Warsaw University of Life Sciences – SGGW
Institute of Economics and Finance
Department of Econometrics and Statistics

QUANTITATIVE METHODS IN ECONOMICS

METODY ILOŚCIOWE W BADANIACH EKONOMICZNYCH

Volume XXV, No. 1

Warsaw 2024

ISSN 2082-792X
e-ISSN 2543-8565

Warsaw 2024, Volume XXV, No. 1
Journal homepage: https://qme.sggw.edu.pl
Published by Warsaw University of Life Sciences Press

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Volume XXV, No. 1, 2024

CONTENTS

QUANTITATIVE METHODS IN ECONOMICS
Volume XXV, No. 1, 2024, pp. 1 – 9

TESTING THE VALIDITY OF THE PURCHASING POWER PARITY (PPP) HYPOTHESIS FOR TÜRKİYE: LINEAR AND NONLINEAR UNIT ROOT TESTS

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Abstract: In this paper, we test the validity of the purchasing power parity (PPP) hypothesis between Türkiye and its trading partners - the European Union, China, and the US - for the period from January 2001 to January 2020. We test the stationarity of the real exchange rates for the validity of the PPP hypothesis by applying linear and nonlinear unit root tests. We also employ Fourier-based tests to account for the structural changes that occurred in the considered period. Test results indicate that shocks are temporary, and the PPP hypothesis is valid for Türkiye.

Keywords: purchasing power parity, real exchange rate, unit root tests

JEL classification: C22, C61, F31

INTRODUCTION

The purchasing power parity (PPP) hypothesis is based upon the idea of the law of one price, which states that the price of an asset in different countries should be the same when expressed in the same currency. The PPP hypothesis might hold more closely in countries experiencing high inflation. This implies that the shocks in real exchange rates are temporary, and it is expected that the rates will return to a constant equilibrium level in the long run. In the literature, there are enormous studies that test the validity of the PPP hypothesis using cointegration and unit root tests. The fact that the real exchange rate series exhibits non-stationary

https://doi.org/10.22630/MIBE.2024.25.1.1

characteristics shows that the PPP theory is invalid. On the other hand, the long-run relationship between the logarithmic transformation of the nominal exchange rate and the logarithmic difference between domestic and foreign consumer price indexes stress the validity of the PPP hypothesis. Although the validity of the PPP hypothesis has been extensively tested for both developed and developing economies, it still maintains popularity in the applied literature, especially for the high inflation countries, like Türkiye. The reason for considering Türkiye as a case is due to its unique features of high inflation and structural changes. Furthermore, the development observed simultaneously in tests and technology makes it possible to re-examine the results of studies such as Telatar and Kazdağlı [1998], Özdemir [2008], Kalyoncu [2009], Sarno [2000], Yazgan [2010], Yıldırım [2017]. However, most of the studies fail to prove the validity of the PPP hypothesis while the other part of the studies prove the stationarity of the real exchange rates through unit root tests and the link between the nominal exchange rate and the consumer price indexes through cointegration tests. For instance; Telatar and Kazdağlı [1998] examined the hypothesis of long-run PPP applying cointegration techniques for Türkiye. The results do not support any long-run bilateral exchange rate and consumer price index rates between Türkiye and France, Germany, the UK, and the USA. Sarno [2000] re-examined the validity of the PPP hypothesis for the period between 1980 and 1997 for Türkiye extending the work by Telatar and Kazdağlı [1998]. The results of the nonlinear modeling techniques provide strong evidence in favor of the PPP hypothesis. Özdemir [2008] re-examined the validity of the long-run PPP hypothesis for Türkiye with the monthly data set from January 1984 to December 2004. The findings provide evidence that the long-run PPP hypothesis is valid for the given period in Türkiye just as supported by Sarno [2000] and Erlat [2004]. Kalyoncu [2009] tested the validity of the PPP between Türkiye and its trading partners namely the USA, Germany, Japan, France, Netherlands, and the UK applying different unit root tests for the quarterly data from 1980 to 2005. The findings support that the validity of the PPP hypothesis is sensitive to the type of tests and the comparison country. Yazgan [2010] found strong evidence of long-run PPP in Türkiye for the period from January 1982 to April 2001 using standard multivariate cointegration techniques. Yıldırım [2017] employed nonlinear unit root tests for the analysis of the PPP hypothesis between Türkiye and its four trading partners. The findings indicate that nonlinear unit root tests give stronger evidence in favour of the PPP hypothesis compared to the classical unit root tests if the nonlinearities of the series are specified correctly.

In this study, we aim to investigate the validity of the PPP hypothesis between Türkiye and its three major trading partners, the US, the European Union (EU), and China for the period from January 2001 to January 2020. Rather than considering the linear behaviour of Turkish real exchange rates within the linear concept, we also utilize the recent development of the nonlinear unit root tests to enhance the reliability of the results. It is also noteworthy that the unit root tests without considering structural breaks and the nonlinear form of the series lose power. Thus, results tend to be biased, and we are less likely to reject an incorrect null hypothesis. In this context, we employ Kapetanios et al. (KSS) [2003], Sollis [2009], Kruse [2011], Kılıç [2011], Christopoulos and Leon-Ledesma [2010], and Güriş [2018] along with the ADF, extended version of Dickey and Fuller [1979], and Zivot and Andrews [1992] tests.

The rest of the paper is organized as follows: Section 2 outlines the methods. Empirical data is detailed in Section 3. Section 4 presents the empirical results. Section 5 provides a summary of the article.

METHODS

In the nonlinear unit root tests, KSS [2003], Sollis [2009], Kruse [2011], and Kılıç [2011] are the most popular tests used in applied econometrics. KSS [2003] unit root test is the first nonlinear unit root test that considers the Taylor series approximation in the testing procedures. The test is known as a nonlinear form of the ADF unit root test. The exponential transition function is used as a transition function to define the nonlinearity in the model following the literature on smooth transition autoregressive (STAR) models.

The model can be written as follows:

$$y_t = \beta y_{t-1} + \rho y_{t-1} G(\gamma, y_{t-d}) + \varepsilon_t, \tag{1}$$

where the transition function $G(\gamma; y_{t-d}) = 1 - e^{(-\gamma y_{t-d}^2)}$. In the function, we assume that $\gamma \geq 0$ and $d \geq 1$. The model can be reorganized by assuming that $\gamma \geq 0$ and $d = 1$ as follows:

$$\Delta y_t = \phi y_{t-1} + \rho y_{t-1} . \left[1 - e^{(-\gamma y_{t-1}^2)}\right] + \varepsilon_t, \tag{2}$$

where $\phi = \beta - 1$. Under the assumption of $\phi = 0$, the model can be formed as follows:

$$\Delta y_t = \rho y_{t-1} . \left[1 - e^{(-\gamma y_{t-1}^2)}\right] + \varepsilon_t. \tag{3}$$

In the KSS [2003] unit root test, the null hypothesis of unit root process $H_0 : \gamma = 0$ is tested against the alternative of stationary exponential smooth transition autoregressive (ESTAR) process $H_1 : \gamma > 0$. However, it is not feasible to directly test the null hypothesis since ρ is unidentified under the null hypothesis. Taylor series approximation is suggested to overcome the nuisance parameter problem, also called as Davies [1987] problem. The suggested model for the stationary test is created based on the first-order Taylor series approximation as follows:

$$\Delta y_t = \delta y_{t-1}^3 + \sum_{j=1}^{p} \rho_j \Delta y_{t-j} + \varepsilon_t. \tag{4}$$

The null and alternative hypotheses are formed as $H_0 : \delta = 0$ and $H_a : \delta < 0$. Critical values are tabulated in the KSS [2003] for the row cases, demeaned, and detrended data. In the Sollis [2009] test, a new unit root test is proposed to test the unit root null hypothesis against the alternative hypothesis that allows symmetric or asymmetric ESTAR nonlinearity unlike KSS [2003]. The suggested model is also known as asymmetric ESTAR (AESTAR) written as follows:

$$\Delta y_t = \phi_1 y_{t-1}^3 + \phi_2 y_{t-1}^4 + \sum_{i=1}^{k} \kappa_i \Delta y_{t-i} + e_t. \tag{5}$$

The rejection of the null hypothesis of unit root $H_0: \phi_1 = \phi_2 = 0$ may address to test the null of symmetric ESTAR nonlinearity $H_0: \phi_2 = 0$ against the alternative of asymmetric ESTAR nonlinearity $H_1: \phi_2 \neq 0$ using a standard F test statistics. Critical values of F tests are tabulated by Sollis [2009] for three cases as KSS [2003] since a standard F test cannot be used to test the unit root null hypothesis.

In the Kruse [2011] unit root test, the model is written based on the Taylor approximation as follows:

$$\Delta y_t = \phi_1 y_{t-1}^3 + \phi_2 y_{t-1}^2 + \sum_{i=1}^{k} \rho_i \Delta y_{t-i} + u_t, \tag{6}$$

where unit root null hypothesis $H_0: \phi_1 = \phi_2 = 0$ is tested against the alternative of globally stationary ESTAR process $H_1: \phi_1 < 0,\ \phi_2 \neq 0$. The asymptotic critical values are tabulated through stochastic simulations.

The unit root test proposed by Kılıç [2011] is similar to the KSS [2003] test process except for the way of dealing with the nuisance parameter problem. To test the null of a unit root against a globally stationary ESTAR process, the following representation of the ESTAR model is considered:

$$\Delta y_t = \rho y_{t-1}.\left[1 - e^{\left(-\gamma \Delta y_{t-1}^2\right)}\right] + \sum_{i=1}^{p} \rho_i \Delta y_{t-i} + \varepsilon_t \tag{7}$$

The null and alternative hypotheses are defined as $H_0: \rho = 0$ and $H_1: \rho < 0$ in Kılıç [2011] test. The test suffers from a nuisance parameter problem only defined under the alternative hypothesis as in KSS [2003]. Thus, the critical values are computed based on the grid research different from the Taylor expansion to overcome this problem as follows:

$$t_{ESTAR} = \inf_{\gamma \in \Gamma_T} \hat{t}_{\rho=0}(\gamma) = \inf_{\gamma \in \Gamma_T} \hat{\rho}(\gamma) / s.e.(\hat{\rho}(\gamma)), \tag{8}$$

where $s.e.$ is the standard error. Γ_T refers to the following equation:

$$\Gamma_T = \left[\underline{\gamma}_T, \bar{\gamma}_T\right] = \left[\frac{1}{100 s_{\Delta y_{t-1} T}}, \frac{100}{s_{\Delta y_{t-1} T}}\right] \in \mathrm{R}. \tag{9}$$

where $s_{\Delta y_{t-1} T}$ is the sample standard deviation of Δy_{t-1}. The asymptotic critical values are provided by Kılıç [2011].

The problems encountered in modelling structural breaks in unit root tests based on linear and non-linear models have raised the question of how to model the breaks in the literature as an alternative way to dummy variables. Gallant [1981], Gallant [1984], Gallant and Souza [1991], Becker et al. [2004], Becker et al. [2006], Enders and Lee [2012a], and Enders and Lee [2012b] have made important contributions to the literature suggested that these problems would be overcome with the Fourier approach. The main idea behind the Fourier-based unit root test is to capture structural breaks regardless of the number, date, and shape of the breaks by using trigonometric functions.

The general form of the Fourier function is as follows:

$$\alpha(t) = \alpha_1 sin\left(\frac{2k\pi}{T}\right) + \alpha_2 cos\left(\frac{2k\pi t}{T}\right), \quad (10)$$

where k is the Fourier frequency chosen for the approximation, α_1 and α_2 represent the measurement of the amplitude and displacement of the frequency component. T is the sample size, t is a trend term, and $\pi \cong 3.1416$. Christopoulos and Leon-Ledesma [2010] and Güriş [2018] developed the KSS [2003] and Kruse [2011] unit root tests by including the $\alpha(t)$ into the model as follows:

$$y_t = \delta_1 + \alpha(t) + e_t. \quad (11)$$

The model is estimated by OLS for the different values of k defined in the range [1,5] and the value that minimizes the sum of squared residuals is selected to compute the OLS residuals as follows:

$$\hat{e}_t = y_t - \delta_1 - \alpha(t). \quad (12)$$

Following, the unit root on the OLS residuals of equation 12 is tested by employing KSS [2003] and Kruse [2011] unit root test models as detailed in equations 4 and 6.

EMPIRICAL DATA

The data used in this study consist of two sets: The first one is nominal bilateral exchange rates for the Chinese Yuan, Euro, and the US Dollar (USD) against the Turkish lira (TL). The second set is the Consumer Price Indexes (CPI) conducted for each domestic and foreign country. All series are taken from the International Monetary Fund's International Financial Statistics and Federal Reserve databases from January 2001 to January 2020 (T = 229). The following equation shows the general structure for the PPP hypothesis, the relation between the nominal exchange rate and relative price levels:

$$s_t = \alpha_0 + \alpha_1(p_t - p_t^*) + u_t, \quad (13)$$

where s_t denotes the logarithm of the domestic nominal exchange rate, p_t^* is the logarithm of the foreign country's price level and p_t is the logarithm of the domestic country's price level, u_t is an error term with zero mean and constant variance. We consider the following equation and transform the variables to explore the validity of the PPP hypothesis between Türkiye and its trading partners by applying unit root tests.

$$r_t = s_t + (p_t^* - p_t), \quad (14)$$

where r_t is the real exchange rate and s_t, p_t^*, and p_t are as defined in equation (13). The nominal exchange rates are computed using the cross-exchange rates TL-US$, Euro €-US$, and Yuan ¥ - US$ and transferred to real exchange rates by using domestic and foreign consumer price indices in equation (14). The real exchange rate series should be stationary at the level following the assumption of the PPP hypothesis. If the r_t, the real exchange rate, is stationary at the level, this emerges that the PPP hypothesis is valid between Türkiye and its trading partners the EU, the US, and China. On the other hand, shocks to the real exchange rates are temporary between two countries. Furthermore, the stationarity of the variables, the rejection of the null hypothesis in unit root tests, would indicate that the changes in the price levels between Türkiye and its trading partners taken into account in the study would be balanced by an equal depreciation or appreciation in the nominal exchange rate [Kalyoncu 2009].

RESULTS

In this part of the study, we present the linear and nonlinear unit root test results to test the validity of the PPP hypothesis in Türkiye. We initially employed the most popular unit root test, ADF and Zivot and Andrews [1992], as reported in Table 1. The results of the ADF test indicate that the null of the unit root is rejected for all cases, TL/USD, TL/EURO, and TL/YUAN. However, the results of the Zivot and Andrews [1992] test provide that the null of the unit root is rejected for the TL/USD and TL/YUAN.

Table 1. Linear Unit Root Test Results

Variables	ADF	Zivot and Andrews (1992)	Break Dates
TL/USD	-3.411** (1)	-5.730***(1)	2006M02
TL/EURO	-3.091** (1)	-4.254 (1)	2013M06
TL/YUAN	-2.883** (0)	-4.823* (5)	2005M04
Critical Values			
1%	-3.459	-5.57	
5%	-2.874	-5.08	
10%	-2.573	-4.82	

Note: Numbers in parentheses are the number of lags chosen by the minimum value of the Akaike Information Criteria. *, **, and *** denote the rejection of the unit root null hypothesis at 10%, 5%, and 1%.

Source: own calculations

The empirical investigation of nonlinear unit root tests is given in Table 2 for the KSS [2003], Kruse [2011], and Kılıç [2011]. Additionally, we include two tests, Fourier KSS [2010] and Fourier Kruse [2018], proposed by Christopoulos and Leon-Ledesma [2010] and Güriş [2018], to model the structural changes regardless of the number of breaks, dates, and form of the breaks. According to the test results in Table 2, we reject the unit root null hypothesis for three cases for all the reported test results with and without considering the breaks in the tests. The results in Tables 1 and 2 provide evidence that the PPP hypothesis is valid for three cases by rejecting the unit root null hypothesis. The results prove that shocks to the real exchange rates are temporary between two countries, Türkiye and the US, China, and the EU.

Table 2. Nonlinear Unit Root Test Results

Variables	KSS (2003)	Kruse (2011)	Kılıç (2011)	Fourier KSS (2010)	Fourier Kruse (2018)
TL/USD	-11.125***	141.147***	-2.715***	-11.273***	141.235***
TL/ EURO	-10.384***	118.942***	-2.908***	-10.412***	110.619***
TL/YUAN	-8.628***	103.789***	-2.937***	-10.048***	106.542***
Critical Values					
1%	-3.48	13.75	-2.98	-4.19	18.82
5%	-2.93	10.17	-2.37	-3.60	14.80
10%	-2.66	8.60	-2.05	-3.29	12.52

Note: The signs *, **, and *** denote the rejection of the unit root null hypothesis at 10%, 5%, and 1%.

Source: own calculations

We also demonstrate the link between the real exchange rates and residuals fitted by the Fourier function for the cases TL/USD, TL/EURO, and TL/YUAN in Figure 1 to show how the Fourier function can catch the possible breaks in the series.

Figure 1. The comparison of the original series and residuals fitted by the Fourier function

*The US, CH, and the EU represent real exchange rates calculated taking into account the US, China, and the EU currencies, respectively. FITUS, FITCH, and FITEU demonstrate the residuals fitted by the Fourier function.

Source: own preparation

SUMMARY

In this paper, we used monthly data to indicate the validity of the PPP hypothesis for Türkiye for the period between January 2001 and January 2020. Instead of working only with the TR-US nominal exchange rates and consumer price index to compute the real exchange rates, we have considered the other major trading partners of Türkiye, China and the EU, to evaluate the results comparatively, different from most of the studies in the literature. Linear and nonlinear unit root tests have been employed to test if the unit root hypothesis is rejected for the variables. Furthermore, we have highlighted the importance of applying unit root tests that include structural changes in the unit root process to enhance the reliability of the test results. The results show that the unit root null hypothesis is rejected for all three cases, and all the series are found stationary. Thus, we can conclude that the rejection of the null hypothesis proves that the PPP hypothesis is valid between Türkiye and its trading partners for the given period and changes in the price levels between Türkiye and the US, China, and the EU would be balanced by an equal depreciation or appreciation in the nominal exchange rate as noted by Kalyoncu [2009].

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QUANTITATIVE METHODS IN ECONOMICS
Volume XXV, No. 1, 2024, pp. 10 – 18

ON JOINT DISTRIBUTION OF BIDS IN SOME k-TH PRICE AUCTION MODEL

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Abstract: In the era of increasingly widespread use of electronic communication tools, constant growth of e-commerce turnover is hardly surprising. One of its mechanisms are online auctions whose turnover, on a global scale, is enormous. This paper proposes a stochastic model of the value of subsequent actual bids appearing in the course of a dynamic internet k-th price auction. On this basis, it was possible to determine unconditional joint probability distributions, as well as univariate marginal distributions of successive bids appearing in the course of the auction type under consideration.

Keywords: auction bids, k-th records, exceedance distribution

JEL classification: C46, C58, C19

INTRODUCTION

Auctions other than first-price took place as early as the end of the 19th century, but the first time they gained a complete theoretical description was in 1961, thanks to [Vickrey 1961]. The original Vickrey-type auctions are second-price auctions. Over time, this type of auction was generalized to the k-th price auction. k-th price auctions, similarly to classic auctions, can be held both in a static and dynamic form.

The latter form began to develop dynamically with the emergence and widespread use of electronic media. The current wide-ranging availability of the Internet means that e-auctions no longer of interest only to a limited group of parties but to billions of customers worldwide. Consequently, they involve enormous financial flows. An introduction to the theory of auctions and a broad review of the

https://doi.org/10.22630/MIBE.2024.25.1.2

literature can be found, among others, in [Klemperer 1999], [Ockenfels et al. 2006], [Hickman et al. 2012].

One may understand static k-th price auction as a game consisting of two stages: 1) auction participants get to know the starting price and submit bids (knowing only their own offer); 2) the participant with the highest bid wins, but pays the k-th highest bid price for the auctioned object. Numerous papers describe this type of auction (especially for $k = 2$). They draw from game theory and aim to study the efficiency, equilibrium points, or limiting properties of the auction with an infinite increase in the number of participants. Occasionally, in the conducted analyses, risk-neutral bidders and risk-averse bidders are discerned or the possibility of collusion between certain bidders, and other specific circumstances, is assumed. See, e.g., [Fibich, Gavious 2008], [Fibich, Gavious 2010], [Mezzetti, Tsetlin 2009], [Gorelkina 2014], [Hickman 2010], [Mihelich, Shu 2020], [Myerson 1981].

By contrast, in dynamic auctions, successive bids of various bidders (including repeated bids) are registered one by one within a predetermined time interval, and the current purchase price changes with successively incoming bids. The bidders know the price at all times. Such an auction is completed at the end of the set time interval, and the last current price becomes the purchase price. Here, one may apply the k-th price auctions.

Following the distinction indicated in [Athey, Haile 2007] and the definitions contained therein, we note that the literature proposes two types of stochastic models of bid flow in dynamic auctions: 1) models taking into account potential bids (bids from potential bidders (e.g. [Haile, Tamer 2003], [Canals-Cerdá, Pearcy 2013])); 2) models taking into account actual bids (bids of actual/active bidders (e.g. [Barbaro et al. 2006], [de Haan et al. 2008], [Namazi, Schadschneider 2006], [Roth and Ockenfels 2000])). In both cases (potential and actual bids), stochastic modeling of the sequence of bid values is reduced to the determination of an appropriately selected (finite) sequence of random variables. Potential bids make it possible to use a sequence of independent and identically distributed random variables.

Although such a sequence does not necessarily accurately describe reality, it makes it easier to use probabilistic tools, e.g., the theory of k-th record values (see [Dziubdziela, Kopociński 1976], [Dziubdziela 2018]). A general model of potential bids for any k was considered by Dziubdziela [2008], who examined limiting properties of relative increment of auction prices, using the results obtained in the papers of [Bieniek, Szynal 2000] and [Bieniek, Szynal 2003]. At the same time, the special case of $k = 2$ was analyzed more thoroughly in [de Haan et al. 2008], [de Haan et al. 2009].

In contrast, the values of actual bids – by their very nature – cannot be modeled with the use of a sequence of independent random variables with the same probability distribution. That is because in this case it is necessary to use sequences of random variables that assume ever greater values from a certain moment.

Consequently, these variables are dependent and have different probability distributions.

The bid value model proposed in the article refers to the principles of conducting eBay on-line auctions which are very similar to dynamic second-price auctions (see [Namazi, Schadschneider 2006]). This model does not consider the minimum bid increment and the possibility of using a proxy bidding system. Although our model describes one aspect of an auction (bid amounts) and uses certain simplifying assumptions, it is innovative on the account of a simple ploy of using exceedance distributions to describe actual bids that are stochastically dependent.

RESULTS

Model definition

Without loss of generality, we assume that a seller price $X_0 = 0$. Let $X_1, X_2, X_3, \dots$ be a sequence of random variables that represent actual bids – announced successively by active bidders – in a dynamic 'k-th-price' auction, for a priorly fixed $k \geq 2$, $k \in \mathbb{N}_+$. Then, $X_k < X_{k+1} < X_{k+2} < \cdots$ almost surely, which means that the auction is ascending from the k-th bid.

Let F be a common cumulative distribution function (cdf) of $X_1, X_2, \dots, X_k$. Since $X_1, X_2, \dots, X_\mathrm{k} \geq 0$, then $F(0) = 0$. We additionally assume that $X_1, X_2, \dots, X_k$ are absolutely continuous, which means that F is differentiable. So let $f = F'$ be probability density function (pdf) of these variables.

With respect to the given cdf F and pdf f, we consider a family of 'exceedance distributions' indexed by all real $a \geq 0$, thus we work on two families $\{F_{\bar{a}}\}_{a\geq 0}$, and $\{f_{\bar{a}}\}_{a\geq 0}$ of cdf's, and pdf's, given by the following formulas:

$$F_{\bar{a}}(t) = \frac{F(t)-F(a)}{1-F(a)} \cdot \mathbb{I}_{(a,+\infty)}(t),$$

$$f_{\bar{a}}(t) = \frac{f(t)}{1-F(a)} \cdot \mathbb{I}_{(a,+\infty)}(t).$$

Note that $F_{\bar{0}} = F$, $f_{\bar{0}} = f$, and $F_{\bar{a}}(0) = 0$ for any $a \geq 0$

This family of distributions was used by [Haile, Tamer 2003], followed by [Canals-Cerdá, Pearcy 2013], which is a very practical approach (Haile and Tamer called these distributions as 'truncated distributions'). However, we must clearly emphasize that these papers considered models of the distributions of private valuations coming from potential bidders, and not, as in our case, the distributions of actual bids.

By our definition, the course of bidding assumes $X_1, X_2, \dots, X_k$ to be independent and identically distributed with a common cdf F, while on the basis of the given family of exceedance distributions, conditional distributions of variables $X_{k+1}, X_{k+2}, \dots$ are:

$$(X_{n+1}|X_n = x_n, X_{n-1} = x_{n-1}, \dots, X_1 = x_1,) \sim F_{\overline{x_{n-k:n-1}}}.$$

Model properties

Let $\mathcal{D}_n^{(k)}$ designate the support of joint distribution of vector $(X_1, X_2, \dots, X_n)$. Therefore:

- $\mathcal{D}_n^{(k)} = \mathbb{R}_+^n$, for $n \in \{1, \dots, k\}$
- $\mathcal{D}_n^{(k)} = \{(x_1, \dots, x_n) \in \mathbb{R}_+^n : \forall_{j \in \{k+1, \dots, n\}}\, x_j \ > \ x_{j-k:j-1}$, for $n > k$

where $x_{1:n} \leq \cdots \leq \ x_{n:n}$ stand for the ordered configuration of numbers $x_1, \dots, x_n$

Let $x \in \mathcal{D}_n^{(k)}$ for a given k and n. Then we define:

$$\Sigma_x^{(k,n)} \ = \ \left\{ \sigma \in \mathcal{P}(n) : \bar{\sigma}(x) \in \mathcal{D}_n^{(k)} \right\},$$

where $\bar{\sigma}(x) = \left(x_{\sigma(1)}, \dots, x_{\sigma(n)}\right)$ and $\mathcal{P}(n)$ designates the permutations' set of $\{1, \dots, n\}$.

Lemma 1.
If $n \geq k$, then for any $x \in \mathcal{D}_n^{(k)}$ we have $\#\Sigma_x^{(k,n)} = k^{n-k}k!$.

Proof: For a fixed value of k we use induction with respect to n.

1. In the basic case, since $n = k$, $\mathcal{D}_n^{(k)} = \mathbb{R}_+^n$, then $\#\Sigma_x^{(k,n)} = k!$.
2. In the inductive step (where $n > k$), assuming that for any $x \in \mathcal{D}_n^{(k)}$ we have $\#\Sigma_x^{(k,n)} = k^{n-k}k!$, we show that for any $x' \in \mathcal{D}_{n+1}^{(k)}$ we obtain $\#\Sigma_{x'}^{(k,n+1)} = k^{n-k+1}k!$.

   For a given $x' = (x_1', \dots, x_n', x_{n+1}') \in \mathcal{D}_{n+1}^{(k)}$, it follows that $x \coloneqq (x_1', \dots, x_n') = \pi_{(1,\dots,n)}(x) \in \mathcal{D}_n^{(k)}$. Let $\sigma \in \Sigma_x^{(k,n)}$ and $(y_1, \dots, y_n) \coloneqq \left(x'_{\sigma(1)}, \dots, x'_{\sigma(n)}\right)$. By definition of $\mathcal{D}_n^{(k)}$, we get $\forall_{j \in \{k+1,\dots,n\}}\, y_j \ > \ y_{j-k:j-1}$. Next, by definition of $\mathcal{D}_{n+1}^{(k)}$, we have $x'_{n+1} \ > \ y_{j-k+1:n}$.

   In such circumstances, there are exactly k possibilities to expand the permutation $\sigma \in \mathcal{D}_n^{(k)}$ to such a permutation $\sigma' \in \mathcal{D}_{n+1}^{(k)}$ that $\sigma'|_{\{1,\dots,n\}} = \sigma$:

   $$\begin{aligned} &\bar{\sigma}_1'(x') = (y_1, \dots, y_{n-k+1}, x'_{n+1}, y_{n-k}, \dots, y_n), \\ &\bar{\sigma}_2'(x') = (y_1, \dots, y_{n-k+2}, x'_{n+1}, y_{n-k+1}, \dots, y_n), \\ &\quad \vdots \\ &\bar{\sigma}_k'(x') = (y_1, \dots, y_n, x'_{n+1}), \end{aligned}$$

   which encompasses exactly all the possible permutations in $\mathcal{D}_{n+1}^{(k)}$.

Steps 1. and 2. justify the proof.

□

Since the number $\#\Sigma_x^{(k,n)} = k^{n-k}k!$ does not depend on the choice of $x \in \mathcal{D}_n^{(k)}$, we denote it hereinafter as $N(k,n)$.

Theorem 2.
The joint unconditional pdf of vector $(X_1, X_2, \dots, X_n)$ has a form:

$$f_n(x_1, x_2, \dots, x_n) = \frac{f(x_1)\cdot f(x_2)\cdot\ldots\cdot f(x_n)}{G(x_{1:k})\cdot G(x_{2:k+1})\cdot\ldots\cdot G(x_{n-k:n-1})} \cdot \mathbb{I}_{\mathcal{D}_n^{(k)}}(x_1, \dots, x_n),$$

where $G(t) = 1 - F(t)$.

Remark 3.
Definition of $\mathcal{D}_n^{(k)}$ yields that if $(x_1, x_2, \dots, x_n) \in \mathcal{D}_n^{(k)}$, then $x_{p:r} = x_{p:s}$ for any ordered entries $x_{p:r}$, $x_{p:s}$ such that $k \le p + k - 1 \le r \le s \le n$. For that reason the pdf in Theorem 2 may be rewritten as:

$$\begin{aligned} f_n(x_1, x_2, \dots, x_n) &= \frac{f(x_1)\cdot f(x_2)\cdot\ldots\cdot f(x_n)}{G(x_{1:n-1})\cdot G(x_{2:n-1})\cdot\ldots\cdot G(x_{n-k:n-1})} \cdot \mathbb{I}_{\mathcal{D}_n^{(k)}}(x_1, \dots, x_n) = \\ &= \frac{f(x_1)\cdot f(x_2)\cdot\ldots\cdot f(x_n)}{G(x_{1:n})\cdot G(x_{2:n})\cdot\ldots\cdot G(x_{n-k:n})} \cdot \mathbb{I}_{\mathcal{D}_n^{(k)}}(x_1, \dots, x_n). \end{aligned}$$

Proof of Theorem 2: On the basis of the proposed definition of k-th-price-auction model it emerges that if number of bids n is smaller or equal to k, then the bids are independent, which leads to joint pdf given by a formula:

$$f_n(x_1, x_2, \dots, x_n) = f(x_1) \cdot f(x_2) \cdot \ldots \cdot f(x_n).$$

So as a special case we have:

$$f_k(x_1, x_2, \dots, x_k) = f(x_1) \cdot f(x_2) \cdot \ldots \cdot f(x_k).$$

Otherwise $(n > k)$, $n - k$ last bids depend on the previous ones. In this case, we derive recursively joint pdf of $(X_1, X_2, \dots, X_n)$ with the use of conditional pdf's:

$$\begin{aligned} f_n(x_1, \dots, x_n) &= f_{n-1}(x_1, \dots, x_{n-1}) \cdot f_{X_n|X_1=x_1,\dots,X_{n-1}=x_{n-1}}(x_n|x_1, \dots, x_{n-1}) = \\ &= \cdots = \qquad\qquad\qquad\qquad (1) \\ &= f_{X_n|X_1=x_1,\dots,X_{n-1}=x_{n-1}}(x_n|x_1, \dots, x_{n-1}) \cdot \ldots \cdot \\ &\quad \cdot f_{X_{k+1}|X_1=x_1,\dots,X_k=x_k}(x_{k+1}|x_1, \dots, x_k) \cdot f(x_k) \cdot \ldots \cdot f(x_1). \end{aligned}$$

Now, since the conditional distributions are modeled by a given family of exceedance distributions, then the direct substitution in (1) completes the proof.

□

The obtained joint distribution formula leads to unconditional marginal distributions of all variables $X_{k+1}, X_{k+2}, X_{k+3}, \dots$, which is stated in:

Theorem 4.
For fixed k and n, $n > k$, the unconditional marginal pdf of X_n has a form:

$$f_{X_n}(t) = \left(\frac{k}{k-1}\right)^{n-k} f(t)\left(1 - G^{k-1}(t) \sum_{i=0}^{n-k-1} \frac{(1-k)^i}{i!} \ln^i G(t)\right).$$

Before we proceed with the proof of Theorem 4, we consider auxiliary integrals that are recursively defined for a fixed $t > 0$ as follows:

$$c_1(x) := \int_x^t \frac{f(s)}{G(s)} G^{k-1}(s)ds, \quad c_{m+1}(x) := \int_x^t \frac{f(s)}{G(s)} c_m(s)ds$$

where $0 < x < t, m \in \mathbb{N}_+$.

Lemma 5.
The auxiliary integrals are given by:

$$c_m(x) = \frac{1}{(k-1)^m} G^{k-1}(x) + \frac{1}{k-1} G^{k-1}(t) \sum_{j=0}^{m-1} \frac{1}{j!} a_{m-1-j}(t) \ln^j G(x), \tag{2}$$

where

$$a_j(t) = -\sum_{i=0}^{j} \frac{(-1)^i}{i!(k-1)^{j-i}} \ln^i G(t).$$

Proof: We use induction with respect to m.

1. In the basic case, the calculation yields that:
$$c_1(x) = \int_x^t f(s) G^{k-2}(s)ds = \frac{1}{k-1}\left(G^{k-1}(x) - G^{k-1}(t)\right).$$
In this case $a_0(t) = 1$.
2. In the inductive step (where $m > 1$), assuming that (2) holds, and knowing that:
$$\int_x^t \frac{f(s)}{G(s)} \ln^j G(s)\, ds = \frac{1}{j+1}\left(\ln^{j+1} G(x) - \ln^{j+1} G(t)\right),$$
for $j \in \mathbb{N}$, we calculate:
$$\begin{aligned}\int_x^t \frac{f(s)}{G(s)} c_m(s)ds &= \frac{1}{(k-1)^{m+1}}\left(G^{k-1}(x) - G^{k-1}(t)\right)\\ &\quad + \frac{1}{k-1} G^{k-1}(t) \sum_{j=0}^{m-1} \frac{1}{j!} a_{m-1-j}(t) \frac{1}{j+1}\left(\ln^{j+1} G(x) - \ln^{j+1} G(t)\right)\\ &= \frac{1}{(k-1)^{m+1}} G^{k-1}(x) + \frac{1}{(k-1)^{m+1}} a_0(t) G^{k-1}(t)\\ &\quad + \frac{1}{k-1} G^{k-1}(t) \sum_{j=1}^{m} \frac{1}{j!} a_{m-j}(t)\left(\ln^j G(x) - \ln^j G(t)\right)\\ &= c_{m+1}(x) - \frac{1}{k-1} G^{k-1}(t) a_m(t) - \frac{1}{k-1} G^{k-1}(t) \sum_{j=1}^{m} \frac{1}{j!} a_{m-j}(t) \ln^j G(t)\\ &\quad + \frac{1}{(k-1)^{m+1}} a_0(t) G^{k-1}(t)\\ &= c_{m+1}(x) - \frac{1}{k-1} G^{k-1}(t) \sum_{j=0}^{m} \frac{1}{j!} a_{m-j}(t) \ln^j G(t) + \frac{1}{(k-1)^{m+1}} G^{k-1}(t).\end{aligned}$$
Now let $S = \sum_{j=0}^{m} \frac{1}{j!} a_{m-j}(t) \ln^j G(t)$. Then $S = -(k-1)^{-m}$, because:
$$\begin{aligned}S &= -\sum_{j=0}^{m} \frac{1}{j!} \sum_{i=0}^{m-j} \frac{(-1)^i}{i!(k-1)^{m-j-i}} \ln^i G(t) \ln^j G(t)\\ &= \frac{-1}{(k-1)^m} \sum_{j=0}^{m} \sum_{i=0}^{m-j} \binom{j+i}{j} \frac{(k-1)^{j+i}}{(j+i)!} \ln^j G(t) \left(-\ln G(t)\right)^i\\ &= \frac{-1}{(k-1)^m} \sum_{l=0}^{m} \sum_{i=0}^{l} \binom{l}{j} \frac{(k-1)^l}{l!} \ln^j G(t) \left(-\ln G(t)\right)^{l-j}\end{aligned}$$

$$= \frac{-1}{(k-1)^m} \sum_{l=0}^{m} \frac{(k-1)^l}{l!} (\ln G(t) - \ln G(t))^l$$
$$= \frac{-1}{(k-1)^m},$$

where we substitute $l = j + i$, and assume that $0^0 = 1$.

□

Proof of Theorem 4: For a given k and n, $n > k$, we calculate:

$$f_{X_n}(t) = \int_{\{(x_1,x_2,\ldots,x_{n-1})\in\mathbb{R}^{n-1}\}} f_n(x_1, x_2, \ldots, x_{n-1}, t) d(x_1, x_2, \ldots, x_{n-1})$$
$$= \frac{N(k,n)}{k!} \int_{\substack{\{(x_1,x_2,\ldots,x_{n-1})\in\mathbb{R}^{n-1}:\\ x_1<x_2<\cdots<x_{n-k},\\ x_{n-k+1},\ldots,x_{n-1},t>x_{n-k}\}}} \frac{f(x_1)f(x_2)\ldots f(x_n)}{G(x_{1:k})G(x_{2:k+1})\ldots G(x_{n-k:n-1})} d(x_1, x_2, \ldots, x_{n-1})$$
$$= k^{n-k} f(t) \int_0^t \frac{f(x_1)}{G(x_1)} \int_{x_1}^t \frac{f(x_2)}{G(x_2)} \ldots \int_{x_{n-k-1}}^t \frac{f(x_{n-k})}{G(x_{n-k})}$$
$$\int_{x_{n-k}}^{+\infty} f(x_{n-k+1}) \ldots \int_{x_{n-k}}^{+\infty} f(x_{n-1}) dx_{n-1} dx_{n-2} \ldots dx_2 dx_1$$

and notice that $f_{X_n}(t) = k^{n-k} f(t) c_{n-k}(0)$ which completes the proof.

□

FINAL REMARKS

This paper proposes a stochastic model of the course of actual bid values in a dynamic internet k-th price auction. The model takes into account the existence of stochastic dependence and different probability distributions describing subsequent bids. Despite conducting an extensive search of the literature on the subject, we were not able to find studies that would explicitly model actual bid values appearing in the course of a dynamic action in a way similar to our proposal. In our opinion, the determination of unconditional joint probability distributions of bid values, and especially marginal probability distributions thereof, is an interesting mathematical problem – even if considered separately from the auction framework.

We observe that in the case of $k = 1$, the probability distributions of subsequent bids in our model are identical to the distributions of maxima of independent and identically distributed random variables. This is so provided that the latter variables are given by the same cumulative distribution function F as the sequence of bids in the model. The mentioned maxima of independent and identically distributed random variables are the records in the sense of [Chandler 1952], and also they are the first record values ($k = 1$) in the sense of [Dziubdziela, Kopociński 1976]. However, we notice that for $k \geq 2$ such an equality of marginal bid distribution in k-th price auction and of k-th record values no longer holds.

Though the presented model describes bid amounts only, it can be combined with models describing two other aspects of dynamic auctions, i.e., time arrivals and the number of bids. For example, an interesting model of bid times has been proposed by [Shmueli et al. 2008]. Simultaneously, the number of bids can be naturally modeled using the Poisson process, as indicated in [de Haan et al. 2009], [Hickman et al. 2012], [Saeedi, Hopenhayn 2015], [Shmueli et al. 2008], [Wang et al. 2008].

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QUANTITATIVE METHODS IN ECONOMICS
Volume XXV, No. 1, 2024, pp. 19 – 30

DOES GENDER DIVERSITY IN MANAGEMENT MATTER? THE CASE BASED ON POLISH COMPANIES FROM RANKING PUBLISHED BY PULS BIZNESU

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Abstract: This article examines the relationship between gender diversity in statutory bodies and woman's odds of being a business leader in Poland. Under the empirical framework we assume that gender diversity in the boardroom can improve the quality of work and lead to better decision-making, and thus affect financial outcomes and increase the probability that a female director will be rewarded by an external institution. The sample consists of 82 firms from the "Business Woman" plebiscite organized by Puls Biznesu in 2012-2021. Our dataset contains information on the members of statutory bodies, i.e.: gender, age, tenure, and function, as well as financial and organizational characteristics of the companies. The variables were collected manually from the InfoVeriti database and EMIS database. We measure gender diversity on boards using Herfindahl – Hirschman Index. Drawing on critical mass theory by measuring gender diversity as levels of female representation in the boardroom, this study finds a positive and significant relationship between gender diversity and the laureate's ranking position.

Keywords: gender diversity, leadership, firm performance, logistic regression

JEL Codes: C50, D22, M14

https://doi.org/10.22630/MIBE.2024.25.1.3

INTRODUCTION

In the history of the Bank of Sweden's Alfred Nobel Prize in Economic Sciences, only three times have women been winners. Elinor Ostrom received the prize for her research on the economic aspects of common property management by user associations, and Esther Duflo was recognized for conducting experimental research to increase understanding and reduce poverty [Nobel Prize, 2023][1]. The 2023 laureate was Claudia Goldin, who is conducting research about women in the labor market and the causes of the wage gap between men and women [NY Times 2023].

Gender pays inequality exists in many countries and does not only affect lower and middle level employees, but also senior management. The size of the wage gap between men and women depends on age, length of service and education profile [Goldin and Polachek 1987], maternity choices [Budig and England 2001], but also on sectoral solutions to employment flexibility and the availability of remote work [Goldin 2014]. One method of indirectly influencing the wage gap has been legislative initiatives in Norway, France and Spain regarding gender parity in statutory bodies and important management positions[2]. As Cohen and Huffman, [2007] have shown, there is a positive relationship between women's participation in decision-making and the reduction of the pay gap, but at the same time, analyses by McKinsey&Company, Boston Consulting Group and Forbes confirm that the inclusion and diversity policies of organizations contribute to their business success.

Our study examines whether a positive relationship between women's involvement in management and business financial performance also increases their chances of personal career success. Considering previous research results, we assume that there is a positive relationship between the diversity of statutory bodies and the business success of women who own or manage businesses. The study was conducted on a sample of 100 winners of the "Business Women" ranking in 2012-2021. The plebiscite is an initiative of the Polish daily newspaper Puls Biznesu, which aims to promote the growing role of women in the management of businesses in Poland and to publicize their impact on the economy. Spreading the positive impact of women's participation in economic life. Based on the estimated logit and probit models, we found that the diversity of statutory bodies is a factor that increases

[1] Both award winners conducted their research in teams. Esther Duflo received the award jointly with Abhijt Banerjeem and Michael Kremer. Elinor Ostrom conducted research with Oliver Williamson.

[2] Norway was the first country to propose and later introduce gender balance regulations for board positions (Odelstingsproposisjon 97, 2002-2003). Nevertheless, within just a few years, France (Copé-Zimmermann Law) and Spain (Equality Act 2007) followed similar paths and introduced some forms of quota regulations (see [Seierstad et al. 2017]). The European Gender Equality Strategy 2020–2025, includes the promotion of gender equality and women's empowerment in firms as one of its key areas [European Commission 2020].

women's chances of professional success, but at the same time we note the limitations of our study related to the selection of enterprises and the methodology adopted.

We have divided our study into five sections. In Section LITERATURE REVIEW, we develop hypotheses that can be tested and discuss the related literature. Section SAMPLE describes our data and explains how we estimated the main variables used in our empirical analysis. In Section EMPIRICAL FINDINGS, we present the empirical results and explore how gender diversity in the boardroom contributes to women's success in leadership positions. We conclude in Section CONCLUSION with a summary of our findings and opportunities for future research.

LITERATURE REVIEW

Different interpretations of the term 'diversity' can be found in the literature [Khatib et al. 2020]. Authors of foreign reports and research studies present diversity from the perspective of age, gender, education, racial and ethnic affiliation, or skin color. In recent years, interest in gender and racial diversity on company boards has intensified. According to this direction, it is not only important because it can affect the company's performance, but also because it is essential for growing and modern companies. The application of diversity in business affects the company's image internally and externally. Magnanelli and Pirolo [2021, 37-38] pointed out that making decisions that contribute to the promotion of diversity in statutory bodies shows that the company is being run responsibly and has a positive impact on the company's image in the eyes of shareholders. Companies whose statutory bodies consist of a multicultural society are better perceived by investors, shareholders, customers, and other stakeholders.

A key element that builds relationships between investors, management, stakeholders in a company is effective corporate governance. The corporate governance structure allows the organisation to achieve its objectives, take appropriate action and monitor performance [OECD 2004]. Both external factors, such as regulations and policies, and internal factors including the relationship between owners, managers and the board of directors affect corporate performance [Jeżak 2010].

The solutions presented are applicable to the domestic and international market, enabling companies to effectively manage their statutory body structure and business processes, which could contribute to success in the economic environment.

The Diversity Matters report [Hunt et al. 2015] analyses organisational diversity in relation to three aspects: gender, ethnicity, and race. Researchers from McKinsey&Company argue that corporations that focus on developing diversity are likely to attract value-adds to their teams in the form of top industry professionals and can improve the image of the organisation in the eyes of the customer. Stahl [2021] suggests that customers feel more attachment and are more likely to buy from companies with ethnically and culturally diverse employees. Companies with diverse teams tend to perform better financially because they can take advantage of broader growth opportunities and effectively reach new markets, leading to increased profits [Stahl 2021]. Lorenzo et al. [2018] reported that companies with more diverse management teams have 19% higher revenues through innovation. In contrast, Ely and Thomas [2020] contend that diversity does not lead to financial improvement, but there are indirect links to financial performance. Diversity enhances work quality, decision-making, team satisfaction, and equality [Ely and Thomas 2020]. Companies with more diversity are 70% more likely to reach new markets and are better at decision-making by as much as 87% than homogeneous companies [Stahl 2021].

The creators of the above initiatives conclude that diversity in the workplace matters. It gives employees the chance to experience a different culture and different experiences. This can result in good relationships between employees and managers, the enrichment of new professionals in the company and better results, not only financially.

Although board diversity is a growing and significant body of literature, there is still much focus on gender diversity only [Khatib et al. 2020]. However, there are differing views on the role of female directors and on the benefits of increased gender diversity on boards for the performance of companies.

Adams and Ferreira [2009] present that companies with at least one woman director tended to be larger, have more business segments, show better ROA performance, and lower volatility. On the other side are results indicating no significant correlation between company performance and gender diversity reported by Fernandez-Temprano and Tejerina-Gaite [2020]. Singh and Dwesar [2022] reviewed the link between gender diversity on boards of directors on corporate performance, and they results are mixed. Some studies showing either positive or negative effect of women on board on firm performance and its risk, whereas others suggesting no relation between the two.

It should be noted that there is a limited number of studies evaluating the influence of boardroom gender diversity on the financial performance of polish companies. For example, Kompa et al. [2023] findings suggest that the presence of women in boardrooms has a limited impact on the financial performance of companies listed on the Warsaw Stock Exchange in Poland. Similarly, Maj [2017] reported that the gender composition of the management and supervisory boards have no influence on the firm financial performance in companies listed on the Warsaw Stock Exchange.

The above discussion on gender diversity and company's performance leads us to posit that gender-diverse boards are linked to business success. However, there is a lack of empirical evidence that gender diversity of boards, particularly the proportion of female board members, is associated with their individual success. Considering previous research results, we assume that there is a positive relationship between the diversity of statutory bodies and the business success of women who own or manage businesses:

Hypothesis: Increased gender diversity on boards is positively and significantly associated with the likelihood of individual success for female board members.

SAMPLE

To test the association between gender diversity of boards[3] and their chances of individual success, we used sample of Polish firms from the "Business Woman" plebiscite organized by Puls Biznesu[4] in 2012-2021. Our dataset includes information on 100 firms, with the top 10 firms selected for each year. Specifically, we collected data on the members of statutory bodies, including their gender, age, tenure, and function, as well as the financial and organizational characteristics of these firms. The data was collected manually using the InfoVeriti and EMIS databases. However, for some companies, we could not obtain financial and corporate governance information. As a result, the sample used in our regression analysis consists of 82 firms.

Panel A of Table 2 presents descriptive statistics for the statutory bodies. Panel B describes the characteristics of the companies, while Panel C provides details on the characteristics of the winners.

The supervisory board typically consists of 1 to 2 members on average, with a maximum of 22 members and a minimum of 0 members. With a maximum of 24 members and a minimum of 1 member, the Board of Directors usually consists of an average of 3 to 4 members. The descriptive data suggest a significant heterogeneity of the sample. The HHI index is 0.42. This means that there is a high diversity of members of statutory bodies in the sample. The average age of the winners is 56. They range from a minimum of 30 to a maximum of 79. The average length of service on a statutory body is 7.85 years, with tenures ranging from 1 year as the shortest to 20 years as the longest.

The value of assets usually determines the size of a company. The average value of a company's assets is just under 0.7 million, with a maximum of 23 billion and a minimum of 19 thousand. Meanwhile, a liquidity ratio serves as an indicator of a company's ability to pay its debts, with an average ratio of 0.53.

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[3] Board gender diversity is calculated with a Herfindahl-Hirschman Index [Rhoades 1993] using the formula $HHI = m^2 + w^2$, where m is the percentage of men on statutory bodies (management and supervisory boards) and w is the percentage of women on statutory bodies. For example, if one firm had 4 female board members ($HHI_{first} = 1.00$) and another firm had 2 female and 2 male executives ($HHI_{second} = 0.50$), using the HHI formula allowed us to credit the second firm as having a more diverse statutory body. An HHI of 1.0 indicates a team where everyone has the gender. Increases in the HHI indicate a decrease in diversity. Decreases in the index indicate an increase in diversity [Hunt et al. 2015].

[4] "Business Woman" plebiscite is a cyclical ranking promoting entrepreneurship of women in the Polish business world.

Table 1. Descriptive statistics

Variable	N	min	mean	sd	median	max
Panel A: Statutory bodies						
HHI	100	0.00	0.41	0.39	0.25	1.00
Management board members	99	1.00	2.44	1.73	2.00	11.00
Supervisory board (binary)	99	0.00	0.28	0.45	0.00	1.00
Number of women on statutory bodies	99	0.00	0.79	0.57	0.67	4.00
Panel B: Firm characteristics						
Assets (ln)	94	11.17	18.55	1.87	18.43	23.90
Assets growth (in %)	88	0.00	1.54	2.96	1.07	26.34
Liquidity ratio	99	0.00	0.73	1.94	0.19	15.24
Panel C: Laureate characteristics						
Age (in years)	95	30.00	55.99	10.03	56.00	79.00
Tenure (in years)	99	1.00	7.85	4.57	7.00	20.00

Source: own elaboration

The dependent variable of the study is the inverse of the ranking in the Puls Biznesu "Business Woman" plebiscite. Rank is therefore a qualitative statistical variable that expresses the quality of a characteristic. For this type of data, standard models can be used to explain the probability. Specifically, we estimate two models: the probit model and the logit model, each with four regressions. In both sets, the rankings were transformed according to the scheme presented in Table 5.

Table 2. Ranking transformation and dependent variable creation rules adopted

Five-level dependent categorical variable		Three-level dependent categorical variable	
Ranking in "Business Woman" plebiscite	models 1, 2, 5, 6	Ranking in "Business Woman" plebiscite	models 3, 4, 7, 8
10-9	1	10-8	1
8-7	2	7-4	2
6-5	3	3-1	3
4-3	4		
2-1	5		

Source: own elaboration

To make the dataset suitable for ordered logistic regression, we use a similar approach like Gruszczyński [2006]. The ranking was converted into a five-point and a three-point categorical variable (a discrete variable ordered from lowest to highest values). In the case of models 1 and 2 and 5 and 6, a five-level categorical variable taking values from 1 to 5 was taken as the explanatory variable (Y). Winners who were ranked in places 1 and 2 were designated as 5, winners from places 3 and 4 were assigned to category 4 and similarly for the remaining places up to category 1. For models 3 and 4 and 7 and 8, a three-stage categorical variable was used in which the laureates ranked 1-3 were labelled as category 3, the laureates ranked 4-7 as category 2 and the laureates ranked 8-10 as category 1. The transformation of ranked places into categorical variables is made possible using ordinal logit modelling and ordinal probit modelling programmed in the statistical package STATA 13 IC.

For the two tables, 8 equations need to be made. These can be first degree equations and second-degree equations. A first degree equation will look like the following:

$$P(winner) = \alpha_0 + \alpha_1 * HHI + \alpha_k * explanatory\ variables_k + \varepsilon$$

If:

P(winner) - probability of winning

$\alpha_{0,} \alpha_1, \dots, \alpha_k$ *- structural parameters*

ε *– random variable*

In contrast, the equation of degree two is of the form:

$$P(winner) = \alpha_0 + \alpha_1 * HHI + \alpha_2 * HHI^2 + \alpha_k * explanatory\ variables_k + \varepsilon$$

If:

P(winner) - probability of winning

$\alpha_{0,} \alpha_1, \dots, \alpha_k$ *- structural parameters*

ε *– random variable*

EMPIRICAL FINDINGS

Tables 3 and 4 below show what influences the placement of a particular winner in each ranking.

Table 3. Results of logit regressions of the probability of success in the "Business Woman" plebiscite

	(1)	(2)	(3)	(4)
HHI	**1.381***	-5.982	**1.721***	-3.979
	(0.814)	(4.072)	**(0.919)**	(4.783)
HHI^2		**6.093***		4.632
		(3.317)		(3.832)
Age	**-0.0532****	**-0.0523****	**-0.0515***	**-0.0494***
	(0.0247)	**(0.0248)**	**(0.0268)**	**(0.0268)**
Tenure	-0.0418	-0.0516	-0.0344	-0.0428
	(0.0586)	(0.0592)	(0.0636)	(0.0640)
Board members	0.0757	0.0462	0.0263	-0.00165
	(0.195)	(0.200)	(0.203)	(0.206)

	(1)	(2)	(3)	(4)
Supervisory Board	-0.617	-1.074	-0.397	-0.796
	(0.776)	(0.817)	(0.889)	(0.952)
Number of women on statutory bodies	-0.0879	0.363	-0.815	-0.428
	(0.602)	(0.629)	(0.776)	(0.805)
Legal form - limited liability company	0.545	0.768	0.345	0.493
	(0.953)	(0.952)	(0.990)	(0.999)
Legal form - cooperative	18.72	18.18	17.61	17.39
	(1117.6)	(1017.1)	(870.8)	(937.3)
Assets	**0.441*****	**0.450*****	**0.430*****	**0.431*****
	(0.149)	**(0.150)**	**(0.158)**	**(0.159)**
Assets growth	-0.0245	-0.0343	0.0184	0.0109
	(0.0579)	(0.0585)	(0.0629)	(0.0635)
Liquidity ratio	**0.217****	**0.201****	0.180	0.165
	(0.100)	**(0.101)**	(0.110)	(0.111)
Pseudo R^2	0.110	0.124	0.154	0.162
Chi^2	29.07	32.52	27.40	28.88
N	82	82	82	82

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Source: own elaboration

The analysis of the estimation results shows that both in the logit function, the HHI, HHI2, the age of the laureate, the natural logarithm of the balance sheet total and liquidity are significant in the model. According to these results, the most significant variables that influence the inclusion of the laureate in the respective ranking include the size of the company. The age of the laureate also influences where she is ranked. The younger the awardee is, the more likely it is to be ranked higher. Companies that had higher cash liquidity were more likely to appear higher in the ranking. The HHI indicates the diversity in the statutory bodies, from the above results if the statutory bodies in a company had greater diversity, then the company was more likely to appear in the ranking at higher positions.

Table 4. Results of probit regressions of the probability of success in the "Business Woman" plebiscite

	(5)	(6)	(7)	(8)
HHI	**0.802***	-3.938	**1.023***	-2.524
	(0.470)	(2.437)	**(0.533)**	(2.741)
HHI^2		**3.923****		2.899
		(1.979)		(2.203)

	(5)	(6)	(7)	(8)
Age	**-0.0295****	**-0.0301****	**-0.0296***	**-0.0295***
	(0.0147)	**(0.0148)**	**(0.0158)**	**(0.0159)**
Tenure	-0.0188	-0.0252	-0.0201	-0.0235
	(0.0349)	(0.0352)	(0.0372)	(0.0374)
Board members	0.0501	0.0314	0.0181	0.00547
	(0.112)	(0.113)	(0.121)	(0.121)
Supervisory Board	-0.376	-0.675	-0.249	-0.487
	(0.475)	(0.501)	(0.533)	(0.561)
Number of women on statutory bodies	-0.0761	0.200	-0.512	-0.269
	(0.346)	(0.376)	(0.459)	(0.485)
Legal form - limited liability company	0.197	0.407	0.166	0.311
	(0.555)	(0.568)	(0.602)	(0.615)
Legal form - cooperative	7.284	7.159	7.052	6.908
	(193.5)	(188.7)	(203.4)	(199.6)
Assets	**0.258*****	**0.271*****	**0.261*****	**0.267*****
	(0.0851)	**(0.0861)**	**(0.0916)**	**(0.0924)**
Assets growth	-0.0150	-0.0220	0.0111	0.00619
	(0.0382)	(0.0384)	(0.0401)	(0.0403)
Liquidity ratio	**0.122***	**0.112***	0.108	0.0996
	(0.0641)	**(0.0645)**	(0.0680)	(0.0685)
Pseudo R^2	0.109	0.124	0.156	0.166
Chi^2	28.66	32.61	27.90	29.64
N	82	82	82	82

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Source: own elaboration

HHI, HHI^2, firm size in terms of total assets and liquidity were found to be significant in the models according to the table above. The interpretation of the results of the probit regression will be like that of the logit regression.

CONCLUSION

Through econometric analysis, the empirical study found a significant impact of the diversity of statutory bodies on the laureates' positions in the rankings. Age plays a key role, with younger awardees more likely to achieve higher rankings. Company size also matters, with larger companies more likely to achieve higher positions in the Puls Biznesu plebiscite. Liquidity is another determinant, as candidates from companies with higher liquidity tend to achieve higher rankings.

The analysis suggests that diversity among statutory bodies can enhance the success of women in business. However, it is important to note the study's limitations, including the lack of a control group, insufficient details on the criteria for selecting female winners of the Puls Biznesu, incomplete financial data and a narrow focus on gender diversity. Future studies could examine longer time series, alternative profitability ratios and dynamics between owners and statutory bodies.

The impact of board diversity on financial performance is a complex issue that requires further research. An increasing number of organisations are beginning to recognise the benefits of diversity and are taking steps to create more diverse teams. This can lead to positive changes in the organisation and in financial performance. While the analysis provides some basis for concluding that board diversity affects financial performance, one should be cautious and aware of the limitations of the study.

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QUANTITATIVE METHODS IN ECONOMICS
Volume XXV, No. 1, 2024, pp. 31 – 42

INCOME LEVEL AND POPULATION AGING IMPACT ON RETIREMENT SAVINGS IN OECD COUNTRIES

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Abstract: Increasing old-dependency ratio has essential impact on pension systems which must be reformed and adjusted to actual demographic situation. Applying linear regression, we look for relationships between retirement savings and levels of incomes and old-age dependency ratios in 36 selected OECD countries. Our findings show that the population aging affects pension assets accumulation differently in developed and developing countries although pension assets are increasing in both groups of countries. Concurrently, the regression models let us divide OECD countries into three groups - Countries where the old-age dependency ratio affects per capita investment positively and statistically significant (26 countries), insignificantly (5 countries) and negatively and significant (5 countries).

Keywords: population aging, retirement savings, OECD countries

JEL classification: G11, G23, G51

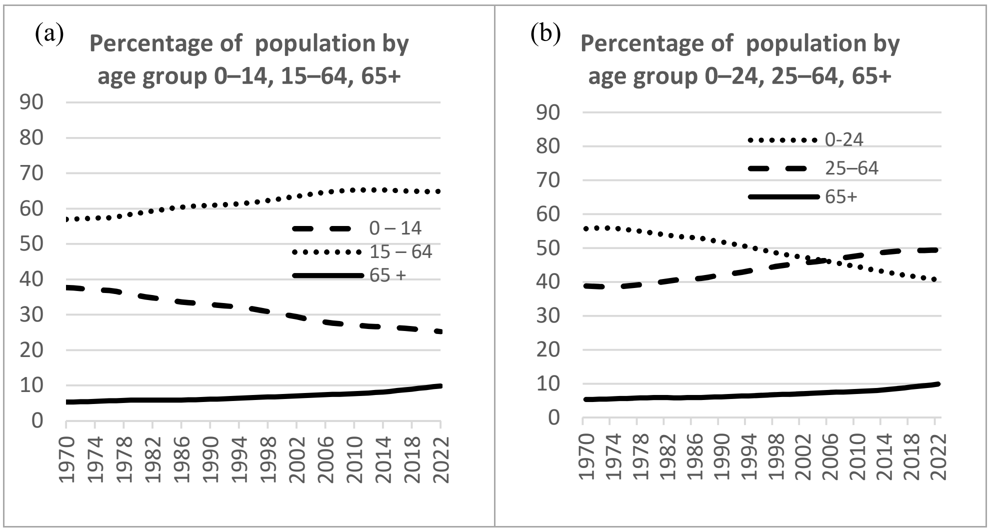
INTRODUCTION

Longer life expectancy and lowering the fertility rate causes ageing of societies all over the world. Therefore, the global population is facing the inverted demographic pyramid. According to the United Nations (2017), the world total dependency ratio of the young, non-working (age 0-14) and elderly (age 65+) to the working age population (age 15-64) dropped to 52.5% in 2015 from 75.2% in 1965. However, there are essential differences in the structure and extent of population aging across countries, particularly between developed and developing countries. The more developed regions show a rising dependency ratios trend. Whereas the less developed regions also show similar trend which is less noticeable due to their higher

https://doi.org/10.22630/MIBE.2024.25.1.4

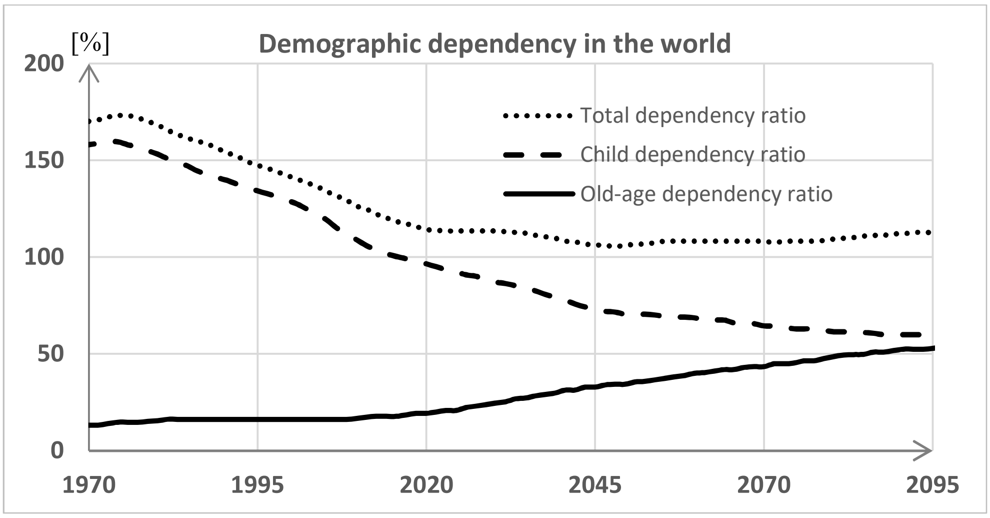
younger population and higher fertility rate. Even among industrialized developed countries, U.S. and Canada have younger population aging than western European countries like Germany, France and Italy.

Figure 1. Percentage of total population by broad age group
(a) working-age population 15–64, (b) working-age population 25–64



Source: prepared in-house based on UN Population Division (2022)

Figure 2. Evolution of the demographic dependency ratio in the world in 1970-2095



Source: quoted from The Social Observatory of the "la Caixa" Foundation [Lee, Mason 2021] as in-house compilation based on Mason et al. [2017]

The old-age dependency ratio is usually defined as the ratio of the number of elderly (65 and older) to the number of adults aged (15 - 64) in a given population (see Fig. 1a). In 2015, the old-age dependency ratio for the developed regions was 26.7% compared to the less developed regions of 9.7%. Using the United Nations probabilistic projections of the aging population to the working age population, the old-age dependency ratio is expected to rise rapidly, especially for the more developed regions, from 2015-2100. The old-age dependency ratio in the more developed regions is expected to rise to 52.97% in 2100 while the ratio for the less developed regions is expected to rise to 34.49% over the same period. The old-age dependency ratio is fundamentally different if the lower working age limit is taken as the upper educational age limit [GUS Methodical Notes] (see Fig. 1b). And in this case, demographic long-term projections indicate a rapid aging of the population and an increase in the global old-age dependency ratio above 50% before 2090 (cf. Fig.2).

Population aging caused unsustainability of the traditional PAYG pension system and resulted in reforms in many (especially developed) countries. The majority of pension systems are hybrid ones with a great variety of pension plans and retirement vehicles. Therefore, it is difficult to compare total retirement saving in different countries since pension assets may be hidden in taxes, treated as budget revenue or hidden government debt, or reported as savings in funded and private pension systems.

The literature suggests that the population aging crisis and its impact on saving behaviors, pension funds accumulation and capital markets and returns is a dynamic process as population demographics keep changing and that they are not the same for developed and developing counties. In our study, we investigate changes in retirement savings (accumulated in funded and private pension systems) in OECD countries in the years 2006-2017 with available, consistent data. We look for relationships between pension savings accumulation behavior and levels of incomes and old-age dependency ratios in the selected OECD considered countries, by applying linear regression models.

This study adds to the literature by linking and comparing the income level and the population aging process and the accumulation of financial assets behavior through institutional investors between developed and developing countries. Developed and developing countries and regions differ by many factors but two of them stand out, the level of income and life longevity. In our research we investigate the influence of both these factors on pension assets. The results would shed light on the ongoing pension system reforms and policies needed to be implemented differently to avert a global population aging crisis.

LITERATURE REVIEW

A consequence of the aging population is the fear of the elderly welfare, the financial sustainability and strains on the fiscal budget. Population aging crisis also rises burden of the working population and future generations, and the drag on economic productivity and growth. These concerns have been well documented in the literature [Börsch-Supan, Ludwig 2009; Bloom et al. 2011] and cause that many countries reformed their pension systems making them the multi-pilar ones. Although countries that focus more on long-term financial sustainability reforms to contain rising government pension spending may increase old-age poverty instead [Hindrichs 2021]. Dalen and Henkens [2023], based on survey among Dutch pension fund participants, show that the level of funding is positively related to the degree of trust of contributors and is strongly positive for older (i.e., 55+) retired participants. Postkute et al. [2022] argue that despite different initiation and inconsistent pension reform paths and objectives undertaken by the Central and Eastern European countries the outcomes of the reforms are similar but still inadequate.

Demographic trends and their savings behavior have a significant impact on capital markets [Liu, Spiegel 2011]. A study by Luhrmann [2003] of 141 countries from 1960-1997 shows that demographic changes, in particular the differences in age structure across countries, affect global capital flows and that future demographic changes have a strong determination on current investment decisions by forward-looking households. As the population aging trends continue, financial institutions will become more important as conduits in channeling funds from savings to capital markets as the working population accumulates funds in their life-cycle for retirement (see [Brav et al. 2010; McCahery et al. 2016; Appel et al. 2016; Lewellen, Lewellen 2018]).

DATA AND METHODS

In our study we look at relationships between pension retirement savings per capita measures as pension funds' assets or all retirement savings (y) and levels of incomes and old-age dependency ratios (x) in the selected OECD countries, using

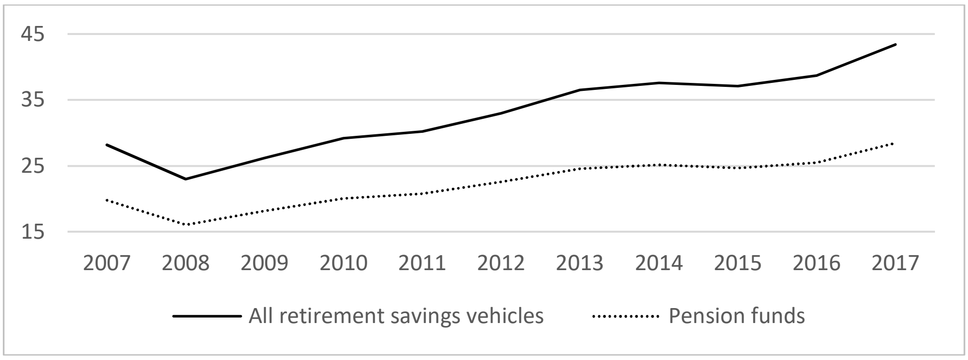
1. simple linear regression models estimated for
   - cross-sectional data:
   $$y_i = \alpha + \beta x_i + \varepsilon_i \tag{1}$$
   - time series:
   $$y_t = \alpha + \beta x_t + \varepsilon_t \tag{2}$$
   $$y_t = \alpha + \beta x_{t-k} + \varepsilon_t \tag{3}$$
2. pooled regression models estimated for panel data:
   $$y_{it} = \alpha + \beta x_{it} + \varepsilon_{it} \tag{4}$$
3. trend function:
   $$y_t = \alpha + \beta t + \varepsilon_t \tag{5}$$

Financial assets in funded and private pension systems for retirement savings are accumulated by workers in countries where incomes are sufficient for personal savings for investment retirement. As a measure of the income levels, we apply gross national income (GNI) per capita converted to international dollars using purchasing power parity (constant 2011 international in thousands of USD). In the model, total pension assets in funded and private pension arrangements are the dependent variable. Models are estimated using four sets of observations i.e., the level of data from years 2006, 2017, 2016-2017, and the growth rate 2017 in comparison to 2006 for OECD countries, except for Greece and Lithuania because of the lack of data for both countries. The missing data for GNI from the World Bank for Iceland, Japan, New Zealand and Switzerland are imputed using real data for the previous year.

The OECD pension data reports on funded and private pension system are based on annual surveys from national authorities. Unless specified otherwise, comparison between different countries is not fully comparable since in some countries like Australia, Canada, Switzerland and USA, the private pension sector dominates, while in Austria, Belgium, France, Germany, Italy, and Japan, the public (state) pension sector dominates. This means that for the latter group of countries only a smaller part of the retirement savings is presented in the OECD reports. There are seven countries (P7) with the largest pension assets which in 2020 totaled US$48,221 billion. This value is equal to 92% of assets from P22 group of countries[1]. Among the P7 countries, USA has the biggest pension market containing nearly 62% of P22 pension assets, followed by Japan and United Kingdom with 6.9% and 6.8%, respectively [Global Pension Assets Study 2021].

The pension market has been growing since 2009, after a small decrease observed in 2008, in comparison to 2007 (Fig. 3). All retirement savings vehicles increased by 54% from 2007-2017, (a 4.4% geometric average growth annually) and pension funds' assets rose by 44% over the same period, or 3.7% annually.

Figure 3. Total amount of assets in retirement savings vehicles (including pension funds) in the OECD countries (USD trillion)

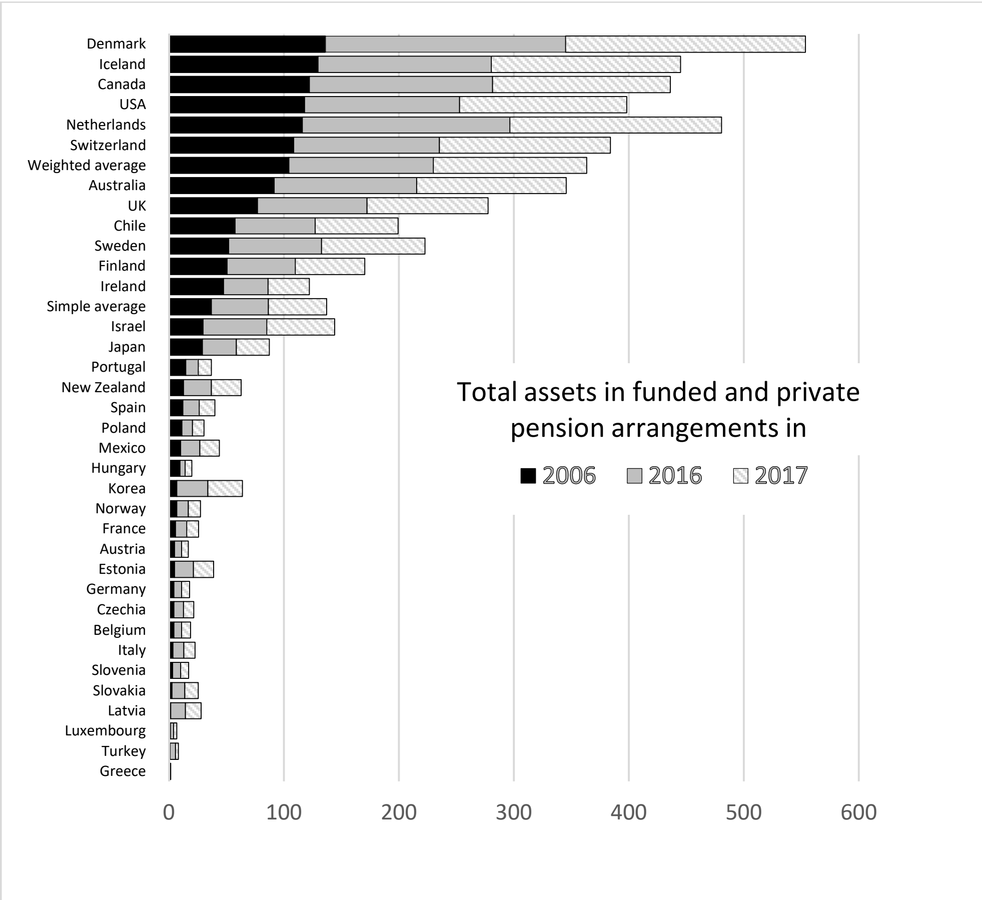


Source: own preparation on the basis of [OECD 2018]

[1] All countries are ranked according to the value of pension assets creating groups of countries P7, P22, etc. For example, P22 contains countries from P7 plus the subsequent 15 countries with the highest pension assets.

In analyzing the changes in total pension assets (Fig. 4), the simple average increase in 2017 is 2% in comparison to the previous year 2016 and 38% in comparison to 2006. The increase of weighted averages in 2017 are 6% and 28%, respectively in comparison to 2016 and 2006. The biggest increase over the eleven years is observed for Latvia and Greece where the pension assets in these countries started small in 2006. However, there are also countries where pension assets decreased: Hungary, Ireland, Portugal, Poland and Japan.

Figure 4. Total assets in funded and private pension arrangements in 2006, 2016 and 2017 (as a % of GDP)



Source: own calculations on the basis of [OECD 2017 and 2018]

RESULTS

The modelling results presented in Table 1 show that the relation between GNI per capita and total pension assets, expressed as percentage of GDP, is insignificant for 2006 but in the models estimated for 2017 and 2016-2017 this relation is significantly positive, meaning that an increase of GNI per capita by one thousand dollar causes the increase of total pension assets by 0.18% of GDP. Regression estimated for rates of changes of total assets and GNI in 2017, in comparison to 2006, shows that changes of earnings per capita significantly influence the pension assets. Pearson coefficient between both variables is not high but positive (from 0.25 in 2006 and 0.34 for 2016 and 2017). The determination coefficients are also small showing that there are other factors influencing pension assets.

Table 1. Parameters of regression models: dependent variable is total assets in funded and private pension arrangements in OECD countries in selected years

Model	Sample	Number of observations	Parameter	Parameter estimate	t-Student	R^2	Pearson coefficient
Dependent variable y_i: total assets in funded and private pension arrangements as % of GDP							
(1)	2006	34	beta	0.7146	1.4846	0.0644	0.2538
			constant	12.0232	0.6324		
(1)	2017	34	beta	0.1761	**2.0597**	0.1171	0.3421
			constant	-14.913	-0.4295		
(4)	2016-2017	68	beta	0.1752	**2.9977**	0.1198	0.3462
			constant	-15.0817	-0.6419		
Dependent variable y_i: growth rate of total assets in funded and private pension arrangements as % of GDP in 2017 in comparison to 2006							
(1)	2017	34	beta	3.141	**1.8091**	0.0928	0.3046
			constant	0.6837	1.9331		

Source: own calculations. Note: bold letters denote values of t-Student statistics bigger than critical value for the significance level α=0.05.

We estimate regression models to explain retirement savings and pension funds' assets by population ageing, measured by population age 65+ as percentage of the total, applying observations from years 2007-2017. All models are estimated using eleven observations[2]. The former dependent variable, pension assets, is explained by the share of the population age 65+ measured not only in the current year, but also in the previous year (explanatory variable is lagged by one year) or five and seven years earlier (explanatory variable is lagged by five and seven years i.e., the first observation of explanatory variable is taken from 2007, 2006, 2002 and 2000, respectively). The results presented in Table 2 show that the determination coefficients are very high and the share of the population age 65+ in the total population significantly influences the amount of retirement savings and pension funds' assets when all OECD countries are considered. This result shows that an

[2] In other words, current relation means that both dependent and independent variables are measured in the same years 2007-2017 whereas for lagged explanatory variables observation periods are: years 2006-2016 for k=1, years 2002-2012 for k=5 and years 2000-2010 for k=7.

increase of the old-age dependency ratio causes an increase of total pension assets and pension fund assets in all OECD countries. It is visible that the aging societies increase their retirement savings early enough by noticing the need to save for old age. Applying trend function, it is noticeable that both variables increase significantly, pension funds' assets by 1.04 and all retirement savings 1.77 every year, and the increase of the former is significantly smaller than the latter.

Table 2. Parameters of regression models - dependent variables: all retirement savings or pension funds' assets in OECD countries and trend function

<table>
<tr><th>Model</th><th>Dependent variable y_t</th><th>Lags</th><th>Para-meter</th><th>Parameter estimate</th><th>t-Student</th><th>R^2</th><th>Pearson coef.</th></tr>
<tr><td colspan="8">Linear regression model</td></tr>
<tr><td rowspan="2">(2)</td><td rowspan="2">All retirement savings in OECD countries</td><td rowspan="2">current</td><td>beta</td><td>6.1647</td><td>9.6999</td><td>0.9127</td><td>0.9553</td></tr>
<tr><td>const.</td><td>-61.2054</td><td>-6.2890</td><td></td><td></td></tr>
<tr><td rowspan="2">(3)</td><td rowspan="2">All retirement savings in OECD countries</td><td rowspan="2">1</td><td>beta</td><td>6.5449</td><td>9.2771</td><td>0.9053</td><td>0.9515</td></tr>
<tr><td>const.</td><td>-65.2670</td><td>-6.1500</td><td></td><td></td></tr>
<tr><td rowspan="2">(3)</td><td rowspan="2">All retirement savings in OECD countries</td><td rowspan="2">5</td><td>beta</td><td>9.4624</td><td>8.8235</td><td>0.8964</td><td>0.9468</td></tr>
<tr><td>const.</td><td>-100.3754</td><td>-6.6332</td><td></td><td></td></tr>
<tr><td rowspan="2">(3)</td><td rowspan="2">All retirement savings in OECD countries</td><td rowspan="2">7</td><td>beta</td><td>10.9747</td><td>9.2631</td><td>0.9051</td><td>0.9514</td></tr>
<tr><td>const.</td><td>-100.3754</td><td>-6.6332</td><td></td><td></td></tr>
<tr><td rowspan="2">(2)</td><td rowspan="2">Pension funds' assets in OECD countries</td><td rowspan="2">current</td><td>beta</td><td>3.6251</td><td>8.2394</td><td>0.8830</td><td>0.9397</td></tr>
<tr><td>const.</td><td>-33.0353</td><td>-4.9032</td><td></td><td></td></tr>
<tr><td colspan="8">Linear trend function</td></tr>
<tr><td rowspan="2">(5)</td><td rowspan="2" colspan="2">Pension funds' assets in OECD countries</td><td>beta</td><td>1.0430</td><td>7.8909</td><td>0.8737</td><td></td></tr>
<tr><td>const.</td><td>16.1200</td><td>17.9813</td><td></td><td></td></tr>
<tr><td rowspan="2">(5)</td><td rowspan="2" colspan="2">All retirement savings in OECD countries</td><td>beta</td><td>1.7706</td><td>9.0021</td><td>0.9000</td><td></td></tr>
<tr><td>const.</td><td>22.4033</td><td>16.7938</td><td></td><td></td></tr>
</table>

Source: own calculations. Note: bold letters denote values of t-Student statistics bigger than critical value for the significance level α=0.05.

Using data from the OECD Pension Statistics and the World Bank, we evaluate the total investment of contributors of funded and private pension arrangements (in thousands of USD) per capita for years 2007-2017, which we treat as proxy of retirement savings per capita and look for the relationship between this variable and the share of the elderly in the population. We estimate several models using different samples. The biggest sample used for estimation the pooled regression model contains 396 data (for 36 OECD countries and eleven years). We also analyze all OECD members for different years for the growth rates where these samples contained 36 observations, and we estimated models for the countries using eleven annual observations.

Table 3. Investment contributors of pension funds and age +65

Model/ sample	Years	Sample country	Parameter	Parameter estimate	t-Student	R^2
Dependent variable y_i or y_t: retirement savings per capita in thousands of USD						
(4)/ 396	2007-2017	OECD countries	beta	0.0295	0.0745	0.0000
			constant	20.3253	**3.1361**	
(1)/ 36	2017	OECD countries	beta	-0.2013	-0.1207	0.0004
			constant	30.1987	1.0122	
(2)/ 11	2007-2017	USA	beta	11.6051	**9.3015**	0.9058
			constant	-93.2605	**-5.4382**	
(2)/ 11	2007-2017	Australia	beta	9.5721	**4.1577**	0.6576
			constant	-76.7139	**-2.3557**	
(2)/ 11	2007-2017	Japan	beta	-0.2772	-1.3614	0.1708
			constant	18.5028	**3.7879**	
(2)/ 11	2007-2017	Finland	beta	-2.7019	**-3.0044**	0.5007
			constant	81.0865	**4.8410**	
(2)/ 11	2007-2017	Poland	beta	-0.1323	-1.0096	0.1017
			constant	3.5695	**1.8736**	
(2)/ 11	2007-2017	Turkey	beta	0.1657	**14.9876**	0.9615
			constant	-1.0922	**-13.2369**	
(2)/ 11	2007-2017	Canada	beta	5.2560	**3.1286**	0.5210
			constant	-17.0303	-0.6729	
(2)/ 11	2007-2017	Hungary	beta	-0.3460	**-1.9579**	0.2987
			constant	6.7951	**2.2846**	
(2)/ 11	2007-2017	Switzerland	beta	23.7013	**9.8114**	0.9145
			constant	-316.5420	**-7.5564**	
(2)/ 11	2007-2017	Austria	beta	0.2691	**3.2613**	0.5417
			constant	-2.3464	-1.5641	
(2)/ 11	2007-2017	Belgium	beta	1.1384	**6.7340**	0.8344
			constant	-17.9582	**-5.9894**	
(2)/ 11	2007-2017	France	beta	0.2644	**2.2264**	0.3551
			constant	-1.1503	-0.5437	
(2)/ 11	2007-2017	Germany	beta	0.6404	**6.1185**	0.8062
			constant	-10.6598	**-4.9186**	
(2)/ 11	2007-2017	Italy	beta	0.5024	**9.4812**	0.9090
			constant	-8.4297	**-7.4577**	
(2)/ 11	2007-2017	Spain	beta	-0.0892	-1.0306	0.1056
			constant	5.5253	**3.5518**	

Source: own calculations. Note: bold letters denote values of t-Student statistics bigger than critical value for the significance level α=0.05

The selected model results presented in Table 3 indicate that it is difficult to identify general relationship between total investment of providers of pension funds and the population age 65+ when the data concerning savings per capita for each country are taken into account. It may be explained by different relationships between both variables in each country. For instance, in Australia, Austria, Belgium, France, Germany, Turkey and U.S.A the increase of percentage share of population age 65+ years causes a significant rise in retirement savings whereas in Finland and Portugal the relation is opposite, and in Japan, Poland and Ireland it is insignificant. Also, the level of the model fitting is completely different for models estimated using different samples.

Table 3. (cont.) Investment contributors of pension funds and age +65

Model/ sample	Years	Sample country	Parameter	Parameter estimate	t-Student	R^2
Dependent variable y_i or y_t: retirement savings per capita in thousands of USD						
(2)/ 11	2007-2017	Portugal	beta	-0.2749	**-2.3902**	0.3883
			constant	7.8169	**3.4729**	
(2)/ 11	2007-2017	Chile	beta	1.9198	**4.6917**	0.7098
			constant	-10.2904	**-2.5772**	
(2)/ 11	2007-2017	Israel	beta	8.4565	**10.8086**	0.9284
			constant	-74.6188	**-8.8492**	
(2)/ 11	2007-2017	South Korea	beta	1.7836	**15.7497**	0.9650
			constant	-15.9974	**-12.0598**	
(2)/ 11	2007-2017	New Zealand	beta	0.6099	**2.3949**	0.3892
			constant	-1.5190	-0.4339	
Dependent variable: growth rate of retirement savings per capita in 2017 in comparison to 2007						
(2)/ 36	2007-2017	OECD countries	beta	-12.4573	-0.8425	0.0205
			constant	4.8121	1.4921	

Source: own calculations. Note: bold letters denote values of t-Student statistics bigger than critical value for the significance level α=0.05

Table 4. Groups of countries with different effects of old age dependency ratio on investment per capita

Countries where old-age dependency ratio affects investment per capita:					
Positively and statistically significant				Insignificantly	Negatively and significant
Australia	Denmark	Israel	New Zealand	Ireland	Finland
Austria	Estonia	Italy		Japan	Hungary
Belgium	France	Korea	Norway	Luxembourg	Mexico
Canada	Germany	Latvia	Slovakia	Poland	Portugal
Chile	Greece	Lithuania	Slovenia	Spain	Sweden
Czechia	Iceland	Netherlands	Switzerland		
Turkey	United Kingdom		United States		

Source: own calculations. Note: the significance level α=0.05

Regression models estimated separately for 36 countries[3] let us construct three groups of countries where old-age dependency ratio affects investment per capita differently, that is (a) the effect is positive and statistically significant (26 countries), (b) insignificant (5 countries) and (c) negative and statistically significant (5 countries). We can observe from Table 4 that there is no direct connection between this classification of countries and the recognition of these countries as developed and developing ones.

SUMMARY

Our paper looks at the population aging crisis and the impact on pension funds accumulation by retirees between the developed and developing countries and the impact on the capital pension funds markets. The results show that the population aging affects developed and developing countries differently and that pension accumulation asset growth is increasing in OECD countries. Gross national income per capita significantly influences the total investment of providers of funded and private pension arrangements in years 2016-2017, and 2017 since the relation is significant and positive. In 2006, this relation was insignificant.

The population age 65+ as percentage of the total population significantly influences total amount of assets in retirement savings choices (including pension funds) in the OECD and pension asset accumulations in years 2007-2017, when we use aggregated data for all OECD countries. The results show that there is a positive relationship between old-age dependency ratio and total pension assets and pension funds' assets growth in all OECD countries. This positive relationship is true for current and lagged relationships, although the increase of old-age dependency ratio 5 and 7 years earlier causes significantly bigger increase of pension assets in the current year than current and lagged by one old-age dependency ratio.

However, in looking at the population age 65+, whether an increase of old-age dependency ratio causes an increase of investment per capita of the total, the results are different in different countries in the eleven years analyzed. For most of the countries, the effect is significantly positive: in Americas – the USA, Canada and Chile; in Western Europe – Austria, Belgium, France, Germany, Switzerland, UK, the Netherlands; in Southern Europe: Italy, Greece, Slovenia; in Northern Europe – Denmark, Iceland, Norway; in Central and Eastern Europe – Czechia, Estonia, Lithuania, Latvia, Slovakia, and in other non-European countries – Israel, South Korea, New Zealand, Turkey and Australia. In Finland, Sweden, Hungary, Mexico and Portugal it is significantly negative, and for Japan, Spain, Luxemburg, Poland and Ireland, it is insignificant. The different results observed in different countries means that in general the relationship is insignificant for all the OECD countries and the analyzed years.

[3] Results for selected countries are presented in Table 3.

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