0.0064
Soft drinks	1.460	-1.470	0.0004	0.0063
Spirits	2.670	3.046	0.0020	0.0271
Wines	0.737	-4.334	0.0035	0.0548
Beers	2.288	1.702	0.0006	0.0084
Tobacco products	3.334	5.698	0.0066	0.0947
$s^2\left(M^*\right)$	X	X	0.0017	0.0258

Source: developed by the author

Following the application of the criterion of maximising the directional variance of the synthetic measure that in this case called for transforming the values of the synthetic measure M into the outcome of the orthogonal projection of objects onto line M^* it turned out that variant II generated "better" rankings of food industry companies with respect to their innovativeness – see Table 2.

Figure 1. Food industry branches in Poland ranked by innovativeness, years 2005-2011



Source: developed by the author

The analysis of the correlation between the diagnostic indicators and the values of the synthetic measure showed that the direction of the correlation (positive) and the strength of the indicators' impact on the value of the synthetic measure were consistent, thus confirming that the set of variables was correctly selected for analysis.

SUMMARY AND CONCLUSIONS

The level of innovativeness in the food industry is specific to its branches, as proven by the results of the presented attempt at constructing and making an empirical verification of a synthetic measure of innovativeness.

The diagnostic indicators used in the analysis show that in the years 2005-2011the most innovative were tobacco companies (5,698), which were followed in the ranking by the producers of edible oils (5,504) and animal feeds(4,553). The spirits companies also showed a relatively high level of innovativeness (3,046). The other branches of the food industry were markedly less innovative. Wine-making companies took the last place in the ranking (- 4,334). The leaders were branches with large shares of direct foreign investments.

For the time being, however, the level of innovativeness of Polish food companies, and thereby their long-term growth, are still lower than in countries that are better developed. Among the biggest weaknesses troubling the domestic system of innovation there are low allocations to R&D activity, their inefficient structure, and a very limited transfer of knowledge between R&D institutions and industry.

The results of this research may serve as an indication as to the future policy of support for innovation activities among Polish food companies.

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APPLICATION OF MIGRATION MATRICES TO RISK EVALUATION AND THEIR IMPACT ON PORTFOLIO VALUE

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Abstract: Migration matrices are widely used in risk management. In particular, quality of financial products due to credit risk is described by assigning them to one of several rating categories. The probability of future rating is determined by a migration matrix. Portfolio's value depends on the rating and on market states. To find an optimal portfolio one should consider migration matrices and the dynamics of market changes. The main goal of our research was to investigate the impact of both risks, market risk and credit risk on portfolio value. On a real portfolio we show that differences in migration matrices that result from the state of economy influence considerably credit risk and portfolio value.

Keywords: migration matrices, portfolio value, market risk, credit risk

INTRODUCTION

The New Basel Accords (Basel II/III) have tightened the rules of managing financial risk and introduced the possibility of using the IRB systems, i.e., internal rating systems to support estimation of economic capital and internal risk. The IRB approach requires estimation of individual PD (probability of default) for each financial position. Within the IRB approach migration matrices are used. They are estimated based on historic data. In credit risk estimation an obligor is assigned to one of several rating classes under the assumption that his future rating is determined by a transition matrix of a Markov chain. The probability that the obligor will migrate to the default state can be read off a migration matrix. The obligors with the same credit quality are assigned to the same risk group. Following S&P the highest and the best rating category is AAA. An obligation rated AAA is judged to be the best quality, with the smallest degree of investment risk. On the other edge of the scale is category D, which is assigned to an obligation where a default has already occurred [Rachev et al. 2009, Crouhy et al. 2001].

Rating related issues were the key factor of the financial crises of 2007. When in 2007 most of investigated bank assets were downgraded by rating agencies an avalanche of bankruptcies was triggered off because the change in credit rating causes a change in the price of rated assets or liabilities. The mechanisms of the economic breakdown were described in [Crouhy et al. 2008].

In the paper we show how the change in rating due to the change of economic state influences credit risk and the portfolio value. By risk we mean here the probability to default. In our research we consider migration matrices estimated for two states of economy and their impact on portfolio value and credit risk.

APPLICATION OF MIGRATION MATRICES TO RISK EVALUATION

Data description

We have based our research on one year migration matrices which were calculated based on quarterly migration matrices for two states of economy sourced from [Bangia et al. 2000]. We denote by E the matrix for the state of economic expansion (Table 1) and by R the annual matrix for the state of recession (Table 2). Comparing the entries of both matrices we can notice that the probabilities of migration to default (PDs) are higher for the matrix R for all states but two highest. On the other hand the entries on the diagonal of the matrix R, i.e., the probabilities that the obligor will stay in the same class, are smaller than the corresponding entries of E. There were no migrations between distant states during recession as positive probabilities for the matrix R are concentrated along the diagonal.

Table 1. Annual migration matrix for expansion (matrix E)

Rating before	Rating after							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,9297	0,0628	0,0057	0,001	0,0008	0,0001	0	0
AA	0,0057	0,9257	0,0609	0,0056	0,0006	0,0012	0,0003	0,0001
A	0,0008	0,0201	0,9265	0,0451	0,0049	0,0025	0,0001	0,0001
BBB	0,0004	0,0031	0,0548	0,8855	0,0448	0,0096	0,0009	0,0011
BB	0,0004	0,0013	0,0086	0,0689	0,8287	0,0793	0,0064	0,0064
B	0	0,0008	0,0030	0,0056	0,0599	0,8511	0,0406	0,0390
CCC	0,0016	0,0002	0,0065	0,0081	0,0177	0,1108	0,5837	0,2716
D	0	0	0	0	0	0	0	1

Source: own calculations based on ¼ year US migration matrix sourced from [Bangia et al. 2000]

Table 2. Annual migration matrix for contraction (matrix R)

Rating before	Rating after							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,9222	0,0653	0,0120	0,0005	0,0001	0	0	0
AA	0,0067	0,8828	0,1009	0,0059	0,0034	0,0001	0	0
A	0,0009	0,0319	0,8680	0,0930	0,0058	0,0002	0	0,0002
BBB	0,0015	0,0021	0,0400	0,8658	0,0818	0,0035	0,0006	0,0047
BB	0	0,0023	0,0031	0,0490	0,8170	0,0936	0,0155	0,0194
B	0	0,0022	0,0023	0,0045	0,0249	0,8173	0,0672	0,0816
CCC	0	0	0	0,0001	0,0004	0,0355	0,5382	0,4258
D	0	0	0	0	0	0	0	1

Source: own calculations based on ¼ year US migration matrix soured from [Bangia et al. 2000]

Application of migration matrices to PD estimation

Credit risk calculations are based on PDs. Risk measures are calculated for a given time horizon. In practice, various risk indicators which are based on migration matrices are calculated for one year ahead. On the other hand risk figures are applied to evaluation of different business measures, e.g., profitability, where one has to take into account migration forecasts for the whole duration of a loan or liability. For time homogeneous Markov chains, changes of transition matrix in given period can be easily calculated by multiplying transition matrices for intermediate periods. In particular, entries of the n-th power of an annual migration matrix denote probabilities of migration in n-th year.

For the purpose of forecasting we have calculated five successive powers of both annual matrices E and R. The values that are essential in risk analysis are shown in Tables 3 and 4. The matrices obtained have decreasing entries on the diagonal and the corresponding values are smaller for the recession than expansion. On the other hand the values in last columns of successive matrices increase. They represent probabilities of migration to default in successive years. The increase is especially significant for lower rating categories in time of recession. In particular, the probability of migration to default from the state CCC within five years is 90%. The high PD values for relatively short time are the reason why the lowest rating categories are described as speculative.

Table 3. Entries of successive powers of an annual migration matrix E

Rating class	Entries on the diagonal					PDs				
	n=1	n=2	n=3	n=4	n=5	n=1	n=2	n=3	n=4	n=5
AAA	0,9297	0,8647	0,8046	0,749	0,6975	0	0	0,0001	0,0002	0,0003
AA	0,9257	0,8585	0,7977	0,7426	0,6927	0,0001	0,0003	0,0007	0,0012	0,0018
A	0,9265	0,8622	0,8057	0,756	0,7122	0,0001	0,0004	0,0009	0,0018	0,0029
BBB	0,8855	0,7897	0,7093	0,6416	0,5842	0,0011	0,003	0,0057	0,0093	0,0136
BB	0,8287	0,6947	0,5894	0,5062	0,4399	0,0064	0,0166	0,0298	0,0451	0,0619
B	0,8511	0,7337	0,6391	0,5617	0,4974	0,039	0,0836	0,1288	0,1722	0,2126
CCC	0,5837	0,3453	0,2083	0,1291	0,0829	0,2716	0,4346	0,5348	0,5986	0,6410

Source: own calculations

Table 4. Entries of the successive powers of an annual migration matrix R

Rating class	Entries on the diagonal					PDs				
	n=1	n=2	n=3	n=4	n=5	n=1	n=2	n=3	n=4	n=5
AAA	0,9222	0,8509	0,7855	0,7255	0,6705	0	0	0	0,0001	0,0003
AA	0,8828	0,7830	0,6977	0,6246	0,5617	0	0,0001	0,0005	0,0011	0,0021
A	0,8680	0,7604	0,6722	0,5996	0,5395	0,0002	0,0009	0,0024	0,0046	0,0079
BBB	0,8658	0,7574	0,6691	0,5966	0,5366	0,0047	0,0109	0,0193	0,0300	0,0433
BB	0,8170	0,6739	0,5612	0,4721	0,401	0,0194	0,0497	0,0874	0,1294	0,1735
B	0,8173	0,6727	0,5569	0,4634	0,3873	0,0816	0,1774	0,2721	0,3591	0,4361
CCC	0,5382	0,2921	0,1604	0,0897	0,0515	0,4258	0,6579	0,7862	0,8586	0,9007

Source: own calculations

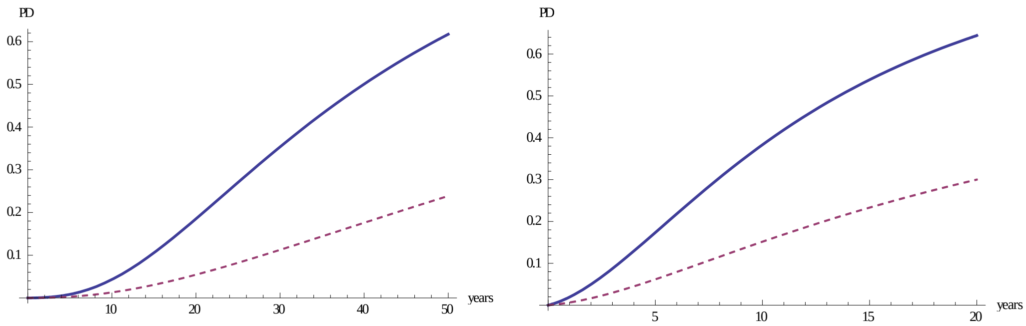
Using infinitesimal generator (or intensity matrix) Q and its eigenvalue decomposition we can make entries of the migration matrix time dependent. By D we denote a diagonal matrix of eigenvalues of Q and by V the corresponding matrix of its eigenvectors. Then for any $t > 0$

$$P(t) = \exp(tQ) = V \exp(tD) V^{-1},$$

where $\exp(tD) = diag(e^{\lambda_1 t}, .., e^{\lambda_n t})$ and λ_i are eigenvalues of matrix Q.

Functional representation of migration probabilities enables calculating PDs for an arbitrary $t > 0$. The graphs of functions representing probability of migration to default from the initial states A and BB are presented on Figure 1. For any $t > 0$ the probability of migration to default is much higher for the state of recession (thick line).

Figure 1. Probabilities of migration to default from BB (right) and A (left); (thick recession, dashed expansion)



Source: own calculations

For both matrices E and R we calculate the average number of years before entering the default state for each initial rating category. In order to obtain the desired result we use the notion of a fundamental matrix of an absorbing Markov chain. The values we have obtained are presented in Table 5. They indicate that in both cases the time before absorption for high initial rating categories is very long. On the other hand, for recession, the average time before absorption for initial class CCC is only 3 years and for initial class B only 12 years.

The results refer to relatively long time horizon that seem to be beyond the time used in practice. Banks however estimate risk for such a period that allows for possible changes in the portfolio structure. Products such as loans require several or a few dozen years.

Table 5. Average numbers of years before migration to the default state

States of economy	AAA	AA	A	BBB	BB	B	CCC
Recession	71	60	51	40	24	12	3
Expansion	162	150	138	120	91	59	27

Source: own calculations

APPLICATION OF MIGRATION MATRICES TO EVALUATION OF PORTFOLIO VALUE

The expected loss is usually measured based on the scoring models, i. e., econometric models of many dependent variables describing the client. There are a few models for calculating the percentile of loss. In the paper we focus on the models that involve migration matrices. One of the models is CreditMetrics™ [Bhatia et al. 1997]. The theoretical background of the model is built on the Merton's theory of firm evaluation. In that model credit quality of the firm can be measured comparing its liabilities and assets [Crouhy et al. 2001].

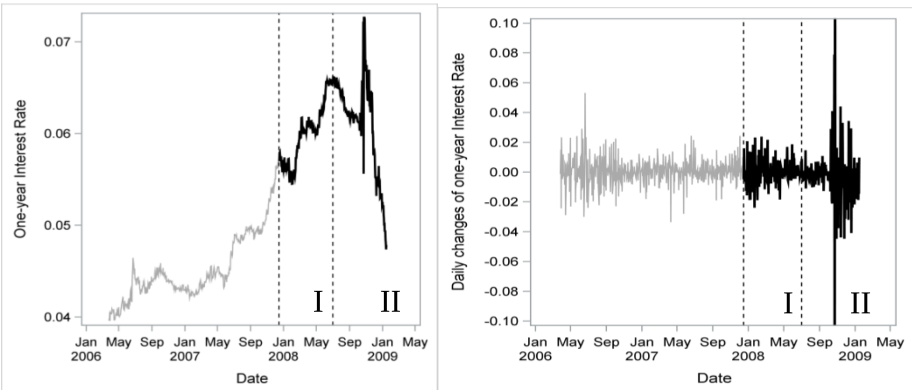
The CreditMetrics™ model goes beyond the approach suggested by Merton. In the model the assets volatility is consistent with the rating system. In other words the asset distribution parameters are modeled based on the migration matrix. In the CreditMetrics™ terminology the assets are called a portfolio. The pricing of bank product depends both on the rating category and the time structure of financial flows. The latter is expressed through the discount coefficients that make it possible to calculate financial flows in various moments. It is especially important when one wants to show differences in portfolio value and risk for different states of economy.

In our research we used two sub-portfolios of bonds sourced from one of Polish bank in year 2008. The first portfolio consisted of 138 corporate bonds issued by telecoms rated BB (according to the internal rating of the bank) and the second portfolio consisted of 164 corporate bonds issued by financial companies, all of them rated A. Default probabilities in both portfolios were higher than 0. In both portfolios there were coupon bonds indexed at fixed or variable interest rates - this has been included in the evaluation functions of the bonds. Duration of both portfolios was about 1.2 years.

Because the portfolios differ, for the purpose of comparison we used a scaling transformation so that the value of each portfolio was equal to 1 at the beginning of the observed period. The value of the portfolios was affected by various factors such as rating categories (A or BB), migration matrix (R or E) and forms of the yield curve.

In order to estimate portfolio value, computer simulations of influence of various factors on that portfolio were used. The results that we have obtained were then compared. The calculations were based on bond portfolio pricing with application of discount rate models estimated based on the market of the so called yield curves.

Figure 2. Annual interest rates and daily changes for expansion (I) and recession (II)



Source: own calculations

In our calculations we have used market data for two states of economy: expansion and contraction. The data used for calculations for the state of economic expansion were the migration matrix E (Table 1) for credit risk simulations and Polish market data covering the period 15.12.2007 – 1.06.2008 for modeling the yield curve. The data used for the state of recession were migration matrix R (Table 2) for credit risk simulations and Polish market data covering the period 15.10.2008 – 15.01.2009 for modeling the yield curve.

The models of interest rate scenarios

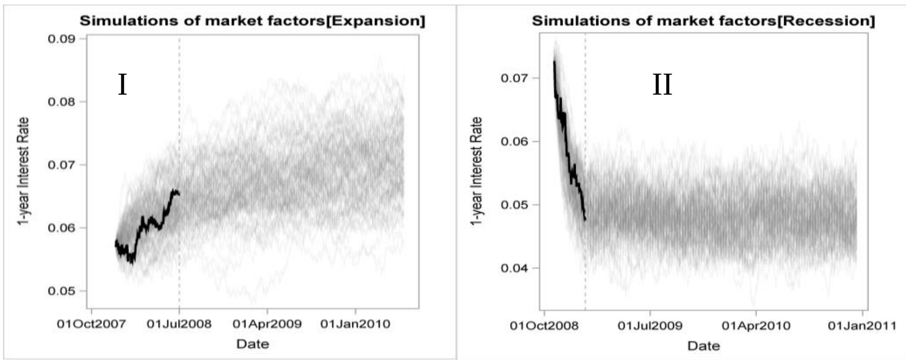
In order to investigate the behavior of portfolio we conducted prices scenarios of interest rates based on real market data. Models applied for generating interest rates were based on the theory of Brownian motion in continuous and discrete case. Brownian motion models satisfy the Stochastic Differential Equation (SDE):

$$dx = \mu dt + \sigma dz$$

where μ is the trend function and dz is Wiener process.

CIR (Cox-Ingersoll-Ross) is a popular model that belongs to the class of Brownian models [Bjoerk 2009]. We used CIR model for both building a generator and forecasts of market changes of rates of return which were then used in calculating the portfolio value.

Figure 3. Historical values and scenarios of interest rates



Source: own calculations

Bond pricing models

Market bond portfolio pricing P_T can be obtained using the equation:

$$P_T = E^M \left[exp\left(-\int_0^T R_t \, dt \right) X \right],$$

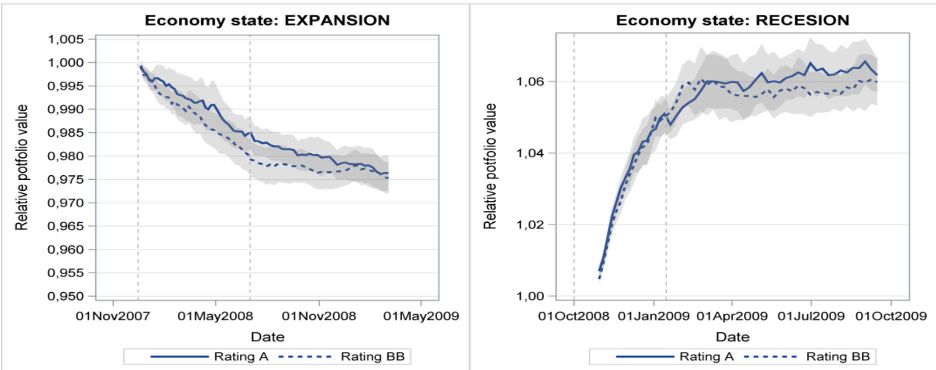
where X denotes the term structure of financial flows for bonds and other contingent debts, subject to default risk. E^M is the expected value for arbitrage free market, where all securities are priced based on a short term interest rate process r_t

on martingale measure M [Weron et al. 1998]. R_t is a yield curve for debt instruments obtained through replacing a process of short term interest rate r_t by a process corrected with respect to the risk of credit default: $R_t = r_t + Q_t L_{t.,}$ Q_t is an intensity function connected with the process of migration between rating categories and L_t is an expected rate of impairment. In the calculations Monte Carlo method was used based on generated scenarios of interest rates. The term structure of X was based on the portfolio description: dates of payments, coupon interest rates. The yield curve R_t was calculated based on a risk free yield curve obtained from treasury adjusted by spread. In order to calculate the spread the intensity matrices Q_E and Q_R obtained from migration matrices E and R were used. Because of the lack of data concerning the recovery rates after default, the rate was assumed to be 50%. It has to be mentioned, that the influence on credit risk of this factor has not been investigated in the paper.

Results

In the first step of our analysis we assumed that the credit spread for both rating classes is equal to 0. The portfolio pricing was performed for two periods reflecting two states of economy (expansion and recession). In figures below the risk parameters for both rating categories A and BB were changed in such a way that credit spread is 0.

Figure 4. Bond portfolio pricing for two states of economy and zero credit spread

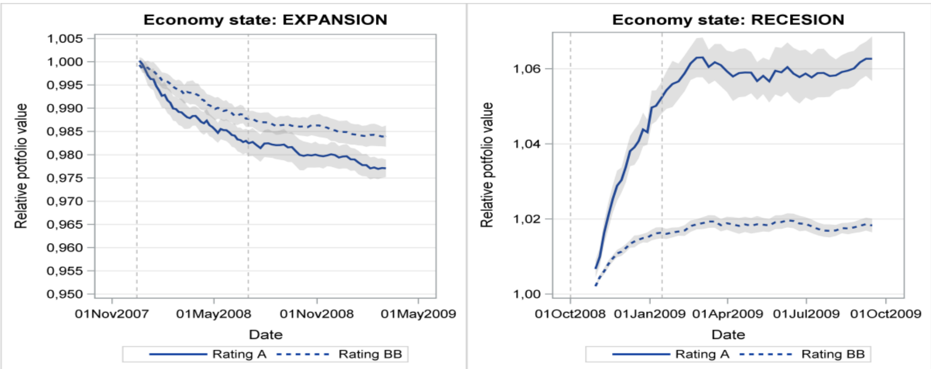


Source: own calculations

It can easily be seen that the portfolio value depends on the state of economy. During economic expansion the portfolio value decreases while during recession it increases. The behavior of the portfolio prices reflects the scenarios of interest rates (Figure 4). The portfolio values fall down during expansion because of the growth of interest rates. This decrease depends on the changes of interest rates (IR) with time. In our example the decline of IR by 1%-1.5% caused a 2% decline of the portfolio value. During recession IR declines about 2% and then it stabilizes. The portfolio value grows by 1% and then it remains constant. Due to the zero spread value the portfolio price does not depend on rating class.

In our further calculations we took into account credit risk captured by migration matrices and next we calculated the portfolio value.

Figure 5. Bond portfolio relative value for two states of economy and credit spread



Source: own calculations

In the case when credit risk is taken into account a visible change in dependence of portfolio value on the rating class can be noticed (Figure 5). One has to remember that economy state influences both IR and migration matrices. IR rates are the input for calculating the yield curve and spread in the pricing function.

During economic expansion the value of the portfolio with rating A rapidly decreases. The decrease is a response on the increase of IR during expansion. Credit risk causes an increase in price because of the so called risk premium, i.e., the price of the bond is modified by adding an additional component for financing a possible default. The difference between prices are due to differences in credit scoring for both portfolios. The difference of migration matrices shown in Table 5 and Figure 1 for ratings A and BB resulted in a 1.0% change of portfolio value within a 1 year horizon.

During recession the situation is reverse. The decrease of IR and the amendment with help of credit spread acts in the opposite direction. The difference between prices due to differences in credit scoring for both portfolios (A and BB) caused a 4.0% change of portfolio value within a 1 year horizon.

CONCLUSIONS

While examining changes in the portfolio value one must take into account not only market risk but also credit risk. In the paper we have shown the application of migration matrices to estimation of both risks: market and credit risk in a given time horizon for two states of economy. The probability of default in relatively short time (one year) is high for lower rating categories during recession.

We have shown the influence of market factors such as interest rates and credit rating on the value of real portfolio in a changing world of the whole economy. The results obtained indicate significant differences in the portfolio value with investing rating category A and speculative rating category BB. During recession the value of portfolio with rating BB is lower than that with rating A.

The value of the credit spread, which is a function of the credibility in credit risk has an important impact on the behaviour of the portfolio. Therefore, while comparing the quantitative models for financial instruments due to market risk it is very important to remember that the observed rates depend on the credit risk. Probability of default in case of rating A grows with the change of the economy state from 0.01% to 0.64%, while in case of rating BB from 0.02% to 1.94%. This entails a change in the portfolio value by 1% and 4%, respectively in one year. At the same time changes related to market risks are equal on average: 1.5% or 2.0% and 2% or 6%, respectively, depending on the rating class and the state of economy. It can be seen that influence of both types of risk on the portfolio value is comparable for rating A. The deterioration of portfolio rating however, brings about the domination of credit risk factors.

It might seem strange but during recession the changes of value of both portfolios is higher than during expansion. In the wake of this the business measure of risk, Value at Risk (VaR), is higher in time of recession. The higher risk is caused both by the form of a migration matrix and the change of interests rates – discount factors.

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EXPORT DIVERSIFICATION AND ECONOMIC GROWTH IN TRANSITION: LESSONS FROM THE 2008 FINANCIAL CRISIS IN CEE

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Abstract: This paper examines the role of export diversification for economic growth in CEE transition economies. The results prove that before the outbreak of 2008 financial crisis export specialization rather than diversification was an important growth factor, especially in those countries which followed more specialized export patterns. However, after the outbreak of the crisis the direction of this causal link changed essentially. All three main aspects of export diversification turned out to play a significant role in reducing the growth slowdown effects of the 2008 financial crisis.

Keywords: export diversification, economic growth, CEE transition economies

INTRODUCTION

The debate on export diversification has focused on the question how developing countries can improve economic performance and achieve higher income. The inspection of export data convince the researchers that currently there are almost no large developed countries with the extremely high levels of export concentration (which in turn is typical for most of developing countries). However, this fact does not supply evidence about the causal relationship between economic growth and export diversification. Another hypothetical explanation is that richer countries are more able to diversify their production structures. In spite of this

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uncertainty many economists draw conclusion that higher diversification affects economic growth positively, especially in developing countries. Economic theory deals with channels through which export diversification might positively affect economic growth, e.g. rise in the number of export industries (i.e. horizontal export diversification) can diminish the dependence on a small number of goods while shift from primary into manufactured exports (i.e. vertical diversification) is associated with faster growth since primary export sectors generally do not exhibit strong spillovers [Herzer and Nowak-Lehmann 2006]. Manufactured exports seems to have greater potential effect for sustained learning than traditional primary exports, because more beneficial spillover effects spread to other activities [Matsuyama 1992]. Thus, in endogenous growth models it is recommended to focus on vertical aspect of export diversification. Some researchers underline that export diversification implies improvement in production techniques through knowledge spillovers, which lead to more efficient management, better forms of organization, labour training, and knowledge about technology and international markets [Al-Marhubi 2000]. Chuang stresses that on international competitive markets it is necessary to gain the knowledge about quality, delivery conditions and foreign buyer's specifications, which may take place through a permanent learning process started by exporting activities, especially the diversification [Chuang 1998]. To summarize, in the economic literature it is mostly taken for granted that horizontal and vertical export diversification may positively affect growth. However, there have been only few empirical investigations on the links between export diversification and growth, none of which, as far as we know, focused on CEE transition economies. Moreover, some recent empirical studies (e.g. [de Pineres and Ferrantino 2000]) failed to confirm the positive growth effect of export diversification. Both these facts are the main sources of the motivation to conduct this empirical study.

In order to check the diversification-led growth hypothesis we use CEE time series data from 1995 to 2011. CEE transition economies are chosen for the analysis because countries from this region, especially Poland, diversified to some extend their exports structures horizontally and vertically on the basis of natural resources. The comparative advantage of CEE countries lies mainly in the production of resource– and agriculture–based products. Thus, our contribution is believed to examine if and how diversification of export on the basis of natural resources can accelerate economic growth.

LITERATURE OVERVIEW

Although economists are deeply interested in the links between export and economic growth, the export-economic growth relation is still not justified empirically and desires further empirical investigation. According to Adam Smith foreign trade allows a country to relocate its given resources to provide new and more effective demand for the output of the surplus resources. At this place it is worth to recall the law of comparative advantages (profits from specialization and foreign trade) which has been formulated by David Ricardo. He investigated in detail advantages and alternative or relative opportunities in the famous example concerning two commodities: wine and cloth, produced by England and Portugal. Myint stresses that a surplus productive capacity suitable for the export market (due to specialization) is a costless way of acquiring imports as well as supporting domestic economic activity [Myint 1958]. It has been proven that marginal factor productivities are significantly higher in the exporting sectors as compared to the non-export sectors [Feder 1982; Kavoussi 1984; Moschos 1989]. In consequence, shifting the existing resources from the less-efficient non-export sectors to the more productive export sectors can accelerate economic growth thanks to more efficient use of resources, the application of technological innovations (which are result of international competition), gains of scale effects following from larger international markets and greater capacity utilization.

Beside discussing the general problem of the role of export in stimulating economic growth, economists for many years have been trying to answer the more detailed question whether by export diversification the developing (and less developed) countries can significantly improve economic performance and achieve higher income. Some recent contributions have checked the impact of export diversification on economic growth [Al-Marhubi 2000; de Pineres and Ferrantino 2000; de Ferranti et al. 2002; Balaquer and Cantavellha-Jorda 2002]. Several cross-sectional studies found evidence supporting diversification-led growth hypothesis [Al-Marhubi 2000; de Ferranti et al. 2002]. Using panel data for Latin American countries, de Pineres and Ferrantino reported positive interrelation between export diversification and per capita income [de Pineres and Ferrantino 2000, Chapter 7]. Balaquer and Cantavellha-Jorda detected (using cointegration and causality tests) a positive relationship between changes in exports and economic growth in Spain [Balaquer and Cantavellha-Jorda 2002].

On the other hand, the study by de Pineres and Ferrantino detected no evidence in favour of diversification-induced growth in Columbia and Chile [de Pineres and Ferrantino 2000, Chapters 4, 5]. Moreover, in the case of Chile export diversification seemed to be even negatively correlated with growth. However, the discussed study suffers from several methodological drawbacks. First of all, the authors do not handle an important problem of cointegration, which is important in terms of long-run relationship between export diversification and economic growth. The contributors take into account the problem of nonstationarity by calculating first differences. However, it is well known that if the variables of interest are cointegrated, the standard procedure of taking first differences may lead to loss of long-run information [Granger and Newbold 1974]. Another drawback is that the contributors do not check the respective time series for the presence of structural breaks when testing for unit roots. Ignoring structural breaks also may imply spurious results of unit root tests. The authors did not conduct standard goodness-of-fit tests, which makes their results less reliable.

Herzer and Nowak-Lehmann also investigated the link between export diversification and economic growth. They came to conclusion that export diversification supports the rate of growth in developing countries, nevertheless diversification occurs in horizontal or vertical dimension [Herzer and Nowak–Lehmann 2006].

The more recent empirical evidence [Imbs and Wacziarg 2003] suggests the existence of a nonlinear relationship between export diversification and income. As income per capita increases, export concentration initially falls, but after reaching a certain level of income, export tends to become more concentrated again. However, in the literature there is still no unique view in respect to the question: what are the main factors of export diversification. Moreover, the literature on this topic is rather poor. Cadot et al. investigate the evolution of export diversification patterns along the economic development path [Cadot et al. 2011]. Using a large database with 156 countries over 19 years, they found a hump-shaped pattern of export diversification similar to what Imbs and Wacziarg established for production [Imbs and Wacziarg 2003]. According to the contributors diversification and subsequent reconcentration took place mostly along the extensive margin. This hump-shaped pattern was consistent with the conjecture that countries travel across diversification cones, as discussed in [Schott 2003, 2004] and [Xiang 2007].

THE DATASET AND ITS PROPERTIES

In this paper we used a dataset consisting of a panel of annual observations for new EU members in transition from the CEE region[2] in the period 1995-2011. The data may be classified into three main categories. The first group includes variables which are related to the measures of economic growth of CEE transition economies and various proxies of main growth factors. Since the existing literature has not yet reached a consensus about a typical set of variables that may affect economic growth, we have followed previous papers which have reviewed the existing literature [Bleaney and Nishiyama 2002; Levine and Renelt 1991; Sachs and Warner 1997, among others] and selected a relatively small subgroup from hundreds of the control variables, which are usually considered as important for economic growth. The second group of variables describes various aspects of export diversification. The last group of variables consists of dummy variables which capture the effects of 2008 financial crisis. Table 1 provides details on all the variables.

[2] In the period 2004-2007 twelve countries joined the EU. These were: Bulgaria, Czech Republic, Cyprus, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Romania, Slovakia and Slovenia. In this paper we focused on all the mentioned economies except Malta and Cyprus since they have never been in a transition phase.

Table 1. Brief description of data used in this paper

Full name	Abbreviation used	Definition	Unit
		ECONOMIC VARIABLES	
Gross domestic product[a]	*Y*	Gross domestic product at constant 2005 prices in US Dollars.	USD
Gross capital formation[a]	*K*	Gross capital formation at constant 2005 prices in US Dollars.	USD
Total labour force[b]	*L*	Total labour force comprises people ages 15 and older who meet the International Labour Organization definition of the economically active population: all people who supply labour for the production of goods and services during a specified period.	-
Tertiary school enrolment[b]	*EDU*	Gross enrolment ratio is the ratio of total enrolment, regardless of age, to the population of the age group that officially corresponds to the level of tertiary education. This indicator is often used to measure the level of human capital [Barro 1995].	-
Government consumption[a]	*GC*	General government final consumption expenditure at constant 2005 prices in US Dollars.	USD
Inflation, consumer prices (annual %)[b]	*INFL*	Inflation as measured by the consumer price index reflects the annual percentage change in the cost to the average consumer of acquiring a basket of goods and services.	%
Foreign direct investment, net inflows (% of GDP)[b]	*FDI*	Foreign direct investment are the net inflows of investment to acquire a lasting management interest (10 percent or more of voting stock) in an enterprise operating in an economy other than that of the investor. It is the sum of equity capital, reinvestment of earnings, other long-term capital, and short-term capital as shown in the balance of payments.	%
Money and quasi money (M2) as % of GDP[b]	*M*2	Money and quasi money comprise the sum of currency outside banks, demand deposits other than those of the central government, and the time, savings, and foreign currency deposits of resident sectors other than the central government.	%
		MEASURES OF EXPORT DIVERSIFICATION	
Concentration index[c]	*CONCE*	Concentration index, also named Herfindahl–Hirschmann index, is a measure of the degree of market concentration/diversification. It has been normalized to obtain values ranging from 0 (maximum diversification) to 1 (maximum concentration).	-
Number of exported products[c]	*NUM*	As suggested in previous studies [Herzer and Nowak–Lehmann 2006] we use natural logarithm of number of exported products to approximate the horizontal export diversification.	-
Share of manufactured exports in total exports[c]	*MANUF*	As suggested in previous studies [Herzer and Nowak–Lehmann 2006] we use this variable to approximate the vertical export diversification.	-
		2008 CRISIS DUMMY VARIABLES	
Indicator of post-crisis period	D_{2009}	This variable was used to capture the possible effects of financial crisis of 2008. It takes the value of 1 starting from 2009 onward and zero otherwise.	-
2009's shock indicator	I_{2009}	This impulse dummy variable captures the negative shock that occurred in 2009. It takes the value of zero everywhere except for a value of 1 in 2009.	-

Source: own elaboration

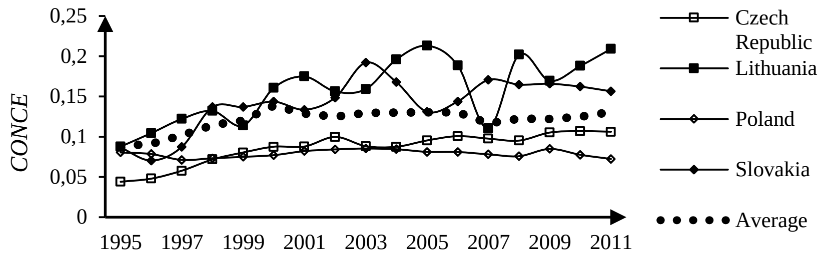
[a] Data gained from National Accounts Main Aggregates Database (http://unstats.un.org/unsd/snaama/introduction.asp)

[b] Data gained from World Development Indicators (http://data.worldbank.org/indicator)

[c] Data gained from United Nations Conference on Trade and Development (UNCTAD, http://unctadstat.unctad.org)

Besides the main group consisting of 10 new EU members in transition we have also decided to consider specific subgroups. In period 1995-2011 some CEE economies followed trade policies characterized with more export diversification while other focused on more specialized export profiles. The highest rise in concentration index was reported for Slovakia and Lithuania, while lowest rise was found for Czech Republic and Poland. The dependencies between levels of concentration index in case of four mentioned economies in comparison to the main group's average are presented in Figure 1.

Figure 1. Concentration index in the group of examined countries



Source: own elaboration

It is worth to note, that almost directly opposite conclusions follow from analysis of share of manufactured exports in total exports and number of exported products (i.e. in period 1995-2011 the levels of these measures in case of Czech Republic and Poland were much higher than the group's average)[3]. To summarize, one may claim that in case of new EU members in transition it is easy to form a subgroup of countries with less diversified export profile (characterized with relatively higher levels of *CONCE* and lower levels of horizontal and vertical diversification) and more diversified export profile (i.e. relatively lower levels of *CONCE* and higher levels of horizontal and vertical diversification). Table 2 contains the details on the groups of countries examined in this paper.

Table 2. Groups of countries examined in this paper

Group name	Symbol	Description
All countries	G_0	The group of 10 CEE transition economies.
More concentrated export profile	G_1	This subgroup consists of all but two countries (Poland and Czech Republic), whose export profiles were found to be most diversified (low values of export concentration indexes, relatively high values of number of exported products and the share of manufactured product on total export).
More diversified export structure	G_2	This subgroup consists of all but two countries (Lithuania an Slovakia), which were found to follow most concentrated export patterns.

Source: own elaboration

In the next section we will briefly present the methodology applied.

[3] Plots of *MANUF* and *NUM* variables are available from the authors upon request.

METHODOLOGY

In this paper we focus on the Solow growth model. In general, the motivation to perform the research in such a framework is twofold. First, we should note that Solow model is relatively easy to evaluate in comparison to a gamut of endogenous growth models [Greiner et al. 2004]. Secondly, as stated in previous papers [Jones 1995; Parente 2001, among others] there is no evidence that endogenous growth models perform better in empirical applications than the Solow's one. Since in the Solow model the steady state growth rate (SSGR) equals total factor productivity, the permanent growth effect of export diversification should be measured by estimating its effect on Total Factor Productivity [Dollar and Kraay 2004]. Following suggestions of previous studies [Herzer and Nowak-Lehmann 2006] we estimate the extended dynamic production function in which TFP depends on selected growth-influencing variables and chosen measures of different aspects of export diversification. To summarize, the main empirical part of our study was based on an application of the following loglinear specification of the Cobb-Douglas production function:

$$\ln\left(\frac{Y_t}{L_t}\right) = \ln\left(A_0\right) + (c_1 + c_2 X_t)t + \alpha \ln\left(\frac{K_t}{L_t}\right), \tag{1}$$

where $X_t = \left[x_t^i\right]_{i=1,\dots,k}$ denotes $k \times 1$ vector of growth-affecting variables (measures of export diversification and a set of control variables), c_2 stands for $1 \times k$ vector of parameters. Moreover, we assumed that first eight coordinates of X_t are fixed and satisfy: $x_t^1 = MANUF_t, x_t^2 = NUM_t, x_t^3 = CONCE_t$, $x_t^4 = D_{2009}MANUF_t$, $x_t^5 = D_{2009}NUMBER_t$, $x_t^6 = D_{2009}CONCE_t$, $x_t^7 = D_{2009}$, $x_t^8 = I_{2009}$. The remaining coordinates of vector X_t are the chosen control variables, i.e. $x_t^i \in \{EDU, GC, INFL, FDI, M2\}$ for $i > 8$[4].

If Y_t / L_t and K_t / L_t are cointegrated the model (1) takes the form:

$$\Delta \ln\left(\frac{Y_t}{L_t}\right) = c_1 + c_2 X_t + \alpha \Delta \ln\left(\frac{K_t}{L_t}\right) + EC_{t-1}, \tag{2}$$

where $EC_{t-1} = D_t + \alpha\left(Y_t / L_t\right) + \beta\left(K_t / L_t\right)$ stands for cointegration equation and D_t denotes deterministic term.

[4] Thus, $k \in \{8, \dots, 13\}$ in case of our dataset. All abbreviations are explained in Table 1.

EMPIRICAL RESULTS

Since both Y_t / L_t and K_t / L_t have experienced significant growth starting from the beginning of transition period, in the first step we have conducted a gamut of panel unit root tests. Table 3 summarizes the outcomes.

Table 3. The results of unit root tests for the group G_0

	Common unit root processes[a]				Individual unit root processes[a]			
	Levin, Lin and Chu[b]		Breitung[b]		Im-Pesaran-Shin[b]		Fisher[c]	
Variable	I.E.	I.E.+T	I.E.	I.E.+T	I.E.	I.E.+T	I.E.	I.E.+T
Y / L	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)
K / L	I(1)	I(1)	I(1)	I(1)	I(0)	I(1)	I(1)	I(1)
CONCE	I(0)	I(0)	I(1)	I(0)	I(0)	I(0)	I(0)	I(0)
NUM	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)
MANUF	I(0)	I(0)	I(1)	I(0)	I(0)	I(0)	I(0)	I(0)
EDU	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)
GC	I(0)	I(0)	I(0)	I(0)	I(1)	I(0)	I(0)	I(0)
INFL	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)
FDI	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)
M2	I(1)	I(1)	I(0)	I(0)	I(0)	I(0)	I(0)	I(0)

Source: own elaboration

[a] All tests were performed at 5% significance level. "I.E." is abbreviation of "individual effects", while "I.E.+T" is abbreviation of "individual effects and linear time trends".

[b] Autocorrelation was corrected via application of additional lags (SIC criterion was used to choose the optimal lag from the set {1, … ,4}).

[c] Autocorrelation was corrected via application of variance estimators based on Bartlett kernel and bandwidth chosen according to [Newey and West 1994].

As one can see the results presented in Table 3 provide solid evidence to claim that Y_t / L_t and K_t / L_t were I(1), nevertheless the type of unit root structure assumed[5]. In general, all remaining variables were found to be stationary (in each case, this was confirmed by at least three out of four conducted tests). In next step we performed cointegration analysis for the pair of nonstationary variables in the group G_0. Suitable outcomes are presented in Table 4.

[5] In further parts of this paper we focus on homogeneous panels (in respect to slope coefficients), since the availability of the data does not allow to draw reliable conclusions from heterogeneous models.

Table 4. The results of Kao residual cointegration test for Y/L and K/L [Kao 1999]

	Lag length selection based on SIC[a]		Lag length selection based on AIC[a]		Lag length selection based on HQ[a]	
Kernel	test statistic	*p*-value	test statistic	*p*-value	test statistic	*p*-value
Bartlett	-2.524	0.005	-3.008	0.001	-3.008	0.001
Parzen	-4.284	0.000	-4.284	0.000	-4.284	0.000
Quadratic Spectral	-4.233	0.000	-4.272	0.000	-4.233	0.000

Source: own elaboration

[a] The null hypothesis refers to lack of cointegration. In each case maximal lag length was set to 3.We assumed that constant is the only deterministic term.

The results presented in Table 4 provide solid evidence to claim that Y/L and K/L are indeed cointegrated around constant. Thus, in the next step we evaluated 2^5=32 different specifications of growth model (2), each of which consisted of one set of control variables. For each specification, we used fixed effects (FE), random effects(RE) and post-Hausman–based[6] estimates to conduct the empirical study in a comprehensive way. Finally, to control for possible impact of heteroscedasticity we also applied robust standard errors[7]. Tables 5-7 contain the empirical results obtained for the groups G_0, G_1 and G_2.

Table 5. The results of estimation of models (2) for the group G_0[8]

	FE			RE			Post-Hausman		
	Coefficients		Percentage of coefficients significant at 10, 5 and 1% levels	Coefficients		Percentage of coefficients significant at 10, 5 and 1% levels	Coefficients		Percentage of coefficients significant at 10, 5 and 1% levels
Measure of export diversification	Mean	% positive		Mean	% positive		Mean	% positive	
CONCE	0.31	100	100, 100, 100 [100, 100, 100]	0.22	100	100, 100, 100 [100, 100, 100]	0.24	100	100, 100, 100 [100, 100, 100]
$D_{2009}CONCE$	-0.20	0	72, 0, 0 [56, 0, 0]	-0.24	0	100, 65, 0 [87, 50, 9]	-0.23	0	75, 28, 0 [65, 34, 9]
MANUF	0.06	100	25, 0, 0 [0, 0, 0]	0.005	65	0, 0, 0 [0, 0, 0]	0.03	71	25(+), 0, 0 [0, 0, 0]
$D_{2009}MANUF$	-0.03	0	9, 0, 0 [0, 0, 0]	-0.02	0	0, 0, 0 [0, 0, 0]	-0.027	0	0, 0, 0 [0, 0, 0]
NUM	0.10	100	75, 50, 12 [85, 60, 25]	0.044	100	43, 0, 0 [47, 3, 0]	0.072	100	60, 31, 0 [68, 43, 9]
$D_{2009}NUM$	0.45	100	25, 0, 0 [12, 0, 0]	0.67	100	75, 50, 18 [97, 21, 0]	0.57	100	40, 18, 9 [50, 3, 0]

Source: own elaboration

6 In the post–Hausman procedure we performed the Hausman test at 5% level to decide whether fixed or random effects model should be estimated for each choice of the set X_t.

7 More precisely, we used the robust Huber/White/sandwich VCE estimator [Wooldridge 2009, Stock and Watson 2008, Arellano 2003].

8 In Tables 5-7 numbers in square brackets refer to results obtained by application of heteroscedasticity robust standards errors. Symbols in round brackets denote the sign of statistically significant coefficients (e.g. “25(+)” denotes finding significant and positive coefficients in 25% of cases).

Table 6. The results of estimation of models (2) for the group G_1[9]

	FE			RE			Post–Hausman		
	Coefficients		Percentage of coefficients significant at 10, 5 and 1% levels	Coefficients		Percentage of coefficients significant at 10, 5 and 1% levels	Coefficients		Percentage of coefficients significant at 10, 5 and 1% levels
Measure of export diversification	Mean	% positive		Mean	% positive		Mean	% positive	
CONCE	0.29	100	100, 100, 97 [100, 100, 100]	0.27	100	100, 100, 100 [100, 100, 100]	0.28	100	100, 100, 97 [100, 100, 100]
D_{2009}*CONCE*	-0.22	0	31, 0, 0 [25, 0, 0]	-0.26	0	85, 6, 0 [54, 6, 0]	-0.23	0	50, 9, 0 [40, 18, 0]
MANUF	0.048	100	0, 0, 0 [0, 0, 0]	-0.007	25	0, 0, 0 [22(-), 0, 0]	0.02	43	0, 0, 0 [9(9-), 0, 0]
D_{2009}*MANUF*	-0.008	50	0, 0, 0 [0, 0, 0]	-0.005	46	0, 0, 0 [0, 0, 0]	-0.013	28	0, 0, 0 [0, 0, 0]
NUM	0.19	100	88, 47, 9 [88, 60, 43]	0.019	60	0, 0, 0 [3(+), 0, 0]	0.058	78	31(+), 21(+), 3(+) [40(+), 25(+),3{+)]
D_{2009}*NUM*	0.85	100	40, 18, 0 [60, 46, 28]	0.92	100	93, 46, 3 [100, 93, 6]	0.91	100	87, 40, 3 [90, 68, 12]

Source: own elaboration

Table 7. The results of estimation of models (2) for the group G_2[10]

	FE			RE			Post–Hausman		
	Coefficients		Percentage of coefficients significant at 10, 5 and 1% levels	Coefficients		Percentage of coefficients significant at 10, 5 and 1% levels	Coefficients		Percentage of coefficients significant at 10, 5 and 1% levels
Measure of export diversification	Mean	% positive		Mean	% positive		Mean	% positive	
CONCE	0.22	100	100, 81, 50 [100, 81, 68]	0.13	100	60, 0, 0 [100, 90, 71]	0.16	100	97, 0, 0 [100, 90, 68]
D_{2009}*CONCE*	0.049	93	0, 0, 0 [0, 0, 0]	-0.03	50	0, 0, 0 [0, 0, 0]	0.004	81	0, 0, 0 [0, 0, 0]
MANUF	0.105	100	84, 40, 21 [25, 0, 0]	0.017	100	0, 0, 0 [0, 0, 0]	0.072	100	53, 37, 21 [18, 0, 0]
D_{2009}*MANUF*	-0.06	0	50, 43, 0 [50, 0, 0]	-0.04	0	0, 0, 0 [0, 0, 0]	-0.058	0	43, 40, 0 [47, 0, 0]
NUM	0.078	100	25, 0, 0 [40, 0, 0]	0.033	78	0, 0, 0 [0, 0, 0]	0.06	93	9(+), 0, 0 [21(+), 0, 0]
D_{2009}*NUM*	0.36	100	0, 0, 0 [31, 0, 0]	0.66	100	50, 31, 0 [87, 50, 12]	0.52	100	25, 6, 0 [53, 21, 0]

Source: own elaboration

The results of our study prove that before the outbreak of 2008 financial crisis export specialization rather than diversification was an important growth factor in CEE transition economies (in Table 5 the mean value of *CONCE* was positive and statistically significant in all research variants at 10% level). This link was

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9 In case of groups G_1 and G_2 the Kao panel cointegration test also pointed at one cointegration vector.

10 We also examined the possible impact of 2004 EU accession using the D_{2004} variable (defined analogously as D_{2009}). However, the results presented in Tables 5-7 turned out to be robust to inclusion of the EU-accession-related component.

especially strong in those countries whose export patterns were most specialized (comp. statistics on *CONCE* variable presented in Table 6). Moreover, the measure of horizontal export diversification (number of exported products) and the measure of vertical export diversification (approximated by the share of manufactured exports in total exports) were in general statistically insignificant in examined growth models (some evidence of statistical significance was found for the group G_2). However, after the outbreak of the 2008 financial crisis the directions of the causal links changed essentially. All three examined aspects of export diversification turned out to play a significant role in reducing the growth-decreasing effects of the crisis. The results proved that this effect was especially strong in case of those CEE transition economies which followed more diversified export patterns (i.e. group G_2).

CONCLUSIONS

This paper is one of the first contributions which provide detailed insights on the role of export diversification for economic growth in CEE transition economies. The examined growth model took into account three different aspects of export diversification: the general measure of the degree of market diversification (calculated on the basis of Herfindahl–Hirschmann index), the measure of horizontal export diversification (number of exported products) and the measure of vertical export diversification (approximated by the share of manufactured exports in total exports). The empirical results prove that the strategy based on export concentration was promoting economic growth in the stable period preceding the outbreak of the global financial crisis in 2008. After the outbreak of the crisis the situation changed – those CEE transition economies which were characterized by more concentrated export structures experienced stronger slowdown of economic growth. On the other hand, the CEE economies which adopted more diversified policies, in both horizontal and vertical dimensions, were confronted with smaller shocks during the crisis. The empirical results lead to formulation of some policy implications. In upcoming years the export diversification pattern seems to be a suitable policy in case of CEE transition economies since the risk associated with distinct export specialisation is still relatively high. In period 1995-2011 the horizontal aspect was a crucial determinant of dynamics of export diversification in CEE transition economies, especially after EU accession. However, further economic growth (and economic development) of this part of Europe seems to depend on the level of vertical export diversification, which is crucial in terms of technological progress and continuously increasing international competition. One cannot forget that specificity of this group of countries (e.g. small size of most of the economies) makes further diversification of exports a quite difficult task.

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THE REACTION OF INTRADAY WIG RETURNS TO THE U.S. MACROECONOMIC NEWS ANNOUNCEMENTS

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Abstract: This paper analyses the reaction of stock returns on the Warsaw Stock Exchange to U.S. macroeconomic news announcements. The study is conducted on the basis of five-minute returns of WIG from January 2004 to December 2012. This nine-year period includes different stages of economic cycle and additionally the global financial crisis. Hence results of our analysis are not limited only to contraction or expansion and can be applied to bull and bear market. The application of event study analysis allows us to measure not only the strength of the impact of information release but also its duration.

Keywords: event study, macroeconomic announcements, intraday data

INTRODUCTION

The reaction of stock prices to various domestic and foreign announcements is an issue that still attracts the attention of many researchers. According to the Efficient Market Hypothesis "stock prices always fully reflect available information" [Fama 1970] and important, unanticipated events should lead to instantaneous market response. It is widely known that macroeconomic news announcements can affect the price formation process but there is still insufficient research about the speed and strength of the reaction of different equity markets and the duration of the reaction's impact. It is especially visible for Warsaw Stock Exchange (WSE) and other emerging markets in Eastern and Central Europe.

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Stock markets are affected by domestics as well as foreign macroeconomic news. Taking into account time zones, trading on WSE is mainly influenced by European and U.S. macroeconomic news. However, most of the European macroeconomic news announcements are released before the opening of WSE, which precludes researchers from precise evaluating their impact. On the other hand, U.S. macroeconomic news is released mainly during trading hours on European stock markets. This fact, together with the importance of the U.S. economy is the reason why U.S. macroeconomic news should be taken into account while investigating investors reaction on WSE.

The issue of European markets reaction to American macroeconomic news has been, so far, the subject of few researches based on daily or intraday data. Using daily data from January 1996 to December 1999 Nikkinen and Sahlström [Nikkinen and Sahlström 2004] show that German and Finnish stock markets react strongly to PPI and Unemployment Rate announcements. While CPI has an impact only on German stock market.

Gurgul et al. [Gurgul et al. 2012] examine the effect of U.S. macroeconomic data announcement on daily WIG20 returns from February 2004 to December 2011. Their research shows, that information about CPI and Industrial Production significantly affects investor reactions, whereas Unemployment Rate announcements have no impact on WSE in the period under study.

In recent years intraday data has been widely used in studies of investors reaction. Andersen et al. [Andersen et al. 2007] take into account the set of 22 U.S. macroeconomic fundamentals released at different intervals (from weekly to quarterly) in the period from July 1998 to December 2002. Their study proves significant impact of unexpected news on high-frequency stock, bond and exchange rate conditional mean in U.S., Germany and UK. However, the reaction of equity markets depends mainly on the state of market.

Using five-minute returns of CAC 40, DAX 30, SMI and FTSE 100 Harju and Hussain [Harju and Hussain 2008] show that intraday seasonality on the European equity markets is affected by U.S. macroeconomic news announcements at 14:30 CET. They also find that the strongest impact is observed in the case of Unemployment Rate and Durable Goods Orders announcements. Moreover, investors reaction to U.S. macroeconomic news is immediate and quickly disappears.

Both abovementioned papers concerning intraday returns behavior focus on developed capital markets. There is little evidence concerning the reaction to U.S. macroeconomic news on emerging markets, such as WSE. This subject is studied by Hanousek et al. [Hanousek at al, 2008], who examine Polish, Czech and Hungarian stock markets. On the basis of intraday data they detect the significant impact of American and E.U. macroeconomic news announcements on returns and volatility of WIG20, BUX and PX50.

Multivariate GARCH models are applied by Będowska-Sójka [Będowska-Sójka 2009], who examines common reaction of German, French and Polish stock markets from November 2007 to April 2009. She confirms results of [Harju and Huissain 2008] that U.S. data announcements explain high volatility of intraday returns about 14.30 CET and have an immediate impact on both returns and volatility.

In this paper we extend research on the impact of U.S. macroeconomic news announcements to intraday stock prices on the Warsaw Stock Exchange. We examine the strength, direction and duration of investors reaction to macroeconomic news. By the event study analysis we are able to describe the behavior of stock prices before as well as after the announcement. Application of the intraday data from January 2004 to November 2012 allows us to examine investor reaction in different phases of business cycle including financial crisis.

The structure of this paper is as follows. Next section presents data and methodology applied in the study. Empirical results are presented and discussed in third section. Short summary concludes the paper.

DATA AND METHODOLOGY

To study the impact of U.S. macroeconomic news announcements on stock prices on the Warsaw Stock Exchange, we choose six important, monthly published macroeconomic indices that appear in literature [Andersen and Bollerslev 1998; Engle and Li 1998; Będowska-Sójka 2010; Harju and Hussain 2008]. These are: Consumer Price Index (CPI), Producer Price Index (PPI), Nonfarm Payrolls (NFP), Industrial Production (IP), Durable Goods Orders (DGO) and Retail Sales (RS). We examine their impact on five-minute log-returns of WIG from January 2004 to December 2012. Five-minute returns are the kind of intraday data that are commonly used in the literature (e.g. Andersen et al. 2007). News release time together with announced values and their forecasts come from www.deltastock.com and www.yahoo.com. We consider only announcements released on trading days on WSE. Most of announcements under study are released at 8.30 EST (14.30 CET). Only Industrial Production is released at 9.15 EST (15.15 CET). Due to differences in introduction of the Daylight Saving Time in the U.S. and Europe in some cases the announcements reach the WSE one hour earlier i.e. at 13.30 CET and 14.15 CET respectively.

For each news release we compare the announced value with its previous market forecasts and thus divide announcements into three clusters: “below consensus”, “in line with consensus” and “above consensus”. In the case of NFP, IP, DGO and RS, the announcement above consensus is expected to have positive impact on stock market, whereas CPI and PPI higher than consensus is expected to have negative impact. Announcement that is in line with consensus is classified as neutral. We focus our attention on unexpected news i.e. news in clusters “below consensus” and “above consensus”. Macroeconomic announcements are released on different days of month, thus there are days when more than one announcement under study is released. To examine the impact of new unexpected information we take into account only first news in such a day. Later announcements are excluded from the sample. When two announcements were published at the same time we take them into account only if they contain the same information, i.e. when they are both positive or both negative. This allows us to avoid or at least to reduce the problem of confounding events.

To investigate the impact of U.S. macroeconomic news on intraday returns on the WSE we use event study methodology. The key issue in the event study is appropriate choice of the lengths of pre-event window (i.e. estimation window) and event window. In this paper we consider four-hour pre-event window (i.e. 48 five-minute returns) and 45-minutes event window. The event window consists of nine returns: three before the event (15 minutes), the return from the time of announcement release and five returns (25 minutes) after the announcement time. Such event and pre-event windows do not contain highly volatile returns from the beginning and the end of trading session.

In this paper we apply the event study test proposed by Brown and Warner [Brown and Warner 1985]. Its advantages are rather mild assumptions. It requires only normality of mean abnormal returns. For each event i in the cluster ("below consensus" or "above consensus") abnormal returns (AR_t) are computed as difference between actual returns and their average in the pre-event window. Then, for each *t* the cross-sectional average abnormal return ($\overline{AR_t}$) is computed as:

$$\overline{AR_t} = \frac{1}{N}\sum_{i=1}^{N} AR_{i,t},$$

where N denotes the number of events in the cluster. To test the significance of mean abnormal returns at time t i.e. to test hypothesis:

$$H_0:\ E(\overline{AR_t}) = 0$$

$$H_1:\ E(\overline{AR_t}) \neq 0$$

we apply test statistic defined as:

$$t_{stat} = \frac{\overline{AR_t}}{\hat{\sigma}_{\overline{AR}}},$$

Where $\hat{\sigma}_{\overline{AR}}$ is the sample cross-sectional standard deviation of $\overline{AR_t}$ for the pre-event window. Under the assumption that mean abnormal returns $\overline{AR_t}$ are normally distributed, t_{stat} has *t*-Student distribution with *N*-1 degrees of freedom. Five-minute returns are in general non-normal but by the Central Limit Theorem the cross sectional mean abnormal returns converges to normality when the number of events in the cluster increases. To study the significance of returns reaction we perform two-sided test because rejection of null hypothesis by two-sided test implies its rejection also by one-sided test if the sign of $\overline{AR_t}$ is appropriate.

Additionally to abovementioned parametric test we apply nonparametric rank test proposed by Corrado and Zivney [Corrado and Zivney 1992] with correction for increased abnormal returns volatility after news release. Its construction and properties are described in details in [Corrado 2011].

If we denote the moment of the news release by $t = 0$ then the pre-event window are $t = -51, \dots, -4$ while $t = -3, \dots, 5$ are event window. For each event abnormal returns are first standardized:

$$SAR_{it} = AR_{it}/SD(AR_i),$$

Where $SD(AR_i)$ is abnormal returns standard deviation in the pre-event window. In the event window, however, increased volatility of abnormal returns is frequently observed [Corrado 2011]. To control for this event-induced shift in the cross-sectional variance we adjust standardized abnormal returns:

$$SAR'_{it} = \begin{cases} SAR_{it} & t = -51, \dots, -1 \\ SAR_{it}/SD(SAR_t) & t = 0, \dots, 5, \end{cases}$$

where $SD(SAR_t)$ is cross-sectional standard deviation and N is the number of events in the cluster. For each t in the event window the significance of abnormal returns can be tested separately. For each *t* from the event window Corrado–Zivney T_{CZ} statistics is defined as:

$$T_{CZ}(t) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \frac{rank(SAR'_{it}) - \frac{n+1}{2}}{\sqrt{n(n+1)/12}},$$

where *n* is the length of the pre-event window and $rank\left(SAR'_{it}\right)$ denotes the rank of SAR'_{it} within the vector consisting of standardized abnormal returns from the pre-event window and SAR'_{it}. T_{CZ} statistics is asymptotically normally distributed. The advantage of nonparametric Corrado–Zivney test is that is does not need any assumption of abnormal returns normality.

EMPIRICAL RESULTS

To verify normality of mean abnormal returns we apply Shapiro–Wilk and Jarque–Berra tests for each cluster of events. The null hypothesis is rejected at 5% significance level in two cases: when announced value of DGO is above consensus and when value of RS is greater than expected. In other cases normality cannot be rejected. Detailed results of event study i.e. mean abnormal returns in the event window together with corresponding test statistics are presented in Table 1 and 2. For each macroeconomic indicator results for two kinds of events are reported: when the announced value of the macroeconomic indicator is below or above market expectations. For each indicator the number of events in each such cluster is also reported.

Results in Table 1 and 2 indicate significant and immediate investor reaction on WSE to U.S. macroeconomic news announcements. In almost all cases mean abnormal returns for the event time ($t = 0$) are significant at 1% level. The exceptions are announcements about PPI (when PPI is below consensus the mean abnormal return is insignificant and when PPI is above forecasts then the mean is significant at 5% level) and about IP lower than forecasts when mean abnormal return is significant at 5%.

Table 1. The reaction of WIG 5-minute returns to CPI, PPI and NFP announcements in the U.S. in the period 2004-2012

	Below consensus			Above consensus		
t	$\overline{AR}$	t-stat	T_{CZ}	$\overline{AR}$	t-stat	T_{CZ}
CPI (31 and 32 events)						
-3	-0.015	-0.87	-1.11	-0.003	-0.14	0.34
-2	0.024	1.39	1.60	-0.016	-0.82	-1.14
-1	-0.018	-1.05	-0.39	-0.007	-0.35	-0.56
0	0.053	3.13*	1.84***	-0.071	-3.61*	-1.54
1	0.040	2.32**	1.96***	-0.013	-0.68	0.33
2	0.020	1.15	0.20	-0.008	-0.40	0.37
3	0.011	0.66	0.92	-0.011	-0.54	0.29
4	-0.003	-0.18	0.62	-0.002	-0.11	0.43
5	0.023	1.36	1.44	0.004	0.23	-0.03
PPI (30 and 41 events)						
-3	0.000	0.01	0.09	0.001	0.10	-0.35
-2	0.015	1.01	2.00**	0.019	1.74	1.20
-1	-0.003	-0.20	-0.08	-0.017	-1.54	-1.12
0	0.006	0.36	0.60	-0.023	-2.11**	-0.31
1	-0.008	-0.52	-0.57	0.004	0.34	-0.12
2	0.003	0.17	-0.50	0.015	1.41	0.13
3	-0.013	-0.87	-0.90	0.023	2.09**	1.40
4	-0.008	-0.52	-0.85	0.020	1.89	1.01
5	-0.005	-0.35	0.05	-0.006	-0.52	0.15
NFP (59 and 42 events)						
-3	0.006	0.61	0.66	0.000	-0.04	0.17
-2	-0.006	-0.62	-0.02	0.001	0.09	0.34
-1	0.010	0.98	1.57	0.018	1.73	1.85***
0	-0.107	-10.5*	-2.82*	0.103	10.1*	2.47**
1	-0.002	-0.24	-0.30	-0.017	-1.68	-0.17
2	0.018	1.75	0.78	0.023	2.27**	2.24**
3	-0.015	-1.45	-0.30	-0.001	-0.12	0.04
4	-0.003	-0.29	1.04	-0.023	-2.31**	-1.54
5	0.023	2.23**	1.66	-0.025	-2.41**	-1.34

Source: own calculation

*,**,*** - significant at 1%, 5% and 10% level respectively

Table 2. The reaction of WIG 5-minute returns to IP, DGO and RS announcements in the U.S. in the period 2004-2012

	Below consensus			Above consensus		
t	$\overline{AR}$	t-stat	T_{CZ}	$\overline{AR}$	t-stat	T_{CZ}
			IP (29 and 13 events)			
-3	-0.022	-1.31	-1.37	0.014	0.66	0.45
-2	0.031	1.81***	1.57	0.030	1.41	1.81***
-1	-0.001	-0.03	1.00	-0.025	-1.18	-0.37
0	-0.036	-2.14**	-1.84***	0.106	5.02*	2.19**
1	-0.003	-0.16	0.19	0.015	0.70	-0.13
2	-0.016	-0.94	-1.21	-0.018	-0.85	-0.58
3	0.006	0.37	1.68***	0.012	0.57	0.35
4	-0.014	-0.83	0.70	0.038	1.79***	1.02
5	-0.029	-1.69***	-0.80	0.033	1.59	1.38
			DGO (53 and 47 events)			
-3	-0.002	-0.15	0.49	-0.012	-1.12	-0.69
-2	-0.004	-0.33	-0.21	0.005	0.53	1.60
-1	-0.004	-0.38	-0.71	0.005	0.51	0.80
0	-0.107	-9.53*	-4.52*	0.074	7.11*	2.63*
1	-0.014	-1.24	-0.53	0.007	0.66	1.70***
2	-0.002	-0.19	-0.20	-0.001	-0.09	-0.61
3	-0.007	-0.65	-0.30	-0.009	-0.85	-0.64
4	-0.007	-0.60	-0.38	0.023	2.16**	1.60
5	-0.008	-0.70	-0.07	0.021	2.01***	1.41
			RS (38 and 32 events)			
-3	-0.030	-2.27**	-1.45	0.013	1.07	0.29
-2	0.021	1.58	1.75***	0.005	0.40	0.35
-1	-0.011	-0.80	-0.49	0.015	1.19	2.11**
0	-0.050	-3.76*	-1.08	0.073	5.97*	2.63*
1	0.008	0.57	0.85	-0.014	-1.14	-0.80
2	0.021	1.63	1.41	0.004	0.35	-0.27
3	-0.034	-2.60**	-0.97	0.008	0.69	0.69
4	0.010	0.79	0.92	-0.024	-1.99***	-0.11
5	0.005	0.42	0.32	-0.004	-0.36	-0.27

Source: own calculation

*,**,*** - significant at 1%, 5% and 10% level respectively

Investor reaction to CPI announcements is in line with earlier results of [Gurgul et. al, 2012] obtained for daily data. CPI below forecast is seen by investors as positive news. It leads to positive mean abnormal returns up to 20 minutes after release and is significant for the first 10 minutes. Similarly, CPI greater than expected is seen as negative information and leads to negative five-minute returns up to 25 minutes after the release. However, only first mean abnormal return is significant. It indicates that investor reaction to CPI is homogeneous across the market and new information about inflation in the U.S. is immediately incorporated into stock prices. However, there are also some its echoes that slowly vanish. Corrado–Zivney test confirms significant reaction to CPI below forecast.

PPI announcements have similar meaning to CPI. Announcement below forecast is good news while PPI greater than expectations is bad news. In the case of PPI, however, changes in abnormal returns are weaker than for CPI. To understand this phenomenon it should be noted that PPI announcements are frequently secondary to CPI and thus investors can update their expectations about true value of PPI.

One of the most complex patterns in average abnormal returns can be seen when Nonfarm Payrolls is announced. NFP below consensus is seen as bad news while NFP above forecast is good news. However, abnormal returns change significantly for $t = 0$ despite the value of unexpected news about NFP. After this immediate and very strong reaction there are observed several significant mean abnormal returns of different signs. This heterogeneous investors reaction is probably due to the fact that Nonfarm Payrolls is only one of a few macroeconomic indicators that are published by the Bureau of Labor Statistics in the Employment Report. The other important indicators are e.g. Unemployment Rate, Average Workweek and Hourly Earnings. Each of them can convey quite different information about employment in U.S. Thus, the changes in stock prices after the event are the reflection of updating investor expectations in the relation to reaction on other financial markets.

The investors reaction to Industrial Production announcements is more straightforward. IP below consensus is seen as bad news and leads to mainly negative abnormal returns in the event window. Differences between test statistics are due to high returns volatility in the event window. IP above consensus is certainly good news for stock market. Investors immediately react homogeneously on it and average stock prices increase about 0.1% in first five minutes after the release. This is the strongest positive reaction to macroeconomic news announcements under study.

The reaction to DGO smaller than expected is clearly negative and very strong just after the event with the highly significant negative mean abnormal return in first five minutes (about -0.11%). All other averages in the event window are also lower than zero. On the other hand, the DGO announcement above forecasts is good news to the market. Averages in the event window are mainly positive with mean for $t = 0$ significant at 1% level. Significance of mean abnormal returns in each cluster is also confirmed by T_{CZ} statistics. In this case it is important due to the non–normality of average abnormal returns in the pre-event window.

Retail Sales below consensus is interpreted by investors as bad news. However, significance of investors reaction cannot be clearly confirmed due to differences between test statistics. In the case of RS above consensus both tests confirm significant changes in stock prices just after the announcement.

CONCLUSIONS

This paper analyses the reaction of intraday stock prices on the Warsaw Stock Exchange to U.S. macroeconomic news announcements. We examine the impact of six macroeconomic indicators: Consumer Price Index, Producer Price Index, Nonfarm Payrolls, Industrial Production, Durable Goods Orders and Retail Sales. All computations are performed on the basis of five-minute WIG returns from January 2004 to November 2012. This period includes different stages of economic cycle and the global financial crisis. Hence results of our analysis are overall and can be applied to bull and bear market. We apply event study analysis do characterize the information content and duration of unexpected news when the announced value is above or below expectations. Results of the performed study confirm that CPI, PPI lower than consensus and NFP, IP, DGO and RS greater than expectations are good news while CPI, PPI greater than expected and NFP, IP, DGO and RS smaller than forecasts are bad news. In each case except for PPI below consensus stock prices react significantly and immediately to news release. The strongest reaction (about 0.1%) in both directions is observed when Nonfarm Payrolls is announced. Strong negative change in abnormal returns is visible when Durable Goods Orders is below forecasts while the highest increase in abnormal returns is caused when Industrial Production is greater than expected. Investors reaction on WSE is not limited to first five minutes after the announcement. Stock prices react significantly not only when U.S. macroeconomic news is announced but also after it.

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STRUCTURAL BREAKS IN FARM AND RETAIL PRICES OF BEEF MEAT IN POLAND

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Abstract: The aim of this study was to analyze the dynamics of monthly prices in the beef marketing chain in Poland in the years 1997-2012. The study showed that in the time series of farm and retail prices of beef meat in Poland structural break points occurred. They are mainly associated with appearance of BSE disease and the Polish accession to the European Union. The farm and retail price series are non-stationary and farm prices Granger-cause retail prices. The estimates of the long-run parameters depend on the assumptions about deterministic variables existence in the Engel-Granger cointegration equation, including structural breaks among them.

Keywords: food chain, beef prices, structural break, price transmission, cointegration

INTRODUCTION

Price development in the agri–food sector is the subject of many studies. The interest of researchers has increased recently due to dynamic evolution of food prices, increase of price volatility as well as food security problems [Abbot et al. 2011, Prakash 2011, Roache 2010]. Specific areas subjected to detailed analysis are the issues of margins' changes and price transmission along food marketing chain. That interest is reinforced by a rapid concentration in the retail trade observed in the last decade. As a result the problems of inefficiency in the market pricing and the market power are quite often raised in the public discussion and in research [Mc Corriston 2002, Seremak Bulge 2012, Kuosmanen, Niemi 2009, Lloyd et al. 2006]. Market power, if confirmed, constitutes a justification for the introduction of different agricultural policy instruments or antitrust regulations.

Economists, who study market efficiency, often focus on vertical price transmission in the marketing chain of food products. It is expected that under competitive market assumption retail prices reflect changes in farm prices, not reversely. Furthermore, retail prices need to respond with the same speed and amplitude (symmetrically) to decreases and increases in the farm prices [Meyer & von Cramon–Taubadel 2004]. Numerous methods have been employed to analyze the nature of vertical price transmission, such as: ARDL models, VAR models, cointegration methods or threshold autoregressive methods. The overview of the methods applied can be found in the works by Meyer & von Cramon–Taubadel [2004] or Frey & Manera [2007].

One of the most important assumptions about models is stability of their parameters over time. Existence of the structural changes in the data has negative influence on the results (bias them, loss of power of tests) of statistical analysis performed with the use of methods which do not take into account structural breaks [Perron 2005]. The presence of structural breaks in the cointegrating relationship between price series in vertical marketing chain leads to over-rejection of null hypothesis of symmetric transmission when standard tests for asymmetric price transmission are applied [von Cramon-Taubadel & Meyer 2001].

It can be assumed that a structural break has occurred if at least one of models' parameters has changed at the break date within the sample period. Structural break could be assumed as immediate or it might seem more reasonable to allow a structural change to take effect over period of time. Structural breaks may be limited to the level shift (LS), trend change (TC) or regime change (RC) when there is allowance for change of structural parameters in different regimes. In a given model there could be one structural break or the combination of a few ones of the same or different categories. In empirical research authors mostly focus on the simple case of an immediate structural break for simplicity and parsimony [Hansen 2001, Zeileis et al. 2003]. One of the research problems is testing for structural breaks in the univariate or the multivariate cases (one break and multiple breaks tests). Timing of structural breaks may be known a priori or the dates need to be estimated therefore many researches concentrate on dating structural breaks. In general, incorporation of structural breaks into a model is challenging task and rarely happens that all mentioned problems are simultaneously studied [Perron 2005, Gregory & Hansen 1996, Carrion-i-Sylvestre & Sansó-i-Rosselló 1996].

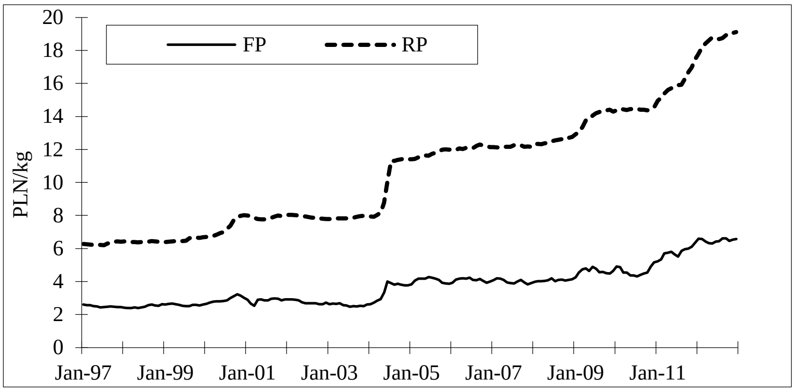
Structural breaks are evident in many economic phenomena. Agricultural prices that are affected by numerous factors, such as weather conditions, animal diseases or changes in agricultural policies, are regarded as those in which the structural changes may occur frequently [Wang & Tomek 2007]. In transition economies like Poland the probability of structural breaks is even higher than in the developed countries. The aim of this paper is to analyze the dynamics of monthly farm and retail prices of beef meat in Poland. Our empirical research is focused on detecting structural breaks in the individual price series as well as in the long-run equilibrium relationship between retail and farm prices. Knowing type and date of structural breaks we can learn more about the relationships between prices in the food chain. It can be also helpful in establishing and estimating the cointegration equation and error correction model more accurately.

DATA AND METHODS APPLIED

Data

Statistical data used in the analysis were monthly information about farm prices (procurement) for live weight beef (FP) and retail prices of sirloin (RP) in Poland (Figure 1). Price series data covered the period from January 1997 to December 2012 (192 observations) and was expressed in PLN/kilo. The source of information was Central Statistical Office in Poland.

Figure 1. Farm prices of live weight beef (FP) and retail prices (RP) of beef sirloin expressed in PLN/kilo



Source: CSO Poland (GUS)

Graphical insight into the data indicates on a high correlation between farm and retail prices. In addition, a widening gap between them is observable over the analyzed period. Beef market belongs to those agri–food markets which were the most influenced by the Poland's accession to the EU. During a few months since May 2004 prices of beef meat on different food chain levels rose by over 50% due to the removal of all trade barriers and restrictions. Less clear is impact of the BSE (Bovine Spongiform Encephalopathy) crisis on prices in 2000-2001.

Methods

The key method applied in the research for data analyzing and structural break detection was TRAMO-SEATS procedure. It belongs to so-called ARIMA--based-method approaches [Gomez & Maravall 2001]. TRAMO (Time series Regression with ARIMA noise, Missing values, and Outliers) is a procedure for estimation and forecasting of regression models with errors that follow nonstationary ARIMA processes, while there may be missing observations in the series, as well as infection of outliers and other deterministic effects. Automatic procedure enable to detect and locate additive outliers, transitory changes and level shifts. Other structural changes, like RAMP effect revealing in linear change during a few periods, can be predefined manually. The regression effect is included if the *t*-value for a given regression variable is higher than 3.5. The model for variable *y* (in logs or original form) can be written as follows [Marraval 2008]:

$$y_t = R_t^{'}\beta + x_t \tag{1}$$

where: $R_t^{'}$ – matrix with *n* regression variables for calendar effects, structural changes and outliers, intervention variables and constant; β is a vector of *n* regression parameters; x_t – stochastic component following ARIMA process:

$$\phi(B)\delta(B)x_t = \theta(B)a_t \tag{2}$$

where: *B* – backward operator; $\delta(B)$ – stationary AR polynomial in B; $\phi(B)$ – non-stationary AR polynomial in B (unit roots); $\theta(B)$ – invertible MA polynomial in B, at white-noise innovations.

SEATS model (Signal Extraction in ARIMA Time Series) allows for decomposition of the series into trend, seasonal, cyclical and irregular components, and provide forecasts for these components. TRAMO-SEATS procedures are implemented in *DEMETRA+* software.

Augmented Dickey Fuller test (ADF) was used in order to verify presumption about existence of unit root in the price series. Two types of data were tested: initial series as well as series transformed via TRAMO procedure for reducing effect of structural brakes. Null hypothesis states that time series is non-stationary (unit root) against the alternative of stationarity. ADF test statistic is based on *t*-statistic of coefficient φ from OLS estimation of the following formula [Enders 2010]:

$$\Delta y_t = \mu_t + \varphi y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \varepsilon_t \tag{3}$$

where: y_t – analyzed price series; μ_t – deterministic term (none, constant, trend); *p* is the number of lags ensuring white noise properties of random component ε_t; δ_i are coefficients describing the short-run persistence of Δy_t. The number of lags *p* was determined with the use of Akaike'a Information Criterion (AIC).

The concept of Granger causality was employed to evaluate the nature of relation between producer and consumer prices. A variable *x* is said to Granger-cause *y* if we can better forecast *y* using lagged values of *x* rather when ignoring them. Test formula presented below was applied for original data and for time series data linearized for structural break effects [Enders 2010]:

$$y_t = a_0 + \sum_{j=1}^{k} \alpha_j y_{t-j} + \sum_{j=1}^{k} \beta_j x_{t-j} + \varepsilon_t \tag{4}$$

where: a_0, α_j, β_j, γ_j are model parameters; y and x are analyzed variables; k – the greatest lag length; ε_t – white noise. Null hypothesis, stating no Granger causality, assumes that $\beta_1=\beta_2= \ldots =\beta_k=0$ against alternative of statistical significance of these coefficients. Determining number of lag length we applied Vector Autoregression Model (VAR) and AIC.

To test existence of the long-term relationship the Engle-Grangers (EG) cointegration framework was applied. The nonstationary time series are cointegrated if there is a stationary I(0) linear combination of these series. The linear combination of two series is referred to as a long-run equilibrium relationship and can be written as follows [Engle Granger 1987]:

$$RP_t = \gamma_0 + \gamma_1 FP_t + u_t \quad \text{or} \quad RP_t = \gamma_0 + \gamma_1 FP_t + \gamma_2 t + u_t \tag{5}$$

where: γ_0 , γ_2 – constant and trend parameter; γ_1 – parameter of the long-run impact; u_t – residuals form EG relationship called as an error correction term ECT.

After testing for structural breaks the basic EG models (5) were extended for set of structural break dummies D_i:

$$RP_t=\gamma_0+\gamma_1 FP_t+\gamma_{3i}D_i+u_t \quad \text{or} \quad RP_t=\gamma_0+\gamma_1 FP_t+\gamma_2 t+\gamma_{3i}D_i+u_t\,. \tag{6}$$

STRUCTURAL BREAKS IN UNIVARIATE PRICE SERIES

Statistical analysis of farm and retail prices was carried out with the use of TRAMO-SEATS method. For each price series two models were estimated: the first one was chosen on pure automatic procedure (denoted as *auto*) implemented in *DEMETRA+* (denoted as RCA3 variant) and the second was selected via automatic procedure with predefined RAMP effects (denoted as *autoRAMP*). RAMP effect is a combination of a few successive LS changes and can be applied when structural change is time-distributed. All studies were carried out on logs of price series. A stable seasonality pattern was confirmed only in farm price series. However, the impact of seasonality on farm prices is relatively low and the amplitude of seasonal variation is around 5 pp.

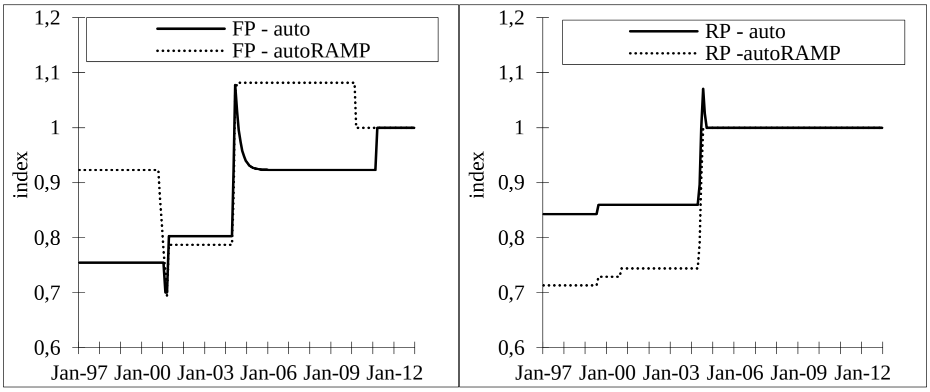
An application of automatic TRAMO procedure for farm and retail prices allows us to estimate models which are presented in Table 1. The visualization of aggregated impact of structural changes on farm and retail prices is presented in Figure 2. We concentrated on dating and recognizing of structural breaks' types. According to automatic TRAMO procedure, breaks are restricted to additive outliers (OA), level shifts (LS) and temporal changes (TC). In case of farm prices statistically significant structural breaks were in 2001 (BSE crisis), in 2004 (EU integration) and in March 2011 (Table 1). Most of them are LS type structural breaks. One can notice an estimated positive impact of BSE crisis on farm prices which is not in line with experts' expectations. It seems that the consequence of BSE should have resulted in the decrease of prices paid for farmers as the demand for beef decreased and restrictions on beef meat import from Poland were imposed by the EU. BSE crisis started in November 2000 and its impact was spread over time which is not reflected in the model. Instead of that the correction of previous declines in prices is estimated (LS[2001.4]) as dominant structural break.

Table 1. TRAMO models for farm and retail prices (all parameters are statistically significant with p-values less than 0.001)

FP - auto		FP - autoRAMP		RP - auto		RP - autoRAMP	
ARIMA model [(0,1,1)(0,1,1)] BIC = -332.3		ARIMA model [(1,1,0)(0,1,1)] BIC = -336.7		ARIMA model [(1,1,1)(0,0,0)] BIC = -433.8		ARIMA model [(2,1,0)(0,0,0)] BIC = -480.2	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
LS[2001.2]	-0.075	rp:2000.10-2001.03	-0.285	LS[1999.8]	0.020	LS[1999.8]	0.022
LS[2001.4]	0.137	LS[2001.4]	0.126	LS[2004.5]	0.042	LS[2000.9]	0.021
LS[2004.5]	0.139	rp:2004.04-2004.07	0.318	LS[2004.6]	0.110	rp:2004.04-2004.07	0.295
TC[2004.6]	0.154	AO[2004.6]	0.089	AO[2004.7]	0.068	AO[2004.5]	-0.039
LS[2011.3]	0.080	LS[2010.3]	-0.079	AO[2004.8]	0.026	x	x
Th(1)	0.351	Phi(1)	-0.264	Phi(1)	-0.751	Phi(1)	-0.458
BTh(1)	-0.770	BTh(1)	-0.815	Th(1)	-0.105	Phi(2)	-0.218

Source: own calculations

Figure 2. Deterministic components effect on FP and RP



Source: own calculations

To overcome this problem RAMP effect (*rp* variable) was introduced from September 2000 to March 2001 for a better description of the BSE crisis. In addition, an effect of Poland's accession to the EU, which distributed over time, was presumed by means of *rp* variable. The result (dotted line in left chart of Figure 2 and in Table 1) seems to be quite different from previous one. In *autoRAMP* model the negative impact of BSE crisis on prices was estimated. Moreover, the permanent effect of EU integration on prices was estimated. There is also a difference in the dating breaks at the end of the time series – instead of LS break in March 2011 (*auto*) there is LS in March 2010 (*autoRAMP*).

The most evident structural breaks in the retail price series are those connected with Poland's accession to the EU. Similarly to the farm prices, pure automatic model and the RAMP effect model differ in terms of break date and type of structural changes. Two additional LS in 2009 and 2011 are of a lesser importance and it is difficult to find any reasonable explanation for them.

STRUCTURAL BREAKS IN THE LONG RUN RELATIONSHIP

The next step of this research paper is to analyze existence of structural breaks in the long run relationship between retail and farm prices of beef meat. Despite the occurrence of the structural changes in particular price series it might have happened that those breaks were not present in the long run relationship or nature of the change was different.

Unit root and causality

Considering the model EG a question about stationarity and casual relations between variables appears. In order to test presumption about existence of unit root the ADF test series was applied for natural logs of original price series as well as for price series linearized via automatic TRAMO procedure and via the automatic procedure with predefined RAMP breaks. As far as FP data is concerned, it was seasonally adjusted via TRAMO-SEATS method. The results obtained (details available on request) indicate the existence of unit root in all price series being considered. The first difference of all price series can be regarded as stationary.

The presumption about Granger non-causality was tested on the basis of pairs of the following price series (all in natural logs): initial data, price series linearized via *auto* TRAMO procedure and prices linearized through *autoRAMP* TRAMO procedure. The Granger-causality test has low power in analyzing data with structural breaks therefore analysis was performed also on pairs of linearized data. The obtained result (details available on request) shows that farm prices become a Granger-cause for retail prices, which is in line with expectations. The null hypothesis stating that past farm prices are not helpful in forecasting current retail prices was rejected for all pairs of price series ($p<0.01$).

Dating structural breaks in the long run relationship

Two of the Engle-Granger cointegration models were estimated on the basic of logs of farm and retail prices (eq. 5). Estimates of models' residuals are as follows: $u_{1t} = RP_t\text{-}(0.90+1.11\cdot FP_t)$ and $u_{2t} = RP_t\text{-}(1.25+0.56\cdot FP_t+0.003\cdot t)$.

There are substantial difference in parameters estimates of the long run relationship. The second model presents more reliable findings. Model with trend is also preferable because trend represent (proxy) other inputs which contribute to the retail prices. Standard error of residuals in the model with constant and trend is

0.041 whereas in model with constant s_e=0.075. Nevertheless, estimates of both models may not be reliable if there are structural breaks.

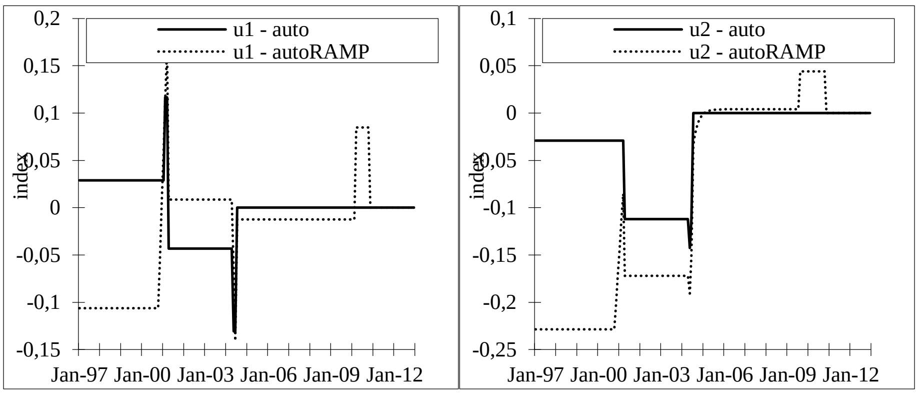
ARIMA models with dummies for structural brakes according to TRAMO methodology were fitted for residuals from above cointegration equations (u_{1t} and u_{2t}) to verify presumption about structural breaks in the cointegration relationships. Two models for each residual were estimated as of initial price series: via automatic procedure (*auto*) and via automatic procedure with predefined RAMP effects (*autoRAMP*). The results are present in Table 2 and in Figure 3.

Table 2. TRAMO models for residuals from EG cointegration model (all parameters are statistically significant with p-values less than 0.001)

u_{1t} - auto		u_{1t} - autoRAMP		u_{2t} - auto		u_{2t} - autoRAMP	
ARIMA model [(0,1,1)(0,1,1)] BIC = -758.4		ARIMA model [(0,1,0)(1,0,0)] BIC = -852.3		ARIMA model [(0,1,1)(0,1,1)], BIC = -993.3		ARIMA model [(1,1,0)(0,1,0)] BIC = -1005.4	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
LS[2001.2]	0.088	rp:2000.10-2001.03	0.274	LS[2001.4]	-0.083	rp:2000.10-2001.03	0.143
LS[2001.4]	-0.160	LS[2001.4]	-0.159	LS[2004.5]	-0.030	LS[2001.4]	-0.087
LS[2004.5]	-0.087	rp:2004.04-2004.07	-0.224	LS[2004.7]	0.112	rp:2004.04-2004.07	0.176
LS[2004.7]	0.130	TC[2004.7]	0.203	x	x	TC[2004.5]	-0.077
x	x	LS[2010.3]	0.097	x	x	LS[2009.8]	0.040
Th(1)	0.236	LS[2010.11]	-0.085	Th(1)	0.220	LS[2010.11]	-0.044
BTh(1)	-0.827	BPhi(1)	-0.277	BTh(1)	-0.857	BPhi(1)	-0.836

Source: own calculations

Figure 3. Deterministic components effect on residuals from long-run cointegration relationships u_{1t} and u_{2t}



Source: own calculations

Estimates of structural breaks differ considerably among the models. More reasonable seems to be those obtained according to *autoRAMP* models. Also the AIC criterion indicates better performance of *autoRAMP* models.

In the final step new cointegration models with structural breaks according to formula 6 were estimated. Deterministic variables included in models are listed in Table 2. The long run equilibrium estimates for models with constant and deterministic variables obtained by *auto* and *autoRAMP* procedures are 0.83 and 0.75 respectively. Estimated coefficients from models with constant, trend and dummies for structural breaks are almost the same for *auto* and *autoRAMP* procedures: 0.49 and 0.50. Comparing above long run equilibrium coefficients with coefficients estimated according equation 5 we can conclude that inclusion of dummy variables for structural breaks changes long run parameters especially in models without trend. Furthermore, it has also affects estimates of ECT and thus may have impact on estimates of transmission model (long and short adjustments).

SUMMARY

The study allows us to formulate the following conclusions:

- The time series of the farm and retail beef prices are non-stationary.
- Farm prices of beef appeared to be the Granger-cause for the retail prices of beef cuts.
- There were structural breaks of different nature present in the time series of farm and retail prices as well as in the long-run cointegration relationship between these prices. The most evident structural breaks are mainly due to the BSE crisis and Poland’s accession to the EU.
- Timing and the nature of the structural breaks estimated on the basis of automatic procedures may be far from reality if structural changes are spread out over time.
- The inclusion of variables for different structural regimes change estimates of the long-term relationship and the nature of the ECT process. It may strongly affects estimates of error correction models for beef prices in Poland.
- To enlarge the field of analysis we can, among other things, estimate error correction models (VECM or EG) with structural breaks and price transmission models (including asymmetric models). Ones can also apply models, which cover other types of structural breaks: trend change and/or transitory change.

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QUANTITATIVE METHODS IN ECONOMICS
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PARAMETRIC AND NON-PARAMETRIC EFFICIENCY MEASUREMENT – THE COMPARISON OF RESULTS[1]

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Abstract: In the paper the author considered estimation of efficiency, which measures the ability of the company to obtain the maximum output from given inputs. The comparison of results obtained by using two approaches: parametric (on the example of the SFA method, Stochastic Frontier Analysis) and non-parametric (on the example of the DEA method, Data Envelopment Analysis) has been carried out. In the paper the data from the companies of a key food processing sector in Poland, namely the meat processing sector, was used. The analysis covered the period 2006–2011, the sample covered from 195 up to 210 enterprises (depending on the analyzed year).

Keywords: efficiency, the SFA method, the DEA method, food processing sector

INTRODUCTION

The aim of the article was the comparison of the parametric (using the SFA method) and non-parametric (using the DEA method) approach to measurement of the basic economic category, which is efficiency. The discussed methods have been applied to the meat processing industry in Poland. This sector was chosen due to the large size of the sample, as well as the strategic importance and significant contribution to the production of the entire agri-food sector. Furthermore, with respect to the meat processing sector there are no comparative analyzes carried out, which justifies the need for their conduction.

[1] Research granted by National Science Centre from the funds for science in years 2011-2013 as a scientific project no 2011/01/B/HS4/02612 „Stopień integracji w łańcuchu dostaw a efektywność przedsiębiorstw przetwórstwa rolno-spożywczego"(„Degree of integration in supply chain and efficiency of food processing enterprises").

The discussion on assessing efficiency of economic entities should start with the precise definition of efficiency, which is not an unambiguous term. M. Bielski claims that there are several different concepts of efficiency, its measurement and expressions. He states that within the framework of the concept of efficiency, many terms of similar meaning may be applied: effectiveness, productivity, profitability [Bielski 2002, p. 54]. However, these concepts are not identical, and the actual concept of efficiency is derived from the structure of the production function, therefore, is conditioned by changes in the productivity of production factors and their remuneration and refers to the allocation of production factors in the most technically efficient way. W. Rembisz presented argument that the growth in efficiency is a function of changes in the productivity of capital and labour productivity and changes in the structure of expenditures (in production technology) [see Rembisz 2011]. Improvement in the efficiency level can lead to the increase in profitability. According to the author of the paper, these three concepts discussed, such as productivity, efficiency and profitability can be a reference point for assessing the degree of achievement of the objectives (*effectiveness*). A broader concept is effectiveness that focuses on the results and the degree of the objectives' achievement.

In the literature, there is likewise a concept of economic efficiency that determines the ratio of outputs achieved and inputs used. Z. Adamowski assumes that economic efficiency may be understood as the ratio of output to input, or cost, or vice versa – input to output. The first case concerns input-oriented efficiency, second one – output-oriented efficiency (capital intensity of output) [Adamowski 1983, p. 70]. The dual approach to efficiency is a result of the existence of two variants of the economic rationality principle. Adherence to this principle is understood as achieving given outputs by using minimal inputs or achieving maximal outputs by using a given level of inputs [Telep 2004, p. 9]. The aspect of efficiency is perceived similarly by C. Skowronek, who claims that maximization of the output/input ratio (or the difference between output and input), as a measure of economic efficiency, can be achieved by maximizing outputs with given inputs, or by minimizing inputs with given outputs [Skowronek 1987, p. 241].

The dual approach to efficiency is also presented by S.C. Krumbhakar and C.A.K. Lovell, according to whom an elementary objective of producers can be avoiding waste, by obtaining maximum outputs from given inputs or by minimizing inputs used in the production of given outputs [Krumbhakar and Lovell 2004, pp. 15], which is defined by the authors as technical efficiency. At a higher level, the objective of producers might entail the production of given outputs at minimum cost or the utilization of given inputs to maximize revenue, or the allocation of inputs and outputs to maximize profit. In these cases, productive efficiency corresponds to what the authors call economic (cost, revenue or profit) efficiency [Krumbhakar and Lovell 2004, p. 16]. They indicate that technical efficiency can be graphically defined in terms of distance to a production frontier, and economic efficiency is defined in terms of distance to a cost, revenue or profit frontier [Krumbhakar and Lovell 2004, p. 17]. Whereas technical efficiency is a purely physical notion that can be measured without recourse to price information, cost, revenue, and profit efficiency are economic concepts whose measurement requires price information.

T.J. Coelli, D.S.P. Rao, Ch.J. O`Donnell and G.E. Battese, that refer to the dual approach in their researches on the efficiency, argue that the efficiency ratio increases by maximizing outputs with given inputs (an output-oriented approach), or by minimizing inputs with given outputs (an input-oriented approach). A company uses materials, labour and capital (inputs) in order to manufacture the final product (output), on the basis of which the authors define efficiency of companies as their ability to transform inputs into outputs [Coelli et al. 2005].

COMPARISON OF SFA AND DEA METHODS

According to the mathematical models of general equilibrium of L. Walras, A. Wald, K.J. Arrow or G. Debreu, a company can be described as a mathematical function depending on the technology applied (without innovation) for transformation of inputs into outputs [Noga 2009, p. 134]. In the literature it is assumed that the production function illustrates available and effectively used manufacturing techniques. It determines the maximum quantity of product (production) (y) that is possible to be obtained by a given of production factor(s): (x). In this sense, the production function is a reflection of the production technique used, the technical relationship of a given state of technology. Related to this are: organization, knowledge and experience (which is accepted on the basis of the *implicite* principle) [Rembisz 2011, p. 10]. In the literature, it is emphasized that the production function is a defined mental structure expressed in an algebraic notation, in which the above-mentioned relationships are included, defining the nature of production in economic and technical terms[2]. The analytic form of the function reflects above all changes in production efficiency (in terms of the relationship between input and output as the factors' involvement and production increase). The production function enables to explain the reasons for changes in technical relationships and the consequent changes in the production efficiency and productivity of individual factors. These are important relationships, as somehow they exemplify the source of changes in production efficiency (related to the changes in manufacturing techniques). They reflect the structural changes.

[2] S.C. Krumbhakar and C.A.K. Lovell assume that producers use a nonnegative vector of inputs to produce a nonnegative vector of outputs. Although the analytical foundations developed by S.C. Krumbhakar and C.A.K. Lovell readily accommodate zero values for some inputs and some outputs; see: Krumbhakar S.C., Lovell C.A.K. (2004) Stochastic Frontier Analysis. Cambridge University Press, United Kingdom, Cambridge.

The method of efficiency measurement, basing on the production function, is the SFA method (Stochastic Frontier Approach). However, in the literature it is also common to use deterministic tools, where the analytical basis is an optimization problem (e.g. DEA method – Data Envelopment Analysis). Here benchmarks (the best objects in an analyzed group) are determined which are de facto the solution/solutions for the optimization problem. Apart from the undeniable advantage of the SFA method, which is using the analytical tool well-established in economic theory (i.e. production function), a number of other advantages may be presented[3].

In the literature, one can find an approach that in a case if there is a random component in the analyzed sample, the application of the SFA method outweighs the DEA method [Krumbhakar and Lovell 2004, p. 1][4]. Considering the random component as inefficiency, as in the DEA method, affects the location of efficiency frontier, and thus the final value of the efficiency ratio. Due to a number of specifics, the analyzed agri-food processing industry is characterized by a certain degree of randomness, which confirms the rightness of the SFA method's use. The application of this method allows conducting the statistical analysis of the significance of the obtained results [Krumbhakar and Lovell 2004, pp. 69].

Due to the infirmities of the deterministic methods (in the context of the validation of the obtained results), the efficiency measurement basing on integrated use of the SFA and DEA method was applied. According to this approach, the specification of the models in the DEA method was made based on the results of parameters' estimation of stochastic frontiers in the SFA method[5]. Hence, the problems associated with the verification of the correctness of variables' selection,

[3] More about the weaknesses of the SFA and DEA methods in the publication: Bezat A. (2009) Comparison of the deterministic and stochastic approaches for estimating technical efficiency on the example of non-parametric DEA and parametric SFA methods, [w:] Witkowska D. (ed.): Metody ilościowe w badaniach ekonomicznych, Vol. 10, Wyd. SGGW, Warszawa, s. 20-29.

[4] S.C. Krumbhakar and C.A.K. Lovell indicate that efficiency may be determined by using a deterministic production function or its stochastic counterpart. The authors state further that because the first model ignores the effect of random shocks, and the second one takes it into account, the preferred approach to the efficiency evaluation is the stochastic frontier; see: Krumbhakar S.C., Lovell C.A.K. (2004) Stochastic…, op. cit., pp. 65-66. This means that the stochastic model is less – in comparison to the deterministic model – vulnerable to the influence of outliers; see: Sellers-Rubio R., Más-Ruiz F.J. (2009) Technical efficiency in the retail food industry: the influence of inventory investment, wage levels and age of the firm, European Journal of Marketing, Vol., 43, No. 5/6, pp. 663.

[5] These conceptions were presented in detail in the monograph: Bezat A. (2012) Efficiency of Polish grain trade companies: an integrated application of SFA and DEA methods, Universität Bonn-ILB Press, Bonn.

orientation of models and economies of scale. The aim of the DEA method's use was to obtain the detailed results for individual companies [Jarzębowski 2011].

EVALUATION OF ENTERPRISES' EFFICIENY USING SFA AND DEA METHODS

The efficiency assessment was carried out on the basis of data collected from meat processing enterprises across Poland (a panel data for the period 2006–2011). The sample covers from 195 up to 210 companies, depending on the analyzed year. The production data is reported as revenue/expenditure denominated in PLN in constant prices. The production frontiers are fitted for a single output and two inputs. The inputs are: value of fixed assets (x_1), operating costs (x_2), and the output is net revenues from sales of goods and materials (y).

Selection of a functional form and specification of the SFA and DEA models

As a parametric approach, the SFA requires assuming a specific functional form that determines the input(s)-output relation a priori [Coelli et al. 2005]. The Cobb-Douglas function is the most commonly used. The adequacy of this function should be tested against a less restricted functional form, which is the trans-logarithmic function [Piesse and Thirtle 2000, p. 474]. Thus, the study involves two functional forms describing the input(s)-output relations, namely the Cobb-Douglas (equation 1) and trans-logarithmic model (equation 2). The tested frontier models take following form:

$$\ln y_i = \beta_0 + \sum_{j=1}^{k} \beta_j \ln x_{ij} + v_i - u_i \tag{1}$$

and

$$\ln y_i = \beta_0 + \sum_{j=1}^{k} \beta_j \ln x_{ij} + \frac{1}{2} \sum_{j=1}^{k} \sum_{l=1}^{k} \beta_{jl} \ln x_j \ln x_l + v_i - u_i \tag{2}$$

where:

i – index indicating objects $i=1,...,I$, where I is a number of objects in a sample,

j – index indicating inputs $j=1,...,l$,

y_i – output of an object i,

x_{ij} – input j of an object i,

β – vector of parameters to be estimated,

v_i – random variable representing the random error, so called statistical noise,

u_i – a positive random variable associated with technical efficiency (TE).

Comparison of the selected functional forms is carried out basing on the likelihood ratio test statistics (LR, Table 1). The LR statistics has the following form:

$$LR = -2[\ln L(R) - \ln L(N)] \quad (3)$$

where:

lnL(R) – logarithm of the maximum likelihood value in the restricted model,

lnL(N) – logarithm of the maximum likelihood value in the non-restricted model.

Table 1. Likelihood ratio statistics and model's selection verification

years	$\ln L(\hat{\theta}_R)$	$\ln L(\hat{\theta}_N)$	LR	result [1]	model
2006	-324,69	-322,25	4,88**	fail to reject of H_0	Cobb-Douglas
2007	-346,47	-344,33	4,28**	fail to reject of H_0	Cobb-Douglas
2008	-329,28	-326,27	6,00**	fail to reject of H_0	Cobb-Douglas
2009	-346,17	-341,15	10,04*	fail to reject of H_0	Cobb-Douglas
2010	-348,03	-342,38	11,30*	fail to reject of H_0	Cobb-Douglas
2011	-327,77	-322,37	10,80*	fail to reject of H_0	Cobb-Douglas

[1] The value of test statistic for $\chi^2(3)$ distribution amounts to 7,82 at the significance level of 0,05 (**) and 11,34 at the significance level of 0,1 (*)

Source: own calculations

The likelihood ratio tests lead to acceptance of the null hypothesis, saying that the Cobb-Douglas function (a model with restrictions on parameters) better describes the inputs-output relations (equation 1). Therefore, the empirical results obtained from estimating only the Cobb-Douglas function are reported in Table 2.

The output-oriented efficiency ratio – in the case of the stochastic frontier function – is measured as a relation between an observed output (value y, equation 1) and maximum output possible to be achieved in environment characterized by *exp*(v_i) (value y^*). Hence, the ratio may be written as:

$$TE_i = \frac{y_i}{y_i^*} = \frac{\exp(\beta_0 + \sum \beta_1 \ln x_{1i} + \sum \beta_2 \ln x_{2i} + v_i - u_i)}{\exp(\beta_0 + \sum \beta_1 \ln x_{1i} + \sum \beta_2 \ln x_{2i} + v_i)} = \exp(-u_i) \quad (4)$$

On the basis of equation (4) it can be stated that the value of the TE ratio varies from 0 to 1, where the unity indicates that this company is technically efficient. Otherwise $TE_i < 1$ provides a measure of the shortfall of observed output from maximum feasible output in an environment characterized by *exp*(v_i), and indicates the inefficiency of this company.

The same sample (data set) and variables, as in case of the SFA method's application, was applied while estimating performance indicators using the DEA method. In this way, the problem of accidental or build upon the insights of experts in the selection of variables in the model has been eliminated. On the basis of the sum of parameters $\sum_{j=1}^{k}\beta_j$ it was stated that the sample of companies operating in the meat processing sector in years 2006-2011 was characterized by decreasing returns to scale, hence the NIRS[6] model was applied with use of the DEA method. The output-oriented models were used in this method. It was the result of the reference to the results obtained using the SFA method, in which the production function may – depending on the progress within the framework of production factors involved – move upwards, which means that by a given input level one can achieve increasing level of output (i.e. orientation on output).

Comparison of the results of efficiency evaluation in terms of the stochastic and deterministic approach

The obtained, by using the SFA method, efficiency ratios are compiled in distinction to size classes (Table 2).

Table 2. The average efficiency ratio calculated by using the SFA method by size of companies in years 2006-2011

Year/size of companies	2006	2007	2008	2009	2010	2011
micro	0,239	0,326	0,266	0,271	0,300	0,307
small	0,378	0,423	0,344	0,362	0,378	0,397
medium	0,493	0,483	0,404	0,494	0,499	0,488
large	0,507	0,483	0,480	0,559	0,564	0,592

Source: own calculations

On the basis of the results presented in Table 3, it can be noticed that the average efficiency ratio increases with an increase in the size of the analyzed companies. Due to the fact that within the framework of the SFA method the relative efficiency is determined there is no possibility of comparison of the results obtained in different models[7]. The micro companies achieved the efficiency ratio

[6] NIRS, Non-increasing Returns to Scale, involves decreasing or constant returns to scale (CRS, *Constant Returns to Scale*). Although there was no constant returns to scale noted in the analyzed sector, in order to keep the complexity of the conducted analyses the possibility of their presence was assumed. For more about the DEA model see [Jarzębowski S. 2011]

[7] Within the framework of the SFA method the creation of a dynamic model for the balanced panel data is one of the approach of possibility to evaluate the changes in

ranging from 0,24 to 0,33; the average ratio for the small companies ranged from 0,34 to 0,42; the ratio for the medium companies ranged from 0,4 to 0,5; the lowest ratio for large companies was equal to 0,48 and the highest to 0,59.
The efficiency ratios obtained by using the DEA method are compiled in Table 3.

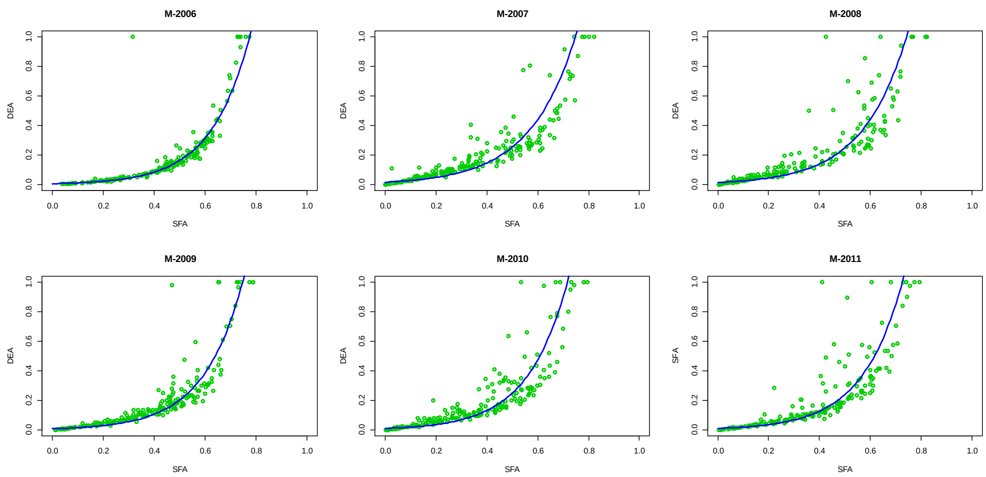
Table 3. The average efficiency ratio calculated by using the DEA method by size of companies in years 2006-2011

Year/size of companies	2006	2007	2008	2009	2010	2011
micro	0,164	0,207	0,185	0,174	0,181	0,185
small	0,182	0,175	0,187	0,171	0,174	0,168
medium	0,226	0,283	0,254	0,223	0,285	0,257
large	0,316	0,226	0,502	0,459	0,428	0,548

Source: own calculations

The efficiency ratios obtained for the stochastic (using the SFA method) and deterministic model (using the DEA method) are compiled according to each year in a form of correlation charts (Figure 1)

Figure 1. The relation between efficiency ratios determined using the SFA and DEA method for companies of the meat processing sector in years 2006-2011



Source: own calculations

efficiency over years; see Bezat A. (2011) Estimation of technical efficiency by application of the SFA method for panel data, Scientific Journal Warsaw University of Life Sciences – SGGW, Problems of World Agriculture 2011, Vol. 11, No. 3, p. 5-13.

The results obtained by using the SFA and DEA methods have been evaluated. On the basis of the correlation graphs it can be stated that the relation between the analyzed variables is best described by the exponential function. The matching of the functional form was based on the value of the coefficient of determination. The determination coefficients for the meat processing sector took values ranging from 0,77 to 0,94.

SUMMARY

A company uses inputs in order to manufacture the output, thus the author defines efficiency of companies as their ability to transform inputs into outputs. In the literature, it is assumed that the production frontier illustrates available and effectively used manufacturing techniques, since the function determines the maximum size of production (*Y*) to be achieved by a given level of production factor(s) (*X*). Thus, the production function is a reflection of the state of technology, including applied technique, organization, knowledge and experience. The production function is defined as the base function for analysing production process, and it was always considered as a kind of the foundation of theoretical analyses in the neoclassical economics.

The SFA method (*Stochastic Frontier Analysis*) is a method of efficiency evaluation. Nevertheless, the deterministic tools are used in the literature as well. Their analytical background is not the production function but the optimization problem (e.g. the DEA method, *Data Envelopment Analysis*). The both methods require all decision making units to have comparable inputs and outputs and both can handle multiple input and multiple output models.

The SFA and DEA methods were applied to evaluate the efficiency of companies of the meat processing sector. The similar results were obtained in case of both methods. On the basis of the conducted analysis it was stated that the results obtained using the DEA method (after the models' specification basing on the SFA method's results) and the results obtained using the SFA method indicate the exponential dependence for the analyzed period.

Basing on the conducted analysis it was claimed that the use of the SFA and the DEA methods integrally - combining advantages of both methods – allows preserving the analogy when comparing the results and formulating reliable conclusions.

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FUNCTIONAL EXPLORATORY DATA ANALYSIS OF UNEMPLOYMENT RATE FOR VARIOUS COUNTRIES

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Abstract:. Functional exploratory techniques are applied in the analysis of an unemployment rate. The rate is smoothed into differentiable function in order to facilitate the analysis. The main aim of the analysis is assigned to find out the unemployment curves which do not follow the same pattern as that of the other ones.

Keywords: B-splines basis system, smoothing with roughness penalty, functional principal component analysis, cluster analysis, depth measures, unemployment rate

INTRODUCTION

Unemployment rate represents unemployed persons as a percentage of the labour force based on International Labour Office definition. The labour force is the total number of people employed and unemployed. Unemployed persons comprise persons aged 15 to 74 who: a) are without work during the reference week; b) are available to start work within the next two weeks; c) have been actively seeking work in the past four weeks or had already found a job to start within the next three months.

Unemployment rate is an important economic indicator with wide range of social dimensions and is one of the primary goals of macroeconomic policy. It is also a key indicator of overall economic performance. A rising rate is seen as a sign of weakening economy that may call for cut in interest rate. A falling rate, similarly, indicates a growing economy which is usually accompanied by higher inflation rate and may call for increase in interest rates. Rapid change of the rate is a strong signal for a feasible grown or drop in a country’s economy [Burgen et al. 2012].

The rate is not a perfect indicator of economic activity or inactivity. There are three main areas of criticisms regarding the measurement of the rate: survey accuracy, discouraged workers and underemployed workers. The criticism reflect the definitional and technical pitfalls involved in the preparation of several unemployment data emanating from different sources of various countries. Thus the interpretability of the rate should come not only from inspecting its level but also from its pace.

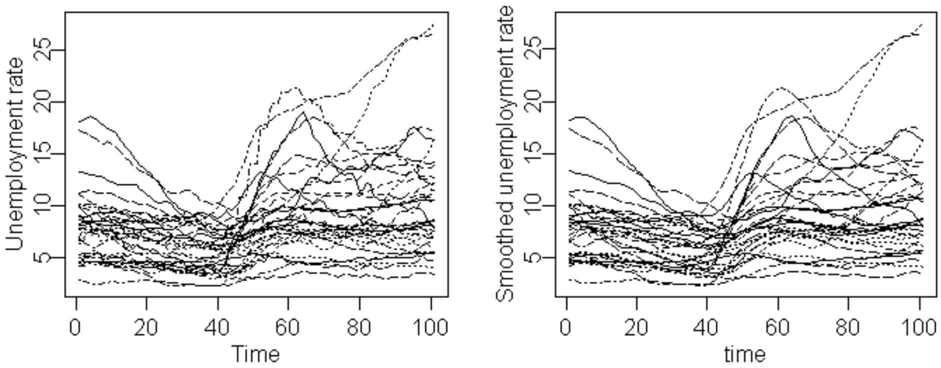
DATA ANALYSIS

The aim of the paper is to summarize main characteristics of the unemployment rate in various countries, mainly in Europe. The investigated data are presented in a seasonally adjusted form. They relate to Belgium, Bulgaria, Czech Republic, Denmark, Germany, Estonia, Ireland, Greece, Spain, France, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Malta, Netherlands, Austria, Poland, Portugal, Romania, Slovenia, Slovakia, Finland, Sweden, United Kingdom, Norway, Croatia, Turkey, United States and Japan. The series are monthly collected since 2005.01 to 2013.05.

Each unemployment curve is presented in a raw and smoothed form in Figure 1. Smoothness, in the sense of possessing a certain number of derivatives, is a property of a latent function. We assume the existence of the function and that it is giving rise to the observational unemployment rate. The approach reflects the idea that an unemployment rate is driven by an overall economic performance and should be filtered out of the noise coming from several shortcomings involved in the preparation of data. Moreover data smoothing allows us to inquire the dynamics of unemployment rates of various countries in a unified way by means of derivative of smoothed unemployment rate with respect to time.

As a method of turning raw discrete data into smooth functions we chose smoothing by a B-spline basis with a roughness penalty (the penalty was based on the integral of the square of the derivative of order 2). A basis is a set of known functions $\{\phi_k\}_{k\in N}$ that any function $X(t)$ could be approximated by a linear combination of a sufficiently large K: $X(t) \approx \sum_{k=1}^{K} c_k \phi_k$ (see details in [Ramsay et al. 2005]. If smoothing penalization is required, apart of $\{c_k\}_{k=1,\ldots,K}$, an additional parameter λ is involved in estimation. The choice of the number of basis K and the smoothing penalty parameter was made according to the cross-validation criteria. The investigated data and method gave $K = 31$ and $\lambda = \mathbf{1.574804}$.

Figure 1. Raw and smoothed unemployment rate for various countries



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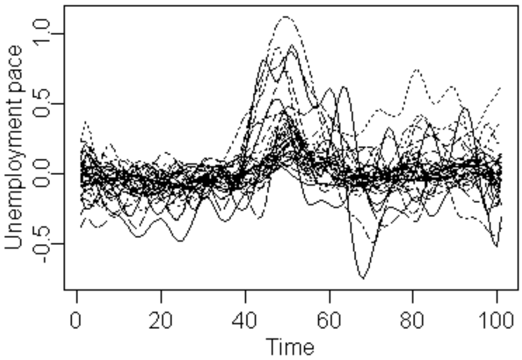
The smoothed unemployment rate was used to represent an unemployment pace as a first derivative. The pace is presented in Figure 2.

Figure 2. The unemployment pace of various countries



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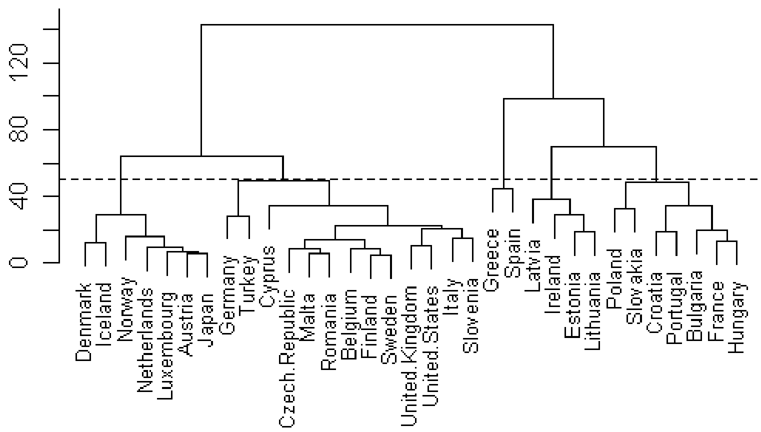
The plots in Figure. 1 and 2 are not particularly informative though some general conclusions can be drawn. First we can find unemployment convergence between countries covering the period 2005.01-2008.05 (Time 0-40 in Figure 1) and unemployment disparities intensified as of 2008.05. Second the corresponding unemployment pace in various countries was relatively low and stable until the first quarter of 2008 and then its level grew and became much more diversed. However, the curves layout in Figure 1. and Figure 2. suggests that there is a low-level unemployment dynamics in several countries.

By the use of metric and semi-metric functions, some additional information about the investigated unemployment rates can be extracted. Let us first consider the L_2-metric $d(X_1,X_2)=\sqrt{\int_{Time}(X_1(t)-X_2(t))^2\,dt}$ and the dissimilarity structure produced by the metric. The structure is presented in Figure 3 (a complete agglomeration method was used).

Figure 3. Dendrogram of the smoothed unemployment rate



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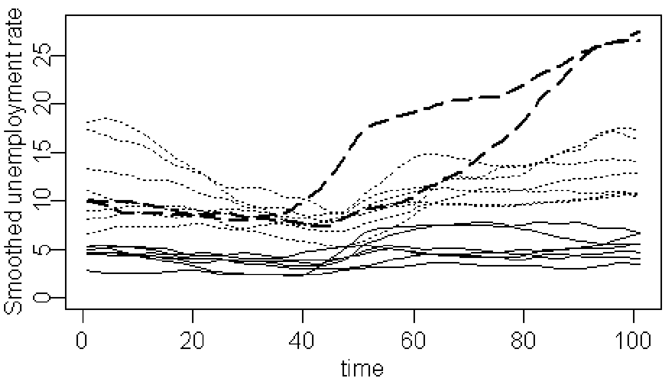
According to the proposed L_2-metric, agglomerated countries reflect similar pattern of unemployment level in the investigated period 2005.01 - 2013.05. For example at the threshold 50 (dashed line in Figure 3) five groups are extracted. Let us consider the three of them: 1. Denmark, Iceland, Norway, Netherlands, Luxembourg, Austria, Japan; 2. Poland, Slovakia, Croatia, Portugal, Bulgaria, France, Hungary; 3. Greece, Spain. Each of the three groups are marked with a different style line in Figure 4. and represents curves which are close to each other over the whole time period. The lower-rate thresholds give finer and more homogenous divisions. For example the second group is divided into two parts at the threshold 40. The first group is not divided in the case because the group is less diverse.

The L_2-metric takes into account all sources of variability between curves. In the case we want to investigate the variability in terms of its main sources, we can use the following semi-norms $d(X_1,X_2)=\sqrt{\sum_{k=1}^{q}\left(\int_{Time}(X_1(t)-X_2(t))v_k(t)dt\right)^2}$, where $v_1,v_2,\ldots,v_q$ are orthonormal eigenfunctions of a covariance operator (see

[Horváth et al. 2012] and [Ferraty 2006]. Here, q is a tuning parameter indicating the resolution level at which the problem is considered. To understand its meaning a functional principal analysis should be applied (the details of the analysis are not presented in the paper; the interested reader is referred to [Furmańczyk et al. 2012]. According to the analysis the first two components (q=2) explain 94% of the total variability of the investigated curves: 78% - the first component and 16% -the second component. The first component reflects the differences between average levels of unemployment rates and the second one the effect of crossing unemployment rate curves in the period 2008-2009. These two sources of variability are depicted in [Furmańczyk et al. 2012], where the same set of unemployment rates is investigated but in the shorter time range, that is for 2005 till 2012.

Figure 4. Unemployment rates of Denmark, Iceland, Norway, Netherlands, Luxembourg, Austria, Japan (solid lines) and Poland, Slovakia, Croatia, Portugal, Bulgaria, France, Hungary (dotted lines), Greece and Spain (dashed lines).



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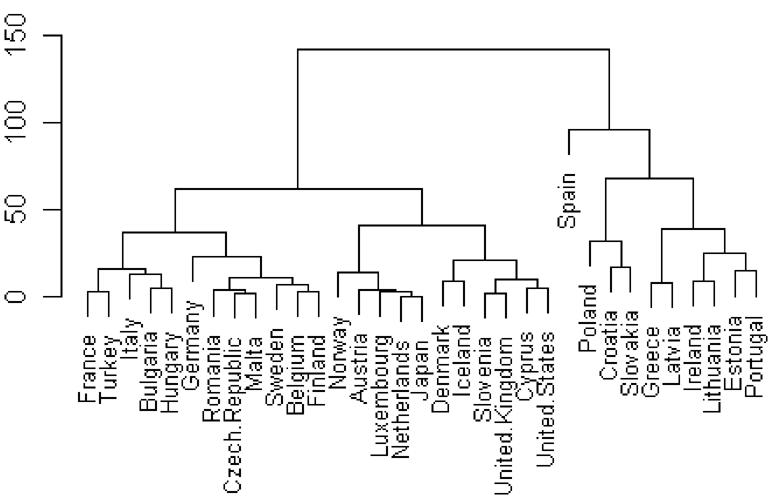
In [Furmańczyk et al. 2012] the weights of the first two sources of variability are accordingly 72% and 20%. It means that the changes of an unemployment rate in 2013 differentiated countries with respect to its first source of variability. The dissimilarity structure produced by the semi-metrics is exhibited in Figure 5. The threshold 50 gives five groups. Although the number of groups is the same as in the case of L_2-metric, the groups are different because the division does not take into account 6% of the remaining variability. For example the curves representing unemployment rates of Greece and Latvia are not close to each other though they are in the same cluster of the dendrogram in Figure 5. The reason is that the integrals $\int_{Time} (X_1(t) - X_2(t)) v_k(t)dt$ for k=1,2 are close to zero in the case of the two curves. In fact the curves are crossing three times in the observed period (see Figure 6).

Greece and Spain, as depicted by Figure 3, are candidates for outliers. By Figure 4, Spain is additionally depicted as an outlier in terms of the main sources of variability. In order to identify other outliers, depth measures can be used (see [Febrero-Bande et al. 2008] and [Cuevas et al. 2007] for theoretical details). A way to detect the presence of functional outliers is to look for curves with lower depth and is based on bootstrap. The result of the approach is presented in Figure 6. As a depth we used a random projection method.

Figure 5. Dendrogram of the smoothed unemployment rate based on first two principal components



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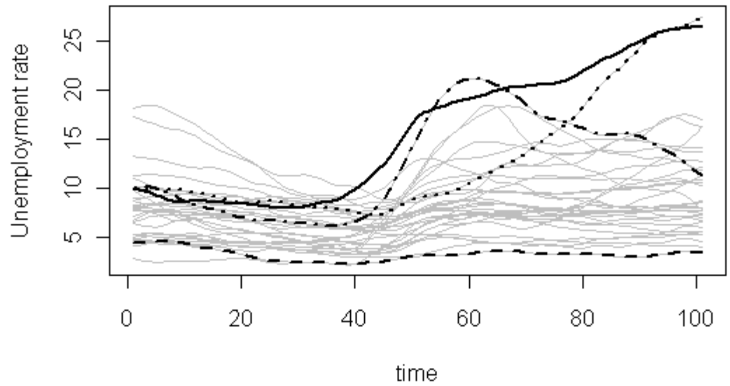
In the method a random direction a and projection of the data along the direction are taken: $\int_{Time} a(t) X_i(t)\, dt,\ i = 1,2,\ldots,n$. Then the sample depth of a datum X_i is defined as the univariate depth of the corresponding one-dimensional projection (expressed in terms of order statistics so that the median is the deepest point). A single representative value is obtained by averaging on a. The direction of a is chosen according to Gaussian distribution. The bootstrap procedure is designed to select C such that, in the absence of outliers, the percentage of correct observations mislabeled as outliers is approximately equal to 1%: $\Pr(D(X_i) < C) = 0.01$, $i = 1,2,\ldots,n$, where $D(X_i)$ denotes depth of the datum X_i. That is the cutoff C in the procedure is found by estimating this percentile, making use of the observed sample curves. In the case of our curves we used the smoothed bootstrap procedure based on trimming described in [Febrero–Bande et al. 2008].

The four exposed outlying curves in Figure 6 are 12% of total number of curves.

Figure 6. Outlying curves (bold lines). Spain (solid line), Norway (dashed line), Greece (dotted line), Latvia (dashed-dotted line)



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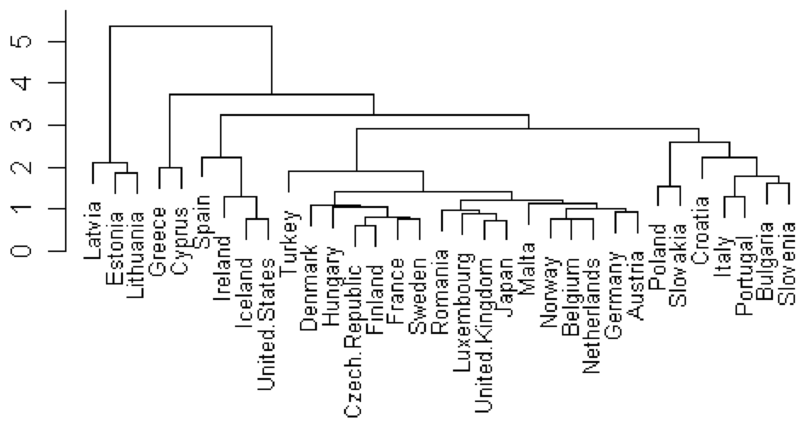
We hinted in *Introduction* that interpretability of an unemployment rate should result not only from inspecting its level but also from its pace. This is the main reason why we represented unemployment rates as smooth differerentiable curves. The unemployment pace is thus well defined and we explored it by the same techniques as in the unemployment rate case (Figures 7, 8, 9).

Figure 7. Dendrogram of the smoothed unemployment pace; being computed by means of the classical L_2-metric



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The placement of Latvia, Estonia and Lithuania in the dendrograms of Figure 7 and Figure 8 indicate that the unemployment pace of these countries is substantially different from the unemployment pace of the remaining countries. These countries are of potential outliers. Same Greece, Spain and Cyprus. Similar conclusions can be drawn from Figure 9. By cutting dendrogram in Figure 8 at level 1.5 we are receiving 5 groups, where the group with Norway is the biggest one and have a stable unemployment rate. The biplot in Figure 10 shows the placement of the extracted groups in terms of principal scores. Note that opposite to outliers the countries with stable unemployment pace are located close to point (0,0).

Figure 8. Dendrogram of the smoothed unemployment pace; being computed by means of the PCA-metric

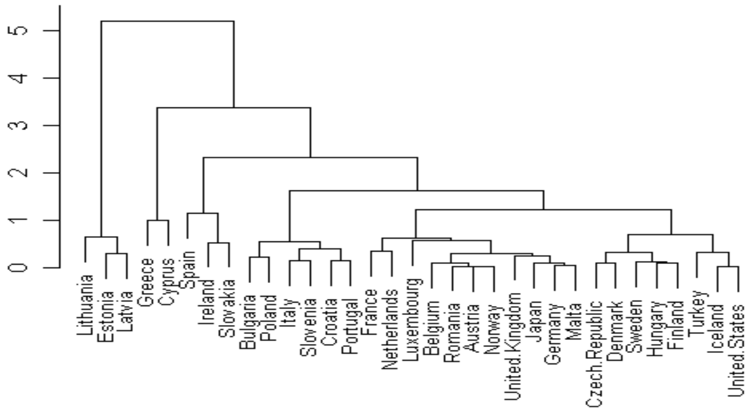

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Figure 9. Outlying unemployment pace curves (bold lines). Estonia (solid line), Latvia (dashed line), Lithuania (dotted line), Greece (dashed-dotted line)

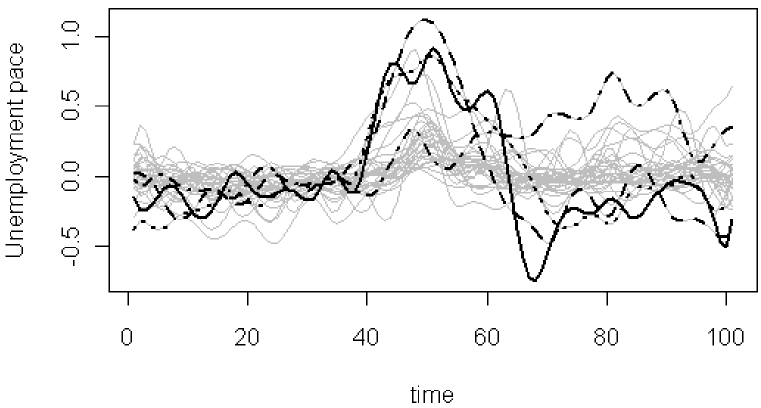

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Figure 10. Biplot of the first two principal scores for unemployment pace. Different symbols are used for the five clusters separated by 1.5 threshold in the dendrogram of Figure 8.

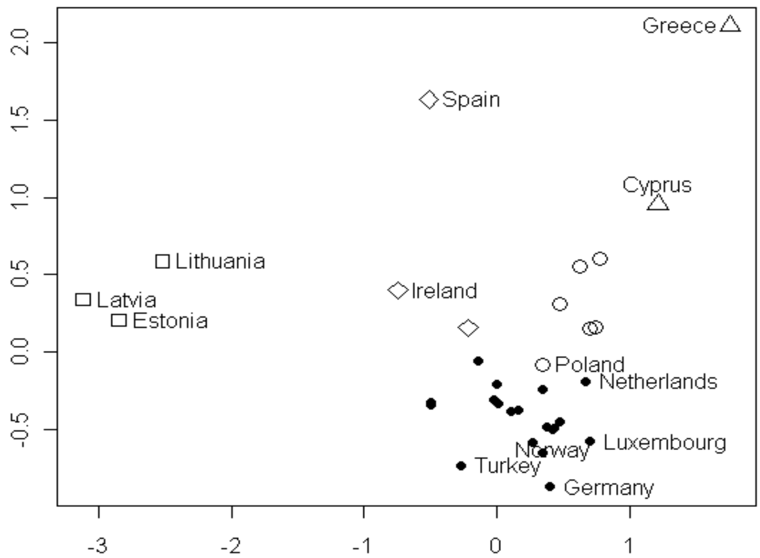

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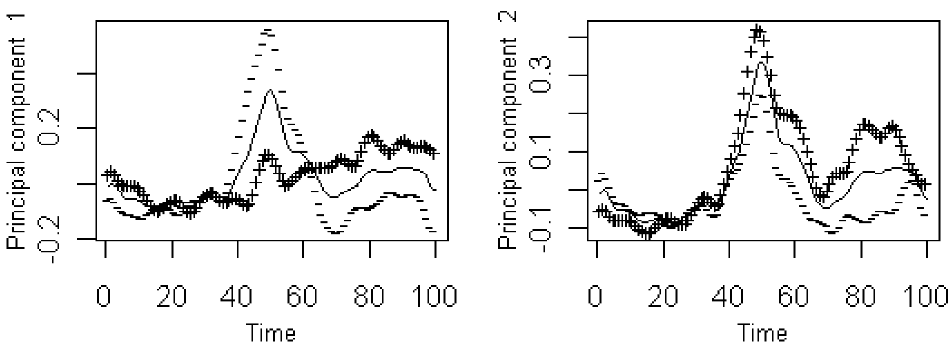
Figure 11. First two principal components as perturbations of the mean unemployment pace.



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According to Figure 10 and Figure 11 we can reflect relative dissimilarity of outliers. Latvia, Lithuania and Estonia have relatively high positive unemployment pace for 2009 till 2010 and negative after the period. Greece, Cyprus and Spain have relatively positive unemployment pace as of 2008.

SUMMARY

Functional exploratory techniques are applied in the analysis of an unemployment rate and pace. The rate is smoothed into differentiable function. The first derivative of the smoothed rate represents the unemployment pace.

An insight into the differences between countries is made by cluster analysis, functional principal components analysis and depth measures analysis. The hierarchical cluster analysis is used on a set of dissimilarities, where a dissimilarity structure is produced by the classical L_2 norm and principal components semi-norm. The depth notion is applied to outlier detection procedure.

The techniques detect Estonia, Latvia, Lithuania, Greece, Norway and Spain as outliers and show countries with comparable unemployment rate and pace. In the case of outliers the analysis reflects the nature of the depicted dissimilarities. It is shown that the dissimilarities begun as of 2008-2009 and still hold, which is a strong signal for a feasible changes in the countries' economy.

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THE USE OF ACCRUAL-BASED AND CASH-BASED APPROACH IN EVALUATING THE OPERATIONAL FINANCIAL THREAT OF ENTERPRISES FROM THE TSL SECTOR - EXAMPLE OF APPLICATION OF THE DISCRIMINANT ANALYSIS

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Abstract: In the reference books one can find information about a lot of models for forecasting financial threat for Polish enterprises. These models vary as they use different measures of evaluation. However, there are very few cases in which measures using data from a cash flow statement are used. The main goal of the paper is to create a model for evaluating of operational financial threat of enterprise from TSL sector using both accrual and cash measures. The accomplishment of this objective is justified by a need for empirical research on mixed models of the evaluation of operational financial threat of enterprises i.e. based on both accrual and cash measures.

Keywords: operational financial threat of enterprise, financial position of enterprise, discriminant analysis, transport, forwarding and logistic sector

INTRODUCTION

Logistics activity that mostly deals with time-spatial transformation of all kinds of goods, capital and accompanying information determines a series of changing conditions for efficient and effective continuation of business activities. In view of a specific character of conducted logistics activity, its subject and the range of services on TSL market this activity may encounter various obstacles, distortions and threats.

Operational financial threat includes real and potential symptoms of a destructive influence of financial factors on enterprise's capability to achieve and maintain a financial condition conducive to a continuation of its activity.

Early recognition of operational financial threat should be then one of the basic tasks of an economic entity's management and also an essential determinant for initiating corrective actions in advance.

The major objective of the paper is to create a model for evaluation of operational financial threat of enterprises from TSL sector using both accrual and cash measures. It is commonly known that there are neither generalized discriminant models nor financial measures acting as universal predictors of financial threat for enterprises of all types and sizes [Malinowska et al. 2005][1].

The accomplishment of this objective is justified by a need for empirical research on mixed models of the evaluation of operational financial threat of enterprises i.e. based on both accrual and cash measures.

The main research hypothesis formulated in order to achieve the objective of the paper took the form of the following statement: the discriminant models for the TSL sector enterprises that use cash measures show a predictive ability that is comparable or higher than in case of models taking into consideration only accrual measures.

ECONOMIC SUBJECTS AND METHODOLOGICAL GROUNDS OF EMPIRICAL RESEARCH

In view of the fact that putting an enterprise into bankruptcy is regulated by specific legal acts[2] and financial threat is a liquid and dynamic category that is placed between the best situation (i.e. „sound" enterprise) and a terminal one (i.e. bankruptcy of an economic entity) [Gruszczyński 2012] in order to set a research sample the author used data about TSL sector enterprises which were

[1] The research carried out by L. Karbownik indicated that in case of TSL sector enterprises the predictive ability of models for forecasting financial threat which are presented in the reference books was significantly lower in relation to its initial level. i.e. assumed by the researchers of the subject [Karbownik 2013].

Predictive ability i.e. accuracy of enterprise classification to group of bankrupt companies or those not at risk of bankruptcy which is evaluated among others on the basis of very often used method, which is called confusion matrix.

The higher classification accuracy of enterprise to one from the two groups i.e. group of bankrupt companies or those not at risk of bankruptcy, the higher predictive ability of discriminant models.

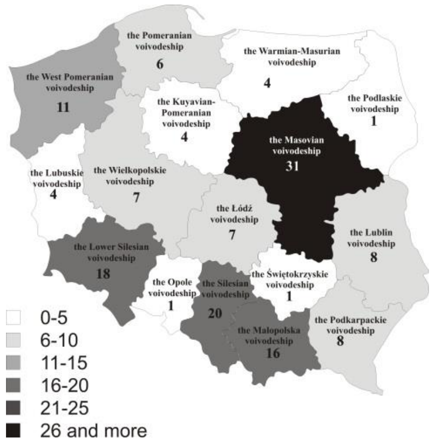
[2] According to Article 10 of the Act of 28 February 2003 on the Law on Bankruptcy and Reorganization (Journal of Laws from 2003 no 60, p. 535 as amended) debtor's bankruptcy is declared when he is proved insolvent.

Article 11 of this act stipulates that debtor is considered insolvent if he does not settle financial liabilities, whereas debtor acting as a legal entity or an organizational unit without legal entity but for which the other act guarantees a legal capacity, is considered insolvent when value of their liabilities exceed the value of its assets, even if liabilities are settled up to date.

declared bankrupt by the court. The majority of models of financial threat are currently based on this kind of data as it is easier to obtain.

According to the data from the Economic Information Centre in Warsaw in the period between 2004-2010 the biggest number of bankruptcies from „Transport and warehouse management" section was noted in the Masovian voivodeship (31 companies - see Figure 1).

Figure 1. Number of enterprises declared bankrupt from „Transport and warehouse management" section in Poland between 2004-2010 by voivodeships



Source: own study on the basis of data from the Economic Information Centre Ltd.

The subsequent selection of the research sample was reduced only to the enterprises that:

- were officially declared bankrupt by the court in the period between 2004-2010 (i.e. in company documents there is a ruling declaring bankruptcy),
- submitted individual financial statements (including cash flow statement) to the National Court Register in Warsaw one year or two and three years before declaring bankruptcy [Platt et al. 2002; Prusak 2005][3],
- as a balance sheet date accepted 31 December in each year of the whole analyzed period[4].

The number of bankrupt enterprises participating in the study was reduced (from the initial number of 31 enterprises from TSL sector that submitted financial statements to the National Court Register in Warsaw to only 5 enterprises). It was related to the fact that 20 bankrupt enterprises did not submit cash flow statement

[3] According to H. D. Platt and M. B. Platt enterprise financial failure can occur within 3 years before the moment of declaring bankruptcy, although in many cases failure happens in the period shorter than 3 years from the moment of declaring bankruptcy.

[4] Lack of financial data prevented conducting a discriminant analysis for a bigger research sample in this period.

in the whole analyzed period and in case of 6 enterprises such statements were not available for at least one of the analysed years.

Discriminant analysis requires taking into consideration two groups of economic entities and that is why it was conducted for 5 enterprises one year, two and three years before declaring their bankruptcy[5] as well as for 5 companies with no risk of bankruptcy [Pogodzińska et al. 1995][6]. The author made an attempt to find for bankrupt companies some economic entities that met specific criteria, namely:

- conducted business activity in the same sector (i.e. TSL sector),
- submitted statements to the same National Court Register (i.e. NCR in Warsaw),
- owned a comparable total size of assets in the analysed year[7].

Therefore, a target research sample[8] comprised 10 entities for each analysed period.

Financial aspect has a specific function in the evaluation of operational financial threat of enterprise as it binds all aspects of its activity that result from making different decisions. In the operational dimension connected with current activity – financial decisions are made in economic units to a very basic extent i.e. they mainly concern financial liquidity, shaping streams of revenues and expenditures or achievement of the best results [Skowronek–Mielczarek 2007; Maślanka 2008].

Taking into consideration high usefulness of cash flow statement for various groups of stakeholders that are interested in survival of a given economic unit it must be stated that it should become an integral part of the evaluation of operational financial threat of enterprise [Duraj 2010; Rutkowski 2000] in order to reflect current economic events suitably.

The evaluation should not be made in isolation, but in mutual relations with other parts of financial statement. Thereby, full dimension of this evaluation will be achieved when data on cash flows are analysed together with other information included in enterprise balance sheet and its profit and loss account.

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[5] Bankruptcy was declared because debtor did not settle due payments and their value exceeded the value of debtor's assets.

[6] M. Pogodzińska and S. Sojak as the first in Poland created – by means of a technique of linear multivariate discriminant analysis a model for only 10 observations.

The model was a good example of using multivariate analysis in empirical research conducted by them, however, it had a very limited practical value for economic practice.

[7] The second part of the sample included carefully selected enterprises in good financial condition, i.e. those which were still functioning (at least after one year since the moment of using data for evaluating parameters of discriminant function).

Financial measures for enterprises not in danger of bankruptcy were calculated respectively for years in which data was collected for bankrupt entities assigned to them.

[8] So-called analytical sample.

Due to the lack of detailed financial data of analysed enterprises from TSL sector the analysis of enterprise operational financial threat was at the initial stage limited to the following 13 diagnostic variables (see Table 1).

Subsequent selection of measures of operational financial threat concerned evaluation of average values of analysed variables, their coefficients of variation and the evaluation of prognostic ability of diagnostic variables, the analysis of correlations between them and evaluation of normality of their distribution. It is of crucial importance as the next phase of the analysis should only include the measures which [Hadasik 1998; Prusak 2005; Maślanka 2008]:

- vary significantly in two groups (i.e. both in bankrupt companies and in those not facing the risk of bankruptcy) – to achieve this intra-group and intergroup coefficients of variation were set down [Maślanka 2008][9];
- are lowly correlated with each other,
- are characterized by a normal distribution [Hadasik 1998; Lipiec-Zajchowska 2003][10].

In order to conduct a discriminant analysis Statistica 10 program was applied, using a stepwise progressive analysis [Stanisz 2007] because in the reference books one can find information that Fisher's linear discriminant function is resistant to lack of normality of distribution [Seber 1984].

NET OPERATING CASH FLOWS IN FORECASTING OPERATIONAL FINANCIAL THREAT OF TSL SECTOR ENTERPRISES

The models of the evaluation of operational financial threat of TSL sector enterprises that are presented in this part of the article were prepared on the basis of financial data from the period comprising one year, two and three years before bankruptcy. To calculate the values of analysed measures data from a balance sheet, profit and loss account and cash flow statement were used. They comprise the activity of 10 analyzed enterprises[11] (analytical sample).

[9] A desirable situation is when values of intra-group coefficients of variation are at the lowest possible level, whereas intergroup coefficients of variation at the highest possible level.

[10] This assumption is crucial from a statistical point of view, however, in the reference books the authors claim that the occurrence of normal distribution is extremely rare, which in fact does not influence on predictive abilities of discriminant functions.

[11] The balanced model stipulates that both groups of companies i.e. bankrupt and with no risk of bankruptcy are of similar sizes (i.e. 5 in each).

Table 1. Selected financial measures of the evaluation of operational financial threat of enterprise*

No	Main areas of the evaluation of operational financial threat of enterprise	Measure		Accrual/cash approach	Symbol
1	2	3		4	5
1	Sales of products (services), goods and materials[12]	net revenues from sales of services, goods and materials	S_t	accrual	m_1
2	Financial liquidity	current ratio	$\frac{CA_t}{CL_t}$	accrual	m_2
3		quick ratio	$\frac{CA_t - In_t - SDE_t}{CL_t}$	accrual	m_3
4		cash ratio	$\frac{C_t}{CL_t}$	accrual	m_4
5		current liabilities coverage ratio with cash from operational activity	$\frac{OCF_t}{CL_t}$	cash	m_5
6	Operating profitability	operating return on sales ratio	$\frac{PoS_t}{S_t}$	accrual	m_6
7		operating return on total assets ratio	$\frac{PoS_t}{\frac{(TA_t + TA_{t-1})}{2}}$	accrual	m_7
8		operating return on equity ratio	$\frac{PoS_t}{\frac{(E_t + E_{t-1})}{2}}$	accrual	m_8
9		cash return on sales ratio	$\frac{OCF_t}{S_t}$	cash	m_9

[12] Analysed companies prepared a cash flow statement by indirect method, so cash measure (OCR – cash revenue from operational activity) is the data that is not included in a financial statement submitted to the National Court Register in Warsaw.

1	2	3		4	5
10	Operating profitability	cash return on assets ratio	$\frac{OCF_t}{\frac{(TA_t + TA_{t-1})}{2}}$	cash	m_{10}
11		cash return on equity ratio	$\frac{OCF_t}{\frac{(E_t + E_{t-1})}{2}}$	cash	m_{11}
12	Short-term financial reserves[13]	hedge ratio no 1	$\frac{SRrb_t}{TA_t}$	accrual	m_{12}
13		hedge ratio no 2	$\frac{OtSR_t}{TA_t}$	accrual	m_{13}

* where:

S_t – net revenues from sales (products (services), goods and materials) at the end of the period,

CA_t – current assets at the end of the period,

In_t – inventory at the end of the period,

SDE_t – short-term deferred expenditure at the end of the period,

C_t – cash and monetary assets at the end of the period,

CL_t – current liabilities at the end of the period,

OCF_t – cash flows from operational activity in period,

PoS_t – profit/loss on sales at the end of the period,

TA_t – total assets at the end of the period,

TA_{t-1}– total assets at the beginning of the period,

E_t – equity at the end of the period ,

E_{t-1} – equity at the beginning of the period,

$SRrb_t$ – short-term reserves for pension benefits and alike at the end of the period ,

$OtSR_t$ – other short-term reserves at the end of the period.

Source: own study on the basis of: [Gmytrasiewicz et al. 1996]; [Sierpińska , Jachna 2004]; [Duraj 2008].

[13] In case of short-term reserves for liabilities only two hedge ratios were accepted as measures of operational financial threat and in their numerator they have either short-term reserves for pension benefits or alike or other short-term reserves.

It turned out that short-term accrued expenses can be included in point B. I. 2 and B. I. 3 as well as in point B. IV. 2. of the balance sheet (here together with deferred revenue). However, the companies did not reveal any detailed information on RMK in additional information and explanations.

On the other hand, cash approach to changes of reserves in a cash flow statement includes change of reserves to total liabilities (i.e. long and short-term), whereas a change of accruals include a total balance of deferred expenditure and accrued expenses.

The results of empirical research presented in the reference books [Largay et al. 1980; Rujoub et al. 1995; Maślanka 2008; Wędzki 2008] provide numerous arguments to justify the need of using measures based on cash flows in order to predict the risk of enterprise bankruptcy. The need is also confirmed by the results of the analysis conducted for the needs of this paper and which involved analyzing values of intra-group and intergroup coefficients of variation of analysed diagnostic variables[14].

Only from the period 3 years before bankruptcy the author eliminated a current liabilities coverage ratio with cash from operational activity (m_5) from the basic measures for distinguishing bankrupt entities from those not facing bankruptcy. It was related to the fact that this measure showed more than 50% total classification error of analysed economic entities from TSL sector.

Two and three years before bankruptcy cash measure was not included in the estimated model for the evaluation of operational financial threat. So, two and three years before bankruptcy it was impossible to verify a research hypothesis formulated in this study. Therefore, in the subsequent part of the paper only discriminant models for one year before bankruptcy of enterprises from the TSL sector will be presented.

In case of one year before bankruptcy - as a result of a subsequent selection - the further discussion took into consideration only 7 measures[15]. This situation was related to a significant correlation between analysed variables, and thus the analysis was restricted only to analyzing those between which a value of correlation coefficient was below 0,8 (at the significance level of 0.05)

Finally, the following measures were qualified for a discriminant analysis: m_1, m_2, m_4, m_5, m_6, m_{12} and m_{13}. By means of the Shapiro–Wilk test it was checked if they are characterized by a normal distribution. As it was supposed in relation to such a small research sample, most of these variable did not have a normal distribution. So one of the most important assumptions from a statistical point of view as far as a discriminant analysis is concerned, was not met. It must be noted, however, that in economic research a normal distribution is really rare. In fact, in practice it does not have any impact on deterioration of prognostic abilities of a discriminant function [Maślanka 2008].

[14] The analyses did not include m_8 and m_{11} because in some analysed enterprises the value of equity capital was negative

The critical value of intergroup coefficient of variation was set at the level v=0,2. All analysed variables met a criterion of sufficient distinction and none of them were eliminated at this stage of empirical research.

[15] Among these measures only m_5 used information from a cash flow statement.

The obtained results (see Table 2) showed that a conducted discriminant analysis was significant (Wilks' lambda = 0,24322, p < 0,0284 [Stanisz 2007][16]). The most useful variable in this analysis at p < 0,05 was only cash ratio. However, at the tendency level (i.e. p < 0,1) also current liabilities coverage ratio with cash from operational activity (m_5) was considered most useful.

Table 2. Summary of function of discriminant analysis (1 year before bankruptcy)

N=10	Variables in model: 3 Grouping: bankruptcy (2 groups) Wilks' lambda: ,24322 approx. F (3,6)=6,2229 p< ,0284					
	Wilks' lambda	Wilks' partial	F-to-remove (1,7)	p	Toler.	1-Toler. (R-Sqr.)
m_4	0,814454	0,298633	14,09153	0,009467	0,575793	0,424207
m_5	0,424579	0,572857	4,47382	0,078811	0,571008	0,428992
m_{12}	0,305514	0,796112	1,53663	0,261403	0,879630	0,120370

Source: own study by means of STATISTICA software

The estimated Model 1 took the following form[17]:

$$Model_1 = - 2{,}3276 + 29{,}3662\ m_4 + 3{,}0424\ m_5 – 48{,}3415\ m_{12}$$

Classification matrix that contained information about the number and proportion of enterprises properly classified in each group proved that all bankrupt enterprises were qualified properly for a specific type of economic entities. There was only one wrongly classified enterprise and it was one of those not in at risk of bankruptcy.

In the subsequent phase of research only 6 accrual measures i.e.: m_1, m_2, m_4, m_6, m_{12} and m_{13} were taken into account. While estimating a model the current liabilities coverage ratio with cash from operational activity (m_5) was excluded. The results of conducted analyses are presented in Table 3.

[16] Wilks' lambda is a standard statistic used to determine statistical significance of discriminant power by currently chosen discriminant variables. Its value falls between 0 (a perfect discriminant power) to 1 (no discriminant power).

[17] Threshold value (threshold point) is 0.

In order to obtain function values on the basis of which classification of objects (enterprises) is made for a specific group – to estimate a model raw data was used i.e. not transformed in any way [see: Prusak 2005].

Table 3. Summary of function of discriminant analysis II (1 year before bankruptcy)

N=10	Variables in model: 2 Grouping: Bankruptcy (2 groups) Wilks' lambda: ,37476 approx. F (2,7)=5,8394 p< ,0322					
	Wilks' lambda	Wilks' partial	F-to-remove. (1,7)	p	Toler.	1-Toler. (R-Sqr.)
m_4	0,629852	0,594994	4,764825	0,065365	0,999988	0,000012
m_2	0,479323	0,781849	1,953139	0,204946	0,999988	0,000012

Source: own study by means of STATISTICA software

Obtained results showed that a conducted discriminant analysis was significant at $p < 0{,}0322$, and a measure that proved to be most useful here, although only at the level of tendency, was a cash ratio. The model that was estimated was model 1a and it took the following form[18]:

$$Model_{1a} = 2{,}7243 – 18{,}6295\ m_4 – 1{,}4773\ m_2$$

Classification matrix showed that all companies not facing a risk of bankruptcy were classified properly for a specific type of economic entities. This group also included one wrongly classified bankrupt entity.

Presented models 1 and 1a (a canonical discriminant function) were also used to predict the bankruptcy of TSL sector companies from a validation sample (e.g. data in the period between 2011-2012 of four enterprise from the TSL sector). In all variants 75% classification accuracy[19] was achieved.

CONCLUSIONS

The major objective of the paper is to create a model for evaluation of operational financial threat of enterprises from TSL sector using both accrual and cash measures.

As a result of applying a discriminant analysis very simple to interpret models of the evaluation of operational financial threat were estimated. Only in the

[18] Threshold value (threshold point) is 0.

[19] Bankrupt companies from TSL sector that were qualified for a validation sample were declared bankrupt by the court between 2011-2012, submitted individual financial statement (including cash flow statement) to the National Court Register in Warsaw one year, two and three years before declaring bankruptcy and as a balance sheet date accepted 31 December in each year of the whole analyzed period.

One of 4 companies from a validation sample was wrongly classified.

Classification errors, regardless of a type of model, concerned the same enterprise from a validation sample (i.e. DSV Solution Ltd.) It may suggest that the reason for bankruptcy was not related to its financial condition [compare: Mączyńska 2004].

first analyzed period (i.e. one year before bankruptcy) a cash measure of operational financial threat of enterprise (i.e. m_5 variable) was included in the estimated model.

Presented models 1 and 1a (a canonical discriminant function) were also used to predict the bankruptcy of TSL sector companies from a validation sample. In all variants 75% classification accuracy was achieved.

The results of conducted empirical research show that one year before bankruptcy of TSL sector enterprises discriminant models using cash measures show a predictive ability comparable with models estimated only on the basis of accrual measures.

Two and three years before bankruptcy cash measure was not included in the estimated model for the evaluation of operational financial threat. So, two and three years before bankruptcy it was impossible to verify a research hypothesis formulated in this study.

The research involves a selected number of TSL sector enterprises, so in view of its 'sectoral' range they do not predispose to formulate general conclusions. Yet, they are to draw attention to the necessity of creating of discriminant models which are dedicated to various enterprises running service activities.

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QUANTITATIVE METHODS IN ECONOMICS
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THE COMPARISON OF RANKINGS CREATED FOR OPEN-END EQUITY MUTUAL FUNDS WITH APPLICATION OF DIFFERENT EFFECTIVENESS MEASURES

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Abstract: The presented work tries to carry out the comparison analysis of eight measures which are used to judge the open-end mutual fund's management. These are: coefficient of Burke, Sterling, Calmar, Omega, Sortino, Sharpe – Izraelsen, Information Ratio and potential of excess rate of return. The above measures were applied to equity mutual funds operating on polish capital market in 2003-2012 years. The investigations were carried out for three periods: 2003-2012 and for two five years sub-periods. Within which of them the ranking of funds were created to classify them from the most effective to the smallest. In order to answer the question about the influence of singled out effectiveness measures on judgment of portfolio management the Spearman rang coefficients were calculated between rankings created with application of different measures. In this way the detailed information was obtained not only about "the best" and "the worst" mutual funds in different periods but also this information was made dependent on market condition. It is because the division of investigation period on two sub-periods harmonizes with bullish and bearish market on polish stock exchange. From one point this work is some recapitulation of the results obtained by authors in previous investigations but from another point it is some kind of its extension and generalization.

Keywords: open-end mutual funds, measures of investment effectiveness, Spearman rang coefficient

INTRODUCTION

Entrusting own savings by the investor to collective investment institutions involves both the choice of the fund as well as the length of time horizon. The investor makes a decision usually on the basis of publicly available rates of return. This may give the wrong picture of profitability of investments, mainly by ignoring the risk taken. The measure that the risk takes into account is Sharpe ratio, constituting the relation of expected premium to an investment risk [Sharpe, 1966]. Its use is conditioned by the assumption of normal distribution of rates of return, not often met in practice[1]. Therefore, in literature, other measures of efficiency of investments can be found, such as Calmar, Sortino, Omega and many generalisations of the Sharpe's ratio. The multiplicity of measures, however, makes it difficult to decide which of them to choose. In foreign literature, one can find the comparison between the rankings of funds based on the Sharpe ratio with the rankings formed on the basis of other measures of performance as well as the latter ones among one another. The research of, among others, Eling and Schumacher [Eling, Schuhmacher 2007] show that, with respect to hedge funds, the choice of measure is not critical. They all lead to similar rankings. Analogous results were obtained by Eling [Eling 2008] for five other groups of funds, including, among others, those investing in shares or bonds.

This paper aims to answer the question: Is the choice of measure on the Polish market important when making a decision on the choice of equity funds. In addition, at work, the issue of stability of rankings is considered by testing the correlation between the two sub-periods, used for dividing the whole study period. The work of Nguyen-Thi-Thanh [Nguyen-Thi-Thanh 2007] is devoted to such an issue. However, it concerns hedge funds, and the studies have indicated less and more stable measures of efficiency. The applied methodology can be easily adapted to open-ended mutual funds.

DATA AND METHODOLOGY

The studies concern the share funds existing in the whole period i.e.in the years of 2003-2012. Sixteen such funds were selected: Arka BZWBK Akcji, BPH Akcji Dynamicznych Spółek, BPH Akcji, AvivaPolskich Akcji (formerly known as CUAkcji Polskich), Investor Akcji (formerly known as DWS Akcji), Investor Akcji Pluo(formerly known as DWS Akcji Plus), Investor Top 25 (formerly known asDWS Top 25), ING Akcji, Legg Mason Akcji, Millenium Akcji, Pioneer Akcji Polskich, PKO/CS Akcji, PZU Akcji Krakowiak, SEB 3, Skarbiec Akcja and Unikorona Akcje. In addition, Investor Top 25 small companies' fund is included in the above group of companies because the authors want to examine whether this type of fund, as compared to those investing in large companies, will be 'standing out' in the generated rankings. As presented below, the applied measures

[1] The studies of the authors conducted for share funds in the period of 1 February 2007-31 August 2011 [Karpio Żebrowska-Suchodolska 2011] also point to the lack of normal distribution of monthly rates of return for 6 out of the 16 studied funds and lack of normal distribution of weekly rates of returns for all (16) funds.

of effectiveness practically do not take into account the fact whether the fund invests in large or small companies. It is important that they are equity funds.

The monthly rates of return of participation units and eight measures have been determined for each fund. The last ones include the following indicators: Burke, Sterling, Calmar, Omega, Sortino, Sharpe-Izraelsen, Information Ratio and the potential for excess rate of returns. They were calculated using the formulas:

Burke ratio:
$$Burke_T = \frac{R}{\sqrt{\sum_{t=1}^{n}\left(MDD_{t,T}\right)^2}} \tag{1}$$

Sterling ratio:
$$Sterling_T = \frac{R}{\frac{1}{n}\sum_{t=1}^{n}\left|MDD_{t,T}\right| + 10\%} \tag{2}$$

Calmar ratio:
$$Calmar_T = \frac{R}{\left|MDD_{t,T}\right|} \tag{3}$$

Omega ratio:
$$Omega = \frac{\frac{1}{T-1}\sum_{t=1}^{T}\max\left(R_t - m, 0\right)}{\frac{1}{T-1}\sum_{t=1}^{T}\max\left(m - R_t, 0\right)} \tag{4}$$

Sortino ratio:
$$Sortino = \frac{\overline{R_x} - m}{\theta(m)} \tag{5}$$

Information Ratio:
$$IR_x = \frac{\overline{R_x} - \overline{R_b}}{TR_x} \tag{6}$$

Sharpe-Israelsen ratio:
$$IR^{SI} = \frac{\overline{R_x} - \overline{R_b}}{\sigma_{R_x - R_b}^{\frac{\overline{R_x - \overline{R_b}}}{\left|R_x - \overline{R_b}\right|}}} \tag{7}$$

potential indicator of excess rate of return:
$$UPR_t = \frac{\frac{1}{T-1}\sum_{t=1, R_t > m}^{T}\left(R_t - m\right)}{\sqrt{\frac{1}{T-1}\sum_{t=1, R_t < m}^{T}\left(R_t - m\right)^2}} \tag{8}$$

where:

$\overline{R_b}$ - average rate of return from the selected benchmark,

$\overline{R_x}$ - average rate of return from the x fund's portfolio,

$TR_x = \sqrt{\frac{1}{T-1}\left(R_{x,t} - R_{b,t} - \left(\overline{R_x} - \overline{R_b}\right)\right)^2}$,

$\sigma_{R_x - R_b}$ =*TR*,

$\theta(m) = \sqrt{\frac{1}{T-1}\sum_{T=1}^{T}\left(\left(R_t - m\right)^-\right)^2}$, where $\left(R_t - m\right)^- = R_t - m$ when $R_t - m \le 0$ and $\left(R_t - m\right)^- = 0$ when $R_t - m > 0$,

m – break-even point, which can assume different values depending on what is the minimal acceptable rate of return for the investor,

R - annual rate of return,

MDD_T - maximum decrease of the rate of return in *T* period.

A detailed description of the above models and their interpretation can be found, among others, in the works of [Karpio, Żebrowska-Suchodolska 2012, 2013].As benchmarks for Information Ratio and Sharpe – Izraelsen ratio, the following indices were used: WIG and WIG 20. Although WIG20 is not the best benchmark for the fund of Investor Top 25small companies, it is just one of the many measures considered. Moreover, these indices are used as benchmarks. Therefore, in the case of small and speculative companies, WIG and WIG 20 can simply be treated as ‘cautious’ benchmarks. To make the calculations for the same threshold of profitability, it is assumed that it has the value of $m = 0$ both in the entire period and the two sub-periods. This means that profitable funds are those that can earn anything.

In the next step, for each measure separately, the ranking of funds was made from the highest to the lowest value of calculated measures. This allowed to create rankings of funds according to their investment efficiency. They have become the starting point to determine the Spearman’s rank correlation coefficient [Luszniewicz, Słaby 2003]:

$$r_s = 1 - \frac{6\sum_{i=1}^{T} d_i^2}{T(T^2 - 1)} \tag{9}$$

where:

d_i – difference between the ranks conferred to both characteristics for the *i*-observation,

T – sample size.

The study of significance of the Spearman's rank correlation coefficient was carried out with the application of null hypothesis: $r_s = 0$,as compared to the alternative hypothesis $r_s \neq 0$. Test statistics

$$t = \frac{r_s}{\sqrt{1-r_s^2}}\sqrt{T-2} \tag{10}$$

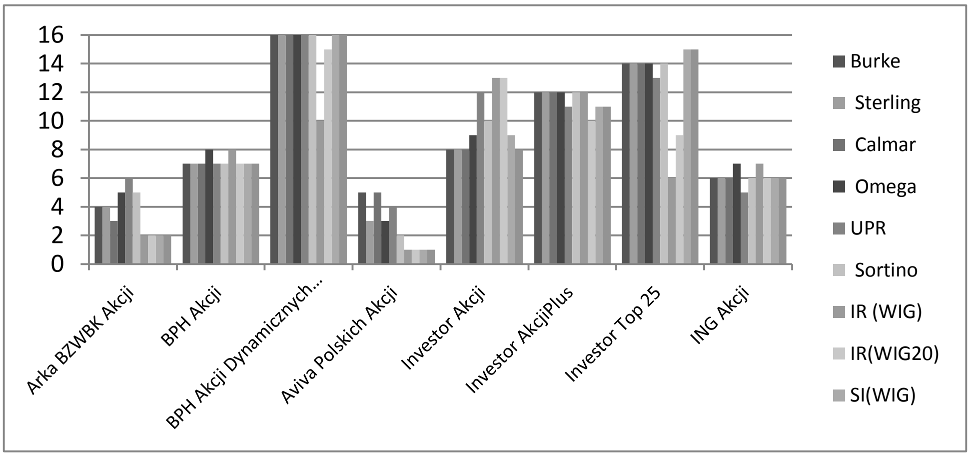
has the schedule of*t*-Student with $T-2$ degrees of freedom.

Analogous calculations were performed for the two sub-periods (2003-2007 and 2008-2013),used for dividing the whole study period. The division is dictated by the moment of the outbreak of financial crisis. In addition, for each measure, the Spearman's rank coefficient was established between the two sub-periods.

RESULTS

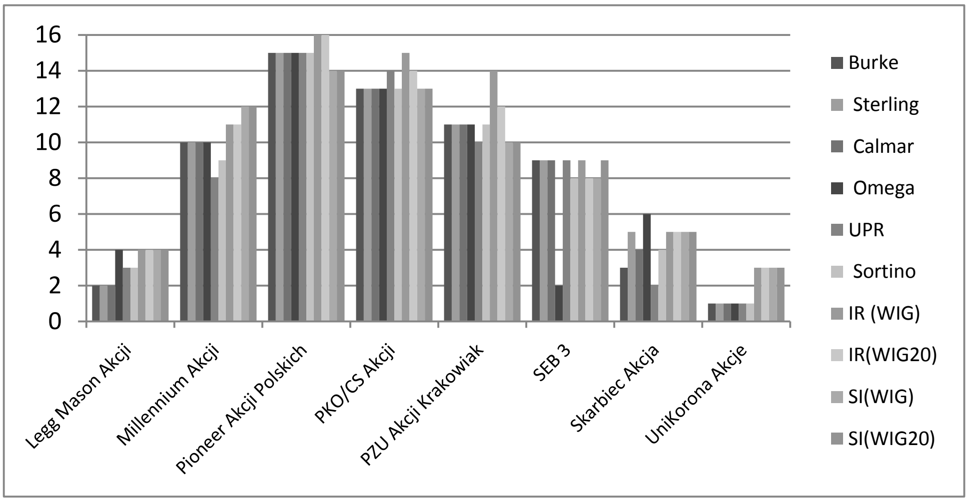
Most of the received rankings of equity funds designated on the basis of 8 measures for the entire study period yield similar results, and some are even exactly the same (Fig. 1). Although the drawing in black and white colour blurs the results obtained for specific measures, the intention of the authors was to show the overall picture of the results. In the case of Investor Top 25 fund, the results obtained with the support of Sharpe-Izraelsen ratio proved to be close to the results obtained, among others, through profit and loss indicators (Burke, Sterling and Calmar), Omega and Sortino. Larger differences in the ranking of Top 25 Investor were shown only by the results obtained with the support of IR.

Figure 1a. The rankings of equity funds for the entire study period obtained on the basis of 8 measures



Source: own study

Figure 1b. The rankings of equity funds for the entire study period obtained on the basis of 8 measures.



Source: own study

The rankings obtained for each sub-period (Tables 1-2) were also analysed. Similar rankings were obtained for each of them. It was especially apparent with the index of profits and losses as well as IR and SI indicators. The reason probably is similar structure of these measures. For the two considered sub-periods, however, the established rankings vary considerably. The differences range from a few to several positions.

Table1.The rankings of equity funds in the first sub-period (the years of 2003-2007)

	Burke	*Sterling*	*Calmar*	*Omega*	*UPR*	*Sortino*	*IR (WIG)*	*IR(WIG20)*	*SI(WIG)*	*SI(WIG20)*
Arka BZWBK Akcji	1	1	1	2	1	1	1	1	1	1
BPH Akcji	9	9	10	10	8	9	8	9	10	9
BPH Akcji Dynamicznych Spółek	16	16	16	16	16	16	11	16	16	16
Aviva Polskich Akcji	6	2	7	3	3	2	2	2	2	2
Investor Akcji	7	10	3	13	13	13	16	15	14	14
Investor AkcjiPlus	10	8	8	9	6	6	7	8	9	8
Investor Top 25	14	13	14	6	11	8	3	5	3	5
ING Akcji	8	7	9	12	9	11	9	7	7	7
Legg Mason Akcji	3	4	5	8	5	5	4	4	5	4

	Burke	*Sterling*	*Calmar*	*Omega*	*UPR*	*Sortino*	*IR (WIG)*	*IR(WIG20)*	*SI(WIG)*	*SI(WIG20)*
Millennium Akcji	15	15	15	15	15	15	14	14	15	15
Pioneer Akcji Polskich	12	12	12	14	14	14	15	11	8	11
PKO/CS Akcji	4	6	4	7	7	7	10	10	11	10
PZU Akcji Krakowiak	13	14	13	1	12	12	13	13	13	13
SEB 3	11	11	11	11	10	10	12	12	12	12
Skarbiec Akcja	2	3	2	4	2	3	6	6	6	6
UniKorona Akcje	5	5	6	5	4	4	5	3	4	3

Sources: own study

Table 2. The rankings of equity funds in the second sub-period (the years of 2008-2013)

	Burke	*Sterling*	*Calmar*	*Omega*	*UPR*	*Sortino*	*IR (WIG)*	*IR(WIG20)*	*SI(WIG)*	*SI(WIG20)*
Arka BZWBK Akcji	11	11	11	9	3	10	6	8	10	10
BPH Akcji	8	8	8	7	9	8	10	9	7	7
BPH Akcji Dynamicznych Spółek	14	14	14	12	14	14	13	12	13	13
Aviva Polskich Akcji	3	3	3	16	10	2	2	2	2	2
Investor Akcji	5	5	5	4	4	5	5	5	3	4
Investor AkcjiPlus	12	12	12	11	12	12	15	13	11	11
Investor Top 25	15	15	15	15	13	16	12	15	15	15
ING Akcji	6	6	6	5	6	6	7	6	6	6
Legg Mason Akcji	2	2	2	2	8	3	3	3	16	3
Millennium Akcji	7	7	7	6	11	7	9	7	5	5
Pioneer Akcji Polskich	16	16	16	14	16	15	16	16	14	14
PKO/CS Akcji	13	13	13	13	15	13	14	14	12	12
PZU Akcji Krakowiak	9	9	9	10	5	11	11	11	8	8
SEB 3	10	10	10	8	7	9	8	10	9	9
Skarbiec Akcja	4	4	4	3	2	4	4	4	4	16
UniKorona Akcje	1	1	1	1	1	1	1	1	1	1

Sources: own study

Spearman's rank correlation coefficients, obtained for the entire study period, indicate a high similarity between the rankings created on the basis of various measures (Table 3). All were found to be statistically significant at the significance level of 0.05. The smallest value (0.74) related to Burke indicators and Information Ratio (WIG). Nevertheless, it points to the similarity of rankings based on these measures.

Table 3. Spearman's correlation coefficients for the rankings of equity funds during the entire study period

	Burke	*Sterling*	*Calmar*	*Omega*	*UPR*	*Sortino*	*IR (WIG)*	*IR(WIG20)*	*SI-(WIG)*	*SI-WIG20)*
Burke	1,00	0,998	0,997	0,897	0,953	0,974	0,744	0,865	0,938	0,941
Sterling		1,000	0,991	0,915	0,941	0,985	0,768	0,888	0,962	0,965
Calmar			1,000	0,900	0,941	0,971	0,753	0,874	0,947	0,950
Omega				1,000	0,868	0,932	0,709	0,844	0,906	0,885
UPR					1,000	0,971	0,768	0,891	0,894	0,885
Sortino						1,000	0,785	0,909	0,956	0,950
IR (WIG)							1,000	0,932	0,762	0,750
IR(WIG20)								1,000	0,906	0,891
S-I(WIG)									1,000	0,997
SI-(WIG20)										1,000

Sources: own study

In the event of correlation between the two sub-periods, most measures suggest that there is no rank stability, as evidenced by statistically insignificant coefficients, shown in bold in Table 4.

Table 4. Spearman's coefficient between the two sub-periods

	Burke	*Sterling*	*Calmar*	*Omega*	*UPR*	*Sortino*	*IR (WIG)*	*IR(WIG20)*	*SI-(WIG)*	*SI-WIG20)*
Spearman's coefficient	0,521	0,518	**0,456**	**-0,056**	**0,488**	**0,424**	**0,432**	**0,415**	**0,009**	**0,182**

Sources: own study

As a result, the funds that do well during good investment times do badly when there is a bear market and vice versa. This indicates a minimal adjustment of fund management investment policy to market conditions.

SUMMARY

The analysis shows that all measures of efficiency of investment in long-term studies (10 years) and over shorter periods, but with 'homogeneous' economic situation, lead to the rankings that are very close to each other. Therefore, we are faced with an almost identical assessment of the efficiency of investment. It would seem that the measure is not critical when choosing a fund. Similar results were obtained in the studies conducted in foreign markets [Eling 2008, Ornelas et al. 2011]. However, this interpretation ends in failure when taking into account the periods differing in terms of economic prosperity. Most of the Spearman's coefficients between the two sub-periods are not statistically significant. Only the results based on Burke and Sterling indicators proved to be stable over time. Equity funds on the Polish market are therefore not able to yield good results in the periods of variable market conditions.

Therefore, the burden to adapt to market situation, for example, by changing the fund, falls on the customers. Yet, the managers charge fees for their activities. Therefore, they should strive to achieve good results in the condition of changing economic situation. It seems that five years is too long period to trust in the abilities of those managing the portfolios of funds, despite the fact that investing in funds is still the long-term process. Considerations related to the length of period guaranteeing the stability of the rankings of funds become interesting here. This will be the subject of future works of the authors. It is worth noting that the fund of small companies does not stand out from the rest. Therefore, the methodology used does not appear to be sensitive to that whether it is used for funds investing in large fundamental or in small speculative companies. Certainly, this question should be the subject of a more detailed analysis. In this place where, the inclusion of DWS Top 25 to the studies was merely of 'exploratory' nature.

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QUANTITATIVE METHODS IN ECONOMICS
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APPLICATION OF LOGISTIC REGRESSION IN THE SUBJECTIVE ASSESSMENT OF THE QUALITY OF LIFE OF POLES

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Abstract: The paper discusses the application of tools of multivariate analytical methods i.e., logistic regression, to model the subjective assessment of satisfaction/dissatisfaction with quality of life achieved. Based on empirical data from a random sampling of households in Podkarpacie province, the statistical significance of variables was estimated, which enabled the estimation of the value of life quality based entirely on qualitative variables, that are characteristic for a given household, with incomes being excluded. The essence of the regression is to seek such household related parameters, that could have varying degrees of impact on the occurrence of either negative or positive values. The models have been designed based on varied classification categories for households, thus allowing for both the quantitative analysis and comparison of the impacts of key cardinal determinants of values of life.

Keywords: logistic regression analysis, households, value of quality of life, qualitative variable, probability

INTRODUCTION

"Quality is a fulfillment of requirements and expectations, the way that leads to satisfaction whilst satisfaction serves as very good and reliable measure of quality".

(E. Skrzypek)

A key phenomenon of contemporary times is the noticeable increase in significance of the quality factor [Skrzypek 2002]. The concept of the quality of life is very complex, both within economic studies and beyond. One of the first definitions of life quality states that "it is the resultant integration of personal traits as well as the subjective and objective indicators of well-being[1]". Quality of life is generally defined as the extent to which material and non-material needs of individuals, families and communities are met. Quality, according to Skrzypek, should be understood as something that can be improved upon. Thus, it can refer to life and be considered a tool for improving all aspects of human endeavors [Skrzypek 2002]. Rampley claims that in speaking about quality of life, it is best to assume that it is a multidimensional concept [Rampley 2003].

The concept of quality of life, in contemporary economic studies, is very complex. It covers conditions construed as objective factors, that include, among other things economic circumstances, free time, social security, living conditions, natural environment, health, and social environment etc. Due to its varied aspects, studies on the quality of life are conducted according to applicable methodologies in given fields, which in economics are categorized by indicators that summarize the common aspects of the measured objective. The best known indicator that reflects standard of living and life satisfaction is the *Quality of Life Index,* published since 2005 by *„The Economist*" magazine for needs of various countries, which combines the results of subjective life satisfaction surveys with objective factors of quality of life in various countries[2]. This, in economic studies, means that the private perception of all individuals impacts on the quality of life. The subjective approach is manifested in self-esteem, which is accounted for in psychological categories, including the feeling of satisfaction, happiness, anxiety, fears, hopes, etc. [Domański et.al. 2000]. Subjective measurements of quality of life have no clearly defined methodology, although attempts are being made to quantify them. R. Kolman postulates, that both theoretical and practical knowledge on quality ought be accurate. He claims that the reason for the poor performance of quality control systems is ignorance in areas of quantitative evaluation methods of levels of quality [Kolman 1992].

The aim of the study is to show the possibilities of applying logistic regression in issues concerning the assessment of subjective quality of life based on households in Podkarpacie province. The essence of modeling is to decide which statistically significant factors determine the divide into two emerging groups of households, i.e., those satisfied or not satisfied with achieved levels of quality of life. The estimated value of certain parameters of the logistic model will be determined on the basis of empirical analysis which should allow the classification

[1] Nowa encyklopedia (1996), PWN, Vol. III, Warszawa, pp. 121.

[2] Economist Intelligence Unit, An annual index that comprises of partial indicators for 9 categories, i.e., cost of living, culture and leisure, economy, environment, freedom, health, safety and risks and climate. http://internationalliving.com/2010/02/quality-of-life-2010/. The Quality of Life Index rating for 2010 places France as first (82 points), with Australia, Switzerland and Germany coming next with 81 points. Poland is placed further in the rating with 70 points.

of households into two separable groups and also to predict, for each group, the chances of positive or negative assessment.

MATERIALS AND RESEARCH METHODS

A survey, using the direct interview method (*mall intercepts method*), was conducted to measure the quality of life. The research period covered 2012. The total sample size was set at $n=1050$, but having taken account of refusals and wrong answers, the final random sample obtained was $n = 835$ responses.

The questionnaire contained a fundamental question of the type: „Are you satisfied with the current level of quality of life of your household?" (of course taking into account material situation, work, health, home, life style etc.)?. Possible variation of yes or no answers were anticipated. Beside the responses to this question, additional information on specifically chosen socio-economic characteristics of the sampled households was collected. The spatial extent of the study covered south-eastern Poland, mainly Podkarpacie province.

The primary issue, for a strictly defined methodology of econometric research, i.e., logistic regression model, was to define the dependent variable as well as the input set of potential independent variables. The dependent variables being nominal variables served as responses to the subjective rating of quality of life, with possible variation of yes or no. The independent variables served as variables that characterized prevailing living conditions of a given household. They were unitary data, which, due to the scope of the study were essentially socio-demographic for purposes of quantitative measurements (income levels, total number of persons per household, number of income earning persons, number of dependent children). The qualitative variables were, on the other hand, types of home management in a household[3], satisfaction with achieved wages levels, the fact of having or not having savings[4] including sex, age, education of head of household, place of residence, membership of any socio-economic group and the biological type of household. Measurements of the qualitative variable was, depending on the question, relative to the nominal or ordinal scale.

The subjective value of the quality of life is strongly linked with material situation, which is a derivative of earned incomes. It was assumed, a priori, that the key, strict economic factor that influences satisfaction or its lack with quality of life is the attainable household income, with improvements in the level of satisfaction with varied aspects of life increasing with rising levels of incomes [Bywalec et al.

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[3] As regards the question *how finance are handled in the household*, the following categories of responses were possible: very modest-not enough for basic needs; modest-very economical in spending; average-enough, but have to save up for major purchases; very good- we can indulge in some luxuries.

[4] The question was: *Does the household have savings?* (with yes/no category of responses) and *Is your remuneration satisfactory?* (with yes/no category of responses).

2002]. The strongest predictor of well-being is the evaluation of material conditions. The more frequent positive responses in the evaluation of quality of life, the higher the state of well-being. The value of the material situation is thus relative to the level of life of the immediate environment as positivism of the comparison increases, the higher the satisfaction. A negative rating for levels of wealth, i.e., foods, incomes, consumption and social contacts, means higher satisfaction [Długosz 2008]. The quantitative variable „income" that is highly correlated with the explanatory (independent) variable was hence deliberately omitted in the model, thus exacting more efforts in seeking other determinants of the quality of life assessments during the analysis.

The binomial logistic regression model (logit model) is applied to explain the dichotomous qualitative variable Y depending on the level of exogenous variables $X_1, X_2, ..., X_k$ (be they qualitative or quantitative). The explained variable is, in such models, quantified using dummy (0-1) variables. The variable Y_i assumes the value of 1 with a probability p_i, and value 0 with probability $1\text{-}p_i$, that can be illustrated thus:

$$P(y_i = 1) = p_i,\ P(y_i = 0) = 1 - p_i$$

Probability is a function of the vector of dependent variables $\boldsymbol{x_i}$ as well as vector parameters $\boldsymbol{\beta}$, hence:

$$p_i = P(y_i = 1) = F\left(x_i^T \beta\right) \qquad \textit{for } i=1,2,...,n \tag{1}$$

In this model, the transform function is the cumulative logistic distribution [Cramer 2004]:

$$p_i = F(x_i^T \beta) = \frac{1}{1 + \exp(-x_i^T \beta)} = \frac{\exp(x_i^T \beta)}{1 + \exp(x_i^T \beta)}, \tag{2}$$

The values of the inverse function of F are known as logits (hence, the adoption of the expression „logit models" in books). Logit is the logarithm of the odds ratio and the occurrence of the incident, which is determined by the dependency [Guzik 2004]:

$$F^{-1}(p_i) = \ln \frac{p_i}{1 - p_i}, \tag{3}$$

Based on this model, one can specify the marginal increase in probability, the so-called marginal effect, which describes how increases in the values of each explanatory variable influences the probability of its belonging to a given group.

$$\frac{\partial p_i}{\partial x_{ji}} = \beta_j p_i (1 - p_i) = \beta_j \frac{\exp(\beta_0 + \beta_1 X_{1i} + ... + \beta_k X_{ki})}{\left[1 + \exp(\beta_0 + \beta_1 X_{1i} + ... + \beta_k X_{ki})\right]^2}, \tag{4}$$

Since $p_i(1\text{-}p_i) > 0$, hence the sign of the parameter by the variable X_j determines the direction of impact of variable X_j on variable Y. If the explanatory variable X_j increases by one unit (*ceteris paribus*) then the odds ratio varies by $exp(\beta_j)$. In situations where $exp(\beta_j)>0$, growth is observed, when $exp(\beta_j)<0$ the odds ratio declines. If X_j is a binary zero-one variable, then $exp(\beta_j)$ provides by how many times the odds ratio $Y_i=1$ for category „1" increases in comparison with the same odds ratio for category „0".

Binomial model parameters are estimated using the maximum likelihood method. If the observations are independent, then the likelihood function of n-element sample should be seen as follows [Chow 1995, Kalouptsidis et al. 2009]:

$$L = \prod_{y=1} p_i \prod_{y=0} (1-p_i) = \prod_{i=1}^{n} [F(x_i^T \beta)]^{y_i} [1 - F(x_i^T \beta)]^{1-y_i}\ , \tag{5}$$

The correctness of the estimated model can be investigated using the likelihood ratio test, aimed at verifying null hypothesis for models with the intercept only (all other parameters of the model except the intercept are equal to zero). The statistical test is expressed with the formula [Gruszczyński 2001]:

$$\chi^2 = 2(\ln L_{UR} - \ln L_R)\ , \tag{6}$$

where:

L_{UR}– value of the likelihood function for a full model,

L_R– value of the likelihood function for models reduced to the intercept.

The statistic has a distribution χ^2 with degrees of freedom equaling the number of explanatory (independent) variables.

Another measure for fitting binomial models is the odds ratio, calculated as the product of properly classified cases to the product of improperly classified cases.

$$\boldsymbol{IS} = \frac{\boldsymbol{n}_{11} \cdot \boldsymbol{n}_{00}}{\boldsymbol{n}_{01} \cdot \boldsymbol{n}_{00}}\ , \tag{8}$$

The appropriate size n_{ij} is illustrated in a table of accuracy of classification of cases:

	$\hat{y}_i = 1$	$\hat{y}_i = 0$
$y_i = 1$	n_{11}	n_{10}
$y_i = 0$	n_{01}	n_{00}

In order to construct the table of accuracy, it is necessary to set theoretical values for the variable y_i. The transformation of probability into a dichotomous variable is achieved keeping with the standard rules of predictions: $y_i = 1$ if $p_i > 0{,}5$ and $y_i = 0$ if $p_i \le 0{,}5$. The model is suitable for prediction since if $IS > 1$, then any classification based on the model is better than random classifications.

EMPIRICAL RESEARCH RESULTS

Based on the empirical material collected, models for several variants of combinations of explanatory variables were constructed. A description of the significant independent variables for the models is presented in table 1.

Table 1. Selected explanatory variables for the models

Variable	Description of variable	Size
Savings	has	425
	does not have	410
Ways of managing finances in household	very modest (not enough for basic needs)	46
	modest – very economical in spending	216
	average - enough, but has to save up for major purchases	337
	good – enough for most needs	203
	very good – can indulge in certain luxuries	33
Satisfaction with remuneration	yes	280
	no	555
Education of head of household (the person that brings in the highest incomes for the household)	elementary (or junior secondary)	26
	vocational	174
	secondary	367
	higher	268
Type of household	marriage	695
	single parent	62
	one-member household	78
Age of head of household	up to 30	148
	30-40	198
	40-50	281
	50-60	169

Source: own studies based on results of a survey

Table 2. Estimation results of the logit model (1)

Items	Independent variable				
	Constant	Savings	Way of managing finances in household	Number of income earning persons	Satisfaction with remuneration
Estimate	-5.915	1.392	1.508	0.300	2.204
Standard error	0.551	0.175	0.210	0.117	0.294
Statistics *t*	-10.727	7.976	7.190	2.561	7.496
Level *p*	0.000	0.000	0.000	0.011	0.000
-95%	-6.997	1.050	1.096	0.070	1.627
+95%	-4.832	1.735	1.920	0.530	2.781
Wald Chi-square	115.074	63.611	51.691	6.558	56.194
Level *p*	0.000	0.000	0.000	0.010	0.000
Odds ratio	0.003	4.024	4.518	1.350	9.059
–95%	0.001	2.857	2.993	1.073	5.087
+95%	0.008	5.669	6.819	1.698	16.131
Number of observations n = *835*, The likelihood function 297.859, χ^2 =*560.83*; p=0.0000					

Source: own studies based on results of a survey

Based on the analyzed parameters of the model, it can be stated that savings, financial management in households, number of income earners as well as satisfaction with remuneration have, from the subjective perspective, statistically significant impacts on the value of quality of life.

Estimates based on random sampling of parameter values has allowed the final form of the logistic model to be written as:

$$p(Y) = \frac{e^{-5.92+1.40\text{ savings}+1.51\text{ financial managt }+0.30\text{ income earners}+2.20\text{ satisfaction with remuneration}}}{1+e^{-5.92+1.40\text{saving }+1.51\text{financial managt.}+0.301.\text{income earners}+2.20\text{satisfaction with remuneration}}}$$

Since the value sign of the parameter next to the independent variables of the estimated model is positive, it means that any increase in the values of these variables increases the chances of being satisfied with current quality of life. In interpreting the odds ratio for each variable, it is worthy of note that *savings* result in fourfold increased (odds) satisfaction with current level of quality of life (odds ratio value for this variable is 4.024). A similar dependency can be observed in respect of the *way of managing finances* variable. The highest value of the odds ratio (9.059) occurs in respect of the *satisfaction with remuneration* variable, which means that people who are satisfied with their remunerations are over nine times more likely to be *satisfied with current level of quality of life.*

Based on the estimated model, it is possible to identify the likelihood for a person to be classified as being *satisfied with its current level of quality of life*. If it is assumed that such a person has savings, that financial management in the household is average and that there are two income earners in the given household, but there is lack of satisfaction with remuneration earned, then the estimated likelihood being classified as *satisfied with current level of quality of life* is 0.288. If for the same person, the fact of being satisfied with remuneration earned improves, then his estimated likelihood will rise to 0.783.

Table 3 illustrates the likelihood of classifying all respondents into groups of „unhappy" or satisfied" with current quality of life. The classification of individual cases indicated the number and fraction of those correctly or incorrectly classified. The number of correct prediction constituted 84.577% of the tested sample. Over 87% of the overall cases accounted for accuracy prediction value of *Y=1* (satisfied with *current quality of life*), while 81.886% was for the accuracy of prediction value of *Y=0* (unhappy with *current quality of life*). The high value 30.986 means that the classification is much better than that obtainable through a completely random classification.

Table 3. Propriety of classification of subjective evaluation of quality of using the logit model (1)

Actual	Predicted		Total	%	
	Y = 0	Y = 1		Odds ratio	30.986
Y=0	330	73	403	Y=0	81.886
Y=1	55	377	432	Y=1	87.269
Sum	385	450	835	Total	84.577

Source: own studies based on results of a survey

Using a different combination of explanatory variables, another model that correctly classifies 78.806% of cases can be evaluated. Estimated results of such a model are presented in table 4.

Based on the parameters assessed, it can be deduced, that key determinants in the subjective assessment of quality of life remains savings, education and type of household.

Table 4. Estimated results using the logit model (2)

Items	Independent variable			
	Constant	Savings	Education	Type of household
Estimate	-2.635	2.531	0.628	-0.377
Standard error	0.393	0.174	0.115	0.143
Statistics *t*	-6.712	14.559	5.461	-2.627
Level *p*	0.000	0.000	0.000	0.009
-95%	-3.405	2.189	0.402	-0.658
+95%	-1.864	2.872	0.853	-0.095
Wald Chi-squares	45.051	211.961	29.821	6.899
Level *p*	0.000	0.000	0.000	0.009
Odds ratio	0.072	12.561	1.873	0.686
-95%	0.033	8.930	1.495	0.518
+95%	0.155	17.668	2.348	0.909
Number of observation $n = 835$, The likelihood function 412.791 $\chi^2 = 330.87; p=0.0000$				

Source: own studies based on results of a survey

A negative parameter value by the variable, *„type of household”,* means that modifying the type of household (e.g., a marriage for a single parent) leads to declining odds (probability) of being *satisfied with current quality of life*, which in other words means that satisfaction with actual level of quality of life in marriage is higher than in single parent families. Increasing education level of the highest income earner in a household by „one level”, for example from secondary to higher, increases the odds of being satisfied with quality of life by almost two-fold (the odds ratio being 1.873). The relevance of case classification, based on this model is slightly lower i.e., 658 cases of accurate classification out of 835.

Table 5. Propriety of classification of subjective evaluation of quality of life using logit model (2)

Actual	Predicted		Total	%	
	Y = 0	Y = 1			
Y=0	318	85	403	Y=0	78.908
Y=1	92	340	432	Y=1	78.704
Sum	410	425	835	Total	78.806

Source: own studies based on results of a survey

SUMMARY

Logit models serve as very good tools in explaining developments in subjective assessment of the quality of life depending on prevailing socio-economic circumstances. Modeling results indicate that the dependence of the value of this variable on such factors as *savings, financial management, number of income earning persons as well as satisfaction with remuneration* is statistically reliable. This is understood to mean, that material factors are basic determinants in the rating of quality of life as either satisfactory or non-satisfactory.

It should be remembered, however, that the fore-going analysis was an attempt to proffer reasons for satisfaction with the quality of life, whilst ratings for the subjective quality of life are driven not only by economic situations but also by non-economic thus making it a complex category that involves both psychological and sociological aspects. The issue of measurement and assessment therefore, remains open and certainly not exhaustive in terms of the issues covered by the studies [Podolec 2008].

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THE USE OF CORRESPONDENCE ANALYSIS FOR DELPHI SURVEY RESULTS INTERPRETATION

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Abstract: In this paper the use of correspondence analysis for interpretation of the Delphi survey results was described. The analysis is based on the data obtained through the double-round Delphi survey carried out within the project 'Mazovia Economic Information Center'. Analyses were conducted on issues related to the on the economic growth of the Mazovia region. The results obtained within the study indicate a noticeable relationship between the location and size of the company headquarters and the assessment of the significance and likelihood of execution of the selected Delphi theses.

Keywords: correspondence analysis, foresight studies, Delphi method

THE USE OF DELPHI METHOD IN FORESIGHT STUDIES

Foresight, understood as a forward-looking study aims to inform policymakers and opinion leaders about the desired direction of long-term development or change [Nazarko 2013]. The results of foresight projects are used for arranging development scenarios and harmonization of the social partners activities (including economic, scientific and government institutions). At the same time, this process leads to the consolidation of various groups and stakeholders. A wide range of foresight objectives requires a variety of methods and research tools. The foresight research procedure is based on a number of methods derived from forecasting, futurology and the future studies, but also from management, economics and planning [Kononiuk et al. 2008]. Due to the dynamic evoulution of foresight the collection of methods mentioned above is extensive and is still open.

The Delphi method is relatively widely used in the foresight studies. It is estimated that it was used in nearly 80% of the Polish foresight initiatives (about

10% world wide) by the year 2012 [Nazarko 2013]. The Delphi method is a variation of the expert research technique, in which intuitive judgments of experts are treated as a final contribution to the formulation of the vision of the future. This method is particularly used when [Kowalewska et al. 2009]:

- there are no reliable data on the anticipated processes,
- the anticipated processes are mainly affected by external factors,
- there are no precise analytical techniques appropriate for forecasting the analysed phenomena.

The repeated surveys on the same group of experts using the Delphi questionnaire containing arguments and challenge questions are one of the stages of the Delphi method. The questionnaire, usually developed by an expert panel, includes a number of research questions related to the future description of the anticipated relations (Delphi theses). Challenge questions are used for getting information needed to achieve the aim of the study, such aseg. the time and the likelihood the thesis realization, push factors, barriers or the expected effects of the thesis. At the first round experts are encouraged to present their judgments on the development of selected factors within the research area. In the next round they answer the same questions, knowing the overall results (distribution of answers) from the previous round of testing. It is also possible to present some comments (reviews) expressed by the respondents in the first round of testing. In order to preserve the anonymity of the study, the authors of individual opinion is not disclosed. Therefore, there is a remote, asynchronous communication between participants of the Delphi study [UNIDO, 2005]. It helps to avoid domination of strong personalities and leads to the better understanding of the shape of the future. Therefore, the accuracy of recruitment of the survey respondents and members of the expert panel are particularly important. There are two basic approaches for the selection of experts suggested by the literature: the selection of a homogeneous [Kotowska et al. 2005] or heterogeneous group [Steinert 2009, Schuckman 2012]. The heterogeneous selection increases the chance of obtaining a large number of different (extreme) opinions, to be agreed in the next rounds of the study. Such an approach seems to be particularly appropriate in the case of foresight studies that involve the participation of potential stakeholders.

CORRESPONDENCE ANALYSIS AS A METHOD OF QUALITATIVE DATA ANALYSIS

Correspondence analysis (CA) is a descriptive, exploratory technique allowing to define the nature and structure of the relationship between qualitative variables, measured in nominal and ordinal scales [Stanisz 2007]. CA belongs to the group of incomplete taxonomic methods [Górniak, 2000]. This technique, as well as multidimensional scaling, principal component analysis and factor analysis, on the cost of losing some of the information leads to an increase in the transparency of data and simplifies their interpretation. The use of statistics and charts specific for that method allows the researcher for easy, intuitive reasoning on the relationships between the analyzed categories of variables [Panek 2009].

In general, the correspondence analysis could be considered as a method to decompose the overall chi-square statistics by defining a system with a small number of dimensions, in which the deviations from the expected values are presented. CA is a multi-step procedure, that starts from the summary of the data in the contingency table [Bendixen 2003]. CA technique includes the following steps [Gatnar et al. 2004]:

1. Determination of correspondence matrix, row and column profiles and masses[1];
2. Calculation of the distance between the rows (columns) using the chi-squared metric;
3. Presentation of row (column) profiles in the space generated by the columns (rows) correspondence matrix;
4. Determination of the average row and column profiles;
5. Reducing the dimension of space;
6. Plotting the correspondence map, as a common row and column profiles chart (bi-plot).

The aim of the correspondence map construction is to reduce the number of analysed space dimensions by choosing such a low-dimensional subspace in which the chi-square distances between points are shown with the best accuracy [Greenacre et al. 1987]. In this process, the SVD (singular value decomposition) algorithm of decomposition of the matrix with respect to specific values is used [Press et al. 2007]. Interpretation of the correspondence map allows the researcher to find the diversity within the analyzed variables profiles, as well as their co-occurrence of different categories.

In this paper the results of the classical and multiple correspondence analysis of the selected Delphi survey results are presented. The survey was carried out in 2013 in the framework of the "Mazovia Economic Information Center"[2] (MEIC), conducted by the Association of Employers of Warsaw and Mazovia. The calculation procedure was carried out using the STATISTICA PL package[3].

RESEARCH PROCEDURE AND RESULTS

The one of the research topics carried out within the MEIC project, was a pilot foresight study covering the following areas: the innovation of Mazoviabusinesses, local development and economic growth. The main aim of the

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[1] The 'mass' term in CA is used to determine the size of records in the table of relative frequencies. Since the sum of all the values in relative frequency equals 1, the table shows the relative frequencies as one 'mass' unit is distributed in the cells of the table.

[2] Detailed information about the project are available on the official website http://mcig.pl/

[3] StatSoft, Inc. (2011). STATISTICA (data analysis software system), version 10. www.statsoft.com.

study was to develop a model and methodology of foresight studies used for projection of alternative scenarios for the region, as well as to test the model by a pilot experiment. In the first phase, a number of expert analyses were made. The studies were based on such methods and techniques as desk research, expert panels, in-depth STEEPVL and SWOT, structural analysis and scenario method. The result of the first stage was the initial presentation of the proposed scenarios for the Mazowieckie region until the year 2025 [Nazarko 2013].

The Delphi method was used in the second stage of the study. The essential objective of this phase was to confront the obtained results with the opinion of entrepreneurs, as the main group of the project's stakeholders. Delphi theses, as well as challenge questions, were developed by expert panels in relation to the scenario-axes factors, trends and unprecedented events. Theprototypeon-line questionnaire was then made and a double-step verification procedure for its correctness, formulation of questions and user-friendliness was conducted. The verification was carried out in a group of 30 experts, who participated in the mini-Delphi survey. The revised questionnaire was used in a double-roundDelphi survey conducted using CAWI method on 120 representatives of companies established in the Mazowieckieregion.

More than 70% of the respondents were the representatives of micro-enterprises (employing less than 10 people). The next largest group were representatives of small enterprises (employing 10 to 49 people), with 14% share of all the surveyed firms. The share of medium and large companies was 7% and 5% respectively. 47% of respondents accounted for the entreprises located in Warsaw. Warsaw East and West subregions were represented by 28% of respondents. The share of companies located in other subregions amounted for a total of 25% [Dębkowska 2013].

In this paper the interpretation of selected results of the Delphi study conducted in the area of economic growth of Mazovia region are presented. There were six following Delphi theses developed for that area[Dębkowska 2013]:

1. The education system forming an innovative and entrepreneurial approach will be a stimmulating factor for the economic growth.
2. Improving the legal system will be one of the most important drivers of economic growth.
3. Development-oriented use of public funds will strengthen the long-term growth.
4. The development of diversified transport infrastructure will improve the accessibility of the region and its internal consistency.
5. Increasing public participation will improve the conditions for economic growth.
6. Deep interference in the international political system will cause an economic crisis in Mazovia region.

The first two theses related directly to the key factors identified as axes of growth scenarios of Mazowieckie region. The next three theses referred to the trends affecting growth. The last thesis represented an unprecedented event, or an event that disrupts trends and occurs for the first time.

For each of the theses above there was the same set of five challenge questions as follows:

1. How do you assess the significance of the thesis for the area of economic growth of Mazoviaregion?
2. When and with what probability, in your opinion, the thesis will be completed or when the processes or phenomena described in the thesis will occure?
3. To what extent the following factors or activities are conducive to the realization of an argument?
4. To what extent the following factors or barriers hinder the execution of the thesis?
5. To what extent it is necessary to take the following actions to implement the thesis?

For questions 3-5, as a result of the work of the expert panel, a set of predefined factors, barriers and the necessary actions required to meet the assessed thesis were developed. In addition, the surveyed entrepreneurs were asked to justify their answers on questions 1 and 2.The respondents in the second round of Delphi study were given the opportunity to view the distribution of responses in the first round as well as the selected justifications.

Initial results of the research

The first question of the questionnaire concerned the significance of each thesis for the area of economic growth of Mazoviaregion. All the theses were highly evaluated in terms of their relevance to the research area and indicated as “very important” or “important” in both the first and in the second round of the Delphi survey. It should be also noted that the percentage of responses indicating theses as important and very important for economic growth of Mazovia region increased in the second round, and simultaneously response rates defining the theses as insignificant or rather insignificant for the area decreased. In the second roundthepercentage of experts who had no opinion in evaluating the significance of the thesis for the area also declined [Debkowska 2013].

The following question of the Delphi survey aimed to determine the most probable period of the execution of each thesis in the future.It was possible to identify three time periods: up to the end of 2013, from 2014 to 2025 or after 2025. In assessing the probabilityan expert could also choose the “never”option.

For the first five theses more than a half of the respondents indicated that the most likely period of their implementation is 2014-2025, and about 30% indicated the year 2025 and after. The nearest future(the end of 2013)was indicated by about 10% of the respondents. In the case of the last thesis, relating to an unprecedented event, in the first round more than 40% of respondents believed that such an event will never happen. In the second round, the percentage of such asnwers has dropped to 30%.

The obtained results have shown the existence of discrepancies in the assessments formulated by the respondents in both the assessment of the significance and probable duration of the theses. That led the authors to use correspondence analysis for in-depth interpretation of the research results. Analyses were performed with respect to the results obtained in the second round of the Delphi survey.

In-depth analysis of the research results

Correspondence analysis was chosen as a research technique regarding the assumption that the size of the company and its location affects the assessment of the significance of theses and alters the projections relating to the probable period of their implementation. The hypothesisabovewas supported by a pre-review of selected comments made by respondents on survey questions. The entrepreneurs comments have confirmed the differences in their opinions with respect to the issues such as i.a. the expected support for the financing of development projects and prospects, in which companies are willing to plan their business activities.

Regarding the observations above the size of the company, measured by the level of employment was additionalytaken into account for all analyzes. As a result, contingency tables and the correspondence matrices for such couples of variables were created: (1) the size of the company –assessment of the significance of the thesis, (2) the size of the company –the duration of the thesis. Subsequently the standard procedure of the classical correspondence analysis was made.

Table 1. Percentage of inertia reproduced in two-dimensional space for various combinations of the analysedvariables

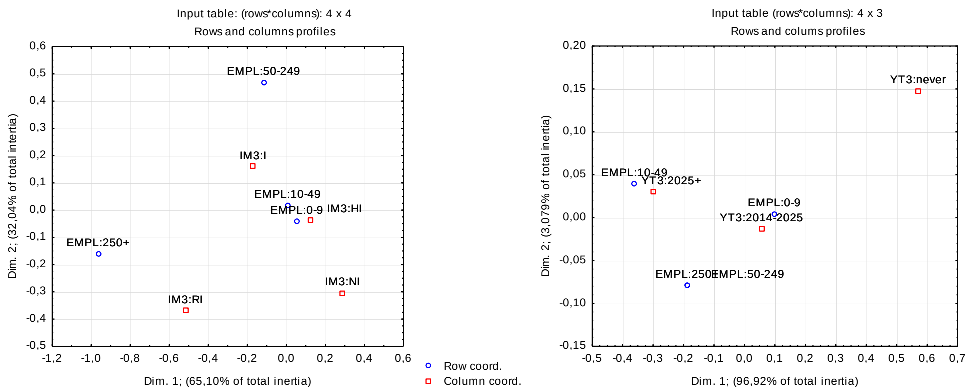
Thesis	Combination of variables			
	(1) size of the company – significance of the thesis		(2) size of the company – duration of the thesis	
	Dimmensions		Dimmensions	
	1	2	1	2
Thesis 1	85,15	96,48	74,84	100,00
Thesis 2	96,89	100,00	91,07	100,00
Thesis 3	65,10	97,14	96,92	100,00
Thesis 4	93,83	98,91	73,13	100,00
Thesis 5	53,00	91,76	92,44	100,00
Thesis 6	71,72	93,59	64,31	95,41

Source: own calculations

It occurred, that projection of row and column profiles in a two-dimensional space allowed us for a very good representation of the original relationship between the variables (Table 1). In allanalyzed cases, the percentage of inertia reproduced in two-dimensional space exceeded 90%. An example of the interpretation of the results obtained in regard to the thesis 3: development-oriented use of public funds will strengthen the long-term growth is presented below. In case of that thesis, the vast majority of respondents indicated the important role that public finance management by the authorities play in stimulating development of the region.

Figure 1.Selected correspondence maps for the 3rd thesis in "economic growth" area

size of the company – significance of the thesis	size of the company – duration of the thesis



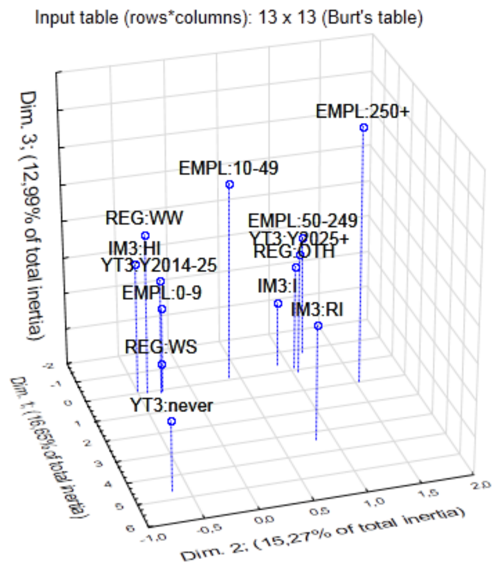
EMPL – employment (0-9; 10-49; 50-249, 250+)
IM*n* – significance of the *n* thesis (HI – very important, I – important, RI – rather important, NI –not important)
YT*n* – duration of the *n* thesis (2013; 2014-2025; 2025+; never)

Source: own calculations

The analysis of two-dimensional correspondence maps relating to the 3rd thesis leads to the conclusion that it was assesesed as important or very important for the processofMazovia region economic growth mainly by micro and small enterprises (employing up to 49 people). The largest enterprises, employing more than 250 employees, were significantly standing out. To some extent itmight be explained by the past experience of entrepreneurs concerning the use of EU funds, support business activities, the beneficiaries of which were mostly small and medium enterprises. With regard to the period in which the phenomenon described in the thesis can be most likely expected companies with no more than 9 employees forecasted earlier scenario exercise (indicating years 2014-2025), while the companies employing 10-49 people indicated a longer-term perspective (after 2025). Definitely distant answer was – inregard to the third thesis –"never" option, which did not identify any of the groups of entrepreneurs.

In order to extend the scope of this interpretation a common analysis including such variables as (1) the size of the company, (2) its location, (3) the assessment of the significance of the thesis and (4) the expected duration of its implementation was made. The projection of variables was performed using a three-dimensional map of correspondence (Fig. 2). For the variables being analysed, it allowed mapping of 44.9% of the total inertia.

Figure2. Three-dimensional correspondence map for the 3rdthesis in “economic growth” area



EMPL – employment (0-9; 10-49; 50-249, 250+)
REG – subregion (WW – Warsaw, WS – surroundings of Warsaw, OTH – other)
IM*n* – significance of the *n* thesis (HI – very important; I – important; RI – rather important; NI –not important)
YT*n* – duration of the *n* thesis (2013; 2014-2025; 2025+; never)

Source: own calculations

It can be observed, that there are two distinct clusters of points on the obtained map (Fig. 2). The first cluster contains representatives of small enterprises (EMPL: 0-9), indicating that the most probable duration of the thesis is 2014-25 period, assessing the 3rd thesis as very important (IM3: HI), located in Warsaw or its immediate vicinity (REG: WW; REG: WS). The second group are larger companies (EMPL: 50-249), located on the outskirts of the region (REG: OTH) evaluating the 3rd thesis as important (IM3: I), indicating that the most probable duration of the thesis is the year 2025 or later (YT3:2025+). The analysis of the relative positions of points on the graph confirms previous findings. It justifies the assumption of a different interpretation of the thesis by the representatives of firms located in Warsaw and its closest neighbourhood and the companies located in the outskirts of the region. It seems that the first group of enterprises, that usually are a kind of natural leaders and located in the area that mostly determines the rate of economic growth of the whole region, expect earlier implementation of the phenomena presented in the thesis.

SUMMARY

Correspondence analysis presented in this paper is a technique that allows for a relatively rapid, cross-sectional analysis of various qualitative data. Graphical presentation of the initial results of the survey allows to find the relationship between the analyzed variables, which is a good starting point for an in-depth interpretation of the results. This technique significantly simplifies the interpretation of the results of Delphi survey, where a vast majority of the obtained data is described by qualitative variables.

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THE APPLICATION OF REGRESSION ANALYSIS IN TESTING UNCOVERED INTEREST RATE PARITY

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Abstract: The aim of the paper is to evaluate and compare two linear regression models proposed by Froot and Frankel (1989) and to show their application in verification of the uncovered interest rate parity (UIP) hypothesis in the selected ten exchange rate markets. The paper shows that both models proposed by Froot and Frankel (1989) are formally equivalent, but they may give different regression results. Many researchers claim that uncovered interest rate parity tends to hold more frequently in emerging markets than in developed economies. The paper is focused on five developed and five emerging economies. It is partly confirmed that developing countries work better in terms of UIP.

Keywords: uncovered interest rate parity, exchange rate, linear regression model

INTRODUCTION

Uncovered interest rate parity (UIP) is a key relationship in international finance. UIP represents the cornerstone parity condition of many fundamental equilibrium exchange rates models such as capital enhanced equilibrium exchange rate model, behavioral equilibrium exchange rate model and monetary model of exchange rate. Uncovered interest rate parity states that high-yield currencies should depreciate and low-yield currencies should appreciate. It needs to be emphasized that UIP parity holds only when investors are risk neutral and have rational expectations. Very often however, we observe the tendency of low interest-yielding currencies to depreciate rather than appreciate as UIP suggests. Deviations from UIP is generally called the forward premium puzzle.

Froot'a and Frankel'a [1989] propose two linear models which may be applied for testing uncovered interest rate parity. The aim of the paper is to assess these models and additionally, to test whether uncovered interest rate parity holds for chosen ten countries. Research is conducted for five developed and five emerging economies. The remainder of the paper is organized as follows. Section UNCOVERED INTEREST RATE PARITY reviews the relevant literature concerning uncovered interest rate parity. Moreover, this section presents two linear models proposed by Froot'a i Frankel'a [1989] for testing uncovered interest rate parity. Section STATISTICAL TESTS FOR REGRESSION MODELS … discusses possible differences in drawing conclusions about UIP on the basis of these two linear models. The empirical results are described in the next section. The last section provides concluding remarks drawn from the empirical research.

UNCOVERED INTEREST RATE PARITY

Covered interest rate parity (CIP) states that the ratio of domestic and foreign interest rates should equal to the ratio of forward and spot exchange rates. CIP can be expressed as follows:

$$\frac{1+r_t}{1+r_t^*}=\frac{F_t^{(k)}}{S_t} \tag{1}$$

where S_t is the price of foreign currency in units of domestic currency in time *t*, $F_t^{(k)}$ is the forward value of exchange rate for a contract agreed in time *t* for an exchange of currencies *k* periods ahead, r_t and r_t^* are domestic and foreign interest rates respectively (with *k* periods to maturity).

Uncovered interest rate parity (UIP) describes the relationship between interest rates and expected exchange rate changes.

$$\frac{1+r_t}{1+r_t^*}=\frac{E(S_{t+k}|\Omega_t)}{S_t} \tag{2}$$

where $E(S_{t+k}|\Omega_t)$ is the expected spot exchange rate at time *t+k*, based on information known at time *t*.

Assuming covered interest-rate parity holds the uncovered interest rate parity can be expressed as follows:

$$\frac{E(S_{t+k}|\Omega_t)}{S_t}=\frac{F_t^{(k)}}{S_t} \tag{3}$$

Equation (2) and (3) cannot be directly testable because market expectations of future spot exchange rates are hardly observable. Therefore, uncovered interest rate hypothesis is tested jointly with the assumption of rational expectations in exchange rate market. Under the assumption of rational expectations, the future value of spot exchange rate in time $t+k$ is equal to expected spot exchange rate at time $t+k$ plus a white-noise error term at time $t+k$.

$$S_{t+k} = E_t(S_{t+k} | \Omega_t) + \eta_{t+k} \tag{4}$$

where η_{t+k} is white-noise error term which is uncorrelated with information available at time *t*.

Uncovered interest rate parity hypothesis may be verified by estimating and testing coefficients in two regression models proposed by Froot, Frankel [1989]. The first regression model (5) refers directly to equation (3) and (4).

$$\Delta s_{t+k} = \alpha + \beta \cdot fd_t^k + \eta_{t+k}^k \tag{5}$$

where $\Delta s_{t+k} = s_{t+k} - s_t$, $fd_t^k = f_t^k - s_t$, s_t denotes the logarithm of spot exchange rate at time *t*, s_{t+k} is the logarithm of spot exchange rate at time $t+k$, f_t^k is the logarithm of the *k*-period forward exchange rate. Under the UIP parity condition, the slope parameter β in equation (5) should be equal to unity ($\beta = 1$) and the coefficient α should be equal zero ($\alpha = 0$). It needs to be emphasized that testing uncovered interest rate parity in this form is tantamount to testing the joint hypothesis that market participants are endowed with rational expectations and risk-neutral.

Empirical studies based on the estimation of equation (5) generally reject the UIP hypothesis. Fama [1984], Froot i Frankel [1989], McCallum [1994] show that coefficient β in equation (5) is significantly less than one, and in fact very often closer to minus unity than plus unity. This finding may imply that the forward exchange rate is biased predictor of the spot exchange rate. The violation of uncovered interest rate parity is traditionally called the forward premium puzzle (forward discount bias). Literature provides several explanations of the phenomenon. One possible reason is the existence of a risk premium. Another explanations involve invalidity of the rational expectations hypothesis, peso problems and market inefficiency [Baillie i Bollerslev 2000].

In the second regression model proposed by Froot'a i Frankel'a [1989] forward rate prediction error (a difference between future spot exchange rate s_{t+k} and forward exchange rate f_t^k) is considered to be a dependent variable and forward premium fd_t^k is considered to be an independent variable (6).

$$fd_t^k - \Delta s_{t+k} = \alpha_1 + \beta_1 \cdot fd_t^k + \eta_{t+k}^k \tag{6}$$

where $\beta_1 = 1 - \beta$ and $\alpha_1 = -\alpha$. Under the UIP parity condition, both the slope coefficient β_1 and the coefficient α_1 in equation (6) should be equal zero ($\alpha_1 = 0, \beta_1 = 0$).

When coefficients β_1 and α_1 in equation (6) are equal zero and coefficients β and α in equation (5) are equal one and zero respectively, then both regression models proposed by Froot'a i Frankel'a [1989] are equivalent. It needs to be emphasized however, that although these models are formally equivalent they may give different regression results.

STATISTICAL TESTS FOR REGRESSION MODELS WHICH VERIFY THE UIP

Let us consider two regressive models:

$$y_i = \alpha + \beta \cdot x_i + \varepsilon_i \tag{7}$$

$$x_i - y_i = -\alpha + (1-\beta) \cdot x_i + \eta_i \tag{8}$$

Where y is the endogenous variable, x exogenous variable, α and β are the structural parameters of the model, and $\eta_i = - \varepsilon_i$

After estimating the coefficients of the models we get:

$$\hat{y}_i = a + b \cdot x_i \tag{9}$$

$$x_i - \hat{y}_i = a + (1-b) \cdot x_i \tag{10}$$

where $\hat{y}_i$ is an estimate of the dependent variable, and a and b are estimates of the structural parameters of model (7).

Let's introduce markings:

$$\begin{gathered} S_{yy} = \sum_{i=1}^{n} (y_i - \bar{y})^2, S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2, S_{yx} = \sum_{i=1}^{n} (y_i - \bar{y}) \cdot (x_i - \bar{x}), \\ SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2, SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 \end{gathered} \tag{11}$$

where n is the number of observations.

The basic statistical test based on F statistics, where the number of degrees of freedom for fraction equals 1 and for denominator equals n-2, verifies the existence of a linear relationship between the dependent variable and explanatory variable. We set the null hypothesis which states that the directional coefficient in the regressive model is insignificantly different from 0 (which means no linear relationship between the variables x and y). In the case of model (7) this hypothesis means that coefficient $\beta = 0$ whereas in the model (8) $\beta = 1$. For model (7) test statistic, which we denote as F_1 is:

$$F_1 = \frac{SSR}{SSE} \cdot (n-2) \tag{12}$$

For the model (8) after translation we obtain statistic F_2:

$$F_2 = F_1 + \frac{S_{xx}}{SSE} \cdot (n-2) - \frac{S_{xy}}{SSE} \cdot 2 \cdot (n-2) \tag{13}$$

F_2 is greater than F_1 if S_{xx} is greater than $2 \cdot S_{xy}$. In this case, the significance of F_1 is greater than the significance of F_2. It may therefore happen that a certain level the significance of the hypothesis about no linear relationship between the variable *x* and *y* cannot be rejected, but you can reject the hypothesis of no linear relationship between the variables *x*-*y* and *x*.

The hypothesis formulated above can also be verified by means of *t* statistic. For model (7) this statistic is determined by the formula:

$$t_1^{\beta} = \frac{b}{S(b)} = \frac{\frac{S_{xy}}{S_{xx}}}{\sqrt{\frac{SSE}{S_{xx}} \cdot \frac{1}{n-2}}} = \frac{S_{xy}}{\sqrt{\frac{S_{xx} \cdot SSE}{n-2}}} \tag{14}$$

where *S* (*b*) is the estimation error of coefficient *b*.

t statistics for model (8) is equal to:

$$t_2^{1-\beta} = \frac{1-b}{S(1-b)} = \frac{1 - \frac{S_{xy}}{S_{xx}}}{\sqrt{\frac{SSE}{S_{xx}} \cdot \frac{1}{n-2}}} = \sqrt{\frac{S_{xx}}{SSE} \cdot (n-2)} - t_1 \tag{15}$$

Statistics t_1^{β} can verify the hypothesis that β parameter is significantly different from zero whereas $t_2^{1-\beta}$ that β parameter is significantly different from *1*. The test based on *F* statistics is equivalent to the test based on *t* statistics are the following equalities are met:

significance F_1 = significance t_1^{β}, significance F_2 = significance $t_2^{1-\beta}$ (16)

If model (7) uses $t_2^{1-\beta}$ statistics (which enables the verification of the hypothesis that β coefficient is significantly different from *1*) it is also necessary to use F_2statistics, rather than F_1, because the F_1 significance will differ from $t_2^{1-\beta}$ significance.

For the tests which verify the significance of intercept α, they are equivalent for both models, except that the empirical values for *t* test have different signs.

If we want to verify the hypothesis of uncovered interest rate parity, first we must verify the hypothesis that the parameter $\beta = 1$, and the parameter $\alpha = 0$. It is better than to use model (8)[1], as the standard regression analysis enables us to determine the value of the relevant statistics. If we use model (7) the results must be transformed according to formulas (13) and (15). In addition, if S_{xx} is greater than $2 \cdot S_{xy}$, it may be that for a certain level of significance, we cannot reject the hypothesis that the coefficient β in model (7) is insignificantly different from zero, while in model (8) we find that β is significantly different from 1.

EMPIRICAL RESULTS

Several explanations for the UIP failure have been put forward in the literature. Flood and Rose [2002] have shown that uncovered interest rate parity works systematically better for countries in crisis, whose exchange rates are considerably more volatile. Clarida et al. [2009] have obtained consistent results. In their opinion the sign of slope coefficient β depends on market volatility. In high volatility regimes, coefficient β occurs to be positive. Bansal and Dahlquist [2000] have noticed that UIP performs better in developing compared to developed countries. Similar researches have been conducted by Ito and Chinn [2007], Frankel and Poonawala [2010]. According to Bansal and Dahlquist [2000] country-specific attributes such as per capita income, interest rates, inflation and country risk rating are important in explaining deviations form uncovered interest rate parity.

The aim of the paper is to test uncovered interest rate parity hypothesis for chosen five developed and five emerging economies and additionally to check whether results are similar to those obtained by Bansal and Dahlquist [2000]. The UIP hypothesis is verified on the basis of regression models (5) and (6) proposed by Froot'a i Frankel'a [1989]. The models are built for ten exchange rates: USD/GBP, USD/AUD, JPY/USD, CAD/USD, CHF/USD, BRL/USD, MXN/USD, MYR/USD, THB/USD, RUB/USD. Countries were selected on the basis of Ghoshray'a and Morley'a [2012]. The uncovered interest rate parity hypothesis is verified for five developed economies such as United Kingdom (British Pound, GBP), Australia (Australian Dollar, AUD), Japan (Japanese Yen, JPY), Canada (Canadian Dollar, CAD), Switzerland (Swiss Franc, CHF) and for five emerging economies such as Brazil (Brazilian Real, BRL), Mexico (Mexican Peso, MXN), Malaysia (Malaysian Ringgit, MYR), Thailand (Thai Baht, THB), Russia (Russian Ruble, RUB). We use monthly data from Bloomberg on spot exchange rates (s_t) and forward exchange rates (f_t^k) for USD/GBP, JPY/USD, CAD/USD, CHF/USD from January 1995 to December 2012, for THB/USD from September 1995 to December 2012, for MXN/USD from November 1997 to December 2012, for BRL/USD from February 1999 to December 2012, for USD/AUD from September

[1] In this model the dependent variable y_i is $fd_t^k - \Delta s_{t+k}$, while the independent x_i is fd_t^k

2000 to December 2012, for RUB/USD from August 2001 to December 2012 and for MYR/USD from April 2005 to December 2012. Time periods for exchange rates are different because of data limitations.

Table 1 presents the obtained results of the UIP test on the basis of regression model (5). Models are marked as M1 (USD/GBP), M2 (USD/AUD), M3 (JPY/USD), M4 (CAD/USD), M5 (CHF/USD), M6 (BRL/USD), M7 (MXN/USD), M8 (MYR/USD), M9 (THB/USD) and M10 (RUB/USD). Table 1 displays coefficients, their standard errors, test statistics and their corresponding significance levels.

Table 1. OLS estimation results of equation (5) for ten exchange rates

Model	*b* S(*b*)	a S(*a*)	F_1	Sig. F_1	t_1^{β}	t^{α}	Sig. t^{α}
M1 - USD/GBP	-1,252 (1,784)	-0,0008 (0,0022)	0,493	0,483	-0,702	-0,391	0,696
M2 - USD/AUD	0,477 (2,681)	0,0056 (0,0072)	0,032	0,859	0,178	0,770	0,443
M3 - JPY/USD	-1,175 (1,240)	-0,0038 (0,0040)	0,898	0,345	-0,948	-0,948	0,344
M4 - CAD/USD	-2,858 (1,854)	-0,0017 (0,0017)	2,376	0,125	-1,541	-1,047	0,297
M5 - CHF/USD	-2,831 (1,675)	-0,0067 (0,0037)	2,858	0,093	-1,691	-1,792	0,075
M6 – BRL/USD	0,523 (0,805)	-0,0047 (0,0083)	0,422	0,517	0,649	-0,560	0,577
M7 - MXN/USD	-0,045 (0,419)	0,0028 (0,0035)	0,012	0,915	-0,107	0,798	0,426
M8 - MYR/USD	0,339 (0,918)	-0,0024 (0,0020)	0,136	0,713	0,369	-1,187	0,239
M9 - THB/USD	1,780 (0,489)	-0,0038 (0,0027)	13,253	0,0003	3,641	-1,397	0,164
M10 - RUB/USD	1,592 (0,299)	-0,0069 (0,0027)	28,376	4,1E-07	5,327	-2,525	0,013

Source: own estimations

The slope coefficient is significantly different from zero in models M5, M9 and M10 with at least 90 percent level of confidence. Furthermore, only in M5 and M10 estimates for α are significantly different from zero. For exchange rates USD/GBP, USD/AUD, JPY/USD, CAD/USD, BRL/USD, MXN/USD and MYR/USD both slope coefficient β and intercept coefficient α are insignificant. Only for CHF/USD, THB/USD and RUB/USD slope coefficient β is significant, which implies that there is linear relationship between dependent variable (Δs_{t+k}) and independent variable (fd_t^k). For CHF/USD coefficient β equals -2,8313, for THB/USD β equals 1,7796 and for RUB/USD β equals 1,59919. These results confirm the findings of Bansal and Dahlquist [2000]. The slope coefficients for emerging economies (Thailand and Russia) are positive and for developed economies (Switzerland) negative. As was mentioned before, UIP is satisfied if the parameter β in the first model (5) is positive, and unsatisfied when β is negative. The estimation signs of β coefficients confirm the results obtained by Bansal and Dahlquist [2000]. For the currencies of emerging countries (Brazil, Malaysia, Thailand, and Russia) parameters β are positive, while for developed countries (United Kingdom, Japan, Canada and Switzerland) negative. Only for Australia and Mexico Bansal and Dahlquist's [2000] thesis has not been confirmed. It needs to be emphasized, however, that for most of the models coefficient β was insignificant. It suggests that presented findings should be treated with some caution.

Table 2 displays regression results of equation (6). Models are marked as M1′ (USD/GBP), M2′ (USD/AUD), M3′ (JPY/USD), M4′ (CAD/USD), M5′ (CHF/USD), M6′ (BRL/USD), M7′ (MXN/USD), M8′ (MYR/USD), M9′ (THB/USD) and M10′ (RUB/USD). The table below contains slope coefficient estimates, standard errors, test statistics and their corresponding significance levels (p-value). Moreover, the value of S_{xx}, S_{xy} and *SEE* are provided, which enable to apply formulas (13) and (15).

The slope coefficient is insignificantly different from zero in models M1′, M2′, M6′, M8′ i M9 with at least 10% significance level. This is equivalent with statement that β in equation (5) is insignificantly different from one, which implies that UIP hypothesis cannot be rejected. In other models slope coefficient is significant which means that forward rate prediction error $\Delta s_{t+k} - fd_t^k$ can be explained by forward premium fd_t^k. Moreover, when we increase the significance level to 0,115, the coefficient β in model M9′ will be also significant.

Estimation results imply that for JPY/USD, CAD/USD, CHF/USD, MXN/USD and RUB/USD exchange rates, uncovered interest rate hypothesis is rejected. However, there are no foundations to reject $\beta_1 = 0$ null hypothesis for USD/GBP, USD/AUD, BRL/USD, MYR/USD and THB/USD with at least 10% significance level.

Table 2. OLS estimation results of equation (6) for ten exchange rates

Model	b S(b)	F_2	sig. F_2	$t_2^{\beta_1}$	S_{xx}	S_{xy}	SEE
M1′ - USD/GBP	2,252 (1,784)	1,595	0,208	1,263	0,0002	-0,0002	0,126
M2′ - USD/AUD	0,523 (2,681)	0,038	0,846	0,195	0,0002	0,0001	0,232
M3′ - JPY/USD	2,175 (1,240)	3,077	0,081	1,754	0,0007	-0,0008	0,228
M4′ - CAD/USD	3,859 (1,854)	4,330	0,039	2,081	0,0002	-0,0005	0,125
M5′ - CHF/USD	3,832 (1,675)	5,234	0,023	2,288	0,0004	-0,0010	0,221
M6′ – BRL/USD	0,478 (0,805)	0,352	0,554	0,594	0,0043	0,0022	0,456
M7′ - MXN/USD	1,045 (0,419)	6,230	0,014	2,496	0,0051	-0,0002	0,161
M8′ - MYR/USD	0,661 (0,918)	0,519	0,473	0,720	0,0004	0,0002	0,034
M9′ - THB/USD	-0,780 (0,489)	2,354	0,112	-1,595	0,0050	0,0088	0,243
M10′ - RUB/USD	-0,592 (0,299)	3,923	0,050	-1,981	0,0087	0,0138	0,104

Source: own estimations

Estimations of equation (6) do not provide unambiguous results. Uncovered interest rate parity holds for two developed economies (United Kingdom and Australia) and two emerging economies (Brazil and Malaysia). However, the UIP hypothesis is rejected for three developed countries (Japan, Canada and Switzerland) and for three developing countries (Mexico, Thailand and Russia). The conclusions were drawn on the basis of evaluation of slope coefficient β_1. The intercept coefficient α has been insignificantly different from zero for United Kingdom, Australia, Japan, Canada, Brazil, Mexico, Malaysia and Thailand. It may imply the existence of transaction costs or non-zero risk premium. Only for Switzerland and Russia α has occurred to be significant.

From a statistical point of view, model (6) seems to be better than model (5). In the first regression model, most slope coefficients β was insignificantly different from zero. Hence, it was not sensible to test whether it is different from 1. The estimation of equation (6) gives insignificant slope coefficient only in four cases. A difference in significance of F-test statistics in models (5) and (6) results from the test structure. This is because the probability distribution of the test statistic is determined by the null hypothesis but not by real variable's distribution.

SUMMARY

The paper verifies the hypothesis of uncovered interest rate parity for the currencies of five developed and five developing countries. The results do not give a clear confirmation or denial of the UIP theory. Bansal and Dahlquist [2000] have found that the UIP works systematically better for developing countries while for developed countries the UIP hypothesis is generally rejected. The parity is satisfied if the parameter β in the first model (5) is positive, and unsatisfied when β is negative.

The estimation signs of β parameter in the first model confirm the results obtained by Bansal and Dahlquist [2000]. For the currencies of developing countries (Brazil, Malaysia, Thailand, and Russia), parameter β has positive values, while for developed countries (United Kingdom, Japan, Canada and Switzerland) negative. Only for the exchange rate of Australian and Mexican currency Bansal and Dahlquist's thesis has not been confirmed. It should be noted, however, that parameter β is insignificantly different from zero for most of the models, which suggests that these findings should be treated with some caution.

The results of calculations presented in the paper confirm the advantage of the second regression model (6) over the first model (5). The first model shows that for most exchange rates parameter β is insignificantly different from 0, and therefore it is not advisable to test whether it is different from 1. However, the results of calculations on the basis of the second model, show that this parameter is insignificantly different from 1 only in four cases.

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AN ATTEMPT AT MEASURING THE DEAGRARIANISATION PROCESS OF POLISH RURAL AREAS AT NUTS4 LEVEL

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Abstract: Deagrarianisation is a complex concept, which denotes a declining economic significance of agriculture and agricultural production in society at large, and rural economy and society in particular. The article is an attempt at constructing a composite indicator of the process of deagrarianisation using variables obtained by official statistics. The proposed composite indicator of deagrarianisation is then used to assess the intensity of deagrarianisation of rural areas between 2002 and 2010 in NUTS4 units (Pol. *powiat*) of the province of Wielkopolska.

Keywords: deagrarianisation, rural areas, composite indicator

THE CONCEPT AND DIMENSIONS OFDEAGRARIANISATION

The origin of deagrarianisation processes is closely related to the origin and development of towns and the concentration of inhabitants engaged in non-agricultural activity. These processes seem to be a natural developmental stage of societies, resulting from a kind of competition between the town and the countryside, between agricultural and non-agricultural activity and between the natural and strongly anthropogenic landscape. Nonetheless, many authors focus on negative effects of deagrarianisation. In addition to a diversity of opinions about the very process of deagrarianisation, the literature contains a number of definitions and competing classifications of its aspects. However, they all seem to agree that deagrarianisation is a complex notion, which is why this author believes its measurement and description should draw on multivariate statistical methods.

M. Halamska defines deagrarianisation as a process of decreasing agrarian development, or the declining influence of agriculture on economy, including rural economy and society [Halamska 2011]. She lists the following basic indicators of agrarian development/deagrarianisation:

- the contribution of agriculture to GDP
- the ration of population employed in agriculture to the whole population[1].

M. Halamska also considers two dimensions in which the process of deagrarianisation can be analysed: the subjective dimension, referring to one's social awareness (respondents' declarations concerning their identification with agriculture)and the objective one, which can be determined by a survey of employment in agriculture or income obtained from agriculture.

W. Musiał points out the complexity of the notion of deagrarianisation and considers various attempts at defining it conducted within different areas of knowledge: agriculture and agricultural sciences, economic politics, sociology, economic geography and spatial management, and economics [Musiał 2007]. From the point of view of economics, deagrarianisation can be defined as a historical process of declining importance of agriculture in the national economy, as well as the declining role of agriculture as a source of income for inhabitants of rural areas. W. Musiał presents different approaches to the economic understanding of deagrarianisation: from a microeconomic one, referring to the structure of income and expenditure in rural households, to the regional and environmental approach, to modern aspects of deagrarianisation, such as multi-functionality of rural development, entrepreneurship and diversification of income sources. W. Musiał also describes a synthetic view of deagrarianisation characterised by 4 dimensions:

- production–related (including a decline in the volume of agricultural output, the increasingly common abandonment of agricultural production, institutionalisation and spread of fallowing, a rise in livestock-free farms and a decline in arable land acreage);
- economic (including a declining contribution of agriculture to GNP, a declining contribution of agriculture-related income in rural households, an absolute and relative decline in agricultural employment, a fall in investment in agricultural production);
- social–cultural (including a fall in the number of the rural population engaged in agriculture, increasing social–occupational diversification of the rural population, abandonment of agricultural employment, falling rates of youth education in agricultural occupations, long-term and shuttle economic migration);

[1]It should be pointed out that defining the so-called agrarian character of population may raise doubts as to the choice of a suitable measure (residence with the use of an agricultural holding, possession of an agricultural holding, employment in agriculture, income obtained from employment in an agriculture, etc.).

- ecological and landscape-related (including growing forest cover, a rise in the acreage of fallow land, a rise in the average acreage of arable fields).

J. S. Zegar defines deagrarianisation as a process which accompanies the development of civilisation and involves a decline in the role of agriculture in rural systems [Zegar 2008]. According to the author, the basic indicators of deagrarianisation are:

- demography (changes in the number of people engaged in agriculture);
- job and sources of income (agricultural sources of income);
- natural environment (participation in land management);
- socio-cultural transformations (lifestyle changes, a farmer's work ethos, folk culture).

The author also identifies the agents of deagrarianisation processes, which include: industrialisation, economic mechanisms, globalisation and European integration.

A DESCRIPTION OF THE RESEARCH PROCEDURE AND DATA SOURCES

The definitions and dimensions of the concept of deagrarianisation presented in the previous section confirm its complexity and multidimensional character. The available literature does not mention any attempts at constructing a composite indicator of deagrarianisation. Articles addressing the problem of transformations related to the declining impact of agriculture typically analyse the various aspects of deagrarianisation separately [Halamska 2011].

To fill this gap, the present article is an attempt at developing a composite indicator of deagrarianisation, relying on statistical data from various surveys conducted by official statistics. It should be stressed that the most complete data coverage regarding aspects of deagrarianisation is only possible for census years (the last two agricultural censuses were held in 2002 and 2010). The two censuses offer the largest scope of data and the lowest levels of aggregation, making it possible to obtain certain data at NUTS 5 level (Pol. *gmina*). It should be pointed out, however, that this is possible only with respect to aspects of deagrarianisation related to such units as persons, families, households, farms, buildings and dwellings of people connected with agriculture. In addition to those aspects of deagrarianisation, there are also those, which official statistics cannot provide at NUTS 5 level – for example the contribution of agriculture to GDP.

The research problem presented in this article is therefore to propose a methodology of describing the process of deagrarianisation at the lowest possible level of aggregation and in the inter-census period. It seems that one way of solving the problem is taking advantage of small area statistics methodology, which draws on auxiliary data from sample surveys (such as Labour Force Survey [LFS], Household Budget Survey [HBS], EU-SILC, a survey of farm structure), agricultural censuses (Agricultural Census 2010) and administrative registers (registered unemployment, social security, residence data, PESEL, POLTAX and others). This is the topic of the author's further work.

To construct a composite indicator of deagrarianisation for *powiats* of the province of Wielkopolska on the basis of data from censuses in 2002 and 2010, a set of potential candidates, consisting of the following variables:

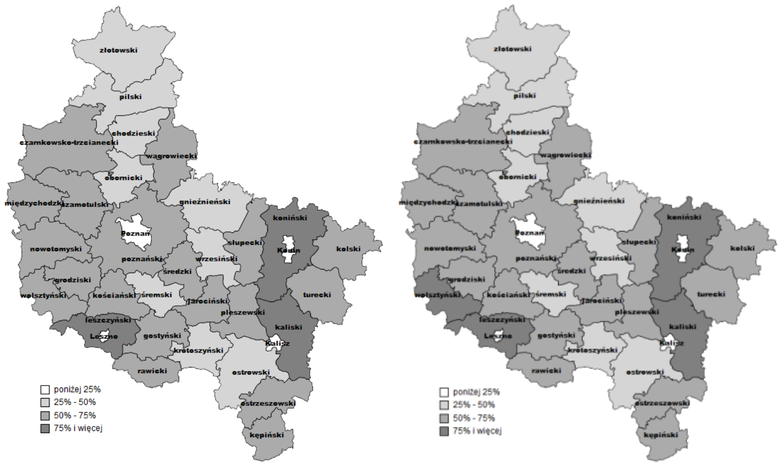
- the percentage of rural population in powiat (X_1),
- the percentage of persons at the pre-productive age residing in a rural area in a powiat (X_2),
- the percentage of population at the productive age residing in a rural area in a powiat (X_3),
- the percentage of population at the post-productive age residing in a rural area in a powiat (X_4),
- the percentage of population residing in a rural area and connected with agriculture in a powiat (X_5),
- the percentage of households in a powiat, which declare income obtained from agricultural activity(X_6),
- the percentage of households in a powiat, which declare income obtained from non-agricultural activity in the powiat (X_7),
- the average acreage of agricultural area in private farms in a powiat (X_8),
- the number of cattle per 100 ha of agricultural area in private farms in a powiat (X_9),
- the number of pigs per 100 ha of agricultural area in private farms in a powiat (X_{10}).

The relevant data come from the Local Data Bank (LDB) published on the website of the Central Statistical Office.

The spatial distribution of selected variables in 2002 and 2010 is presented in Figures 1-3 below[2].

[2] In the figures the cartogram on the left depicts a given phenomenon in 2002 and the one on the right, in 2010.

Figure 1. The percentage of rural population in *powiats* of the province of Wielkopolska in 2002 and 2010



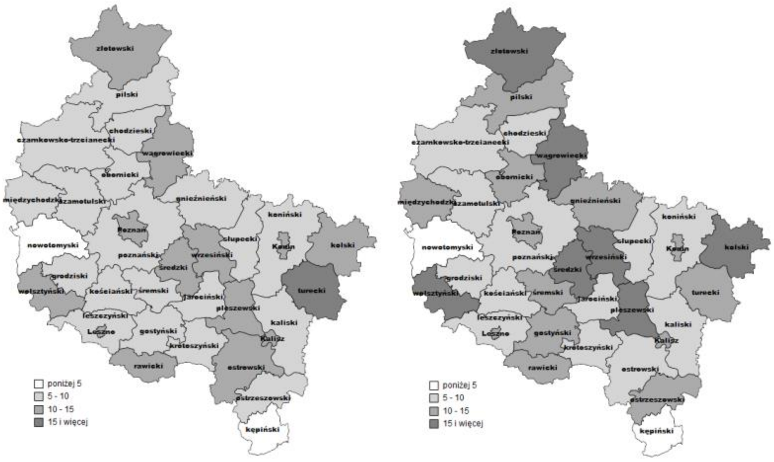
Source: own calculations based on LDB data

Figure 2. The percentage of population connected with agriculture in rural areas in *powiats* of the province of Wielkopolska in 2002 and 2010

Source: own calculations based on LDB data

The above cartograms (Figs. 1-3) show that the biggest changes between 2002 and 2010 involved the percentage of rural population connected with agriculture. In 2002 in three powiats of Wielkopolska this number exceeded 75% (the powiats of Leszno, Kościan and Grodzisk), while in four powiats it was at the level of 50-75% (the powiats of Złotów, Chodzież, Czarnków-Trzcianka and Międzychód). In contrast, in 2010 not a single exceeded the 50% watermark of rural population connected with agriculture. This fact reflects very serious changes that took place in rural areas during the inter-census period, changes whose intensity we are not always aware of.

Figure 3. The average acreage of agricultural area (in hectars) cultivated by private farms in *powiats* of Wielkopolska in 2002 and 2010



Source: own calculations based on LDB data

THE STRUCTURE OF A COMPOSITE INDICATOR OF DEAGRARIANISATION

The process of constructing a composite variable in the study consisted of the following stages:

- specifying a set of potential diagnostic variables,
- reducing the vector of potential diagnostic variables,
- differentiating diagnostic variables into stimulants, destimulants and nominants),
- choosing an aggregation formula

During the first stage, the specified set of potential diagnostic variables (variables X_1–X_{10}) was analysed to reduce the vector of potential diagnostic variables. This stage involved:

- eliminating variables with little variation (coefficient of variation < 10%),
- applying a method based on variable orthogonality (analysis of diagonals of an invertible matrix of coefficients of correlation between variables)

Since all the variables display sufficient variation (the coefficient of variation for all the variables exceeded 10%), the method of analyzing diagonals of an invertible matrix of coefficients of correlation between variables. The results are presented in Tables 1 and 2[3].

Table 1. An analysis of the coefficient of variation for specified diagnostic variables

Variable	Coefficient of variation (%)	
	2002	2010
$\mathbf{X_1}$	46.01	45.57
$\mathbf{X_2}$	44.18	43.79
$\mathbf{X_3}$	46.76	45.74
$\mathbf{X_4}$	46.61	48.04
$\mathbf{X_5}$	69.04	74.48
$\mathbf{X_6}$	43.96	29.68
$\mathbf{X_7}$	47.03	47.90
$\mathbf{X_8}$	32.97	34.45
$\mathbf{X_9}$	46.88	66.04
$\mathbf{X_{10}}$	49.33	107.19

Source: own calculations

To construct a composite indicator of deagrarianisation the following variables were selected: the percentage of population residing in a rural area and connected with agriculture (X5), the percentage of households, which declare income obtained from agricultural activity (X6), the percentage of households, which declare income obtained from non-agricultural activity(X7), the average acreage of agricultural area in private farms (X8), the number of cattle per 100 ha of agricultural area in private farms(X9) and the number of pigs per 100 ha of agricultural area in private farms(X10)[4]. The same set of variables for both target years was characterised by statistical properties that made them suitable to construct the composite indicator. In addition, since all variables (except X7) are destimulants, to ensure that all variables are homogeneous, the X6 variable was converted by means of formulas (1) and (2), for stimulants and destimulants respectively, using the Zero–Unitarisation Method [Kukuła 2000].

Unitarisation for stimulants

$$z_{ij} = \frac{x_{ij} - \min\{x_{ij}\}}{\max\{x_{ij}\} - \min\{x_{ij}\}} \qquad (1)$$

[3]Figures shaded grey represent variables whose values on the main diagonal exceeded 10, and hence were regarded as variables, which duplicated information supplied by other variables from the list of potential variables; consequently and they were not used to construct a composite indicator. Variables in cells with bold borders are part of the proposed composite indicator of deagrarianisation.

[4]It should be noted that though there are some arguments for treating the last two variables (X9 and X10)as stimulants with some veto threshold values, determining these threshold values was beyond the scope of the paper.

Unitarisation for destimulants

$$z_{ij} = \frac{\max\{x_{ij}\} - x_{ij}}{\max\{x_{ij}\} - \min\{x_{ij}\}} \tag{2}$$

The arithmetic mean was used as an aggregation formula.

Table 2. An invertible matrix of coefficients of correlation for specified diagnostic variables for 2002

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
X_1	653592	-165992	-396051	-92587	-305	54	149	-433	76	96
X_2	-165992	42414	100407	23447	77	-24	-50	108	-18	-26
X_3	-396051	100407	240211	56055	185	-24	-86	264	-46	-56
X_4	-92587	23447	56055	13228	43	-7	-14	61	-12	-13
X_5	-305	77	185	43	2	0	0	0	-1	1
X_6	54	-24	-24	-7	0	2	0	0	0	0
X_7	149	-50	-86	-14	0	0	3	1	0	0
X_8	-433	108	264	61	0	0	1	2	1	-1
X_9	76	-18	-46	-12	-1	0	0	1	3	-3
X_{10}	96	-26	-56	-13	1	0	0	-1	-3	3

Source: own calculations

Table 3. An invertible matrix of coefficients of correlation for specified diagnostic variables in 2010

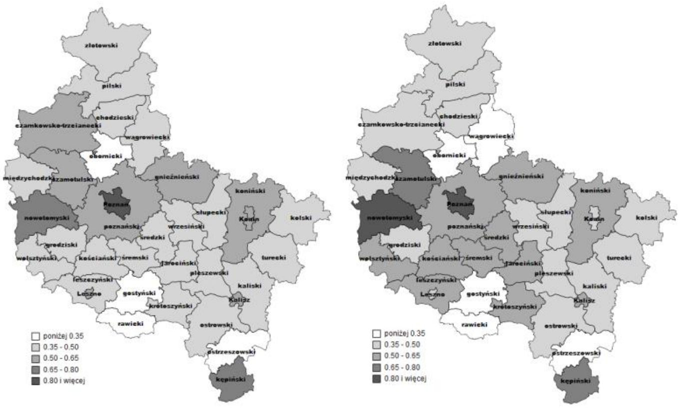
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
X_1	315249	-63929	-207637	-43998	-62	63	-1	-125	-86	160
X_2	-63929	13215	41805	8978	16	-13	3	27	13	-24
X_3	-207637	41805	137230	28805	38	-36	0	77	61	-114
X_4	-43998	8978	28805	6261	8	-15	-2	22	14	-22
X_5	-62	16	38	8	1	0	0	0	0	0
X_6	63	-13	-36	-15	0	7	3	-5	-2	-1
X_7	-1	3	0	-2	0	3	3	-2	0	0
X_8	-125	27	77	22	0	-5	-2	4	2	1
X_9	-86	13	61	14	0	-2	0	2	2	0
X_{10}	160	-24	-114	-22	0	-1	0	1	0	2

Source: own calculations

RESULTS OF CLASSIFICATION– CONCLUSIONS

The resulting classification of deagrarianisation is shown in Figure 4.

Figure 4. The spatial distribution of the value of the composite indicator of deagrarianisation in powiats of Wielkopolska in 2002 and 2010



Source: own calculations based on LDB data

The resulting classification of powiats in the province of Wielkopolska in terms of the degree of deagrarianisation shows that the powiat most affected by deagrarianisation processes is the city of Poznań, and, interestingly enough, the powiat of Nowy Tomyśl. In 2010 it is possible to recognise a certain spatial pattern of deagrarianisation processes: namely, powiats situated close to the city of Poznań and in its south-western neighbourhood are less connected with agriculture. On the other hand, powiats located in the south of the province (the powiats of Gostyń, Rawicz and Ostrzeszów, with the exception of Kępno) and north of Poznań (the powiats of Oborniki and Wągrowiec) are still strongly connected with agriculture.

The results of classification provide a good starting point for further work on the analysis of deagrarianisation processes in rural areas. Further stages of research should involve an attempt at estimating of deagrarianisation processes at a similar level of aggregation, during the inter-census period. Particular attention should be paid to data about the demographic structure and processes. The fact that the proposed composite indicator does not account for them should be considered as a deficiency. Another thing worth pointing out is the need to homogenise variables (a clear distinction between the rural and urban parts of powiats) so that the results can refer precisely to rural parts. Unfortunately, these types of cross-classifications are not always available in data provided by official statistics. That is why the author considers it worthwhile to use the methodology of small area statistics in his future studies. The use of information not contained in the sample, relying on the effects of "borrowing strength" in time and space, could provide a solution to the problem of incomplete data coverage encountered when studying certain complex phenomena. It should also be stressed that the characteristic patterns observed in deagrarianisation processes seem to confirm the need to account for spatial autocorrelation in the analysis of deagrarianisation in rural areas.

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QUANTITATIVE METHODS IN ECONOMICS
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REAL OPTIONS FOR AGRICULTURAL INVESTMENTS

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Abstract: We present an overview of the real options methodology and its applications in agricultural decision making. We examine the current state of research on real options in agriculture as well as propose a range of possible extensions and further applications of diverse types of real options in agricultural investments and their policy implications.

Keywords: real options, agricultural investments, agricultural policy

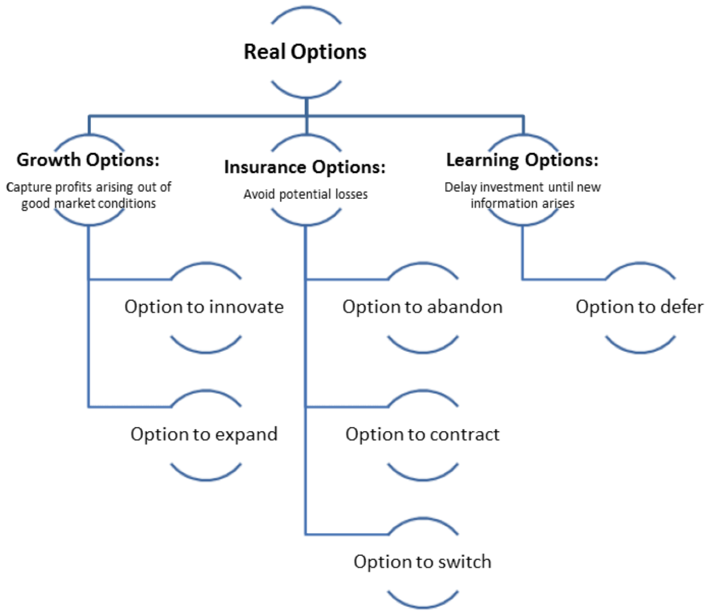
INTRODUCTION

Farmers nowadays are facing important strategic investment decisions (such as, for example switching from traditional to organic farming methods), which are to be made in uncertain environments, in which market growth, price moves, costs of development of new technologies or weather conditions may be unknown or difficult to assess [Dixit and Pindyck, 1994; Pindyck, 1991]. Thus, evaluating the adoption of any investment plan in agriculture must be accompanied by an investigation of the effect of uncertainty and risk. The classical methods of valuation require tools relying on forecasts of future cash flows, which often do not reflect the immanent uncertainty. The second problem is that the decisions about undertaking a particular project are taken once and for all - traditional discounted cash flow approach does not allow for recapitulation at a later stage. Although, in agricultural decisions it is often the case that investments are irreversible or difficult to be changed once have been made i.e. cutting the forest and using land for agricultural purposes instead cannot be easily changed (the forest can be even lost forever if the top soil is destroyed), but one needs a valuation method which is flexible enough to be applied to a broad range of agricultural problems. Theoretical advances in real options methodology have progressed very rapidly and have been assimilated in several empirical applications. Real options have been identified and valued in projects with high uncertainty, and a growing body of literature provides various examples of flexible investment strategies [Tzouramani and Mattas, 2004]. In recent years, real options have been adopted to analyze diverse branches of the economy: R&D investments [Childs and Triantis, 1999; Huchzermeier and Loch, 2001], intellectual capital investments [Bose and Oh, 2003; Kossovsky, 2002], patent valuation [Laxman and Aggarwal, 2003; Ming-Cheng and Chun-Yao, 2006] etc. all of which reveal similar characteristics to agricultural investment decisions, namely the high level of uncertainty. Only a limited number of studies, however, have implemented real options in agriculture. In this work, we focus on practical examples of how real options methodology can facilitate investment decision making in agriculture. The rest of the work is structured as follows. Section REAL OPTION METHODOOGY presents the overview of the real options valuation method. Section REAL OPTIONS IN AGRICULTURE presents an extensive survey of the literature related to the use of real options in agricultural decision–making processes. Section FURTHER APPLICATIONS presents further applications, for which we claim that the real options approach should be used. The last section concludes.

REAL OPTIONS METHODOLOGY

Based on Black and Scholes [1973], Myers [1987] developed the idea of a real option, as the right but not the obligation to purchase a real asset. In analogy to financial options, a real option which gives the owner the right, but not the obligation to make a specific investment at a specific price in the future. The two most common styles of options are European and American options. The former may be exercised only at the pre–specified expiration date of the contract, whereas the latter can be exercised at any time before the expiration date. Each style reflects a specific investment situation: the former works for investments that need to be taken at a specific time (e.g. investment in pest control of crops, investment in planting a specific type of a crop), the latter for opportunities that can be exercised in a less specified future (e.g. switching o organic farming methods, building a greenhouse etc.) By construction, the real options approach allows explicit inclusion of the uncertainty in the decision-making process. The real options approach considers multiple decision pathways as a consequence of uncertainty in choosing optimal strategies or options along the way when new information becomes available [Mun, 2006]. Figure 1 presents the overview of types of real options.

Figure 1: Main types of real options



Source: own work

Each type of a real option corresponds to a analogous financial option e.g. option to delay investment to the future can be seen as an American call option from the perspective of the investor e.g. a farmer: like the owner of a call, the farmer has the right, but not the obligation to pay a fixed sum I and to receive a stochastic cash flow with a discounted value V (t). Standard discounted cash flow approach prescribes to conduct the investment as long a $V - I > 0$, whereas this formula does only account for the intrinsic value of the investment, and neglects the continuation value[1]. To include the uncertainty associated with the investment, the real option approach would prescribe that the value of the investment changes over time according to

$$(dV(t))/V = \mu dt + \sigma dz, \quad (1)$$

a standard Brownian motion, where σ is the variance, and *dz* is the increment of the Wiener process, for which it holds that

$$dz = \varepsilon_t \sqrt{dt}, \quad (2)$$

[1] For additional details, please refer to financial options literature e.g. Black and Scholes [1973], or real options literature e.g. Trigeorgis [1996].

where ε_t ~ (0, 1). Therefore, changes in the value of the investment are a function of μ and σ, which is governed by the increment of the Wiener process *dz* [Dixit and Pindyck, 1994]. The new decision rule, taking into account the future changes of value is a solution to the corresponding Bellman equation

$$F(V,t) = max\left\{V(t) - I, \frac{1}{1+kdt} * E(F(V + dV, t + dt)|V(t))\right\} \quad (3)$$

where *F* denotes the value of the option to invest, *k* is a discount rate, and *E* indicates the expectations operator. Since finding the maximand of (1) requires solving a partial differential equation, in case of complicated real options, in particular most of the American style options, an analytical solution might not be possible to find, in which case, one can refer to numerical methods, such as the binomial lattice method or Monte–Carlo simulation.

Let us turn to a detailed description of the real options by types. According to classification found in Figure 1 ”a growth option” allows a decision–maker to secure profits if the market conditions occur to be better than expected. This can be accomplished by reinvesting the capital e.g. expanding the scale of production or entering new market sectors. Additional sources of the growth options comprise R&D and innovation, intellectual property and change in the market position. Growth options are the equivalent to a financial call option. This kind of option can be used whenever a farmer decides to expand his operations. Once a project is undertaken, he may have the flexibility to alter it in various ways at different times during its life. When a farmer buys vacant, undeveloped land, or when he builds a new facility in a new location to position itself to develop a large market, he essentially acquires an expansion option.

”Insurance options” allow the management to scale down or abandon certain investments in order to avoid potential losses. Therefore, it is mainly seen as risk–reducing option. The value of the option stems from the opportunity to postpone or abandon unprofitable investments. Insurance options are equivalent to financial put options. In agriculture, insurance options can be found in various situations. A prominent example for an insurance option would be a pre–payment strategy in the wine industry or long–term contracts for delivery of certain types of crops. From a progressive point of view, insurance options are used in situations in which it becomes obvious that variable costs exceed market prices. The option to switch can also be seen as an insurance options. In this, they include situations in which one can use agricultural land in diverse way e.g. switch to a different type of a crop or plant a coppice instead of growing crops.

Finally, ”learning options” allow the decision-maker to ”wait and see”. In other words, they allow the decision-maker to defer decisions regarding investments. The value of the option stems from the opportunity to wait for the resolution of uncertainties before committing resources to investment. A specific type of learning options, the deferral option, or option of waiting to invest, derives its value from reducing uncertainty by delaying an investment until more information has arrived. These types of options are of particular importance in agricultural investments. The scope of applications of the deferral option in agriculture is described in more details in the next two sections.

REAL OPTIONS IN AGRICULTURE

In this section we provide an overview of recent applications of the real option method in agricultural decision–making. As the literature on real options in agricultural investments is fairly scarce, the following survey is structured chronologically rather than subject to the topic covered. It can be, though, seen that the first strand of real options applications for agriculture concentrates on the irreversible investments, whereas more recent works broaden the scope into more diverse applications. One of the first works that applied the real option approach in agriculture is Purvis et al. [1995] who analyze the technology adoption of a free-stall dairy housing under irreversibility and uncertainty and its implications for the design of environmental policies. The free-stall investment involves significant start–up costs and limited potential for recouping the investment capital quickly should it become necessary to disinvest [Purvis et al. 1995]. Authors use data from early adopters in central Texas and calculate expected returns from investing in a 1000-cow free-stall facility. Subsequently, they compare the expected returns from the technology change as compared to the prevailing technology, open-lot dairying, assuming that the returns are influenced by two stochastic factors: milk production and feed costs. Authors conclude the analysis with relevant policy implications: they identify that adoption of the new technology can be precluded by the risk associated with investment and uncertainty regarding the investment cost. Moreover, authors quantify the magnitude of these factors and provide policy recommendation of subsidizing producers willing to adopt the new technology in order to obtain an optimal level of investment.

Winter-Nelson and Amegbeto [1998] analyze present a model of investment under uncertainty to analyze the effect of variability of prices on the decision to invest in soil conservation and apply it to the case of adoption of Fanya Juu terraces in eastern Kenya. Authors claim that changes in policy that lead to increase in the output prices encourage agricultural investment, however, simultaneous increase in price volatility could reduce incentives to invest. Real options model is in this case the most appropriate method of analysis as it directly incorporates the underlying investment risk. They test the hypothesis and find confirmation of the assumptions, which then leads to policy implications. Empirical study shows that commodity market liberalization changes the value of options enough to influence terrace adoption in Kenya. This result also highlights the need of economic institutions to moderate the price movements during and after market reforms.

Yet another application of real options for the case of irreversible investments with high sunk costs comes from Price and Wetzstein [1999], who analyze the market for perennial crops. In particular, production of perennial crops such as peaches requires a large sunk cost of orchard establishment and high uncertainty of future yields and prices. Similarly to previous cases, addition of the uncertainty to the model results in different decisions than ones arising from a standard NPV approach. Incorporating price and yield uncertainty, the irreversibility of the investment decision, and the value of the option to delay production enters into the decision, results in a 120% increase in the entry threshold and a 3% decrease in the exit threshold from the thresholds found conventionally [Price and Wetzstein, 1999]. Another agricultural application of a model of irreversible investment under uncertainty can be found in Tegene et al. [1999], who analyze landowner's decision to convert farmland to urban use. Application of the real option method leads to a conclusion that even under certainty about future returns to urban use, it may be optimal to delay conversion in order to realize returns to agriculture until they are exceeded by urban returns net of conversion costs, even if urban value already exceeds agricultural value Tegene et al. [1999]. Moreover, it seems that conservation easements are currently underpriced and that farmland owners might be reluctant to sell easements at the prices offered. In fact, conventional valuation procedures may also systematically overprice easements by inadvertently exaggerating the urban returns (based on comparisons with already-converted parcels) that could be realized on yet-to-be converted parcels of agricultural land.

Finally, Khanna et al. [2000] applies an option-pricing model to analyze the impact of uncertainty about output prices and expectations of declining fixed costs on the optimal timing of investment in site-specific crop management (SSCM). It also analyzes the extent to which the level of spatial variability in soil conditions can mitigate the value of waiting to invest in SSCM and influence the optimal timing of adoption and create a preference for custom hiring rather than owner purchase of equipment. Numerical simulations show that while the net present value (NPV) rule predicts that immediate adoption is profitable under most of the soil conditions considered here, recognition of the option value of investment indicates that it is preferable to delay investment in SSCM for at least 3 years unless average soil quality is high and the variability in soil quality and fertility is high. The use of the option value approach reveals that the value of waiting to invest in SSCM raises the cost-share subsidy rates required to induce immediate adoption above the levels indicated by the NPV rule [Khanna et al. 2000].

The second strand of research initiated in the beginning of 2000s, further developed the idea of real options in agriculture, beyond the cases of irreversible investments. Among miscellaneous topics, current agricultural and environmental topics such as organic farming, fisheries sustainability and pest control are covered. Work by Kuminoff and Wossink [2005] is the first attempt to analyze the farmer's decision to convert from traditional to organic farming using the real options approach. Using county data on organic and conventional corn and soybean production in the U.S., authors develop a theoretical model to assess the dollar compensation required for the conversion to organic farming. Authors relate their result to policies that can ease adoption of organic farming, and estimate with the example of the Conservation Security Program adopted in the U.S., the size and the duration of the necessary payments to the farmers.

Optimal pesticide control of crops is a subject of works by Saphores [2000] and Mbah et al. [2010]. The former paper formulates an optimal stopping model for applying pest control measures when the density of a pest population varies randomly. A delay between successive pesticide applications is introduced to analyze the farmer's expected marginal cost of reentry. This model is applied to the control of a foliar pest of apples via a pesticide, and solved numerically. A sensitivity analysis shows that the pest density that should trigger pesticide use can vary significantly with the pest density volatility. Incorporating pest randomness into decision rules helps better manage the chemicals applied to soils and crops [Saphores, 2000]. The latter work by Mbah et al. [2010] who develop a framework to examine the economically optimal timing of pest treatment. The real option analysis suggests that the decision to treat should only be undertaken when the benefits exceed the costs by a certain amount and not if they are merely equal to or greater than the costs as standard net-present-value (NPV) analysis would prescribe. This criterion leads to a large reduction in fungicide use which is associated with significant economic and environmental benefits. Authors then investigate the effect of the model for disease progress on the value required for immediate treatment by comparing two standard models for disease increase. Analyses show that the threshold value of benefits required for immediate release of treatment varies significantly with the relative duration of the agricultural season, the intrinsic rate of increase of the disease and the level of uncertainty in disease progression.

A fistful of other topics has been additionally analyzed up to now; Musshoff [2012] examines the decision to grow short rotation coppice on agricultural land in Germany, Fenichel et al. [2008] analyses precautionary fisheries management; in Odening et al. [2005] option–pricing theory is applied to an investment problem in hog production; finally Luong and Tauer [2006] explain with the use of the real option theory a significant growth in coffee production in Vietnam in the recent years.

FURTHER APPLICATIONS

In the previous section we examined the current strand of research of real options for agricultural applications. The presented overview highlights the broad scope of possibilities of application of the real options method in this context. This part of the work is dedicated to presenting possible further applications and research that can help modernize decision making in agriculture. The previous section has also highlighted the main advantage of the real options methodology over other valuation methods, as it allows to make predictions which include the value of risks associated with investments. Table 1 presents an extract of main uncertainty and risk factors facing agricultural production and types of insurance procedures against each of them, which will be also described in more detail later on. As a matter of fact, whenever any of these risks/uncertainties appear as a factor of a valuation of opportunities stemming from investments, one should refer to real options methodology, as it allows to directly incorporate them into the value of the decision. In the traditional NPV approach, estimation of the discounted stream of profits does not explicitly include the volatility of the stream of profits, resulting from the identified risks. In the presence of substantial risks, that affect agricultural production, as identified in Table 1, exclusion of this information from the model should be considered inappropriate and may results in wrong decisions. An example of how real options incorporate directly the issue of uncertainty, has been presented e.g. in Winter-Nelson and Amegbeto [1998].

The following overview is structured according to the main types of real options as presented in Figure 1. Among the presented literature items, what can be qualified as ”growth options” are applications presented in Winter-Nelson and Amegbeto [1998], Price and Wetzstein [1999], Khanna et al. [2000] and Luong and Tauer [2006]. All of these works deals with the value of irreversible investments which can, but not necessarily will, lead to an increase in productivity of the farmer. This type of reasoning can be applied to many agricultural decisions, in which a farmer is presented with an opportunity to build his strategy on a substantial investment, often associated with high sunk costs, but leading to an increase in productivity. Other than straightforward options to expand production, an obvious example of growth options in agriculture are innovations. A farmer can decide to adopt a multitude of innovations, which would increase his productivity, whereas we concentrate here on embodied innovations, for which it is feasible to quantify their impact. Agricultural innovations can be categorized into certain classes, each of which can be valued with real options analysis. Product innovations, though not happening very often, happen occasionally e.g. safflower which was introduced in the 1950s. With biotechnology, one may expect innovations that will be new, value–added final products that can be produced by the agricultural sector. More commonly, agriculture is often impacted by process innovations. A yield–

increasing innovation, that is introduction of new high-yield varieties, is the most often encountered. Among these, one can distinguish between innovations that increase the mean–yield and those that influence the variability of the yield. Both of these changes, as seen in Equation (1), will be clearly reflected in the value of the real option. Notice, the substantial advantage of the real option method over the traditional NPV approach, for which the value of the variability altering innovation will not be captured at all. Cost–reducing innovations, such as a new and improved type of harvesting equipment which may be most noted for its labor-saving effect or a new irrigation technology having a water–saving effect, can be both quantified and used as input data for the growth option analysis. Given the inelastic demand for the main agricultural commodities and some products, one way to increase value added in agriculture, which can be quantified and valued with the real options methodology is improvement of product quality.

Table 1: Types of risks in agriculture.

Type of risk	Risk	Insurance Instrument
Political	Direct subsidies Biofuel Price setting (e.g. intervention prices) Environment regulations	
Production	Diseases Weather conditions Yield Pest	Insurance Insurance
Market	Sales Commodity prices Product quality Production factors prices	Commodity forward exchange
Personal	Absence of staff	
Assets	Fire Theft Damages	Insurance Insurance Insurance
Soil / Environment	Environmental damages Contamination	

Source: own work

New genetic engineering varieties are expected to significantly augment product quality, for example, by enhancing shelf life, improving the nutrient content, and improving appearance, and a growth option to invest in the improved technology, can in this case be valued with a model that accounts for all these factors, as well as their variability. Finally, the public is increasingly concerned about food safety, worker safety, ground water contamination by pesticides, and other types of negative external effects of agriculture. The development of technologies that improve environmental quality or at least reduce damages relative to existing technologies is becoming a major research and policy priority. In order to fully describe the economic value of investment in the environmental innovations, policy makers as well as researchers could refer to real options methodology of assessment.

Learning options arise whenever a farmer is facing a decision that is affected by such a high uncertainty of future events, that it might be profitable to defer the investment until new market information is available. These cases were also highlighted in the cited literature e.g. in Saphores [2000] and Mbah et al. [2010], who adopt the idea that pest control should be adapted, always when new information about the previous round of a spraying has arrived. Of particular importance in agricultural investments are political decisions, which strongly affect the environment in which a farmer operates. Among these, typical examples include production quotas for agricultural products set by the European Union, the effects of the financial crisis on the future composition of the direct subsidies to agriculture, the current values of intervention prices at which national intervention agencies in the EU are obliged to purchase commodities, and others. Real options analysis offers, in each case, a methodology to find the value of deferring any investment decisions until the crucial information is available.

Finding insurance options in agricultural applications, other than the straightforward options to abandon investments or switch the type of activity, is a bit trickier. In financial terms, owning an insurance option is equivalent to a form of hedging behavior. There exist various financial instruments, which can be used as diverse forms of hedging also in agricultural context. For example, it is a common practice of wine producers, to sell their product before it actually starts to exist. Another example of a typical insurance behavior in agricultural applications is the use of weather derivatives. There are, however not many obvious agricultural examples of 'real' rather than financial hedging behavior, but identification of such opportunities would be of great value to any farmer. A typical example of a real insurance option stems from crop fertilization. A farmer, who over fertilizes his crops is insuring himself against a variety of possible weather conditions that may arise: whenever the weather conditions are good, the plants will use the additional portion of a fertilizer, and if they are bad, the excess of fertilizer will not harm the yield (it creates an externality on other farmers, but this is not important for an individually rational decision). Further example of a real insurance option of great importance in modern agriculture, in particular in the organic farming sector, is vertical integration. Vertical integration allows the farmer to fix the prices for his product, and in this sense it insures him against unfavorable market conditions.

CONCLUSIONS

In this work, we have presented an overview of the real options methodology and its applications in agricultural decision making. We also examined the current state of research on real options in agriculture as well as proposed a range of possible extensions and further applications of diverse types of real options in agricultural investments. Each of the identified options corresponds to either an individual farmer's decision making, or an agricultural problem which could be considered a policy issue. Among the latter category, the cited works of Kuminoff and Wossink [2005], Fenichel et al. [2008] and Musshoff [2012] raise vast topics in modern agricultural policy making: organic farming, fisheries management, the adaptation of farming land to different uses and so on. In each of these examples, real options methodology highlights how misleading policy decisions are, when they are solely based on the traditional net present value approach in an industry, which is exposed to so many factors of uncertainty and high volatility of outputs. Policy makers should, in any case, modernize the way they think about cost–benefit analysis in a way that directly allows for inclusion of the flexibility and uncertainty in the decision–making process.

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APPLICATION OF CLASSIFICATION TREES TO ANALYZE INCOME DISTRIBUTION IN POLAND

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Abstract: The aim of presented research is to identify factors that determine wages in Poland and to find out if gender pay gap exists, applying classification trees. For the grouping variable i.e. net income from the main place of employment we construct clusters of respondents that are created due to such features as: gender, education, employment contract, economic, occupation, additional job, size of enterprise, measured by the number of employees, age and job seniority in years. Investigation is provided applying data from the Polish Labor Force Survey in the years 2003, 2006 and 2009.

Keywords: labor market, gender segregation, classification tree

INTRODUCTION

In modern economics income distribution has been concerned as the distribution of income across individuals and households. Important theoretical and policy concerns include the relationship between income inequality and economic growth. The transformation of economies from centrally planned toward market-oriented that has been taking place in Central and Eastern Europe involved significant changes in labor market institutions. Constraints on layoffs and redundancies were significantly reduced but unemployment – the unknown in communist era phenomenon - appeared. Situation on labor market in transitional economies has been discussed by: Adamchik and Bedi (2003), Grajek (2001), Kot (1999), Keane and Prasad (2006), Newell and Reilly (2001), Newell and Socha (2005) and (2007), Witkowska (2012) among others.

There are many factors influencing wages that are widely discussed in literature ([Kot 1999] may be an example). These features are either connected with the individual attributes of employees or describe the general situation at the labor market and characterize the particular place (institution or enterprise) of employment. The former may be the subject of potential wage disparities. Inequalities at the labor market concern different aspects and social relations such as [Cain 1986, p. 693]: gender, sexual orientation, age, race, disabilities, religion, etc. Labor market discrimination by gender, race and ethnicity is the world-wide problem and estimation of these types of discriminations has become routine [Neuman and Oaxaca 2003].

Gender discrimination at the labor market may appear in a variety of forms such as: wage discrimination, discrimination in hiring, human capital discrimination (educational gender segregation) and occupational segregation (see [McConnell and Brue 1986, p. 289 –290], [Kot 1999, p. 225 – 226], [Livanos and Pouliakas 2009]). To explain causes and mechanisms of gender discrimination is very difficult however it seems to be easier to define it than to measure such inequalities [Kot 1999, p. 225]. Literature offers variety of theories about how and why women face discrimination in the labor market: Becker (1957), Madden (1975), McConnell and Brue (1986), Thurow (1975), Arrow (1973) and Bergmann (1971) among others.

The aim of our research[1] is to analyze income distribution to detect factors influencing wages and to answer the question if gender pay gap exists in Poland. Our investigation is based on data from Polish Labour Force Survey in years 2003, 2006 and 2009, and is conducted applying classification trees.

CLASSIFICATION TREE

Classification trees are used to predict membership of cases or objects in the classes of a categorical dependent variable from their measurements on one or more predictor variables. Classification trees are a powerful alternative to the more traditional statistical models. This model has the advantage of being able to detect non-linear relationships and showing a good performance in presence of qualitative information. Classification tree analysis is one of the main techniques used in so-called data mining. Description and examples of classification trees may be found in Breiman et al. (1984), Gatnar and Walesiak (2004) among others.

The entire construction of a tree consists of 3 elements: (1) the selection of the split; (2) the decisions when to declare a node terminal or to continue splitting it; (3) the assignment of each terminal node to a class. In the tree structures, leaves represent class labels and branches represent conjunction of features that lead to those class labels. Since classification trees are used to recognize homogenous groups, we apply them to find out major factors that create these classes.

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In our research we use QUEST (*Quick Unbiased Efficient Statistical Tree*) algorithm developed by Loh and Shih (1997) that employs a modification of recursive quadratic discriminant analysis and includes a number of innovative features for improving the reliability and efficiency of the classification trees that it computes. QUEST is fast and unbiased. It's lack of bias in variable selection for splits is also a distinct advantage when some predictor variable have few levels and other predictor variables have many levels since predictors with many levels are more likely to produce "fluke theories," which fit the data well but have low predictive accuracy(see [Doyle 1973]). Finally, QUEST does not sacrifice predictive accuracy for speed, (see [Lim, Loh, and Shih 1997]).

DATA DESCRIPTION

In our research we apply data from Polish Labor Force Survey in the fourth quarters of the years 2003, 2006 and the first quarter of 2009. Employing data from selected years is due to assumption that the structure of the labor market changes essentially in the longer period than one year (thus we do not compare situation at the Polish labor market year by year). Analysis is provided only for respondents who inform about their incomes. In order to create data base for further investigation we removed respondents who (see Table 1): (a) declared that is not employed, (b) did not inform about incomes, (c) did not know how many employees work in the enterprise (that he is working for).

Table 1. Comparison of number of respondents in PLFS and our data base

Quarter Year	Number of respondents in		Percentage share of PLFS observation in the data base
	PLFS	our data base	
Q4 2003	39893	9288	23.28
Q4 2006	54665	9498	17.37
Q1 2009	54665	12919	23.63

Source: own elaboration

In our investigation we employ several variables that describe qualitative and quantitative features. Three of them are dichotomous variables: *gender* (men or women), *additional job* (yes or no), and type of the *ownership* of the enterprise where respondent is employed (private or state). Other variables that are characterized by four and more variants are described in Table 2. In addition, in experiments for years 2006 and 2009 we employ job seniority that was introduced as quantitative variable with integer values.

Among 11 mentioned above variables we select the grouping variable as *net income* from the main place of employment while others are discriminant variables that are used to construct clusters of respondents.

EMPIRICAL RESULTS

Searching for factors that influence wages has been provided for very long time. Usually regression analysis or econometric modeling are employed in such investigation. In the paper we present different approach using classification method (see also Matuszewska-Janica, Witkowska, 2013).

Table 2. Description of variables

<table>
<tr><th>Variable</th><th colspan="6">Description</th></tr>
<tr><td rowspan="2">Education level</td><td>tertiary education</td><td colspan="3">post-secondary and vocational secondary</td><td colspan="2">general secondary</td></tr>
<tr><td colspan="2">basic vocational</td><td colspan="4">lower secondary and below that level</td></tr>
<tr><td>Economic sectors</td><td>agricultural</td><td>industrial</td><td colspan="2">services</td><td colspan="2">others</td></tr>
<tr><td rowspan="3">Occupation</td><td>army</td><td>managerial</td><td colspan="2">professional</td><td colspan="2">technical</td></tr>
<tr><td>clerical</td><td>sales & services</td><td colspan="2">farmers, fishers, etc.</td><td colspan="2">industry workers</td></tr>
<tr><td colspan="2">skilled workers</td><td colspan="4">unskilled workers</td></tr>
<tr><td>Employment contract</td><td>permanent job</td><td>temporary - training</td><td colspan="2">temporary since cannot find permanent job</td><td colspan="2">temporary since that form is suitable</td></tr>
<tr><td>Size of firm [number of employees]</td><td><10</td><td>11-19</td><td>20-49</td><td colspan="2">50-99</td><td>>100</td></tr>
<tr><td>Age [years]</td><td><29</td><td>30 – 39</td><td>40 – 49</td><td colspan="2">50 – 59</td><td>>60</td></tr>
<tr><td>Incomes [PLN]</td><td>< 1000</td><td>1001 – 1400</td><td>1401 – 1800</td><td colspan="2">1801 – 2200</td><td>> 2200</td></tr>
</table>

Source: own elaboration

In our experiments we construct classification trees for each year employing the same rules such as minimal number of cases in superior node is 100, for inferior node – 50, and maximal length from the starting node to the leaves is 5. In all trees but one for 2003 the set of discriminant variables is the same.

The results are presented in tabular format. Table 3 contains counts of all nodes and leaves (clusters) in each tree together with detailed description of participation of each variable in splitting procedure. As one can see in the tree constructed for the year 2003 job seniority is missing since data were not available. Among distinguished variables only one, informing if the respondent had *additional job*, does not create any classes. We may notice essential changes in factors influencing wages in analyzed years. For instance *economic sector* seems to be important factor only in the year 2006 while *ownership* of the firm, where respondent is employed, together with *age* participate in splitting only in the year 2003. However the last mentioned variable is probably replaced by *job seniority* in experiments provided for following years. We must also realize that "economic sector" is represent only by 4 variants of the variable and in such a case there is no good representation of economic branches, and at least NACE classification (***N**omenclature statistique des **A**ctivité séconomiques dans la **C**ommunauté **E**uro-péenne*) is recommended[2].

Table 3.Participation of discriminant variables in the construction of classification trees

Years	2003		2006		2009	
Count of:	Count of all nodes and clusters in the entire tree					
	nodes	clusters	nodes	clusters	nodes	clusters
	27	14	43	22	49	25
Variables	Number of nodes in the tree:					
	splitting	terminal	splitting	terminal	splitting	terminal
Education level	2	5	14, 22, 26	27, 28, 37, 38, 41, 42	17, 23	33, 34, 39, 40
Economic sectors			24	39, 40		
Occupation	0, 1, 3, 16	23, 24	2, 4, 5, 8, 9, 15, 17, 20,	18, 29, 30, 31, 32, 35, 36	0, 1, 6, 7, 24	41, 42
Employment contract			12	25	4, 8, 11, 13, 27	9, 18, 21, 47, 48
Size of firm	8, 14	19, 20			10, 15, 16, 20,	19, 29, 30, 31, 32, 35, 36
Age	9	18				
Job seniority	×	×	0, 1, 3, 7	16	2, 5, 12, 22,	37, 38
Gender	4, 6, 7, 15	10, 11, 12, 13, 21, 22	6, 10, 11, 19	13, 21, 23, 33, 34	3, 14, 25, 26	28, 43, 44, 45, 46
Additional job						
Ownership	17	25, 26				

Source: own calculation

Synthetic description of all factors that take part in creation of groups of respondents is presented in Table 4.Variables that create splitting the most often are: *occupation* (24%), *gender* (23%), *education level* (14%), *size of firm* (13%) and *employment contract* (10%). Taking into account percent of objects in terminal nodes we see that 41% in 2003, 21% in 2006 and 28% in 2009 of them are created for *gender* as discriminant variable. Therefore this factor influences wages the most. Other important variables are: *occupation* although it seems to become less

[2] As it was done in the paper [Matuszewska–Janica and Witkowska 2013].

important in 2009 than before, size of firm in years 2003 and 2009, and *education* in 2006.

Table 4. Participation of discriminant variables in the cluster construction

	Count of all nodes			Percentage share in terminal nodes			Count	Percentage
							of splits	
Years	2003	2006	2009	2003	2006	2009	2003 - 2006	
Education level	2	9	6	2.1	35.8	19.2	17	14.41
Economic sectors		3			6.7		3	2.54
Occupation	6	15	7	22.0	26.6	14.2	28	23.73
Employment contract		2	10		4.2	13.6	12	10.17
Size of firm	4		11	25.5		22.9	15	12.71
Age	2			1.2			2	1.69
Job seniority*	×	5	6	×	6.0	2.2	11	12.09
Gender	10	9	9	41.3	20.5	27.6	27	22.88
Ownership	3			7.9			3	2.54
Sum	27	43	49				118	102.77

Source: own calculation.
* Percentage share for job seniority is calculated for 2 models only.

In Tables $5 \div 9$ we present groups of respondents recognized by classification trees as homogenous, separately for selected variables that participate in terminal splits. In column "%" we present the percentage share of all respondents that are classified to the particular leaf, i.e. 0.8 means that 0.8% of the whole sample from the certain year creates the terminal node which number is given in the second column (for instance, in Table 5, cluster generated by the node 11 in 2003 contains 0.8% of the sample i.e. 74 respondents). Structure of incomes is represented by percentage share of respondents from each cluster who obtain wages belonging to five groups of incomes from the main place of work (see Table 2).

In Table 5 we describe income distribution in clusters selected by *gender*. As one can see incomes are essentially lower in "woman nodes". However one should notice that gender structure in terminal nodes is not symmetric. In the year 2003, there are 2636 women and 1200 men in terminal nodes, in 2006 and 2009 this proportion is the opposite 513 to 1424, and 815 to 2751, respectively. Nevertheless in 2003 in the highest income group there are only 1% of women from the nodes: 11, 13 and 21, while 11% of men from the nodes: 10, 12 and 22. In 2006 only 0.4% of women from the node 34, but 17% of men belonging to nodes: 13, 21, 23 and 33 earn more than 2200PLN. The women situation seems to be better in 2009 since 9% of women from terminal nodes belong to the highest income class although this share equals 27% for men.

Table 5.Analysis of wages for respondents who created leaves according to: *gender*

Year	Structure of sample				Structure of incomes in the node				
	No. of node	Count	Gender	%	<1000	<1400	<1800	<2200	>2200
2003	11	74	women	0.8	32.4	35.1	14.9	13.5	4.1
	13	1808	women	19.5	95.5	3.4	0.9	0.0	0.2
	21	754	women	8.1	58.6	25.9	10.3	2.9	2.3
	10	401	men	4.3	22.4	22.2	20.7	13.0	21.7
	12	138	men	1.5	14.5	15.9	33.3	22.5	13.8
	22	661	men	7.1	35.7	31.0	20.7	8.2	4.4
2006	34	513	women	5.4	81.9	12.3	4.9	0.6	0.4
	13	357	men	3.8	7.6	8.4	23.2	20.7	40.1
	21	126	men	1.3	4.8	11.9	18.3	19.8	45.2
	23	425	men	4.5	72.7	13.6	8.0	2.8	2.8
	33	519	men	5.5	50.9	26.0	14.1	4.2	4.8
2009	43	102	women	0.8	43.1	33.3	12.7	8.8	2.0
	45	713	women	5.5	16.0	31.0	29.3	13.3	10.4
	28	477	men	3.7	2.5	4.8	12.4	19.5	60.8
	44	328	men	2.5	21.0	33.5	24.1	10.1	11.3
	46	1946	men	15.1	6.2	21.7	31.1	20.1	21.0

Source: own calculation

In Table 6 we analyze wages considering *employment contract*. It is worth mentioning that there are two nodes containing respondents with permanent job which show completely different wage distribution. In the node 48 respondents earn much better than employees classified to the node 18.

Table 6.Wages analysis for respondents created leaves according to: *job contract*

Year	Structure of sample			Structure of incomes in the node				
	No. of node	Contract	%	<1000	<1400	<1800	<2200	>2200
2006	25	temporary: training or since one cannot find permanent job	4.2	65.8	20.1	8.2	2.7	3.2
2009	47	temporary: all situations	0.5	28.4	28.4	14.9	11.9	16.4
	48	permanent	6.6	2.1	12.1	30.3	27.4	28.1
	18	permanent	4.9	22.6	32.6	25.9	9.9	9.1
	21	temporary: training	0.9	59.8	22.1	11.5	4.1	2.5
	9	temporary: training	0.7	53.6	31.0	11.9	2.4	1.2

Source: own calculation

Incomes of temporary employed because of training are identically distributed while including all reasons for nonpermanent employment changes the distribution of earnings toward higher wages.

Influence of education is analyzed in Table 7. It is visible that incomes of employees with tertiary education were on the highest level in the year 2003, while wages dramatically decreased in 2006. It may be connected with depreciation of higher education since, due to demographic decline, every year higher percentage of young people in age 19 – 24 study at universities. It causes that university alumna are worse and worse educated and "overproduction" of people with tertiary education in "soft disciplines" every year. Therefore they cannot find better paid jobs. In 2003 2% of respondents were recognized as employees with tertiary education in the node 5 while in 2006 there were 8% in the nodes: 27, 37 and 41. Among them in 2003 nearly 40% belong to the highest income class while in 2006 there were only 13% of the ones from the terminal nodes. In 2009 there is no cluster containing employees with tertiary only. It may be interpreted that level of education became less important factor in wage determination.

Table 7. Wages analysis for respondents created leaves according to: *level of education*

Year	No. of node	Education	%	<1000	<1400	<1800	<2200	>2200
2003	5	tertiary	2.1	6.0	10.6	22.1	21.6	39.7
2006	27	tertiary	4.6	8.2	24.9	38.7	14.0	14.2
	37	tertiary	2.2	11.6	28.5	35.7	14.5	9.7
	41	tertiary	1.1	19.6	25.2	22.4	16.8	15.9
	28	general, and post-secondary basic and secondary vocational	2.7	36.5	36.5	15.4	6.5	5.0
	38	general, and post-secondary basic and secondary vocational	0.8	59.5	27.0	8.1	5.4	0.0
	42	all variants except tertiary	24.4	34.0	31.7	20.0	9.3	5.0
2009	40	general secondary, basic vocational and below that level	5.1	23.2	30.6	27.1	11.2	7.8
	34	general secondary, basic vocational and below that level	2.3	52.2	29.6	9.0	5.3	4.0
	39	tertiary, post- and vocational secondary	10.9	8.6	22.0	27.9	17.5	23.9
	33	tertiary, post- and vocational secondary	0.9	35.0	38.3	17.5	7.5	1.7

Source: own calculation

One of the main factors influencing wages is *occupation* that is multi-variant variable. Analysis of such variables is not precise because - due to assumed rules of splitting in order to obtain the reasonable size of the tree – different variants are "aggregated". In Table 8 we can see that three groups of occupation: army, managerial and professional earn the best and their wages has been increasing.

Table 8. Analysis of wages for respondents who created leaves according to: *occupation*

Year	No. of node	Occupation	%	<1000	<1400	<1800	<2200	>2200
2003	23	clerical and skilled workers	13.0	70.5	20.2	6.8	1.7	0.7
	24	technical	9.0	56.5	22.6	12.8	4.8	3.2
2006	18	army, managerial	0.5	15.4	21.2	15.4	23.1	25.0
	30	army, managerial, professional	1.4	41.0	33.6	11.2	7.5	6.7
	29	all except army, managerial and professional	9.4	81.6	12.7	3.9	1.1	0.7
	31	all except army, managerial, technical and professional	7.3	69.4	17.3	8.8	3.2	1.3
	32	professional, technical	3.3	29.6	28.3	21.1	10.7	10.4
	35	skilled workers	2.5	49.8	28.7	14.3	3.4	3.8
	36	technical	2.2	26.7	22.4	22.9	19.0	9.0
2009	42	army, managerial, professional	5.3	2.2	10.0	22.5	20.6	44.8
	41	technical, industry and skilled workers	8.9	14.8	27.6	27.6	15.1	14.9

Source: own calculation

Table 9. Wages analysis for respondents created leaves according to: size of firm in 2009

No. of node	Size of firm	%	<1000	<1400	<1800	<2200	>2200
36	<10	1.1	35.2	37.2	20.0	6.2	1.4
32	<10	5.2	57.5	33.0	8.0	1.3	0.1
35	11 - 49	2.7	16.8	39.6	32.4	6.6	4.6
30	1 - 100	4.0	65.5	28.8	5.2	0.4	0.2
31	>10	3.9	39.3	41.9	15.0	2.8	1.0
19	>50	4.4	12.0	31.5	29.6	15.4	11.5
29	>101	1.6	45.3	38.7	13.2	1.9	0.9

Source: own calculation

In Table 9 we look at incomes obtained in different *sizes of firms*. This factor is essential only for 2009, and as one may see the highest wages are observed in institutions with more than 50 employees. Although in firms with more than 100 employees earnings are much smaller.

CONCLUSIONS

It is worth noticing that direct comparison of clusters does not give reliable results unless the samples are not characterized by similar structure. Since splitting was provided under constrains, that let us create reasonable size of the tree, it is difficult to compare income distribution in the time span especially for discriminant variables that are characterized by many variants (i.e. occupation, education or employment contract) because leaves represent group of respondents due to “aggregated feature”.

Application of classification trees let us distinguish the most important variables that create homogenous classes of earnings. It also proves that during transition period determinants of wages has been changed. However gender, occupation and education seem to be the most important in the whole period of analysis. While influence of employment contract and size of the institution becomes more and more essential.

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QUANTITATIVE METHODS IN ECONOMICS
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STOCHASTIC EQUIVALENCE SCALES IN LOG-NORMAL DISTRIBUTIONS OF EXPENDITURES

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Abstract: In the paper, the properties of the stochastic equivalence scales (SES) are analysed when expenditure distributions are log-normal. The SES provides the equivalent distribution of expenditures when the population of households is heterogeneous with respect to such attributes as household size, demographic composition, etc. For log-normal expenditure distributions, the non-parametric SES deflators are proportional to the ration of geometric means in compared distributions. The statistical analysis of expenditure distributions for Poland in the years 2005-2010 shows that these deflators perform quite well.

Keywords: equivalence scales expenditure distributions, log-normal distribution

INTRODUCTION

The main purpose of this paper is to develop the formulae and estimators of nonparametric stochastic equivalence scales when expenditure distributions are log-normal. We also estimate such scales using data from Polish Household Budget Surveys for the years 2005-2010.

When households differ in all aspects other than expenditures[1], e.g., the size and the composition of the household, the age of the adults, the age of the children, the disabilities of the household members, etc., serious problems arise when making judgments and decisions to address inequality, welfare and poverty. Traditionally, equivalence scales have been used to homogenize heterogeneous household populations [Buhmann et al. 1988, Jones and O'Donnell, 1995].

[1] In this paper, we confine ourselves to expenditures; however, a significant amount of our considerations also relates to incomes.

Muellbauer (1977) defines equivalence scales as budget deflators that are used to calculate the relative amounts of money two different types of households necessary to attain the same standard of living. The distribution of the expenditures or incomes of heterogeneous household populations is adjusted by such deflators. As a result, the initial heterogeneous population becomes homogeneous. It is this adjusted distribution of expenditures in this now homogeneous population that is used to assess welfare, poverty, inequality, etc.

However, various practical advantages of equivalence scales are offset by a significant disadvantage, as the specification of an equivalence scale requires strong assumptions concerning the relationship between income and needs, and there may not be wide agreement concerning the validation of the appropriate assumptions. Furthermore, numerous severe identification issues arise in the estimation of equivalence scales (see, in particular, [Pollak and Wales, 1979, 1992], [Blundell and Lewbel, 1991], [Blackorby and Donaldson, 1993], and the surveys of [Lewbel, 1997, and Slesnick, 1998]). Moreover, there are evidences that the results of distributional comparisons are sensitive to the choice of the equivalence scale (Coulter et al. 1992a,b).

The aforementioned approaches to the problem of equivalence scales seem to be unsatisfactory. Many economists maintain that, "There is no single 'correct' equivalence scale for adjusting incomes - a range of scale relativities is both justifiable and inevitable" [Coulter et al. 1992a]. Jäntti and Danziger [2000, p.319] remark that, "There is no optimal method for deriving an equivalence scale". Indeed, without additional assumptions, there is no way of selecting the basis for choosing an equivalence scale. The *independence of base* (IB) (or *exactness of equivalence* scale) is one such assumption. Several papers have tested this assumption and have ultimately rejected it [Blundell and Lewbel, 1991, Blundell et al. 1998, Dickens et al. 1993, Pashardes, 1995, Gozalo, 1997, Pedankur, 1999].

The concept of the stochastic equivalence scale (SES)offers the method for the adjustment of expenditure distributions in heterogeneous populations [Kot, 2012]. The SES is any function that transforms the expenditure distribution of a specific group of households in such a way that the resulting distribution is stochastically indifferent from the expenditure distribution of a reference group of households. The stochastic indifference criteria are also used in developing the method of the estimation of the *SES*.

In this paper, the *SES* is applied when theoretical expenditure distributions are log-normal. The formula for such scales is developed. Then the *SES* are estimated using data from Polish Household Budget Surveys for the years 2005-2010.

The rest of the paper is organized as follows. Section THEORETICAL BACKGROUND offers a theoretical background concerning the *SES* and the formula of deflators when log-normal distribution is assumed. In Section EMPIRICAL RESULTS FOR POLAND 2005-2010, the results of estimation of deflators are presented. Section CONCLUSIONS summarize the paper.

THEORETICAL BACKGROUD

Suppose that a society is composed of heterogeneous households and each household is distinguished by two attributes: expenditures and a type. The household type may be interpreted in various ways, e.g., as an index of neediness which increases with family size.[2] We also allow for an analysis of the household types, which may not necessarily reflect household needs. We assume that there exists a given and finite number ($m+1 \geq 2$) of types of household groups that differ in many respects other than their expenditures.

We arbitrarily chose certain type of households as the 'reference group'. The expenditure distribution for this group will be called the 'reference distribution'. This distribution will be described by a positive continuous random variable Y with the distribution function $G(y)$ (abbreviated $Y \sim G(y)$). The remaining m groups of households will be called the 'evaluated groups'. The corresponding *evaluated distribution* of the expenditures will represent the set of the positive continuous random variables $X_1,\dots,X_m$ with the distribution functions $F_1(x),\dots,F_m(x)$, respectively. The random variables Y, $X_1,\dots,X_m$ describe the distribution of expenditures per household.

Formally, the *SES* is defined as follows. Let $\boldsymbol{s}(\cdot) = [s_i(\cdot),\dots,s_m(\cdot)]$ be a vector function for which the inverse function $\boldsymbol{s}^{-1}(\cdot) = [s_i^{-1}(\cdot),\dots,s_m^{-1}(\cdot)]$ exists and is differentiable. Let the random variable $Z_i = s_i(X_i)$ with the distribution function $H_i(z)$ be the transformation of evaluated expenditure distribution X_i. Hereafter, the random variable $Z_i \sim H_i(z)$ will be called the 'transformed expenditure distribution'.

Definition 1. With the above notations, the function $\boldsymbol{s}(\cdot)$ will be called *the stochastic equivalence scale* (*SES*) if and only if the following equality holds:

$$\forall z>0,\ i=1,\dots,m;\ H_i(z) = G(z). \tag{1}$$

When the function $\boldsymbol{s}(\cdot)$ is the *SES*, $Z_i = s_i(X_i)$ will be called 'the equivalent expenditure distribution'.

Definition 1 of the *SES* is axiomatic in the sense that it only postulates the criterion for a function to be recognized as an *SES*. This definition does not describe how an *SES* should be constructed or its conditions of existence. In other words, *any* function $\boldsymbol{s}(\cdot)$ that fulfils condition (1) has to be recognized as an *SES*.

The validation of condition (1) can be verified by Kolmogorov-Smirnov *K-S* test. The *K-S* statistic can be also used as the loss function in developing estimators of parametric or nonparametric *SES.* Details of the statistical procedures of testing and estimating the *SES* are presented in Kot (2012).

[2]We follow Ebert and Moyes (2003) in associating the household type with family size for convenience. However, this framework can be extended by taking into account the vector of the household attributes comprising the number of adults, the number of children, the age of household members, etc.

The relative *SES* can be defined as follows. Let $\boldsymbol{d} = [d_i]$, $i = 1,\ldots,m,$ be the vector of positive numbers called 'deflators' which transforms the evaluated expenditure distributions $X_1,\ldots,X_m$ as follows:

$$Z_i = X_i/d_i \sim H_i(z), i = 1,\ldots,m. \quad (2)$$

Definition 2. Under the above notations, the vector $\boldsymbol{d}$ will be called the *relative SES* if and only if the deflators $d_1,\ldots,d_m$ are such that equality (1) holds.

The following corollary summarizes the properties of *SES*:

Corollary 1. Let X be the distribution of expenditures of the evaluated group of households, Y the distribution of expenditures of the reference group of households, and $Z = s(X)$.If s is the *SES*, then the following equivalent conditions hold:

- Z is stochastically indifferent to Y, or
- Social welfare in Z is exactly the same as in Y, for all von Neuman–Morgenstern utility functions, or
- Poverty in Z is exactly the same as in Y for all poverty lines, or
- Inequalities in Z are exactly the same as in Y.

[Davidson, 2008].

One may ask what kind of homogeneity *SES* provides. If an initial heterogeneous population of households consists of $m+1$ subpopulations (including reference subpopulation), then the adjustment of each m different expenditure distribution by the *SES* will give new fictitious subpopulations which are homogeneous with respect to utilitarian social welfare, inequality and poverty.

Let expenditure distributions be two-parameter log-normal $X \sim \Lambda(\mu, \sigma)$ with the density function given by

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{(\ln x - \mu)^2}{2\sigma^2}\right\}, \quad x > 0 \quad (3)$$

[Kleiber, Kotz, 2003, p. 107]. The k*th* moment in this distributions is given by

$$E[X^k] = \exp\left\{k\mu + \frac{k^2\sigma^2}{2}\right\} \quad (4)$$

and the geometric mean (x_{geom}) is given by

$$x_{geom} = e^{\mu} \quad (5)$$

which coincides with the median [Kleiber, Kotz, 2003, p. 112].

Let $Y \sim \Lambda(\mu_y, \sigma_y)$ and $X \sim \Lambda(\mu_x, \sigma_x)$ denote the expenditure distributions of the reference group and the evaluated group respectively. If we adjust X by the d deflator, as in (2), then the transformed distribution $Z=X/d$ will be log-normal $\Lambda(\mu_z, \sigma_z)$,where $\mu_z = \mu_x - \log d$, and $\sigma_z = \sigma_x$.

It is easy to see that the *d* deflator will provide the *SES* if compared distributions *Z* and *Y* have the same parameters, i.e., $\mu_y = \mu_x - \ln d$ and $\sigma_y = \sigma_x$. Then the *d* deflator is given by

$$d = \exp\{\mu_x - \mu_y\} = \frac{x_{geom}}{y_{geom}}. \qquad (6)$$

In other words, the deflator *d* of the relative *SES* (2) in log-normal expenditure distributions will be simply the *ratio of geometric means* (*RGM*) if the condition $\sigma_y = \sigma_x$ holds. The validation of this condition can be easily checked using standard statistical test.

It might happen, however, that statistical test rejects the aforementioned condition $\sigma_y = \sigma_x$. In order to assess how the violation of this condition affects the *d* deflator we use the fact that the *SES* implies the equality of moments (4) for all *k*. After simple algebra, we can get the following formula for an adjusted *d**

$$d^* = \frac{x_{geom}}{y_{geom}} \exp\left\{k\,\frac{\sigma_x^2 - \sigma_y^2}{2}\right\}. \qquad (7)$$

Obviously, (7) coincides with (6) if $\sigma_y = \sigma_x$. Because of that $\sigma_y > \sigma_x$ in practice, the greater disparity between σ_x and σ_y, the lower *d*, for all *k*. This means that the violation of the $\sigma_y = \sigma_x$ condition leads to underestimation of the *d* deflator.

EMPIRICAL RESULTS FOR POLAND 2005-2010

We will use expenditure distributions for estimating the relative *SESs*. The monthly micro-data come from the Polish Household Budget Surveys for the years 2005-2010. The expenditures are expressed in constant 2010 year prices. The household groups are distinguished according to the number of members (household size). The households of single childless persons are chosen as the reference group.

The chi-square test rejects the null hypothesis that expenditure distributions are log-normal. This result is not uncommon in applications involving large sample sizes. In fact, all theoretical models of income or expenditure distributions have been usually rejected at conventional level of significance [McDonald, Xu, 1995].

Table 1 presents estimates of *RGM d* and *d** deflators given by eq. (6) and (7) respectively. These deflators are estimated separately for each household group. We calculate the *d** deflator assuming *k*=1.

Table 1. The estimates of *RGM* deflators for Poland 2005-2010

Year	2005		2006		2007		2008		2009		2010	
Size	d	d*	d	d*	d	d*	d	d*	d	d*	d	d*
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.617	1.609	1.604	1.609	1.672	1.668	1.699	1.693	1.667	1.650	1.678	1.676
3	1.889	1.875	1.928	1.921	2.038	2.031	2.103	2.078	2.066	2.036	2.063	2.039
4	2.049	2.019	2.075	2.056	2.239	2.215	2.292	2.246	2.245	2.185	2.220	2.172
5	2.028	1.964	2.108	2.051	2.257	2.179	2.348	2.277	2.272	2.175	2.251	2.163
6 or more	2.183	2.105	2.299	2.212	2.477	2.380	2.495	2.387	2.460	2.330	2.505	2.405

Source: own calculations using data from Polish Household Budget Surveys

An analysis of the results presented in Table 1 shows that differences between *d* and *d** are rather small and they can be neglected if one decimal place is taken into account. This means that the violation of the assumption $\sigma_y = \sigma_x$ does not seriously affect the estimates of deflators. Because of that variances σ^2 of logarithms of expenditures within each year have turned out non-homogeneous, the adjusted *d** deflator seems to be more adequate approximation of the *SES* than *d* deflator.

CONCLUSIONS

The possibility of estimating non-parametric *SES*s opens new interesting perspectives for applications when expenditures have log-normal distribution. The estimation of non-parametric deflators is very easy because it requires estimates of geometric means of compared distributions only.

Empirical results exhibit two important features. First, equivalence scale varies over time. This means that one definite form of the scale does not exists. Second, Polish equivalence scale are very flat. This is an indication of the economies of scale enjoyed by Polish households in the years 2005-2010.

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ESTIMATION OF POPULATION PARAMETERS USING INFORMATION FROM PREVIOUS PERIOD IN THE CASE OF OVERLAPPING SAMPLES – SIMULATION STUDY

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Abstract: The paper concerns the problem of estimating population parameters for repeated rotating surveys. Coefficients required for theoretical BLUE estimator for rotating surveys are for actual real surveys usually not known. There are no theoretical papers relating to this problem. It is therefore necessary to conduct suitable simulation studies. Broad simulation analyses conducted in the paper are carried out on the basis of two populations: generated from a multivariate normal distribution and based on real data derived from agricultural censuses.

Keywords: survey methodology, rotating surveys, repeated surveys overlapping samples

INTRODUCTION

Theory connected with overlapping samples, also known as theory of rotating surveys or theory of rolling samples, started with the papers [Jessen 1942, Eckler 1955]. The theory was growing in 20th century [Patterson 1950, Rao and Graham 1964, Kordos 1967, Scott and Smith 1974, Jones 1980, Binder and Dick 1989, Fuller 1990, Szarkowski and Witkowski 1994] and it remains of utmost importance in 21 century [Feder 2001, Fuller and Rao 2001, Kowalczyk 2003a, Kowalczyk 2003b, Kowalczyk 2004, Steel 2004, Nedyalkova et al. 2009, Steel and McLaren 2009, Berger and Priam 2010, Wesołowski 2010, Ciepiela et al. 2012, Kordos 2012, Kowalczyk 2013].

The established role of rotating surveys theory is connected with the role of repeated rotating surveys in central statistical offices. Many of the most important surveys, both in Poland and other countries, are rotating surveys, e.g.

labour force surveys, household budget surveys. Repeated surveys are usually of multi-purpose nature. They aim to estimate population parameters on each current occasion, to estimate difference between two successive population means (i.e. net changes), ratio of two population means, various components of individual changes, combined population means from several periods etc. Additionally repeated surveys also aim to aggregate sample in time, which is of particular importance in measurement of rare events and rare populations. To take into account conflicting aims of repeated surveys they are often conducted in rotating manner, which means that they are based on overlapping samples. More precisely, a sample on each occasion consists of two parts: a part that has been also examined on previous occasion (matched part) and a part that is new in the sample, i.e. has not been examined on previous occasion (unmatched part). For more than two occasions rotating scheme becomes more complicated.

THE BASIS OF THE PROBLEM

Among many problems connected with rotating surveys the following one is of particular importance: no auxiliary information is available, we base only on a sample (overlapping) from all previous occasions and we want to increase precision of the population mean estimation on the current occasion by using all information from the sample, also from prior occasions. The problem for model approach for rotating scheme without holes was solved by Patterson 1950, in randomized approach it was given by [Kowalczyk 2002]. Model approach for rotating schemes with holes under different assumptions was considered by [Kowalski 2009, Kowalski and Wesołowski 2010, Wesołowski 2010, Ciepiela et al. 2012].

In the present paper we first give theoretical results for two periods in randomized approach to introduce general problem and divergence between theory and practice of rotating surveys. Kowalczyk [2002] has proved that for two periods the best linear unbiased estimator of the population mean on the second occasion for rotating surveys of a finite population is the estimator of the form:

$$e_2 = a\bar{y}_{1U} - a\bar{y}_{1M} + c\bar{y}_{2U} + (1-c)\bar{y}_{2M}, \tag{1}$$

where

$$a = \frac{\dfrac{n_M}{n_2}\dfrac{n_{1U}}{n_1}}{1-\rho^2(Y_2,Y_1)\dfrac{n_{2U}}{n_2}\dfrac{n_{1U}}{n_1}}\frac{C(Y_2,Y_1)}{S^2(Y_1)},$$

and

$$c = \frac{\frac{n_{2U}}{n_2}\left(1-\rho^2(Y_2,Y_1)\frac{n_{1U}}{n_1}\right)}{1-\rho^2(Y_2,Y_1)\frac{n_{2U}}{n_2}\frac{n_{1U}}{n_1}}.$$

Its variance is given by:

$$D^2(e_2) = \left(\frac{1}{n_2}\frac{1-\rho^2(Y_2,Y_1)\frac{n_{1U}}{n_1}}{1-\rho^2(Y_2,Y_1)\frac{n_{2U}}{n_2}\frac{n_{1U}}{n_1}} - \frac{1}{N}\right)S^2(Y_2) .$$

Notation used here is the following:

n_t – sample size on the t-th occasion, $t = 1,2$,

n_M – matched sample size,

n_t – unmatched sample size on the t-th occasion, $t = 1,2$,

N – population size.

We have:

$$n_t = n_M + n_{tU}, \ t = 1,2 .$$

As it can be seen, coefficients a and c in formula (1) include population parameters, namely correlation coefficient $\rho(Y_1, Y_2)$ and regression coefficient $C(Y_1, Y_2)/S^2(Y_1)$, which in real surveys are usually not known. That problem is common for rotating surveys theory in general, also for model approach. The same applies for analogous estimators considered by [Patterson 1950, Kowalski 2009, Kowalski and Wesołowski 2010, Wesołowski 2010 and Ciepiela at al. 2012].

So important question arises. What happens if we substitute in formula (1) unknown population correlation coefficient and unknown population regression coefficient by its estimates given on the basis of the sample? Does this procedure still increase precision of the estimation? No mathematical theory is given relating to this problem because of the complicacy of coefficients a and c.

Moreover, as most rotating surveys are of multi-purpose nature, what happens to other population parameters estimation? Kowalczyk [2013] gave the following theoretical results for net changes estimation:

- if $\rho(Y_1, Y_2) > 0$, then for all n_M we have:

  $D^2(e_2 - \bar{y}_1) \le D^2(\bar{y}_2 - \bar{y}_1)$,

- if $\rho(Y_t, Y_{t+1}) < 0$, then we have:

$$D^2(e_2 - \bar{y}_1) \le D^2(\bar{y}_2 - \bar{y}_1) \Leftrightarrow \frac{n_{2U}}{n_1} \le \frac{-S^2(Y_2)}{2C(Y_2,Y_1)}$$

and for combined population means from two successive periods:

- if $\rho(Y_1, Y_2) > 0$, then we have:

$$D^2\left[(e_2 + \bar{y}_1)\right] \le D^2\left[(\bar{y}_2 + \bar{y}_1)\right] \Leftrightarrow \frac{n_{2U}}{n_1} \le \frac{S^2(Y_2)}{2C(Y_2, Y_1)}, \tag{2}$$

- if $\rho(Y_1, Y_2) < 0$, then for all n_M we have:

$$D^2\left[(e_2 + \bar{y}_1)\right] \le D^2\left[(\bar{y}_2 + \bar{y}_1)\right].$$

Still the question has to be answered. Are the results valid if we substitute in estimator e_2 given by formula (1) unknown population correlation and regression coefficients by their estimates based on a sample?

To answer all the questions broad simulation study will be presented in the next section.

SIMULATION STUDY

Description of the population

For simulation study two finite populations[1] are considered. Population 1 is finite population generated from a multivariate normal distribution. Generated finite population parameters look as follows:

$$\begin{bmatrix} \bar{Y}_1 \\ \bar{Y}_2 \end{bmatrix} = \begin{bmatrix} 4{,}916 \\ 9{,}833 \end{bmatrix}, \quad S = \begin{bmatrix} 6{,}109 \\ 7{,}031 \end{bmatrix}, \quad \rho = \begin{bmatrix} 1 & 0{,}601 \\ 0{,}601 & 1 \end{bmatrix}, \ N{=}10000$$

Population 2 is based on real data taken from agricultural censuses of 2002 and 1996. The population consists of 1575 rural areas and variable under study are:

Y_1 – sawn area of spring wheat in 1996,

Y_2 – sawn area of spring wheat in 2002.

Finite population2 parameters look as follows:

$$\begin{bmatrix} \bar{Y}_1 \\ \bar{Y}_2 \end{bmatrix} = \begin{bmatrix} 19743{,}24 \\ 19515{,}67 \end{bmatrix}, \quad S = \begin{bmatrix} 21387{,}08 \\ 23511{,}31 \end{bmatrix}, \quad \rho = \begin{bmatrix} 1 & 0{,}7964 \\ 0{,}7964 & 1 \end{bmatrix}, \ N{=}1575$$

Description of the sample

Two different sample sizes were considered in the simulation study:

$$n_1 = n_2 = n = 100 \ \text{ and } \ n_1 = n_2 = n = 50.$$

[1]Details of populations and of a sampling scheme are given in Kowalczyk B. (2013). In the book the same populations were discussed but the problems considered were of different nature.

For a given sample size different matched fraction were taken into account:

$$p = \frac{n_M}{n_2} = \frac{n_M}{n_1} = \ 0{,}1;\ 0{,}2;\ 0{,}4;\ 0{,}6;\ 0{,}8;\ 0{,}9\ .$$

For instance, $n = 100$ and $p = 0{,}1$ means that on both occasions 100 element samples were examined, out of which only 10 elements were examined on the first and second occasions together. Analogously, $n = 100$ and $p = 0{,}9$ means that out from 100 elements examined on the first occasion, 90 were also examined on the second occasion and 10 were additionally resampled. Unknown correlation and regression coefficients used in estimator e_2 given by formula (1) are estimate on the basis of 10 elements only in the first example and on the basis of 90 elements in the second example, although in both examples total sample sizes are the same.

For every sample size and every matched fraction sampling was repeated 1000 times.

Simulation results

In Tables 1-3 average absolute differences in percentage are juxtaposed. Average absolute difference for correlation coefficient ρ, regression coefficient β and estimator e_2 is defined respectively as:

$$\frac{|\hat{\rho}-\rho|}{\rho}\cdot 100\%\ ,\ \frac{|\hat{\beta}-\beta|}{\beta}\cdot 100\%\ ,\ \frac{|\hat{e}_2-e_2|}{e_2}\cdot 100\%\ ,$$

where ρ, β are real population values, $\hat{\rho}, \hat{\beta}$ are values assessed on the basis of a sample, e_2 is theoretical estimator given by formula (1), $\hat{e}_2$ is available in practice estimator constructed in such a way that the population correlation and regression coefficients that appear in formula (1) are substituted by their estimates on the basis of the sample.

Table 1. Average absolute difference in % for population 1, n = 100

p	0,1	0,2	0,4	0,6	0,8	0,9
Correlation coeff.	29,5	20,6	13	11,2	9,7	8,8
Regression coeff.	38,6	26,2	16,8	14,4	12,5	11,2
e_2	1	0,7	0,4	0,3	0,2	0,1

Source: own calculations

Table 2. Average absolute difference in % for population 2, n = 100

p	0,1	0,2	0,4	0,6	0,8	0,9
Correlation coeff.	17,6	13,5	10,6	8,9	8,1	7,9
Regression coeff.	34,4	26,9	20,7	17,8	16,3	15,3
e_2	2,7	1,9	1,2	0,7	0,5	0,3

Source: own calculations

Table 3. Average absolute difference in % for population 2, n=50

p	0,1	0,2	0,4	0,6	0,8	0,9
Correlation coeff.	24,8	17	13,5	11,2	10,3	9,8
Regression coeff.	48,6	33,9	26,3	22,4	20,6	20,5
e_2	6,1	3,4	2,1	1,4	0,9	0,6

Source: own calculations

Although correlation and regression coefficients assessed on the basis of the sample can differ substantially from real population values (from 7,9% up to 48,6%), substituting that real values by their assessments based on the sample in formula (1) does not influence estimator e_2 substantially (it changes the value of the estimator from 0,1% up to 6,1%).

In tables 4-6 efficiency of the estimation of estimatorse_2 and $\hat{e}_2$ compared to common sample mean is presented for different populations, sample sizes and matched fractions of the sample. Efficiency of the estimators e_2 and $\hat{e}_2$, i.e. theoretical estimator and estimator available in practice are defined respectively as:

$$eff\,(e_2) = \frac{MSE(\bar{y}_2)}{MSE(e_2)},\ eff\,(\hat{e}_2) = \frac{MSE(\bar{y}_2)}{MSE(\hat{e}_2)}.$$

Table 4. Efficiency of mean estimation on the second occasion for population 1, n = 100

p	0,1	0,2	0,4	0,6	0,8	0,9
$eff(e_2)$	1,060	1,089	1,094	1,083	1,049	1,032
$eff(\hat{e}_2)$	0,977	1,078	1,094	1,081	1,047	1,028

Source: own calculations

Table 5. Efficiency of mean estimation on the second occasion for population 2, n = 100

p	0,1	0,2	0,4	0,6	0,8	0,9
$eff(e_2)$	1,166	1,227	1,250	1,240	1,130	1,079
$eff(\hat{e}_2)$	1,089	1,172	1,241	1,230	1,132	1,084

Source: own calculations

Table 6. Efficiency of mean estimation on the second occasion for population 2, n = 50

p	0,1	0,2	0,4	0,6	0,8	0,9
$eff(e_2)$	1,169	1,165	1,234	1,200	1,152	1,085
$eff(\hat{e}_2)$	0,888	0,990	1,206	1,162	1,147	1,084

Source: own calculations

Substituting unknown correlation coefficient and regression coefficient in estimator e_2 by their assessments based on the sample in most cases increased efficiency of the population mean estimation on the second occasion compared to common sample mean. Efficiency of the estimation decreased only in the case of very low number of elements examined on both occasions, namely not greater than 10.

In Tables 7-9 efficiency of the estimation of net changes is presented. Efficiency of the estimation for estimators $e_2 - \bar{y}_1$ and $\hat{e}_2 - \bar{y}_1$ compared to difference of two usual sample means is defined respectively:

$$eff\,(e_2 - \bar{y}_1) = \frac{MSE(\bar{y}_2 - \bar{y}_1)}{MSE(e_2 - \bar{y}_1)},\ eff\,(\hat{e}_2 - \bar{y}_1) = \frac{MSE(\bar{y}_2 - \bar{y}_2)}{MSE(\hat{e}_2 - \bar{y}_1)}.$$

Table 7. Efficiency of net changes estimation for population 1, n = 100

p	0,1	0,2	0,4	0,6	0,8	0,9
$eff\,(e_2 - \bar{y}_1)$	1,072	1,096	1,131	1,113	1,081	1,035
$eff\,(\hat{e}_2 - \bar{y}_1)$	1,054	1,088	1,139	1,114	1,081	1,032

Source: own calculations

Table 8. Efficiency of net changes estimation for population 2, n = 100

p	0,1	0,2	0,4	0,6	0,8	0,9
$eff\,(e_2 - \bar{y}_1)$	1,024	1,359	1,453	1,364	1,248	1,138
$eff\,(\hat{e}_2 - \bar{y}_1)$	1,219	1,417	1,472	1,368	1,252	1,155

Source: own calculations

Table 9. Efficiency of net changes estimation for population 2, n = 50

p	0,1	0,2	0,4	0,6	0,8	0,9
$eff\,(e_2 - \bar{y}_1)$	1,072	1,096	1,131	1,113	1,081	1,035
$eff\,(\hat{e}_2 - \bar{y}_1)$	1,054	1,088	1,139	1,114	1,081	1,032

Source: own calculations

In the case of net changes estimation for multi-purpose surveys, substituting unknown correlation coefficient and regression coefficient in estimator e_2 given by (1) by their estimates in all cases increased efficiency of net changes estimation, even for low number of elements examined on both occasions.

In Tables 10-12 efficiency of the estimation of combined sample means from two periods is presented. Efficiency of the estimation for estimators $e_2 + \bar{y}_1$ and $\hat{e}_2 + \bar{y}_1$ compared to summing usual sample means is defined respectively:

$$eff\,(e_2 + \bar{y}_1) = \frac{MSE(\bar{y}_2 + \bar{y}_1)}{MSE(e_2 + \bar{y}_1)},\ eff\,(\hat{e}_2 + \bar{y}_1) = \frac{MSE(\bar{y}_2 + \bar{y}_2)}{MSE(\hat{e}_2 + \bar{y}_1)}.$$

Table 10. Efficiency of the net changes estimation for population 1, n = 100

P	0,1	0,2	0,4	0,6	0,8	0,9
$eff\,(e_2 + \bar{y}_1)$	1	1,013	1,006	1,015	1,09	1,014
$eff\,(\hat{e}_2 + \bar{y}_1)$	0,934	1,010	1,001	1,013	1,008	1,012

Source: own calculations

Table 11. Efficiency of the net changes estimation for population 2, n = 100

p	0,1	0,2	0,4	0,6	0,8	0,9
$eff\,(e_2 + \bar{y}_1)$	0,990	0,958	1,008	1,043	1,049	1,034
$eff\,(\hat{e}_2 + \bar{y}_1)$	0,806	0,872	0,990	1,021	1,044	1,034

Source: own calculations

Table 12. Efficiency of the net changes estimation for population 2, n = 50

p	0,1	0,2	0,4	0,6	0,8	0,9
$eff\,(e_2 + \bar{y}_1)$	0,995	0,965	0,999	1,055	1,031	1,026
$eff\,(\hat{e}_2 + \bar{y}_1)$	0,926	0,907	0,989	1,048	1,032	1,027

Source: own calculations

According to formula (2) applying estimator e_2 does not always increase precision of the combined population means estimation. The main problem considered in the paper is the influence of substituting unknown population coefficients by its estimates based on the sample. So we focus only on cases in which the effect of using estimator e_2 is different from that of using $\hat{e}_2$. This is the case of population 1, n = 100, p = 0,1 and population 2, n = 100, p = 0,4 only.

CONCLUSIONS

Substituting unknown population correlation and regression coefficients by its estimates on the basis of the sample and applying estimator that uses information from previous period caused decrease of the population mean estimation in three extreme cases only, namely for $np \leq 10$. In all other cases efficiency of the mean estimation on the second occasion increased compared to applying usual sample mean. In the case of multi-purpose surveys using previous information and estimator $\hat{e}_2$ increased efficiency of net changes estimation in all considered cases. Estimation of combined population mean from two successive periods posed more of a problem. But this population parameter is rarely used in practice. Population mean on each current occasion and net changes are of utmost importance in real surveys.

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A TAXONOMIC ANALYSIS OF DIVERSIFICATION IN THE CONSUMPTION STRUCTURE IN HOUSEHOLDS IN THE EU COUNTRIES

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Abstract: The aim of the article was to identify differences in the consumption structure in households in the EU countries in 2010. Ward's method was used to identify the types of the EU countries with different structures of consumption in households and to present the diversification. The research problem in question was analysed on the basis of the data from Eurostat and the International Statistics Yearbook, published annually by the Central Statistical Office. As a result of the research with Ward's method eight types of the EU countries with different consumption structures in households were distinguished.

Keywords: consumption, households, Ward's method

INTRODUCTION

In the microeconomic approach referring to the household consumption[1] is understood as the process of using acquired, produced or shared goods to satisfy the joint and individual needs of its members [Kramer 1997, Piskiewicz 2009]. The scale and structure of consumption in households in the EU countries is clearly diversified, which is caused by such reasons as regional differences in the socioeconomic development. Historical, geographical and cultural factors are of equal importance.

[1] The Polish economic literature uses two terms referring to consumption – 'konsumpcja' and 'spożycie'. Both terms are synonymous and interchangeable [Zalega 2012].

By analysis of the structure of expenditures in households it is possible to grasp the similarities and differences in their living standard and in consequence, to assess the living standard of entire society [Szumowicz 1995]. At present the issues related with people's living standard are becoming one of the key terms in politics and social development strategies.

The main goal of the article was to identify differences in the structure of consumption in households in the EU countries in 2010. Studies on the spatial diversification of phenomena more and more often use taxonomic methods, which enable the identification of homogenous groups of regions with similar traits, which are subject to analysis [Wysocki 2010]. Therefore, the diversification in the structure of consumption in households in the EU countries was assessed by means of cluster analysis - Ward's method.

RESEARCH METHODOLOGY

In order to determine diversification in the consumption structure in households in the EU countries data from national accounts were used. They present the consumption structure of the entire household sector as groups of commodities and services. The presentation of the structure of consumption expenditures on individual purposes eliminates the problem of diversification in the level of prices and income in households in individual countries.

The research problem in question was analysed on the basis of the data from *Eurostat* and the *International Statistics Yearbook,* published annually by the Central Statistical Office.

By means of Ward's method the EU countries were classified in the following stages according to the consumption structure in households in 2010 [Wysocki 2010]:

Stage 1. Partial measures of the consumption structure in households were selected on the basis of factual premises:

X_1 – the share of expenditures on food and soft drinks in total expenditures (%),
X_2 – the share of expenditures on alcoholic beverages and tobacco products in total expenditures (%),
X_3 – the share of expenditures on clothing and footgear in total expenditures (%),
X_4 – the share of expenditures on the use of lodgings and energy carriers in total expenditures (%),
X_5 – the share of expenditures on furnishings in total expenditures (%),
X_6 – the share of expenditures on health in total expenditures (%),
X_7 – the share of expenditures on transport in total expenditures (%),
X_8 – the share of expenditures on communication in total expenditures (%),
X_9 – the share of expenditures on recreation and culture in total expenditures (%),
X_{10} – the share of expenditures on education in total expenditures (%),
X_{11} – the share of expenditures on restaurants and hotels in total expenditures (%).

In order to eliminate excessively correlated traits an invertible matrix was formulated for the correlation coefficients between them. No trait was rejected on the basis of the analysis of the diagonal elements of the matrix.

Stage 2. The values of the traits were classically standardised according to the following formula:

$$z_{ik} = \frac{x_{ik} - \bar{x}_k}{s_k} \tag{1}$$

where:

x_{ik} - the value of the *k*-th trait in the *i*-th object,
$\bar{x}_k$ - the arithmetic mean of the *k*-th trait,
s_k - the standard deviation of the *k*-th trait.

Stage 3. Ward's method[2] was applied for cluster analysis, which consists in combining the closest units with each other until one cluster is made. It uses the analysis of variance to estimate the distance between units, aiming at minimisation of the sum of squared deviations within clusters [Stanisz 2006]. At each stage of the method, the pair of clusters which results in a minimally diversified combination is chosen from all combinable pairs of clusters [Everitt et al. 2011]. The desirable number of classes of the EU countries with a similar consumption structure in households was determined upon analysis of the diagram with the course of agglomeration.

Stage 4. The typological classes were interpreted and described. The identification of types consists in describing them by means of selected descriptive statistics (intra-class means) of selected traits. In order to identify the characteristic traits in each class the *difference-of-means measure* was calculated according to the following formula:

$$z_{ck(d)} = \frac{\bar{x}_{ck} - \bar{x}_k}{s_{k(w)}} \tag{2}$$

where:

C – the number of classes $(c = 1, \ldots, C)$,
K – the number of traits $(k = 1, \ldots, K)$,
$\bar{x}_{ck}$ - the mean of the *k*-th trait in the *c*-th class,
$\bar{x}_k$ - the overall mean of the *k*-th trait in a group consisting of *N* objects,
$s_{k(w)}$ - the average intra-class diversification in the *k*-th trait value, which is calculated according to the following formula:

$$s_{k(w)} = \left[\frac{1}{N-C} \sum_{c=1}^{C} (N_c - 1) \cdot s_{ck}^2 \right]^{1/2} \tag{3}$$

where:

[2] Statistica 10.0 software was used in the computing process.

s_{ck}^2 - the intra-class variance in the *c*-th class calculated in relation to the *k*-th trait.

$z_{ck(d)}$ - values were the basis for distinguishing characteristic traits in typological classes with the following scale of values, where [Wysocki 2010]:

1. $z_{ck(d)} \in (-\infty;-3> \vee <3;+\infty)$ - there is very high intensity of the *k*-th trait in the *c*-th class, the trait is highly characteristic (positively or negatively),
2. $z_{ck(d)} \in (-3;2> \vee <2;3)$ - there is high intensity of the *k*-th trait in the *c*-th class, the trait is averagely characteristic (positively or negatively).
3. $z_{ck(d)} \in (-2;2)$ - there is average intensity of the *k*-th trait in the *c*-th class, the trait does not stand out and it is not characteristic.

RESEARCH RESULTS – THE TYPOLOGY OF THE EU COUNTRIES ACCORDING TO THE CONSUMPTION STRUCTURE IN HOUSEHOLDS IN 2010

Spatial diversification in the consumption structure in households in the EU countries was presented by means of cluster analysis. Standardised data concerning the consumption structure in households in the EU-27[3] countries were classified by means of Ward's method. In order to determine the optimal number of clusters factual premises and conclusions from the analysis of the diagram with the course of agglomeration were taken into consideration. As a result, the community under investigation was divided into eight clusters (Fig. 1).

Tables 1 and 2 present a description of the identified types of countries with a similar consumption structure in households. Table 1 includes the intra-class values of simple traits describing the consumption structure in households in the identified typological classes. Two passive traits were used for the description (characterisation) of the identified types, i.e. the GDP value (euros per head) and the value of disposable income in the household sector (PPS[4] per head). Table 2 includes the values of the *difference-of-means measure* for the traits under analysis in the group of 26 EU countries according to the identified classes. Specific and non-specific traits in the specified typological classes were identified on the basis of these values.

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[3] Bulgaria was not included in the research due to the absence of statistics published by *Eurostat.*

[4] PPS - *Purchasing Power Standard* is a common contractual currency adopted in the European Union. Purchasing power parities are a kind of exchange rates which are used for the conversion of economic indexes expressed in national currencies into the common contractual currency.

Class I consists of two countries – Romania and Lithuania, which are characterised by the *food consumption model*[5]. This fact is proved by a very high share of expenditures on food and alcoholic beverages in the household budget of these countries, i.e. 27.8%. It is necessary to stress the fact that it was about 18% higher than in highly developed countries, which formed typological Classes VII and VIII (Fig. 1, Table 1). It is necessary to note the fact that Romania and Lithuania also had the highest share of fixed expenditures[6] in the total household expenditures, i.e. 47%, whereas the European mean was 36.5% (Table 1). The share of necessary expenditures in total expenditures is a determinant of the living standard in societies. In highly socioeconomically developed countries the share of these expenditures in household budgets is low and does not exceed 35% of total expenditures (e.g. in the UK – 33.9%, in Luxembourg-33.5%, in Germany-35.6% in 2010). On the other hand, in less socioeconomically developed countries this share is much higher and limits the capacity to satisfy other needs in the household, especially higher-order needs (e.g. it was 52.6% in Romania in 2010) [*International Statistics Yearbook* 2012].

Apart from that, Class I countries are distinguished by the low share of expenditures on recreation and culture (5.4%) and those on restaurants and hotels (3.9%) in the consumption structure in households (Table 1). This may have been caused by such factors as the poorer economic situation of those households, because the average disposable monthly income in Romanian and Lithuanian households in 2010 amounted respectively to 40.3% and 56.0% of the average income in the household sector in the EU-27 [*International Statistics Yearbook* 2012].

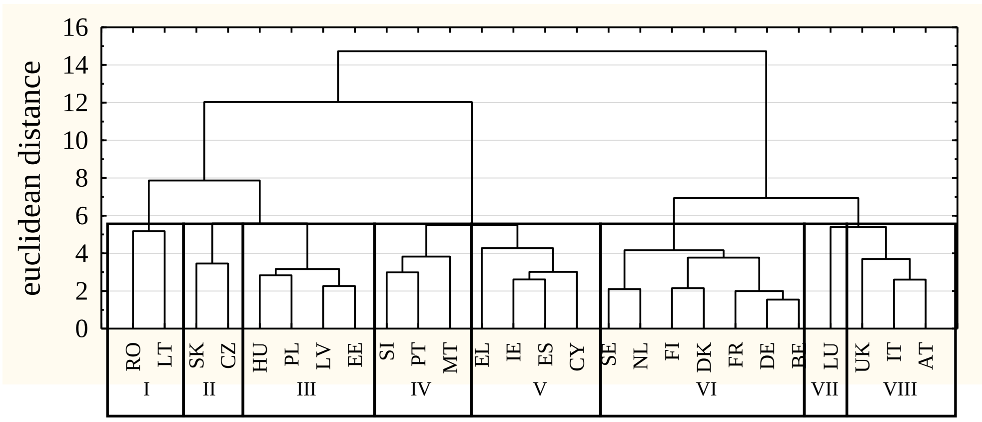
Class II consists of two countries – Slovakia and the Czech Republic. This group of countries was characterised by an average share of expenditures on food and soft drinks and a high share of expenditures on alcoholic beverages and tobacco products in the consumption structure of households, which in 2010 reached 15.6% and 7.1%, respectively. The high rate of the share of expenditures on alcoholic beverages and tobacco products in the household budgets in this group may be related with such factors as the tradition of beer consumption [Smoluk-

[5] We can talk about the food consumption model when there is a high share of expenditures on food in total expenditures, which affects the capacity to satisfy other demands [Kuśmierczyk, Piskiewicz 2012].

[6] The group of fixed (necessary) expenditures include the expenditures related with satisfying basic household needs, which cannot be postponed in time. Most researchers include expenditures on food and soft drinks and those related with the use of lodgings and energy carriers into this group. The other expenditures are free-choice expenditures (free decision fund). The division line between fixed expenditures and free-choice expenditures is fluid and disputable, especially as far as expenditures on transport, furnishings and health are concerned [Skrzypczak 2008]. The aim of comparison of the structure of consumption expenditures in households, including this division, is to assess the degree of modernity of structures in the typological groups under investigation.

Sikorska, Kalinowski 2011]. The countries in this class were also distinguished by the lowest share of expenditures on transport and the highest share of expenditures on communication in the total expenditures in households, i.e. 8.2% and 3.5%, respectively (Fig. 1, Table 1).

Figure 1. The classification of the EU countries [a)] by means of Ward's method according to the consumption structure in households in 2010.



[a)] AT - Austria, BE - Belgium, CY – Cyprus, CZ-Czech Republic, DE - Germany, DK – Denmark, EE – Estonia, EL – Greece, ES-Spain, FR – France, HU – Hungary, IE – Ireland, IT – Italy, LU - Luxembourg, LT - Lithuania, LV - Latvia, NL – Netherlands, PL – Poland, PT – Portugal, RO - Romania, SE – Sweden, SI – Slovenia, SK - Slovakia, UK - United Kingdom.

Source: own compilation based on the data from the *International Statistics Yearbook 2012*, Central Statistical Office

Class III consists of four countries - Hungary, Poland, Latvia and Estonia, which are situated in Central and Eastern Europe (Fig. 1). The countries in this class had a high share of expenditures on food and soft drinks in the total expenditures in households, but this share was almost 9% lower than in Class I (Table 1). The high share of expenditures on food in the total expenditures in households in the countries belonging to Classes I and III shows certain disproportions in the economic development of those countries, as compared with the other ones.

In the countries that were grouped as Class III it is also possible to notice a relatively low share of expenditures on furnishings and those on restaurants and hotels in household budgets, i.e. 4.2% and 5.2%, respectively. This may have resulted from the lower socioeconomic development of those countries, which is measured with the GDP per head. This value affects the income in households, which chiefly influences the consumption level and structure (Table 1). The households in the countries which formed Class III, i.e. Hungary, Poland, Latvia and Estonia, were in a much worse financial situation, because in 2010 they reached 57.2%, 62.6%, 46.5% and 56.0% of the average disposable income in the household sector in the EU-27, respectively [*International Statistics Yearbook* 2012].

Table 1. The inter-class diversity of the EU countries in the consumption structure in households in 2010

Specification	Typological group								Total
	I	II	III	IV	V	VI	VII	VIII	
	RO, LT	SK, CZ	HU, PL, LV, EE	SI, PT, MT	EL, IE, ES, CY	SE, NL, FI, DK, FR, DE, BE	LU	UK, IT, AT	
Food and soft drinks (%)	27,8	15,6	19,2	15,2	13,2	12,3	9,2	10,2	12,9
Alcoholic beverages and tobacco products (%)	5,5	7,1	7,5	3,1	4,1	3,6	8,6	3,4	3,5
Clothing and footgear (%)	5,6	3,6	4,6	5,2	5,0	5,0	3,7	6,0	5,3
Use of lodgings and energy carriers (%)	19,2	26,1	23,4	15,6	21,3	25,5	24,3	22,2	23,6
Furnishings (%)	5,0	5,8	4,2	6,4	4,9	5,8	6,8	6,6	5,8
Health (%)	4,3	3,3	4,0	4,3	4,7	3,8	2,1	3,0	3,7
Transport (%)	14,8	8,2	12,8	12,7	12,0	12,1	17,1	13,1	13,0
Communication (%)	2,3	3,5	3,3	3,1	3,2	2,7	1,7	2,1	2,7
Recreation and culture (%)	5,4	10,0	7,8	9,2	7,1	10,0	8,2	10,4	8,9
Education (%)	1,5	1,2	1,4	1,3	2,0	0,6	0,9	1,0	1,1
Restaurants and hotels (%)	3,9	6,6	5,2	11,0	14,2	5,8	7,0	9,9	8,6
GDP per head (euros)	7100	13150	9500	16200	21700	33300	79500	27500	24400
Disposable income per head in household sector (by PPS)	9318	13517	10961	16034	19288	21331	28882	21919	19359

Source: own compilation based on the data from the *International Statistics Yearbook 2012*, published by the Central Statistical Office (2012) and on the data from *Eurostat*.

Class IV consists of three countries - Slovenia, Portugal and Malta (Fig. 1). The countries were characterised by an average share of expenditures on food and soft drinks and a very low share of expenditures on the use of lodgings and energy carriers in the consumption structure in households in 2010, i.e. 15.2% and 15.6%, respectively. The low share of expenditures on the use of lodgings and energy carriers in households in those countries may be chiefly related with the climatic conditions. The group was also characterised by a relatively high share of expenditures on restaurants and hotels in the total expenditures in households, i.e. 11.0% (Table 1). The diversified values of the index of the share of expenditures on restaurants and hotels in the households of the EU countries can be explained with such factors as the diversified financial situation, the prices of gastronomic and hotel services, the hierarchy of consumer needs and different lifestyles in societies.

Class V includes four countries - Greece, Ireland, Spain and Cyprus (Fig. 1). The countries have a relatively high level of socioeconomic development because in 2010 the average gross domestic product in this group was 21,700 euros per head. The countries also had a relatively low share of fixed expenditures in household budgets, i.e. 30.8% in 2010 (Table 1). This resulted from a relatively good financial situation of households in those countries. They were also characterised by a high share of expenditures on education (2.0%) and those on restaurants and hotels (14.2%) in the consumption structure of households, i.e. expenditures on higher-order needs (Table 1). In comparison with the other EU countries the differences in the share of higher-order expenditures in household budgets in this group may have been strongly influenced not only by economic factors but also by cultural factors related with the tradition of a particular country.

Table 2. The values of the *difference-of-means measure* [a)] for the traits describing the consumption structure in households in the EU countries in 2010 according to the typological classes identified

Specification	Typological group							
	I	II	III	IV	V	VI	VII	VIII
	RO, LT	SK, CZ	HU, PL, LV, EE	SI, PT, MT	EL, IE, ES, CY	SE, NL, FI, DK, FR, DE, BE	LU	UK, IT, AT
Food and soft drinks (%)	8,0	1,4	3,4	1,2	0,1	-0,3	-2,0	-1,5
Alcoholic beverages and tobacco products (%)	1,6	3,0	3,3	-0,3	0,5	0,1	4,3	-0,1
Clothing and footgear (%)	0,3	-1,6	-0,7	-0,1	-0,3	-0,3	-1,5	0,7
Use of lodgings and energy carriers (%)	-1,8	1,0	-0,1	-3,3	-1,0	0,8	0,3	-0,6
Furnishings (%)	-1,3	0,0	-2,8	1,0	-1,5	0,0	1,7	1,3
Health (%)	0,6	-0,4	0,3	0,6	0,9	0,1	-1,5	-0,7
Transport (%)	1,4	-3,7	-0,2	-0,2	-0,8	-0,7	3,2	0,1
Communication (%)	-0,8	1,6	1,1	0,8	1,0	0,0	-2,0	-1,2
Recreation and culture (%)	-2,7	0,8	-0,8	0,2	-1,4	0,8	-0,5	1,1
Education (%)	1,0	0,1	0,8	0,5	2,3	-1,3	-0,5	-0,3
Restaurants and hotels (%)	-2,4	-1,0	-1,7	1,2	2,8	-1,4	-0,8	0,7

[a)] The shades of grey refer to the medium and high absolute values of the difference-of-means measure $(z_{ck(d)})$, which were the basis for highlighting specific traits in individual classes (light grey – relatively high intensity, dark grey - very high intensity).

Source: own compilation based on the data from Table 1, processed with Statistica 10 computer software.

The most numerous Class VI was formed by seven countries - Sweden, the Netherlands, Finland, Denmark, France, Germany and Belgium, which are situated in northern and western Europe (Fig. 1). The consumption structure in households in those countries was the closest to the EU-27 average. These countries are characterised by a high level of socioeconomic development and high living standard of their inhabitants. This fact is proved by such indexes as the low share of expenditures on food and soft drinks in total expenditures, which amounted to 12.2% in this class. The characteristic trait in this typological class was a relatively low share of expenditures on education (0.6%) and those on restaurants and hotels (5.8%) (Table 1). When making comparisons and assessments of the indexes of the share of expenditures on basic goods and those on higher-order goods in the structure of expenditures related with consumption in the typological groups under analysis it is also necessary to bear in mind that the state finances some services related with health and education. Depending on the policy of public expenditures in a particular country households can benefit from free medical or educational services to a greater or lesser extent. Their tendency or need to spend extra funds on this purpose is low. In consequence, this implicates the low share of expenditures in these categories in total expenditures. The correction of expenditures in these categories by the share of the state in these expenditures makes a significant change in the relations between the expenditures on basic goods and those on higher-order goods.

Class VII was formed by one country, i.e. Luxembourg (Figure 1). The high level of socioeconomic development in this country influences very good financial situation of households, which were characterised by the lowest share of expenditures on food and soft drinks in total expenditures (9.2% in 2010) and a high share of expenditures on alcoholic beverages and tobacco products (8.6%), transport (17.1%) and furnishings (6.8%) (Table 1).

The last class – VIII was formed by three countries – the United Kingdom, Italy and Austria, which belong to the group of highly socioeconomically developed countries (Fig. 1). In 2010 the disposable income in households in those countries amounted to 113.2%, 103.5% and 118.3% of the average income in the household sector in the EU-27. The good financial situation of households in the countries in class VIII resulted in lesser burdening of their budgets with fixed expenditures, which reached 32.4% in 2010 (Table 1). The characteristic trait of the structure of consumption in those households was also a high share of expenditures on recreation and culture and those on restaurants and hotels, i.e. 10.4% and 9.9%, respectively.

The results of the assessment of diversification in the structure of consumption in households in the EU countries obtained by means of cluster analysis were compared with the results of the research by [Kuśmierczyk and Piskiewicz 2012]. The authors made a multidimensional analysis and identified six clusters with a similar consumption structure. As a result of our investigations eight clusters were distinguished for the purposes of this article. The components ofindividual classes were largely identical.

CONCLUSIONS

Upon the analyses conducted by means of Ward's method eight types of the EU countries differing in the consumption structure in households were distinguished. The findings of the research lead to the following conclusions:

- Romania and Lithuania (Class I) and Hungary, Poland, Latvia and Estonia (Class III) were characterised by a high share of expenditures on food and soft drinks in the total expenditures in households. This fact points to certain disproportions in the economic development of those countries in comparison with the other countries.

- Slovakia and the Czech Republic (Class II) were characterised by an average share of expenditures on food and soft drinks in total expenditures and a high share of expenditures on alcoholic beverages and tobacco products in the structure of consumption in households.

- Slovenia, Portugal and Malta (Class IV) were distinguished by a very low share of expenditures on the use of lodgings and energy carriers in the structure of consumption in households, which chiefly resulted from the climatic conditions, and there was also a relatively high share of expenditures on restaurants and hotels in total expenditures in households.

- Greece, Ireland, Spain and Cyprus (Class V) were characterised by a relatively low share of fixed expenditures in household budgets and by a high share of expenditures on education and those on restaurants and hotels, i.e. higher-order goods and services. The differences in the share of higher-order expenditures in household budgets, as compared with other EU countries, may have been strongly influenced not only by economic factors but also by those related with the tradition of a particular country.

- The consumption structure in households in Sweden, the Netherlands, Finland, Denmark, France, Germany and Belgium (Class VI) was the closest to the average in the EU-27. These countries are characterised by a high level of socioeconomic development and a high living standard of their inhabitants.

- Luxembourg (Class VII) was characterised by the lowest share of expenditures on food and soft drinks in total expenditures and by a high share of expenditures on alcoholic beverages and tobacco products, transport and furnishings.

- The United Kingdom, Italy and Austria (Class VIII), which are classified as highly socioeconomically developed countries, were characterised by low burdening of household budgets with fixed expenditures and by a high share of expenditures on recreation and culture and those on restaurants and hotels in the structure of consumption in households.

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QUANTITATIVE METHODS IN ECONOMICS
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HISTORICAL AND IMPLIED VOLATILITIES: A REVIEW OF METHODOLOGY

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Abstract: Volatility is a subject of numerous studies. Many of them focus on predictive power of different sources of volatility. Most often, the Black-Scholes implied volatility is believed to outperform historical volatility, although some research demonstrates that implied volatility is a biased forecast of future volatility. Taken into account different opinions, the paper aims at presenting alternative methods for estimating volatility.

Keywords: historical volatility, Black-Scholes implied volatility, model-free implied volatility

INTRODUCTION

In 1973 Black and Scholes developed an option pricing model that depends upon five variables: stock price, strike price, time to maturity, risk-free rate, and the standard deviation of returns from the underlying stock – volatility. Of the five variables that are necessary to specify the model, all are directly observable except the last one (the risk-free rate of interest may be closely approximated by the rate of return on short term government securities, however Beckers [1981] states the model is not very sensitive to an exact specification of the risk-free rate). Thus, the most important is to estimate the standard deviation of the stock's rate of return over remaining life of the option.

There are two basic ways to assess the volatility: the first one uses historical data on underlying asset prices, and the second technique uses option prices to find the option market's estimate of the stock's standard deviation. This estimate of the stock's standard deviation drawn from the options market is called an implied volatility [Kolb, Overdahl 2007]. When evaluating volatility using historical data, there is no general rule how far back in the history the data should be used to estimate the parameter. According to Hull [2012] more data generally lead to more accuracy. On the contrary, too old data may not be relevant for predicting the future volatility. He suggests to use closing prices from daily data over the most recent 90 to 180 days.

There are also more sophisticated approaches to estimating historical volatility involving exponential weighted average or GARCH model. In practice, traders usually work with implied volatilities. They are used to monitor the market's opinion about the volatility of a particular stock. Whereas historical volatilities are referred to as backward looking, implied volatilities are referred to as forward looking. The implied volatility can be interpreted as the average volatility that the underlying asset will have from now to the option's expiration time, or it can be used to forecast the change of underlying asset price in a short term [Zahng 2006].

The volatility implied from option prices is widely believed to be informationally superior to the historical volatility of the underlying asset. Musiela and Rutkowski [2007] present studies, both confirming and negating the superiority of implied volatility over the historical one. They quote Latané and Rendleman [1976], Schmalensee and Trippi [1978], Beckers [1981] who found that estimates of the actual volatility based on market implied volatilities outperform, at least in terms of their predictive power, more straightforward estimates based on historical data. Contrary to these findings, subsequent studies of stock index options, reported in Canina and Figlewski [1992], Day and Lewis [1992], and Lamoureux and Lastrapes [1993] suggest that the implied volatility has virtually no correlation with future volatility. Moreover, Jiang and Tian [2005] note in their paper that nearly all research on the information content of implied volatility are focused on implied volatility derived from at-the-money options. By concentrating on at-the-money options alone, these studies fail to incorporate the information contained in other options. Taking into account all those pros and cons, the paper aims at presenting alternative methods for estimating, both historical and implied volatilities.

HISTORICAL VOLATILITY

To estimate volatility using historical data, several techniques could be used. In his book Haug [2007] presents historical volatility from close prices, high-low volatility, high-low-close volatility, and exponential weighted historical volatility.

Historical volatility from close prices

The most widely used method for estimating historical volatility is calculation of the annualized standard deviation given by the formula (1):

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\ln\left(\frac{Close_i}{Close_{i-1}}\right)^2 - \frac{1}{n(n-1)}\left[\sum_{i=1}^{n}\ln\left(\frac{Close_i}{Close_{i-1}}\right)\right]^2}\ , \tag{1}$$

where n is the number of observations. When assuming 252 trading days in a year, the annualized close volatility is obtained by multiplying σ from formula (1) with the square root of 252.

High-Low volatility

Parkinson [1980] suggests estimating the standard deviation by:

$$\sigma = \frac{1}{2n\sqrt{\ln(2)}}\sum_{i=1}^{n}\ln\left(\frac{High_i}{Low_i}\right). \tag{2}$$

The result should be also multiplied with the square root of 252. The high-low method is statistically much more efficient than the standard close method (in terms of number of observations needed to get the same interval compared with the standard close method). However, it assumes continuous trading and observations of high and low prices. Thus the method can underestimate the true volatility. The same is true for the high-low-close volatility [Haug 2007].

High-Low-Close volatility

Garman and Klass (1980) propose using a volatility estimator of the following form:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}\left[\ln\left(\frac{High_i}{Low_i}\right)\right]^2 - \frac{1}{n}\sum_{i=1}^{n}\left[2\ln(2)-1\right]\left[\ln\left(\frac{Close_i}{Close_{i-1}}\right)\right]^2}\ . \tag{3}$$

The annualized high-low-close volatility is obviously got by multiplying the result with the square root of 252.

Exponential weighted historical volatility

Exponential weighted volatility often referred to as exponentially weighted moving average (EWMA) puts more weight on more recent observations. It can be calculated as:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\left[\ln\left(S_t / S_{t-1}\right)\right]^2, \tag{4}$$

where σ_t is the current volatility and σ_{t-1} is the volatility as calculated one observation ago (see also Hull [2012]). If one uses daily data, the volatility is annualized by multiplying the result with the square root of the number of trading days per year. In most markets λ should be between 0.75 and 0.98. The RiskMetrics developed by J.P. Morgan uses an EWMA with λ=0.94 [Haug 2007].

GARCH-based volatility

A more sophisticated approach to estimating volatility is based on the GARCH (1,1) model. Since 1982 when Robert Engle first introduced the ARCH (autoregressive conditional heteroscedasticity) model, whose generalized version - GARCH was proposed by Bollerslev in 1986, a number of their extensions have arisen. Nevertheless, the simplest and the most widely used is the GARCH (1,1) model. It may be useful in predicting future volatility [Rouah, Veinberg 2007]:

$$\sigma_{t+1}^2 = \omega + \alpha r_t^2 + \beta \sigma_t^2 , \tag{5}$$

where $\alpha + \beta$ measures the volatility persistence, and $\alpha + \beta < 1$ is required for the variance to be mean-reverting. Moreover, the closer the value of $\alpha + \beta$ to one, the more volatility will persist, and the closer $\alpha + \beta$ is to zero, the faster volatility will revert to the long run variance. In the model (5) the forecast variance for time t+1, σ_{t+1}^2, is a weighted average of the squared return at time t, r_t^2, and the time-t estimate of the variance, σ_t^2 (the GARCH(1,1) model puts more weight on the most recent squared return). The long-run variance in the GARCH (1,1) model (5) is:

$$\sigma^2 = \frac{\omega}{1 - \alpha - \beta} . \tag{6}$$

To annualize the volatility, one must multiply it with the square root of 252.

Although the methods presented in this section are often easy to implement, Rouah and Veinberg [2007] note they constitute a retrospective estimate of asset volatility, since they are based on historical prices. Hence, many authors advocate the use of implied volatility as it reflects future expectations about volatility rather than reflecting past realization.

BLACK-SCHOLES IMPLIED VOLATILITY

The most popular type of implied volatilities are Black-Scholes implied volatilities. They are obtained by equating an observed market price with a given strike price and maturity to the Black-Scholes formula with the same strike price and maturity. The value of volatility in the Black-Scholes formula that yields the observed option price is the implied volatility. To find implied volatilities, one begins with established values for the stock price (S), the exercise price (X), the interest rate (r), the time until expiration (T), and the option price (C). Although it is impossible to solve the Black-Scholes equation directly for the standard deviation, one can use numerical search to closely approximate the standard deviation by any given option price. To do this, some iterative methods are applied. Chriss [1997] describes two of them: the method of bisections, and the Newton-Raphson method.

The method of bisections

The method of bisections works as follows:

1. Step 1. Choose a first guess for the implied volatility that must be greater than the actual implied volatility, and write σ_0 for this guess (if the Black-Scholes value computed using σ_0 is greater than the market price of option, σ_0 is greater than implied volatility).
2. Step 2. This step produces the next implied volatility guess. As we have already ensured that σ_0 is greater than the actual implied volatility, we need to make our next volatility guess lower. We set σ_1 to σ_0 reduced by 50% ($\sigma_1 = \sigma_0 - \sigma_0 / 2 = 0{,}50\sigma_0$). This is the first "real" guess – σ_1.
3. Step 3. In this step, we produce the next guess σ_2. First, we need to compute the Black-Scholes value with given input parameters *C*(*S*, *X*, *T*, σ_1, *r*). If it is larger than the actual option price, then we set the next guess by reducing σ_1 by half as much as we did last time ($\sigma_2 = \sigma_1 - \sigma_0 / 4$). If *C*(*S*, *X*, *T*, σ_1, *r*) is smaller than the actual option price, we increase σ_1 by $\sigma_0 / 4$ ($\sigma_2 = \sigma_1 + \sigma_0 / 4$).
4. Step 4. Iterate the process: compute σ_k from σ_{k-1}, compute the Black-Scholes value using σ_{k-1}, the volatility guess from the previous step. If it is larger than the market price, form the next guess by reducing σ_{k-1} by $\sigma_0 / 2^k$ ($\sigma_k = \sigma_{k-1} - \sigma_0 / 2^k$). Otherwise, rise σ_{k-1} by $\sigma_0 / 2^k$ ($\sigma_k = \sigma_{k-1} + \sigma_0 / 2^k$).

The most obvious place to stop is when a volatility guess produces a Black-Scholes price exactly equal to the market price. We can also have a preset "error tolerance". The only drawback of the method is its speed. It converges rather slowly. A faster technique is the Newton-Raphson method.

The Newton-Raphson method

The Newton-Raphson method works in a following way:

1. Step 1. The first step is to guess what the correct implied volatility is and call this guess σ_1. Haug [2007] presents an efficient seed value when the Newton-Raphson method is used to compute the implied volatility. The seed value developed by Manaster and Koehler in 1982 will guarantee convergence (if the implied volatility exists) for European Black-Scholes stock options. The seed value is: $\sigma_1 = \sqrt{\left|\ln(S/X) + rT\right| \cdot 2/t}$. Next we compute the Black-Scholes

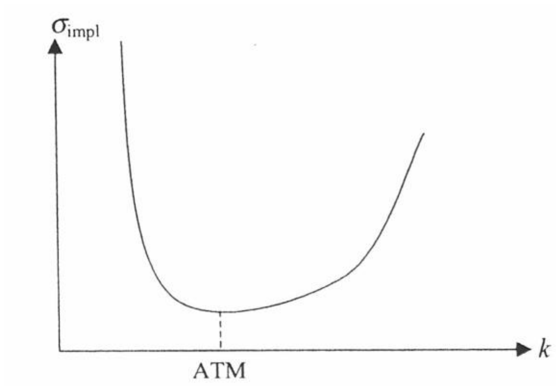
value of the option with σ_1, denoted $C(\sigma_1)$, and the vega of this option, denoted $V(\sigma_1)$.

2. Step 2. We compute the value of our next volatility guess: $\sigma_2 = \sigma_1 - (C(\sigma_1) - C)/V(\sigma_1)$, where C is the market value of the option.
3. Step 3. The n^{th} volatility guess is given by: $\sigma_n = \sigma_{n-1} - (C(\sigma_{n-1}) - C)/V(\sigma_{n-1})$.

The Newton-Raphson method requires knowledge of the partial derivative of the option pricing formula with respect to volatility (vega) when searching for the implied volatility. For some options (exotic and American options in particular), vega is not known analytically and Haug [2007] suggests using the bisection method to estimate implied volatility when vega is unknown.

Normally, implied volatilities are larger for options with shorter time to maturity. This is the term structure of volatility that describes how implied volatility varies with time until expiration. Another important feature of Black-Scholes implied volatilities is that they resemble a smile or smirk when plotted against the strike or moneyness[1]. Figure 1 presents an example of volatility smile. It shows that, all else equal, at-the-money options have a lower implied standard deviation than options that are in- or out-of-the money (the pattern can differ across option markets and across related options that differ only in their expiration dates).

Figure 1. Volatility smile



Source: Geman [2007], p. 101

As the implied volatility of a European call option is the same as that of European put option when they have the same strike price and time to maturity[2],

[1] Plotting implied volatility as a function of moneyness and maturity produces a three-dimensional graph called an implied volatility surface.

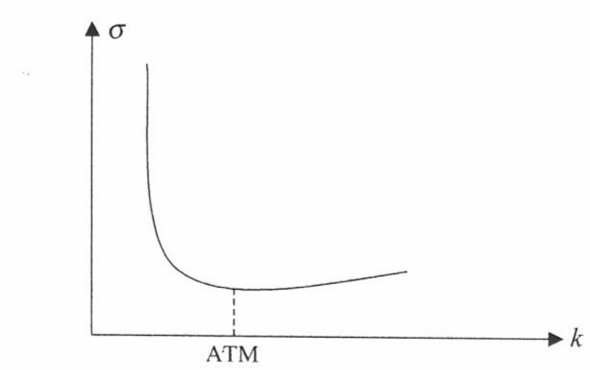
[2] Zahng [2006] presents examples showing that implied volatilities can be different for put and call options even with the same strike price and time to maturity. The difference may

this means that the volatility smile for European calls with a certain maturity is the same as that for European puts with the same maturity [Hull 2012]. In classical practice, call options are used to build the right part of the smile (i.e. out-of-the money calls), and out-of-the money put options are used for the left part. According to Rouah and Veinberg [2007] one explanation of a volatility smile is that the true distribution of asset prices has fatter tails than the distribution assumed in option pricing models. Smiles can occur, because returns show greater kurtosis than stipulated under normality, so that extreme returns are more likely. This implies that deep-in-the money options and deep-out-of-the money options are more expensive relative to the Black-Scholes price.

Kolb, Overdahl [2007] note that sometimes volatility smiles are an artifact of the exchange settlement procedure. For most listed options, at- or near-the-money options are the most liquid and quotations for this options are representative of market opinion. For deep in- or out-of-the money options, trades take place less frequently. For these illiquid options with stale prices, exchange settlement committees may set the price for clearing purposes only. This means that implied standard deviations obtained from these prices are not reflecting the market consensus.

In some markets, the volatility pattern resembles a smirk. Figure 2 presents an example of a volatility smirk with no symmetry between upward and downward movements.

Figure 2. Volatility smirk



Source: Geman [2007], p. 102

Rouah and Veinberg [2007] explain that smirks can occur because returns often show negative skewness, which of course the normal distribution does not

imply the imperfection of the actual market which violates the assumptions of the Black-Scholes model. The imperfect factors may include taxation, transaction costs, liquidity, and others.

allow. This implies that large negative returns are more likely, leading to implied volatilities for in-the-money calls that are higher than implied volatilities for out of the money calls. Similarly, implied volatilities for out-of-the money puts are higher than implied volatilities for in-the-money puts. They note that smiles and smirks are more pronounced for short-term options, and less pronounced for long-term options, which is synonymous with long-term returns being closer to normally distributed than short term returns.

At the market, there are several options on the same underlying with different strike prices and expirations traded at once. Each of them might have a different implied volatility. How to obtain a "collective assessment of volatility", then? Trippi [1977] calculated an arithmetic average, although Latané and Rendleman had previously labeled the use of such an arithmetic average as "unreasonable". In their paper [1976], they employed a weighted average of implied standard deviations as a measure of market forecasts of return variability. Their weighting system was:

$$WISD_{it} = \left[\sum_{j=1}^{N} ISD_{ijt}^{2} \cdot d_{ijt}^{2} \right]^{0.5} \cdot \left[\sum_{j=1}^{N} d_{ijt} \right]^{-1}, \tag{7}$$

where $WISD_{it}$ = weighted average implied standard deviation for company i in period t, ISD_{ijt} = implied standard deviation for option j of company i in period t, n denotes the number of options analyzed for company i and is always greater than or equal to 2, d_{ijt} = partial derivation of the price of option j of company i in period t with respect to its implied standard deviation using the Black-Scholes model.

Chiras and Manaster [1978] notice that Latané and Rendleman's weighted average is not truly a weighted average since the sum of the weights is less than one. Therefore, the weighted average implied standard deviation (*WISD*) for Latané and Rendleman is biased towards zero. Furthermore, the bias increases with an increase in the sample size even if every option was observed to have the same *ISD*. They relate Latané and Rendleman intended to weight the *ISD*s by the partial derivatives of the Black-Scholes model with respect to each implied standard deviation. That is equivalent to weighting *ISD*s according to the sensitivity of the dollar price change for the options relative to the incremental change in the implied standard deviations. A rational investor measures returns as the ratio of the dollar price change to the size of the investment, but Chiras and Manaster point out that reasoning of Latané and Rendleman emphasizes the total dollar return without regard to the size of the investment (a one-dollar price change on a one-dollar stock is considered equivalent to the same price change on a fifty-dollar stock). They give an opinion that in order to be consistent with a rational measure of returns, the price elasticity of options with respect to their *ISD*s must be considered. One must be concerned with the percentage change in the price of an option with respect to the percentage change in its *ISD*. To obtain the weighted implied standard deviation of the options on one stock for each observation date, they use the following equation:

$$WISD = \frac{\sum_{j=1}^{N} ISD_j \frac{\partial W_j}{\partial v_j} \frac{v_j}{W_j}}{\sum_{j=1}^{N} \frac{\partial W_j}{\partial v_j} \frac{v_j}{W_j}}, \tag{8}$$

where N = the number of options recorded on a particular stock for the observation date, $WISD$ = the weighted implied standard deviation for a particular stock on the observation date, ISD_j = the implied standard deviation of option j for the stock, $\frac{\partial W_j}{\partial v_j} \frac{v_j}{W_j}$ = the price elasticity of option j with respect to its implied standard deviation (v).

In an efficient market prices will fully reflect all available information. Therefore, estimated variances calculated from option prices should reflect not only the informational content of stock price history but also any other available information. Thus one may suspect that the *WISD* values reflect future standard deviations more accurately than do the historic sample standard deviations [Chiras, Manaster 1978].

Beckers [1981] used an alternative weighting scheme that concentrates mainly on the implied standard deviations (*ISD*s) for at-the-money options. Specifically, on any single observation day the following loss function was minimized:

$$f(ISD) = \sum_{i=1}^{I} w_i \left[C_i - BS_i(ISD)\right]^2 / \sum_{i=1}^{I} w_i, \tag{9}$$

where C_i = market price of option i, BS_i = Black-Scholes option price as a function of the *ISD*, I = total number of options on a given stock with the same maturity, w_i = weight for the i-th option= $\delta BS_i(ISD)/\delta ISD$ (i.e., the first derivative of the Black-Scholes option pricing formula with respect to the standard deviation).

This procedure comes down to minimizing the weighted sum of the squared deviations between market value and the corresponding Black-Scholes price. The actual weights used in the procedure are proportional to the squared values of the Latané-Rendleman's weights. This method therefore tends to put more weight on the options that are highly sensitive to an exact specification of the standard deviation. Beckers believes his measure tends to outperform Latané-Rendleman's *WISD*. In contrary to the Latané-Rendleman's study suggesting that the best predictive performance could be obtained by using the information available in all options, he concludes that most of relevant information is reflected in the price of at-the-money options. However, he admits it is not clear whether this result is due solely to the fact that the other options are not as sensitive to an exact specification of the underlying variance. Their prices could also have a higher tendency toward distortion. Systematic biases in the *ISD*'s due to the fact that the Black-Scholes model does not hold exactly for in-the-money or out-of-the money options could also influence this result [Beckers 1981].

The smile-shaped pattern, which constantly appears in volatilities extracted from a wide variety of options, has provided evidence against the constant volatility assumption inherent in the Black-Scholes model (if returns were normal, then implied volatility would be constant across moneyness and maturity). As a result, estimation procedures that use the Black-Scholes model to estimate implied volatilities may produce biased estimates. Moreover, using in forecasting experiments at-the-money implied volatility only, discards all potential information contained in the rest of option prices, especially that in practice options rarely trade exactly at-the-money. Britten-Jones and Neuberger [2000] derived a model-free implied volatility measure that incorporates the whole cross-section of option prices, not only at-the-money prices.

MODEL-FREE IMPLIED VOLATILITY

Under the assumptions that the underlying asset does not make dividend payments and the risk-free rate is zero, Britten-Jones and Neuberger [2000] derive the risk-neutral expected sum of squared returns between two dates (T_1, T_2) as:

$$E^Q\left[\int_{T_1}^{T_2}\left(\frac{dS_t}{S_t}\right)^2\right] = 2\int_0^{\infty}\frac{C(T_2,X)-C(T_1,X)}{X^2}dX \;, \tag{10}$$

where $E^Q[\;]$ refers to expectation under the risk-neutral measure Q, $C(T, X)$ is an observed call price with maturity T and strike price X, and S_t is the asset price at time t. The asset return variance $(dS_t/S_t)^2$, which is also the squared volatility, is a function only of observed call prices at one point in time. No model for the underlying asset price is required in the derivation of (10), hence it is a "model-free" measure of variance. It only requires two cross sections of call prices with varying X, one with time to maturity T_1 and the other with time to maturity T_2. Since (10) is the model-free implied variance, the model free implied volatility is obtained as its square root [Rouah, Veinberg 2007].

As Britten-Jones and Neuberger derived the model-free implied volatility under diffusion assumptions, it was unclear whether it was still valid when the underlying asset price process included jumps. This could be a serious limitation since random jumps are an important aspect of the price dynamics of many assets. Jiang and Tian [2005] extend the model and demonstrate that it is still valid even if the underlying asset price process has jumps. They also show how to relax original assumptions of no dividends and a zero risk-free rate. Results of their empirical tests conducted using the S&P 500 index options traded on the Chicago Board of Trade (CBOE) ensure the generality of the model-free implied volatility.

The volatility index VIX published by the CBOE constitutes one important application of model-free volatility. The index was originally defined in terms of Black-Scholes implied volatilities calculated from at-the-money options on the S&P 100 index. The revision of the VIX uses options on the S&P 500 index, on a wide range of moneyness, not only at-the-money. It also uses model-free implied volatility rather than the Black-Scholes implied volatility [Rouah, Veinberg 2007].

CONCLUDING REMARKS

The volatility of an asset is a measure of our uncertainty about the returns provided by the asset. There has been extensive research regarding the prediction of future volatility. In particular, researchers have examined what sources of information are the best predictors of volatility. According to Chriss [1997], some obvious candidates are: historical volatility, implied volatility, some combination of the first two. The aim of the paper was presenting those alternative approaches to estimate volatility as in the literature there are different studies supporting different opinions on practical usefulness of separate methods.

Most often, the Black-Scholes implied volatility is believed to be superior to the historical volatility of the underlying asset, since it is derived from options prices that reflect market participants' expectations of future movements of the underlying asset. Even though early studies found that implied volatility was a biased forecast of future volatility and contained little incremental information beyond historical volatility, more recent studies present evidence that implied volatility is a more efficient forecast for future volatility than historical volatility. Research on the information content of implied volatility usually focuses on the Black-Scholes implied volatility from at-the-money options. Being more actively traded than other options, they can be a good starting point. However, by concentrating on at-the-money options alone, one omits the information embedded in other options.

An important departure from previous research is the model-free implied volatility derived in 2000 by Britten-Jones and Neuberger. Their model is not based on any specific option pricing model. It is derived entirely from no-arbitrage conditions and utilizes the whole cross-section of option prices. Although Britten-Jones and Neuberger had derived the model-free implied volatility under diffusion assumptions, Jiang and Tian [2005] extended their model to asset price processes with jumps and developed a simple method for implementing it using observed option prices. Their results obtained from the S&P 500 index options suggest that the model free implied volatility subsumes all information content in the Black-Scholes implied volatility and past realized volatility, and is a more efficient forecast for future realized volatility. As they write, their findings also provide theoretical and empirical support for the CBOE decision to modify its VIX index.

Now, it is based on the model-free implied volatility instead of the Black-Scholes volatility of at-the-money options.

In Poland, one of the first to study the subject was Piontek, who in 1999 published his paper presenting historical and the Black-Scholes implied volatilities obtained from currency options on PLN/USD exchange rate. He tried to assess the predictive power of historical and implied volatilities and concluded they both faild to forecast future volatility of the exchange rate as option market in Poland was small and illiquid at that time. Krawiec and Krawiec [2002] analyzed volatilities implied in commodity options traded at Poznań Exchange[3] and at the Warsaw Commodity Exchange. In their opinion, implied volatilities derived from the options under consideration could not be a reliable source of information on realized volatilities. The Warsaw Stock Exchange, that introduced options on WIG20 index in September 2003, does not publish any own implied volatility index, although there have been already proposed some concepts [Ślepaczuk, Zakrzewski 2007, Rudzki 2008].

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QUANTITATIVE METHODS IN ECONOMICS
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THE APPLICATION OF NASH GAME FOR DETERMINE OF CONTROLLING MULTIDIMENSIONAL OBJECTS

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Abstract: The article presents the approach of decentralization of calculations related to the synthesis of controlling through division of an optimalization problem into lesser dimensionality subproblems. Applied was differential Nash game for objects of serial structure. The main idea of the method is to assign individual functionals of quality to subobjects and conduct the synthesis of regulators in a sequential way, beginning with the first subobject. Thanks to the sequential structure of the object it is possible to obtain a solution by decentralized optimizations.

Keywords: complex systems, dynamics equations, differential game, functionals of quality, Riccati equation, decentralized calculation of regulator gain

INTRODUCTION

The synthesis of complex systems controlling is a complicated task. Technically, it can be made in two ways. The first, a natural one, requires constructing decentralized control system consisting of local regulators, possibly equipped with interaction compensators. However, compensation is sensitive to differences between a model and an object. In the case of greater factors it can even lead to instability. No method that would in a decentralized way allow to construct a stable system with coupling for a system of arbitrary structure has been so far created [Golnaraghi and Kuo 2010], [Kaczorek 1999], [Han 2009] . All successful results so far eventuate from intuition of a designer who is familiar with the process [Kociemba et al. 2013] , [Wenwu et al. 2013].

The second way bases on designing a regulator for the entire system using classic method, i.e. with the use of location of poles or basing on solving linear-quadratic problem. In practice, when dimensionality exceeds 10, it turns problematic.

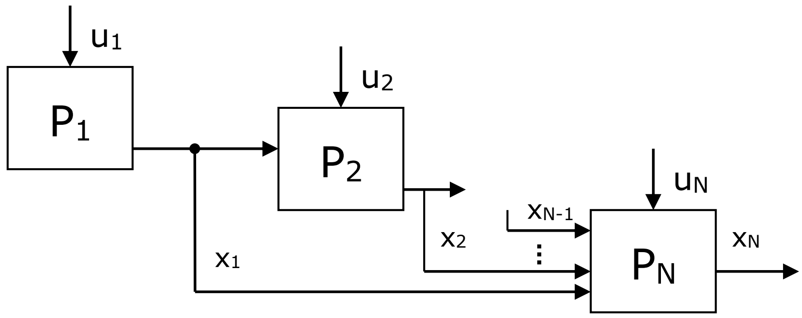
Limiting speculations to linear structure only, a method of decentralized designing of a feedback system is presented. The system is stable and thus more efficient than local regulators with interaction compensator and control system created with the method of coupling disorders. It was initially proposed by Özgüner and Perkins [Kwater 1987]. It bases on assigning individual quality indicators to subobjects and conducting synthesis of regulators in a sequential manner, beginning with the first subobject. Thanks to cascade structure of an object, solution is made by decentralized optimization and therefore the issue of dimensionality is made irrelevant.

DYNAMIC OBJECTS OF A LUMPED PARAMETER SYSTEM

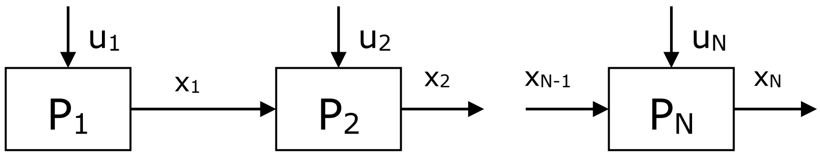
A large proportion of composite objects is composed of objects whose each subsequent subobject is linked to the preceding ones. (Figure1).

Figure 1. Objects of: a) cascade structure, b) linear structure

a)



b)



Source: own work

Considered is a linear stationary cascade object, composed of N subobjects. Dynamics of i-subobject is expressed in the following equation:

$$\frac{d}{dt}\mathbf{x_i}(t) = \mathbf{A_i x_i}(t) + \mathbf{A_{i,i-1} x_{i-1}}(t) + \mathbf{B_i u_i}(t),$$
$$\mathbf{x_i}(t_0) = \mathbf{x_0}, \tag{1}$$

where: $\mathbf{x_i}$, $\mathbf{u_i}$, $\mathbf{A_i}$, $\mathbf{B_i}$ – vectors and matrices of state and control of i-subobject. $\mathbf{A_{i,\,i+1}}$ is a matrix accommodating links between neighboring subobjects.

Collective mathematical model of a cascade object is:

$$\frac{d}{dt}\tilde{\mathbf{x}}(t) = \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}\tilde{\mathbf{u}}(t),$$
$$\tilde{\mathbf{x}}(t_0) = \tilde{\mathbf{x}}_0, \tag{2}$$

where: $\tilde{\mathbf{x}}(t) = col[\mathbf{x_1,...,x_{i-1},x_i,...,x_N}]$, $\tilde{\mathbf{u}}(t) = col[\mathbf{u_1,...,u_{i-1},u_i,...,u_N}]$,

$\mathbf{B} = diag[\mathbf{B_1,...,B_{i-1},B_i,...,B_N}]$ **-** Block diagonal matrix. Matrix of state $\mathbf{A}$ is:

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A_1} & \phi\cdots & & 0 \\ \mathbf{A_{21}} & \mathbf{A_2}\ddots & & \vdots \\ 0 & \mathbf{A_{i,i-1}} & \mathbf{A_i} & \\ \vdots & & \ddots & . \\ 0 & \cdots & \mathbf{A_{N,N-1}} & \mathbf{A_N} \end{bmatrix} \tag{3}$$

Submatrices on diagonal in (3) concern particular subobjects, whereas the rest presents interaction between them.

Models of type (2) present quite a wide range of technological processes. Typical examples include rolling of metal sheets, reactors in a cascade of distillation columns and multizone pusher-type furnace. For the majority, division into subobjects is a natural consequence of mechanical construction, locally dominating chemical reactions or methods of controlling. Non-technological processes constitute a considerable group of objects of linear structure. In the group one may include a biologically contaminated river and transport system consisting of a stream of vehicles, where subobjects are segments between bigger tributaries and distances between crossroads respectively.

TASK OF CONTROLLING

One may consider the issue of a feedback system in reference to model (2). The effective synthesis of a multi-parameter regulator when objects's dimensionality exceeds 10 poses a complex computational problem. Local regulators designed individually for subobjects regardless of mutual connections may, in turn, not provide sufficient quality of regulation of the entire system.

Solving the problem of the synthesis of regulators of state in large systems may be conducted in multiple ways. One of the solutions is to assign individual quality functional to each subobject. Assuming that the first subobject operates under the control of a regulator, one approaches the second subobject and determined controlling for a group of subobjects (the first one with a regulator and the other subobject without one). In a similar manner, designed are subsequent regulators up until the end. As a result, obtained is a group of local regulators with additional cross-coupling from preceding subobjects. Such technique makes it possible to conduct calculations of all regulator gains in a decentralized way, while preserving the accurate succession. Firstly, based on solutions of Riccati equations for subobjects defined are local gains. Further on, cross-coupling gains are defined, basing on certain bilinear equations parallel to matrix Lapunow equation. The whole procedure employs lower dimentionality equations adequate to subobjects, which allows to avoid computational problems for an object treated as inseparable body.

Such natural conduct is justified on the grounds of game theory.

One may assume that controls $\mathbf{u_i}(t)$ in equations (1) are created by N partners (players) in a differential game. Every player interacts with an object through "their" control $\mathbf{u_i}(t)$ in order to minimise given functional of quality J_i assigned to objects:

$$J_i = \int_T [\mathbf{x_i}(t)^T \mathbf{Q_i} \mathbf{x_i}(t) + \mathbf{u_i}(t)^T \mathbf{R_i} \mathbf{u}_i(t)]dt \tag{4}$$

Due to occurring interactions, the consequences of one player's decision in general influence the achievement of goals of other players. Hence, J_i depends on $\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_N}$. Nash equilibrium is defined by $\mathbf{u_i}^*$ control that assures the inequality:

$$J_i(\mathbf{u_1^*},\ldots,\mathbf{u_{i-1}^*},\mathbf{u_i^*},\mathbf{u_{i+1}^*},\ldots,\mathbf{u_N^*}) \le J_i(\mathbf{u_1^*},\ldots,\mathbf{u_{i-1}^*},\mathbf{u_i},\mathbf{u_{i+1}^*},\ldots,\mathbf{u_N^*}), \quad \forall \mathbf{u_i} \in U_i ,$$

$$i=1,2,\ldots,N, \tag{5}$$

where U_i is a space of allowable controls for i-player.

Assumption that none of the partners takes dominant position protects players from one-sided withdrawal from declared $\mathbf{u_i}^*$ control. Such partner can only lose.

It should be noted that despite the occurrence of only local variables of state $\mathbf{x_i}$ in the functional (4), it also depends on controls of preceding subobjects, due to the interactions expressed in matrix $\mathbf{A_{i,\,i-1}}$.

One may also assume that subobjects are asymptotically stable, i.e. matrices $\mathbf{A_i}$ have eigenvalues in the left half-plane. From the cascade structures, (4) satisfies the inequalities:

$$J_i(\mathbf{u_1^*,...,u_i^*}) \le J_1^*(\mathbf{u_1^*,...,u_{i-1}^*,u_i}) \text{ for } i = 1,...,N \,. \tag{6}$$

Hence, one obtains formula for optimal control of an object of cascade structure:

$$\mathbf{u_i}(t) = -\sum_{j=1}^{i} \mathbf{G_{i\,j}} \mathbf{x_j}(t), \qquad i = 1,\dots,N. \tag{7a}$$

Gains are:

$$\mathbf{G_i} = \mathbf{G_{ii}} = \mathbf{R_i^{-1}B_i^T P_{ii}^i} \tag{7b}$$

$$\mathbf{G_{ij}} = \mathbf{R_i^{-1}B_i^T P_{ij}^i},$$

where $\mathbf{P_{ii}^i}$ is a solution of differential Riccati equation

$$\mathbf{P_{ii}^i A_i} + \mathbf{A_i^T P_{ii}^i} - \mathbf{P_{ii}^i B_i R_i^{-1} B_i^T P_{ii}^i} + \mathbf{Q}_i = \frac{d\mathbf{P_{ii}^i}}{dt} \tag{8}$$

whereas $\mathbf{P_{ii}^i}, i > j$ is a solution of bilinear equation

$$\mathbf{P_{ij}^i A_i^*} + \mathbf{A_i^{*\,T} P_{ij}^i} - \sum_{k=j+1}^{i-1} \mathbf{P_{ik}^i A_{kj}^*} - \mathbf{P_{ii}^i A_{ij}} = \frac{d\mathbf{P_{ij}^i}}{dt}$$

$$\mathbf{A_{kj}^*} = \mathbf{A_{kj}} - \mathbf{B_k R_k^{-1} B_k^T P_{kj}^k} \tag{9}$$

$$\mathbf{A_i^*} = \mathbf{A_i} - \mathbf{B_i R_i^{-1} B_i^T P_{ii}^i}$$

$$(\mathbf{A_{kj}} = 0 \quad dla \quad k \supset j+1).$$

Equation (9) is similar to matrix Lapunow equation.

The same results may be obtained as a consequence of sequential (heuristic) procedure which is based on determining optimal control for subsequent subobjects, beginning with the first one. Control of *i*- subobject is obtained when assuming that all preceding, i.e. $1,\ldots,i-1$ have their controls determined.
Considering the abovementioned procedure in details, one subsequently gets:

Stage 1. Optimal control of the first subobject is expressed in a formula:

$$\mathbf{u}_1^*(t) = -\mathbf{R}_1^{-1}\mathbf{B}_1^{\mathbf{T}}\mathbf{P}_{11}^{1}\mathbf{x}_1(t),$$

where $\mathbf{P}_{11}^{1}$ is a solution of Riccati equation (8), here inafter local. Having $\mathbf{u}_1^*$ we close feedback loop.

Stage 2. Considered is a system consisting of a first subobject (controlled optimally) and the second. Following dependencies are in operation:

$$\frac{d}{dt}\tilde{\mathbf{x}}_2(t) = \tilde{\mathbf{A}}_2\tilde{\mathbf{x}}_2(t) + \tilde{\mathbf{B}}_2\mathbf{u}_2(t);\quad \tilde{\mathbf{x}}_2(t_0) = \tilde{\mathbf{x}}_{20},$$

$$\text{where}: \tilde{\mathbf{x}}_2 \quad col[x_1, x_2],\quad \tilde{\mathbf{B}}_2 = col[\phi, \mathbf{B}_2],$$

$$\tilde{\mathbf{A}}_2 = \begin{bmatrix} \mathbf{A}_1^* & \phi \\ \mathbf{A}_{21} & \mathbf{A}_2 \end{bmatrix},\quad \mathbf{A}_1^* = \mathbf{A}_1 - \mathbf{B}_1\mathbf{R}_1^{-1}\mathbf{B}_1^{\mathbf{T}}\mathbf{P}_{11}^{1}$$

and

$$J_2 = \int_T [\mathbf{x}_2(t)^T\tilde{\mathbf{Q}}_2\tilde{\mathbf{x}}_2(t) + \mathbf{u}_2(t)^T\mathbf{R}_2\mathbf{u}_2(t)]dt,\quad \tilde{\mathbf{Q}}_2 = \begin{bmatrix} \phi & \phi \\ \phi & \mathbf{Q}_2 \end{bmatrix}.$$

Optimal control is:

$$\mathbf{u}_2^*(t) = -\mathbf{R}_2^{-1}\tilde{\mathbf{B}}_2^{\mathbf{T}}\mathbf{P}^2\tilde{\mathbf{x}}_2(t),$$

where $\mathbf{P}^2 = \begin{bmatrix} \mathbf{P}_{11}^{2} & \mathbf{P}_{21}^{2} \\ \mathbf{P}_{21}^{2} & \mathbf{P}_{22}^{2} \end{bmatrix}$. Due to the matrix $\tilde{\mathbf{B}}_2$, to determine $\mathbf{u}_2^*$ it is enough to determine $\mathbf{P}_{22}^{2}$ from the equation (8), and $\mathbf{P}_{21}^{2}$ from (9).

Stage 3 and subsequent stages. For the rest of subobjects, optimalization problems are defined in formulas:

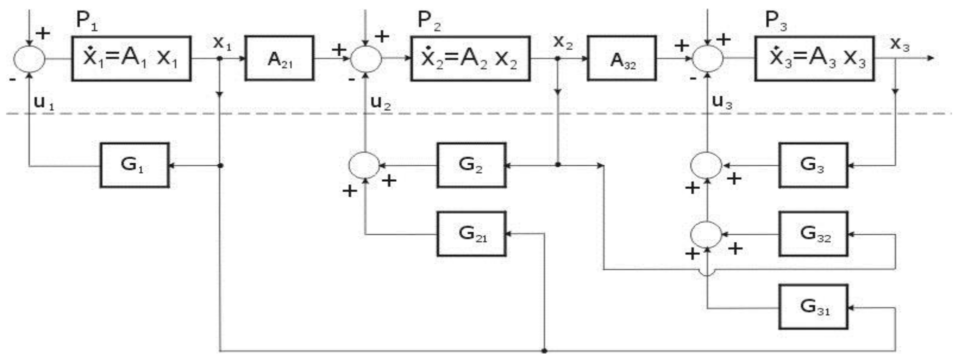
$$
\begin{aligned}
&\frac{d}{dt}\begin{vmatrix}\tilde{\mathbf{x}}_{\mathbf{i-1}}\\ \mathbf{x_i}\end{vmatrix} = \begin{vmatrix}\tilde{\mathbf{A}}_{\mathbf{i-1}} & \phi\\ \mathbf{A_{i,i-1}} & \mathbf{A_i}\end{vmatrix}\begin{vmatrix}\tilde{\mathbf{x}}_{\mathbf{i-1}}\\ \mathbf{x_i}\end{vmatrix} + \begin{vmatrix}\phi\\ \mathbf{B_i}\end{vmatrix}\mathbf{u_i},\\
&J_i = \int_T \left\{ \left[\tilde{\mathbf{x}}^{\mathbf{T}}_{\mathbf{i-1}}, \mathbf{x}^{\mathbf{T}}_{\mathbf{i}}\right]\begin{vmatrix}\phi & \phi\\ \phi & \mathbf{Q_i}\end{vmatrix}\begin{vmatrix}\tilde{\mathbf{x}}_{\mathbf{i-1}}\\ \mathbf{x_i}\end{vmatrix} + \mathbf{u}^{\mathbf{T}}_{\mathbf{i}}\mathbf{R_i}\mathbf{u}_i \right\} dt, \qquad (10)\\
&i = 3,\ldots,N,
\end{aligned}
$$

where $\tilde{\mathbf{x}}_{\mathbf{i-1}} = col[\mathbf{x_1},\ldots,\mathbf{x_{i-1}}]$. $\tilde{\mathbf{A}}_{\mathbf{i-1}}$ is a triangular matrix obtained as a result of closing feedback for subobjects stage by stage. Finally, control of i-subobject is in a form:

$$
\mathbf{u}^{*}_{\mathbf{i}}(t) = -\mathbf{R}^{-1}_{\mathbf{i}}\mathbf{B}^{\mathbf{T}}_{\mathbf{i}}\sum_{j=1}^{i}\mathbf{P}^{\mathbf{i}}_{\mathbf{ij}}\mathbf{x_j}(t) \qquad (11)
$$

One should note the importance of the order in which equations are solved. Firstly, one should solve all local Riccati equations (8). Subsequently, basing on $\mathbf{P}^{\mathbf{i}}_{\mathbf{ii}}$ one should solve bilinear (9), starting with $j = i-1$ and ending with $j = 1$. Such decentralized manner allows to avoid computational problems related to dimensionality. To emphasize such possibility of synthesis, regulator is called sequential. For an object composed of three subobjects, the structure of a regulator is presented in Figure 2.

Figure 2. Cascade of three subobjects with a system of regulators determined in a equential manner



Source: own work

As is apparent, despite local gains $\mathbf{G_i}$ there are gains $\mathbf{G_{ij}}$ resulting from the interaction among subobjects. Controlling i-subobject is generated basing on own state and states of preceding subobjects.

CONCLUSION

In the presented approach received the composed regulator, which consists of the local regulators and the regulators that use the feedback from the previous subobjects. The feedback is the result of interaction between subobjects obtained. Parameters of local regulators are determined on the basis of individual Riccati equations o for subobjects. Cross-coupling gains are determined by the bilinear equations solutions in a form similar to the Lapunov equation. As a result of this approach received is a decentralized method of determining gains in a regulation system, i.e. so called system of sequence regulators.

Qualities of the presented method:

- Realization of a regulator requires knowledge of vector of state only of the preceding subobjects.
- Gains may be determined in a sequential manner, solving problems of small dimensionality.
- Local feedback are calculated basing on solution of Riccati equation, and cross basing on bilinear equations.

The presented method leads to obtaining structure adequate to classic linear-quadratic regulator. The way of synthesis of a regulator for lumped parameters systems can be naturally extended to systems of linear structure expressed in partial equations of transport type.

The method may be also adapted to other objects of cascade or linear structure. The advantage of the method are relatively simple calculation algorithms.

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A COMPARISON OF THE USEFULNESS OF WINTERS' AND SARIMA MODELS IN FORECASTING OF PROCUREMENT PRICES OF MILK IN POLAND

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Abstract: In the paper the Winters' model has been studied as one from adaptive models based on exponential smoothing methods as well as seasonal autoregressive integrated moving average model SARIMA. The aim of the paper is the assessment of accuracy of short-term forecasts of procurement prices of milk in Poland. Empirical verification of ex post forecasts of monthly procurement prices of milk on the basis of 109 time series with 12-month forecast horizon was conducted. Forecasts constructed with the use of SARIMA model are more often exact than when additive and multiplicative Winters' model are used.

Keywords: forecast, forecast accuracy, forecast error measures (*ex post*)

INTRODUCTION

In Poland after 1989 annual milk production was decreasing greatly and fell from about 16 billion litres to its lowest value of 11.3 billion litres in 1995. In the years 2000-2004 the production remained constant at about 11.5 billion litres and it was increasing slightly from 2005. The growth of production was limited by the quota system to about 12.1 billion in 2011. The largest purchaser of milk has been the milk industry that bought 71.4 % of production in 1989 and this percentage decreased over the following years down to 51.8% in 1994. The reconstitution of milk procurement to the level from the end of 1980s took 15 years. The procurement exceeded 60% only in 2002, and 70% in 2005. It reached 74.8% of milk production in 2011 [Urban 2011].

Procurement and prices of milk industry vary between seasons in a given year; it is connected with different levels of milk production. The level of milk production, in turn, first of all depends on the cattle feeding system and gestation chart [Iwan 2005; Majewski 2006].

The aim of the paper is to assess the accuracy of short term forecasts of procurement prices of milk after Poland's accession to the EU, obtained with regard to seasonal differences and trend on the basis of additive and multiplicative Winter's model and SARIMA model. Monthly expired forecasts of procurement prices obtained on the basis of 109 time series of 12-month forecast horizon were subjected to empirical verifiability. The analysis of forecasts will make it possible to search for a better forecast model [Makridakis and Hibon 2000]. The research material constituted mean monthly procurement prices of milk in Poland from January 1990 to December 2012 on the basis of the Statistical Bulletin of the Central Statistical Office (1990-2012).

RESEARCH METHODOLOGY

In time series made up of mean milk procurement prices the following elements may be distinguished: trend, seasonal and irregular component. In practice two decomposition formulas of observed time series values are used with the condition of elements being independent. The first formula is based on the application of additive model of the elements of the series, and the second one on the application of the multiplicative one. Therefore, to forecast the procurement prices on the basis of time series both additive and multiplicative models of Winter's exponential smoothing methods can be used [Winters 1960; Stańko 1999, Tłuczak 2009, Stańko 2013]. Moreover, factors influencing the price with certain delay, through the use of seasonal autoregressive integrated moving average model SARIMA, may be taken into account [Zeliaś et al. 2003; Cieślak 2008, Hamulczuk 2011].

To assess the accuracy of forecasts of procurement prices of milk the basic forecast error assessment methods were used [Zeliaś et al. 2003]: percentage error (PE), mean squared error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). For errors $|\mathrm{PE}|$ (absolute value) and MAPE it was assumed that the forecasts made with the adopted model are very accurate if they do not exceed 3%, are good in the range of $(3\%, 5\% >$, are acceptable (the recipient may decide they are adequately accurate or reliable) in the range of $(5\%, 10\% >$ and are unacceptable for error above 10%. In the case of forecast error: MSE and MAE forecasts are the more accurate the more their values approach 0.

Average monthly procurement prices of milk in Poland from January 1990 to December 2012 were grouped in 109 time series. The first series included 168 time units reflecting monthly milk procurement prices from the years 1990-2003 and the consecutive time series were one element longer from the previous one, moreover, the last time series included 276 units, e.g. consecutive months from the period of 1990-2012.

For monthly procurement prices both additive and multiplicative models of Winter's exponential smoothing methods with linear form of trend, with MAPE as adopted minimizing criterium. Additionally, seasonal model of autoregression and moving average SARIMA $(0,1,3)\times(0,1,1)_{12}$ was used. Its construction process was based on single differentiation of adjacent elements in a series. For a stationary series, Box-Jenkins procedure was used to determine the order of autoregression and wandering mean on the basis of autocorrelation (ACF) and partial autocorrelation (PACF). During statistical verification of the model, the significance of its parameters was confirmed.

To find the best forecasting model prediction criteria were used. Every time series had smoothed procurement prices of milk attributed with the use of studied models and 12-month forecasts were constructed. Next, procurement prices and respective smoothed values were used to determine percentage errors and summary accuracy measures: MSE, MAE and MAPE. In turn, actual procurement prices from the forecast horizon and *ex post* forecasts were used to calculate percentage errors. Moreover, analysis of *ex post* forecasts of procurement prices were based on blocked one-way ANOVA, where the factor was the type of model and consecutive time series were blocks. After rejecting the null hypothesis of equal mean forecasts values of procurement prices obtained on the basis of studied models, multiple comparisons procedure based on Tukey method for all pairwise comparisons was used. Necessary numerical calculations were made on the basis of *STATISTICA 10* package and *gretl* programme.

THE RESULTS OF EMPIRICAL RESEARCH

During the first stage of research percentage errors were analysed, then measures of ex post forecast accuracy. For the first time series made of consecutive months from the period of 1990-2003 the Winter's models yielded 168 percentage errors. In the case of SARIMA$(0,1,3)\times(0,1,1)_{12}$ model, 155 percentage errors were obtained as the stationarity of time series required single differentiation both of seasonal fluctuatons and trend. As a result, the model was 13 elements shorter.

In every series one more error was obtained, compared with the precedent time series. In turn, for the last series containing monthly procurement prices from the period of 1990-2012 respectively 276 and 263 percentage errors were found. Analysing the median of ex post percentage errors shown on Figure 1 for Winter's model especially their large values, e.g. above 10% or below -10% may be noted for the years 1990-2012. In the case of Winter's additive model those errors were observed in January (-12,2%), May (12,6%), September (-18,2%), October (-16,7%) and November (-11,2%) of 1990 as well as January (13,3%) 1991. For the multiplicative model, however, errors were observed only in January (25,5%) and May (-12,7%) 1990. High value of the first percentage error (January 1990) may be connected with the starting point, and the other ones with considerable fluctuations of procurement prices in the first years after the transformation in Poland. Only for the SARIMA(0,1,3)×(0,1,1)$_{12}$ no particularly high values of percentage errors were found, as they ranged from -4,8% in April 2012 to 5,3% in October 1991.

Table 1 shows the basic characteristics for ex post forecast errors calculated on the basis of milk procurement prices and their smoothed values for 109 time series. On the basis of mean values of these errors it can be stated that for the SARIMA(0,1,3)×(0,1,1)$_{12}$ model the obtained values were the lowest, the next was Winter's multiplicative model, and the highest error values were observed for additive version of Winter's model. Therefore, for SARIMA(0,1,3)×(0,1,1)$_{12}$ model the mean variation from forecast error amounted to about 1.05 PLN for 1 hl of milk, forecast bias (as for the absolute value) on the level of about 0.68 PLN per hl and relative total adjustment of the model to procurement prices – about 1.17%. Respective values of forecast errors for Winter's multiplicative model amounted to 1.11 PLN/hl, 0.77 PLN/hl and 1.89%, and for Winter's additive model – 1.22 PLN/hl, 0.80 PLN/hl and 2,02%.

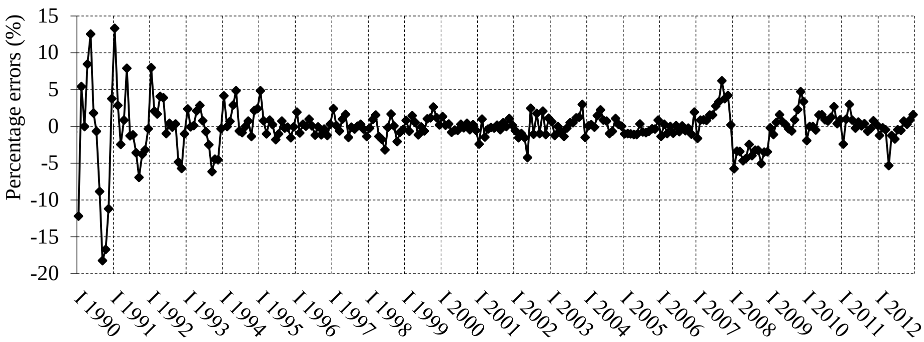
Table 1. Descriptive statistics of ex post forecasting errors

Ex post forecast errors	Minimum	Lower quartile	Median	Upper quartile	Maximum
Additive Winters' model					
MSE [zł/hl]^2	0,5378	0,6182	1,4904	2,4693	9,9006
$\sqrt{\text{MSE}}$ (zł/hl)	0,7334	0,7863	1,2208	1,5714	3,1465
MAE (zł/hl)	0,5482	0,5773	0,8026	1,0582	2,1685
MAPE (%)	1,60	1,69	2,02	2,44	4,69
Multiplicative Winters' model					
MSE [zł/hl]^2	0,4493	0,9391	1,2217	2,2206	2,2870
$\sqrt{\text{MSE}}$ (zł/hl)	0,6703	0,9691	1,1053	1,4902	1,5123
MAE (zł/hl)	0,4719	0,7062	0,7680	0,9799	1,0331
MAPE (%)	1,34	1,85	1,89	1,94	1,99
SARIMA(0,1,3)×(0,1,1)$_{12}$					
MSE [zł/hl]^2	0,5389	0,5908	1,1059	1,3170	1,3981
$\sqrt{\text{MSE}}$ (zł/hl)	0,7341	0,7686	1,0516	1,1476	1,1824
MAE (zł/hl)	0,5135	0,5424	0,6806	0,7646	0,7945
MAPE (%)	1,09	1,12	1,17	1,19	1,20

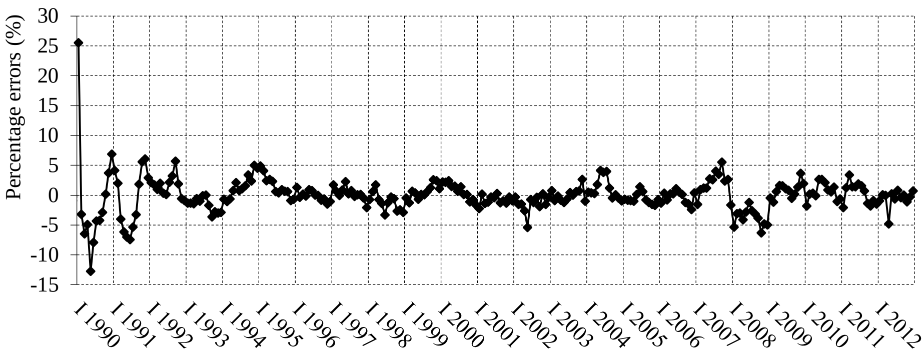
Adopted acronyms: mean squared error (MSE), mean absolute error (MAE), Mean absolute percentage error (MAPE).
Source: own study based on Statistical Bulletin from the period of 1990-2012.

Figure 1. Median of ex post percentage errors for individual months from 1990 to 2012[1)]

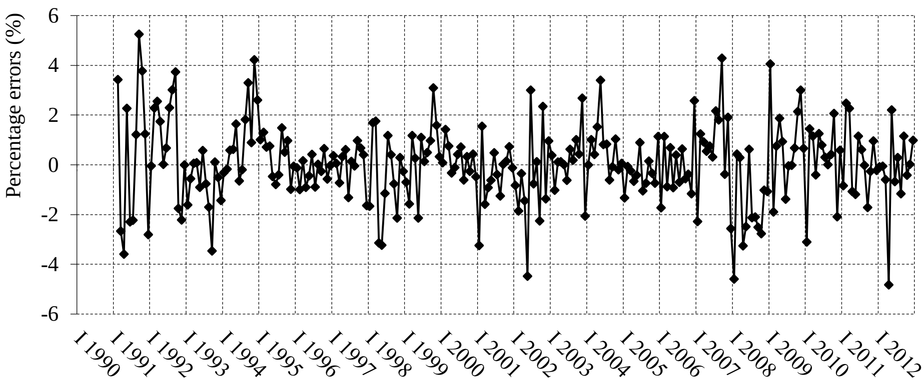
a) additive Winters' model



b) multiplicative Winters' model



c) SARIMA$(0,1,3)\times(0,1,1)_{12}$



[1)] From February 1991 for model SARIMA$(0,1,3)\times(0,1,1)_{12}$.

Source: see Table 1.

In the second stage of the study ex post forecasts were subjected to empirical verifiability. For every time series obtained due to grouping milk procurement prices, 12-month forecast horizon was adopted. In the case of the first series, constructed forecasts included consecutive months from January to December 2004. For every next time series the forecast horizon was moved by one month, e.g. from February 2004 to January 2005 (the second series), from March 2004 to February 2005 (the third series), ..., from January to December 2013 (the last series, no 109). In total 1308 forecasts were constructed while only ex post forecasts from January 2004 to December 2012 were subjected to empirical verifiability i.e. 1230 ex post forecasts, including 108 forecasts with one-month advance, 107 with two-month advance and 97 with 12 months of advance. In Table 2 basic characteristics concerning percentage errors are presented for SARIMA$(0,1,3)\times(0,1,1)_{12}$ model and both additive and multiplicative Winter's model. The structure of these errors is, in turn, presented in Table 3.

Table 2. Characteristics of percentage errors for ex post forecasts and milk procurement prices in Poland (%)

Step-ahead of forecast	Lower quartile			Median			Upper quartile			Percentage of positive errors PE (%)		
	AMW	MMW	SAR	AMW	MMW	SAR	AMW	MMW	SAR	AMW	MMW	SAR
1	-0,83	-1,11	-0,72	0,01	0,09	0,18	0,86	1,27	1,07	51,9	50,9	54,6
2	-1,86	-1,93	-1,22	0,19	-0,14	0,34	1,56	2,19	2,00	50,5	49,5	54,2
3	-2,35	-3,45	-1,93	0,21	0,08	0,08	2,62	3,58	3,33	51,9	50,9	50,9
4	-3,21	-4,34	-2,60	0,62	0,20	0,37	3,64	4,77	4,98	56,2	53,3	52,4
5	-3,57	-4,99	-2,98	0,70	1,10	0,54	4,82	5,33	5,68	57,7	53,8	52,9
6	-4,66	-5,56	-3,52	0,95	0,38	0,54	5,35	6,17	6,42	57,3	52,4	55,3
7	-6,04	-5,66	-4,38	1,08	0,37	0,73	6,80	6,77	7,58	55,9	52,0	51,0
8	-7,35	-5,78	-5,63	1,35	0,38	1,10	6,69	7,56	8,65	55,4	52,5	54,5
9	-9,09	-5,80	-6,59	0,91	-0,55	0,94	7,76	8,08	9,15	56,0	47,0	51,0
10	-10,16	-6,47	-6,85	1,45	-0,05	0,21	8,69	8,49	9,16	55,6	49,5	51,5
11	-14,35	-6,75	-7,26	3,06	0,45	0,77	10,17	9,42	10,88	55,1	50,0	52,0
12	-14,53	-6,31	-7,00	3,33	1,76	-0,23	10,63	9,75	11,21	54,6	54,6	49,5

Explanation: AMW – additive Winters' model; MMW – multiplicative Winters' model, SAR – SARIMA$(0,1,3)\times(0,1,1)_{12}$.
Source: see Table 1

Forecasts constructed on the basis of Winter's additive model were more often undervalued for every forecast advance from 0.5 pp (two months) to 7.7 pp (five months). The mean value of percentage error ex post increased from 0.01% for the advance of one month to 3.33% for the advance of 12 months. The highest percentage of very good forecasts was obtained for the advance of one to three months, respectively: 84.3%, 67.3% and 54.7%. Moreover, for this model it was discovered that for the advance of 11 and 12 months the percentage of inadmissible forecasts exceeded 50%. More than a half of both good and very good forecasts were noted for the advance of one (95.4%) to five months (51.9%).

Table 3. Structure of percentage errors for ex post forecasts and milk procurement prices in Poland (%)

Percentage errors *ex post*	Step-ahead of forecast (months)												
	1	2	3	4	5	6	7	8	9	10	11	12	total
Additive Winters’ model													
(-3%; 3%)	84,3	67,3	54,7	44,8	39,4	32,0	22,5	18,8	17,0	15,2	11,2	7,2	35,3
(-5%; -3%> and <3%; 5%)	11,1	16,8	16,0	16,2	12,5	16,5	17,6	17,8	12,0	8,1	8,2	8,3	13,5
(-10%; -5%> and <5%; 10%)	3,7	12,2	18,9	25,7	28,9	26,2	27,5	25,8	32,0	34,3	28,6	27,8	24,1
≤-10% and ≥10%	0,9	3,7	10,4	13,3	19,2	25,3	32,4	37,6	39,0	42,4	52,0	56,7	27,1
Multiplicative Winters’ model													
(-3%; 3%)	85,2	62,6	45,3	37,2	28,8	22,3	23,5	17,8	15,0	17,2	14,3	11,3	32,4
(-5%; -3%> and <3%; 5%)	11,1	17,8	20,8	17,1	20,2	22,3	15,7	16,8	16,0	9,1	8,2	11,3	15,6
(-10%; -5%> and <5%; 10%)	3,7	15,9	19,8	28,6	27,9	28,2	31,4	32,7	36,0	37,4	37,7	35,1	27,5
≤-10% and ≥10%	0,0	3,7	14,1	17,1	23,1	27,2	29,4	32,7	33,0	36,3	39,8	42,3	24,5
SARIMA$(0,1,3)\times(0,1,1)_{12}$													
(-3%; 3%)	92,6	73,8	53,8	43,8	40,4	29,1	26,5	19,8	17,0	15,2	15,3	10,3	37,2
(-5%; -3%> and <3%; 5%)	7,4	9,4	19,8	15,2	11,5	21,4	14,7	16,8	14,0	14,1	9,2	13,4	13,9
(-10%; -5%> and <5%; 10%)	0,0	16,8	17,0	24,8	26,9	18,4	26,5	30,7	32,0	31,3	34,7	27,8	23,7
≤-10% and ≥10%	0,0	0,0	9,4	16,2	21,2	31,1	32,3	32,7	37,0	39,4	40,8	48,5	25,2

Source: see Table 1

Analysing ex post forecasts obtained with the use of Winter's multiplicative model it may be observed that the forecasts were more often overvalued for advance of 2, 9 and 10 months and they were overvalued by, respectively, 0.5 pp, 3.0 pp and 0.5 pp. For the advance of 1, 3-8 and 12 months, however, undervalued forecasts were more frequent. The mean value of percentage error for advance of one to 12 months ranged from 0.55% (for 9 months) to 1.76% (for 12 months). More than half of very good forecasts for this model were observed only for the advance of one (85.2%) and two months (62.6%), and analysing both good and very good forecasts, for the advance of one (96.3%) to four months (54.3%).

Forecasts obtained on the basis of SARIMA$(0,1,3)\times(0,1,1)_{12}$ model were more often undervalued for the advance of one to 11 months and overvalued only by 0.5 pp for 12 months. In the case of one-month advance 100% of both good and very good forecasts were obtained, and for the advance of 6 months there were more than 50% of good and very good forecasts. In total 25% of inadmissible forecasts were found.

CONCLUSION

Winters' model based on exponential smoothing methods both in additive and multiplicative versions as well as seasonal integrated autoregression model SARIMA allow to construct short-term forecasts of agricultural produce prices subject to seasonal variations. Those forecasts inform agricultural producers of possible level of prices in consecutive months in a year and may influence their actions on the agricultural market.

The lowest mean overall values of ex post forecasts errors were obtained for SARIMA$(0,1,3)\times(0,1,1)_{12}$ model, that showed the best fit of this model for the empirical data. The next place was taken by Winter's multiplicative model and Winter's additive model followed. For SARIMA$(0,1,3)\times(0,1,1)_{12}$ model, compared with Winter's additive model, the mean forecast error for milk procurement prices for 1 hl was 0.17 PLN less, forecast bias (as to absolute value) was 0.12 PLN less and relative total adjustment of the model to procurement prices – 0.85 pp.

The highest accuracy of forecasts for advance of one to three months was obtained for Winter's additive model and SARIMA$(0,1,3)\times(0,1,1)_{12}$ model, and in the case of Winter's multiplicative model for one (85.2%) to two (62.6%) months. In turn, more than half of both good and very good forecasts were obtained for the advance of one (100%) to six (50.5%) months for the SARIMA$(0,1,3)\times(0,1,1)_{12}$ model; from one (95.4%) to five (51.9%) months for Winter's additive model, and one (96.3%) to four (54.3%) months for Winter's multiplicative model.

Forecasts constructed with the use of SARIMA$(0,1,3)\times(0,1,1)_{12}$ model are more often exact than when additive (for the advance of 1, 3, 6-7 and 9-11 months) and multiplicative (for the advance of 1-8 and 10-12 months) Winters' model based on exponential smoothing methods are used. In turn, the additive Winters' model based on exponential smoothing methods gives more frequently more exact forecasts than its multiplicative version, as for the advance of two to six months there were more very good forecasts, respectively 4.7 pp, 9.4 pp, 7.6 pp, 10.6 pp and 9.7 pp. However, the percentage of both good and very good forecasts for SARIMA$(0,1,3)\times(0,1,1)_{12}$ model was 51.1% and exceeded by 2.3 pp the percentage of those forecasts for Winter's additive model and by 3.1 pp their percentage for Winter's multiplicative model. For every studied model it can be stated that the smaller the variability of milk procurement prices in forecast horizon, the higher the accuracy of the forecasts.

After rejecting the null hypothesis of the same mean values of milk procurement prices constructed on the basis of Winter's additive and multiplicative models as well as SARIMA$(0,1,3)\times(0,1,1)_{12}$ model (statistics F=7.3 and probability p = 0.001) it was stated on the basis of Tukey method for all pairwise comparisons only significant differences in mean values of forecasts for both Winter's models.

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ASSESSMENT OF THE RISK OF MONETARY POVERTY IN POLISH HOUSEHOLDS

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Abstract: Traditionally, the financial status of households is ascribed by categorizing them as either poor or nonpoor. This study does not make use of such categorization when assessing monetary poverty. Instead, it employs elements of the theory of fuzzy sets. Thus, each household was given a value from the interval [0,1] that indicated the degree of financial poverty according to its equivalised income. The analysis was based on data from household budget survey conducted by the Central Statistical Office in 2010.

Keywords: identification of the poor, monetary poverty, risk of poverty, poverty in Poland

INTRODUCTION

Monetary poverty is defined as poverty indicated by the financial status of households. Traditionally, households are categorized by ascribing them to either of just two subsets: poor or nonpoor. Such a restrictive categorization does not take into account differences in achieved values of monetary indicators. Moreover, a particular poverty line needs to be used, which affects any obtained results. Therefore, instead of dividing households into two categories, one can determine the risk of poverty. This solution was first developed by Cerioli and Zani, who, following the theory of fuzzy sets, proposed a linear membership function [Cerioli, Zani 1990]. A membership function ascribes each household a value that indicates the risk of poverty. Publications on the subject provide different forms of the membership function [Dudek, Landmesser 2010]. The most popular forms include the linear function [Cerioli, Zani 1990], the empirical distribution function [Cheli, Lemmi 1995], the function that limits the effect of ranks on the degree of risk of poverty [Betti, Verma 1999], and the function that takes into account the empirical distribution function together with the limited effect of ranks [Betti, Verma 2008].

This study assessed the risk of monetary poverty in Polish employees' households by using a risk of poverty indicator based on a membership function developed by Cheli and Lemmi. Values of the membership function were obtained based on equivalised incomes according to the $OECD_{70/50}$ scale. The obtained values of the risk of poverty indicator were compared in terms of selected characteristics of a household. The analysis did not include nonmonetary dimensions of poverty.

METHODOLOGY

Comparability of Monetary Indicators Between Households

Income and consumption expenditure are the most frequently used monetary indicators. A comparison of household wealth based on nominal values of these indicators does not include demographic differences through which a household's income is divided among all its members. Income is usually divided unequally between the members [Rusnak 2007]. Furthermore, multimember households benefit from economies of scale, meaning that the individual cost of living decreases with the increase of the number of members in a household[1].

Uneven distribution of income between household members and the economies of scale phenomenon require the use of equivalence scales [Dudek 2011]. Equivalence scales are relative, which means that the obtained equivalised income is compared to the income of a household with a particular demographic composition, called the reference household. Usually, the reference household is assumed to be a single adult person or two adult persons [Rusnak 2007]. Nowadays, one of the most frequently used equivalence scales in the European Union are the OECD scales. There are two types of OECD scales: $OECD_{70/50}$ (also called the OECD original scale) and $OECD_{50/30}$ (known as the OECD modified scale). The former ascribes a greater value to large households. The Central Statistical Office of Poland still uses it to determine the poverty line [*Poverty*… 2011]. The value of the $OECD_{70/50}$ scale is calculated according to the following formula:

$$OECD_{70/50} = 1 + 0.7 \cdot (d - 1) + 0.5 \cdot c \tag{1}$$

where:
d – number of adult persons in a given household,
c – number of children[2] in a given household.

[1] This is especially true for expenditures on heating, electricity, and access to the Internet.
[2] A child is usually defined as a person aged below 15 years [Panek 2007],[*Poverty*… 2011].

The equivalised income of a household is obtained by dividing the nominal income by value of the equivalence scale. For instance, a household comprising two adult persons and one child will have 2.2 on the $OECD_{70/50}$ scale. If the nominal income in a household is PLN 4,400, then its equivalised income will be PLN 2,000.

Risk of Monetary Poverty Indicator

The financial situation of a household is one of the most important factors considered when identifying its degree of poverty. Traditionally, this identification involves determining whether the household is poor or not. Thus, each household is divided into one of two subsets: poor or nonpoor. This requires one to determine a threshold value, called the poverty line. A household is considered poor if its income or equivalised expenditures are lower than the poverty line. This method has its limitations. One needs to determine the particular value of a monetary indicator that should constitute the poverty line. The higher the value of the poverty line, the more households are considered poor. Moreover, this approach does not take into account the differences in values of monetary indicators achieved by households. These differences are especially important for households in which the value of income (expenditures) is close to the poverty line[3].

To include differences in equivalised income (or expenditures), elements of Zadeh'sfuzzy set theory are used. A fuzzy subset *A* of a set *X* is defined as ordered pairs $\left[x, f_A(x)\right]$ [Zadeh 1965]:

$$A = \{x, f_A(x)\}, x \in X \tag{2}$$

The $f_A(x)$ function is called the membership function of member *x* of a subset *A* of a set *X*. The function takes values from the interval [0,1]. In terms of assessing poverty, set *X* constitutes all households in the sample. Subset *A* includes poor households, while the values of the membership function $f_A(x)$ indicate the degree to which the household belongs to the poor households subset. If:

- $f_A(x) = 0$, then the household does not belong to the poor households subset;
- $f_A(x) = 1$, then the household fully belongs to the poor households subset;
- $f_A(x) \in (0,1)$, then the household partially belongs to the poor households subset. The membership of a given household is fuller the closer the value of the membership function is to 1.

Membership functions have been introduced to the analyses of poverty by Cerioli and Zani, who proposed that the values of a linear membership function be

[3]Assuming that the poverty line is equal to the minimum subsistence, which amounted to PLN 443 in 2010 [*Poverty*… 2011], a household with an income of PLN 442 would be considered poor, whereas a household with an income of PLN 444 would be considered nonpoor. Despite the two households being in a very similar situation, they are classified as belonging to different subsets.

calculated once two threshold values are determined: the value below which poverty is definitely present and the value above which poverty is definitely absent [Cerioli, Zani 1990].

Cheli and Lemmi, to evade arbitrarily defining threshold values, proposed a membership function that used an empirical distribution function [Cheli, Lemmi1995]. The function is calculated according to the following formula:

$$f(x_i) = f\left(x^{(r)}\right) = \frac{1 - F\left(x^{(r)}\right)}{1 - F\left(x^{(1)}\right)} \tag{3}$$

where:

x_i – values of equivalised income (or expenditures) for the *i*-th household,

$x^{(1)}, x^{(2)}, \ldots, x^{(n)}$ – values of equivalised income (or expenditures) in nondecreasing order, where $n \le N$,

n – number of different values of equivalised income in households within the sample,

N – total number of households in the sample,

$F\left(x^{(r)}\right)$ – value of the empirical distribution function for the variable.

The membership function allows one to assess the degree to which a household belongs to the poor households subset according to a characteristic represented by the *x* variable. When assessing monetary poverty, only equivalised income or expenditures of households are taken into account. The risk of monetary poverty indicator can be calculated based on the membership function.

DATA AND RESULTS

The analysis concerns the assessment of monetary poverty based on data from household budget survey conducted by the Central Statistical Office in 2010. A total of 18,422 employees' households were included in the analysis. Households with negative income, which constituted 0.09% of the sample, were not included in the analysis. Disposable income was chosen as the monetary indicator. Due to a varied demographic structure of households in the sample, values of nominal income were scaled by the $OECD_{70/50}$ scale. Values of the scale for each household were calculated according to Formula 1. A child was defined as a person aged 14 years or below. The membership function, Formula 3, was used to determine the value of the risk of poverty indicator. Table 1 shows a comparison of the values of the indicator according to selected characteristics of the household head[4].

[4] The head of household is defined as the person who provides all or most of the financial resources needed to maintain the household. When such a person cannot be determined, the head of household is considered to be the person who manages most of these resources.

Table 1. Values of the risk of monetary poverty indicator for selected characteristics of the household head

Characteristic	Mean	Median	Standard deviation	Min	Max	Size
Total	0.498	0.495	0.281	0	1	18,422
Household head’s occupation						
Manual labour position	0.622	0.650	0.243	0.002	1	9,794
Non-manual labour position	0.357	0.310	0.253	0	1	8,628
Gender of the household head						
Female	0.476	0.462	0.283	0	1	6,060
Male	0.509	0.510	0.279	0	1	12,362
Level of education of the household head						
Lower secondary or less	0.705	0.750	0.224	0.025	1	1,505
Basic vocational	0.624	0.650	0.240	0.001	1	6,450
Secondary general	0.477	0.469	0.251	0.003	1	1,368
Secondary vocational	0.493	0.488	0.253	0.003	1	4,042
Post-secondary	0.426	0.406	0.243	0.002	0.996	455
Bachelor's degree or engineer	0.329	0.283	0.235	0	1	846
Master or equivalent	0.265	0.211	0.218	0	1	3,601
University	0.134	0.097	0.150	0.001	0.875	155

Source: own calculations based on the household budget survey conducted in 2010

The mean value of the risk of monetary poverty indicator for all employees’ households was 0.498. The median was slightly lower and equaled 0.495. This means that the risk of monetary poverty for half of the assessed households was over 0.495. The values of the indicator were considerably dispersed around the mean; the dispersion amounted to 56% of the mean. This was due to differences in equivalised income and variation of income between the households. There were 14,674 different values of equivalised income within the analyzed sample.

When the values of the indicator were analyzed in terms of the household head’s occupation, the risk of monetary poverty was on average twice as high for persons performing physical work than for persons performing nonphysical work. Standard deviation for both types of households was similar. Therefore, the risk of monetary poverty was much greater for households in which the reference person performed physical work.

Households in which the reference person was male were twice as numerous and usually showed a higher risk of monetary poverty than households with a female head. However, these differences were small in terms of the mean and the median.

The most significant differences were found for the education level of the household head. Households in which the reference person had no higher than lower secondary education were found to be in the worst situation. The mean value of the indicator for this group amounted for 0.705 and was 42% higher than the mean value for the entire sample. The lowest risk of monetary poverty in this group equaled 0.025. The mean and median values of the indicator decreased with the increase in the education level. Households in which the reference person had university education and a scholarly title showed the lowest risk of monetary poverty; their median value of the indicator equaled only 0.134. Moreover, the indicator for half of the households in this group was lower than 0.097. Dispersion of values around the mean was also lower than in other types of households.

The characteristics of a given household affected its risk of monetary poverty. Table 2 presents the values of the risk of monetary poverty indicator according to the place of residence.

Table 2. Values of the risk of monetary poverty indicator according to the place of residence

Characteristic	Mean	Median	Standard deviation.	Min	Max	Size
Total	0.498	0.495	0,281	0	1	18,422
Place of residence						
Cities over 500 thous.	0.287	0.221	0.241	0	1	2,517
Cities 200-499 thous.	0.419	0.392	0.261	0	0.994	1,747
Towns 100-199 thous.	0.449	0.425	0.262	0.003	1	1,350
Towns 20-99 thous.	0.472	0.460	0.261	0.001	1	3,215
Towns below 20 thous.	0.527	0.532	0.264	0	1	2,246
Rural	0.601	0.63	0.263	0	1	7,347
Voivodeships						
dolnośląskie	0.473	0.451	0.277	0.001	0.998	1,439
kujawsko-pomorskie	0.535	0.544	0.267	0.006	0.999	983
lubelskie	0.579	0.598	0.274	0	1	930
lubuskie	0.503	0.491	0.266	0.008	1	504
łódzkie	0.507	0.511	0.270	0.002	1	1,220
małopolskie	0.517	0.522	0.269	0.002	1	1,612
mazowieckie	0.369	0.314	0.288	0	1	2,828
opolskie	0.519	0.521	0.272	0.013	1	495
podkarpackie	0.615	0.651	0.257	0	1	995
podlaskie	0.528	0.543	0.273	0.007	0.996	451
pomorskie	0.479	0.462	0.289	0.001	1	1,063
śląskie	0.503	0.500	0.265	0	0.998	2,281
świętokrzyskie	0.580	0.602	0.265	0.008	1	597
warmińsko-mazurskie	0.535	0.541	0.281	0.001	0.993	653
wielkopolskie	0.521	0.520	0.261	0.001	1	1,577
zachodniopomorskie	0.502	0.492	0.280	0.001	0.998	794

Source: own calculations based on the household budget survey conducted in 2010

The class of the place of residence had a considerable effect on the mean value of the indicator. Usually, households in rural areas were in the worst situation, with the risk of monetary poverty of half of them exceeding 0.63. The mean and median values of the indicator decreased as the size of the place of residence increased, with households in cities with a population of over 500,000 being in the best situation. The mean value of the indicator was over twice as low for these households than for households in rural areas. No significant differences in the standard deviation were observed. The size of the place of residence had a considerable effect on the risk of monetary poverty but had no influence on the dispersion of the values of income.

Significant differences in the risk of monetary poverty were found between the voivodeships. Usually, the highest values of the indicator were found in Lubelskie and Podkarpackie voivodeships. Mazowieckie voivodeship showed the lowest values of the indicator.

CONCLUSIONS

The analysis showed that the risk of monetary poverty was affected by various characteristics of the household. The risk of monetary poverty was higher for households whose reference person performed physical work than for nonphysical workers. The greatest differences were found when the risk of poverty was compared to the education level of the household head. The greatest values of the risk were observed for households whose reference person had no more than lower secondary education. The risk of monetary poverty decreased with the increase of the education level. Gender of the household head usually did not affect the risk of monetary poverty. The class of the place of residence was tied to considerable differences in mean and median values of the risk of monetary poverty. Households in large cities showed the lowest values of the risk, while the highest values were observed for households in rural areas. On the voivodeship level, the Podkarpackie and Lubelskie voivodeships showed the highest level of the risk, while the Mazowieckie voivodeship showed the lowest level.

In this study, the risk of monetary poverty indicator was calculated based on a relative approach to measuring poverty, in which the results depend completely on the dispersion of the values of income. Future assessments of monetary poverty should limit the effect of ranks on the obtained results and provide indicators of monetary poverty based on an econometric model. Many authors emphasize that poverty is a multidimensional phenomenon. Therefore, a comprehensive analysis of poverty should include nonmonetary dimensions.

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